The Swing Voters' Blessing

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Abstract

Can democracy work well if the electorate is neither fully informed nor fully rational? I provide an affirmative answer in a model with quality differences between two ideological candidates running for office. The candidates commit to policy platforms before the elections take place. All voters care about the quality of the candidates as well as the policies they offer; however, the quality differences are only observable to a limited number of informed voters. I consider two different scenarios. In the first one, all uninformed voters are fully rational and follow an optimal strategy of making their voting decisions dependent only on the position of their own policy bliss point relative to the median policy bliss point. As in the standard case with only informed voters, the candidate who is preferred by the median voter wins. In equilibrium, this is the higher quality candidate and the policy implemented is the same as if all voters had been fully informed. In the second scenario, I show that the existence of boundedly rational uninformed 'swing voters' increases the welfare of the majority of voters. These swing voters do not consider the fact that their vote influences their utility only when their vote is pivotal. Consequently, they always support the candidate whose policy platform they prefer. In this scenario, the winning high quality candidate's policy is closer to the median voter's bliss point than in the first scenario with only fully rational voters. This is the "Swing Voters' Blessing" that ends up making the majority of voters better off.

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1 Introduction

Can democracy work well if the electorate is neither fully informed nor fully rational? I develop a model in which some voters are uninformed about the quality of the candidates running for office to provide new insights into this question.

When most political economists model elections, the focus is on aggregating individual preferences. Voters disagree on questions of distribution or ideology and elections are a way of deciding which policies are implemented. This literature goes back to the seminal contributions of Black (1948) and Downs (1957).[1] Here, the problem is that voters want different things and elections decide whose preferences prevail. If candidates can commit to policy platforms, as is often assumed in the literature, elections become a way of aggregating conflicting preferences.

A different approach to modeling voting and elections goes back to the Jury Theorem by the Marquis de Condorcet (1785). The idea is to model elections as an information aggregation device.[2] Voters' interests and preferences are aligned and if all voters were fully informed they would support the same proposition or candidate. Here, the problem is not that voters want different things, but that limited information creates uncertainty about the consequences of a particular election outcome. Therefore, voters who maximize their expected utility need to understand that their vote has an impact on the election results only if both sides obtain exactly half of the votes and their vote is pivotal for the outcome of the elections (Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1996, 1997). Voters who do not consider this when making their voting decision can suffer from what Feddersen and Pesendorfer (1996) call "the swing voter's curse". Whenever such a voter actually decides an election with her vote, it is likely to turn out that her voting decision makes her worse off.

In reality, elections aggregate preferences as well as information. There is little disagreement about the fact that voters and candidates have different preferences, for example about the amount of income redistribution. However, voters also have common interests, for example in having a President who is a good crisis manager. Candidates who are aware of their superior abilities might be tempted to let the electorate "pay" by choosing relatively extreme policy positions close to their own bliss

[1] For an overview of this literature, see Persson and Tabellini (2000).

[2] Condorcet himself was more interested in the verdict of a jury in a court. For an overview of the information aggregation approach, see Piketty (1999).

point, knowing that they will beat their opponent nonetheless. Therefore, separate modeling of preference and information aggregation might conceal important insights.

1.1 Outline of the model and the results

This paper combines the information aggregation as well as the preference aggregation aspects of elections. In contrast to a similar attempt by Feddersen and Pesendorfer (1999), I allow policy offers to be freely decided by the candidates. As in the information aggregation literature, there is a dimension on which voters agree when they are fully informed and, as in the preference aggregation literature, there is a policy dimension over which voters disagree. Specifically, I model elections with quality or valence[3] differences between two ideological candidates who can commit to policies before the elections. Incomplete information plays a crucial role because the valence differences are only observed by a limited number of voters. However, given the true quality difference, voters agree on which candidate they prefer to win the election for a given policy position. Voters may prefer the candidate who is ideologically further away from their ideological bliss point if his quality advantage over the other candidate is sufficiently large.

I show that if uninformed voters follow a simple equilibrium strategy of basing their voting decisions on their own ideological position relative to that of the median voter, the candidate who is preferred by the informed median voter wins. Thus, the uninformed voters effectively ignore the policy platforms of the candidates.

As an example, consider the problem of an uninformed conservative voter deciding between Republican John McCain and Democrat Barack Obama in the 2008 United States presidential elections. Obama's unobserved valence advantage could be so large that the uninformed conservative voter would prefer him if she were fully informed. However, in this case, Obama would not need the conservative voter's support to win the elections because his great appeal to informed voters would ensure his victory even without her vote. The elections are only a close call if McCain has a relatively high valence as compared to Obama. Therefore, the conservative voter knows that she must prefer McCain in case her vote is pivotal.

Uncertainty among uninformed voters makes no difference for the implemented

[3] In the political science literature, quality differences between politicians are often referred to as valence differences. For an early use of the term "valence" in the literature, see Stokes (1963). I use both terms, valence and quality, interchangeably throughout the text.

policy, compared to a situation where all voters are fully informed. The candidate with the support of the median voter wins in both cases. Therefore, the candidate with the valence advantage wins. He announces the platform that is as close as possible to his own bliss point without giving the other candidate the opportunity to win the support of the median voter. If the median voter is uninformed, it can be shown that the results for the informed median case provide a good approximation for the uninformed median case, as long as informed voters are located close to the median voter.

The proposed strategy requires the uniformed voters to have a certain amount of sophistication that not every reader might find credible. Therefore, I introduce unsophisticated swing voters into my model to check for the robustness of the results. These uninformed voters do not take into account that their vote makes a difference only if the elections are decided by just one vote. Thus, they vote for the candidate they prefer given the unconditional distribution of the valence difference. Because they can only observe the different policy offers by the candidates, their decision is always in favor of the candidate whose policy offer is ideologically closer to their own preferences. They are not only "swing voters" in the sense of Feddersen and Pesendorfer (1996), that is voters who make their voting decision without considering the fact that their vote only makes a difference when they are pivotal. They are also swing voters in the more common use of the term in the political science literature, that is voters who are likely to switch their support from one party or candidate to a different one. In the terminology introduced by Austen-Smith and Banks (1996), the swing voters in my model vote "sincerely", not "strategically".

It turns out that the majority of voters are better off, in expectation, if such boundedly rational uninformed voters exist. This somewhat surprising result is an application of the second-best principle that introducing an additional distortion into a model may bring the equilibrium closer to the equilibrium without distortion and increase welfare rather than reducing it further (Lipsey and Lancaster 1956). The existing valence differences between candidates "distort" political competition on the policy dimension and lead to results that are different from normal Downsian Competition. The additional distortion of boundedly rational voting brings the results closer to Downsian competition. But just as Downsian competition will not necessarily lead to welfare maximizing results, there is no guarantee that swing voters bring the outcome closer to the utilitarian optimum.

It is illuminating to consider the consequences of swing voters in the Obama versus McCain example mentioned above. An unsophisticated uninformed voter with a bliss point close to, but left of the median voter votes for the centrist McCain if Obama chooses a position far to the left. The existence of such voters forces Obama to stay closer to the median voter than he would otherwise have to to win the election.

Unsophisticated voters make irrational voting decisions, but this turns out to be a blessing and not a curse. They can play a strategy that a rational voter could not commit to because it would not be time-consistent to do so and she would want to deviate after the candidates have chosen their positions. No unsophisticated swing voter needs to regret her vote because the candidate with valence advantage wins nonetheless. Her vote could only make her worse off if it were not foreseen by the candidates. But because the candidates know about the existence of swing voters, they adjust their positions. The candidate with the valence advantage wins, but with a more moderate policy position than in the case of full rationality. I call this force of moderation the "swing voters' blessing".

1.2 Related approaches and literature

Feddersen and Pesendorfer (1999) combine motives from the information aggregation with the preference aggregation literature in an attempt to explain abstentions in a setup where voters' interests are not perfectly aligned. Their main example is a plebiscite over the decision of whether to build a bridge. The main difference compared to my setup is that the details about the building plans are exogenously given. Feddersen and Pesendorfer also mention the example of different candidates for office, but their framework is ill suited to this application since the policies proposed before the election are not exogenously given. Thus, what is missing to make the model in Feddersen and Pesendorfer an adequate framework for the analysis of elections is a stage of the game in which candidates or parties endogenously decide on policies. With exogenous policy proposals the swing voters' blessing cannot occur.

My model is similar to that in Groseclose (2001) in combining a policy dimension with a candidate quality dimension. However, I focus on uncertainty in the electorate about the quality of the politicians, while Groseclose focuses on uncertainty among candidates about the preferences of the electorate.

In a series of papers by McKelvey and Ordeshook (1985, 1986), uninformed voters use a sequence of opinion polls to infer the truth about candidate positions. However, McKelvey and Ordeshook ignore the strategic aspects of being a pivotal voter that are central to my basic model. Voters simply try to vote for their favorite candidate given their best estimate of the candidates' positions just as is done by the swing voters in the generalization of my model. If the McKelvey and Ordeshook model were formulated as a game, an uninformed voter would have to condition her estimate of the candidates' positions on herself being pivotal. Moreover, the answers to opinion poll questions may be given strategically. McKelvey's and Ordeshook's assumptions could nonetheless be a good description of how boundedly rational voters actually make their voting decision, but there is no discussion of this issue in their papers.

Another paper in the same tradition is Cukierman (1991) whose model is very similar to mine with respect to voters' preferences and information. In contrast to the papers by McKelvey and Ordeshook, voters do not only care about policy, but also about valence. Just as in my approach, some of the voters do not directly observe valence. However, as in the McKelvey and Ordeshook approach, uninformed voters try to gauge some information from opinion polls and, once more, their voting decisions lack game-theoretic foundations.

An idea related to mine can be found in the recent paper by Bond and Eraslan (2010). These authors endogenize proposals in a Feddersen-Pesendorfer setup. However, they do not model political competition, but rather decision making within a committee as in Feddersen and Pesendorfer (1998), and there is only one offer by an agenda setter, not two offers by competing candidates. Just as in my setup, however, endogenizing positions leads to important differences in the results.

1.3 Structure of the paper

The paper proceeds as follows. In Section 2, the basic model is introduced and discussed. Section 3 allows for some generalizations. In Section 4, swing voters are introduced. The welfare implications of their existence are discussed in Section 5 and Section 6 provides an example with a continuum of voters. The paper ends with a conclusion. A technical appendix contains most of the proofs.

2 The model

Consider a polity with a one-dimensional ideological policy space on the real line $[0, 1]$, two candidates L and R and an odd number n of voters denoted by $i = 1, 2, 3, ...n$. Candidates have quality (also called valence) q_L and q_R, respectively, and a bliss point for implemented policy b_L and b_R, respectively.[4] The candidates' utility is decreasing in the distance of implemented policy to their bliss point and it is given by:

$$U_J(p) = -(p - b_J)^2, \tag{1}$$

with $J = L, R$, and where p is implemented policy.

Just as the candidates, every voter i has a bliss point b_i on the policy space. By assumption, they are all distinct and no two voters have exactly the same preferences. Voters are ordered by their bliss points so that $b_1 < b_2 < b_3$ and so on.[5] Besides the policy dimension, voters care about the quality of candidates and voter i has the utility function:

$$U_i(b_i, p, q) = -(p - b_i)^2 + q, \tag{2}$$

where $q \in \{q_L, q_R\}$ is the quality of the candidate who wins the elections.[6] Assuming that the (dis)utility from policy does not interact with the quality of the winning candidate as is done here is the most straightforward way of modeling information and preference aggregation in one election. However, the results also hold for more general utility functions where the possible interaction of quality and distance is not ruled out. This is shown in Section 3.3 where generalizations of (1) as well as (2) are discussed.

The median bliss point of the voters is denoted by b_m with $m = \frac{n+1}{2}$. By assumption $b_L \leq b_m \leq b_R$; I thus call candidate L the left and candidate R the right candidate. The difference in quality of the two candidates is denoted by the variable $v = q_R - q_L$, which hence measures the quality advantage of the right candidate. If the left candidate has a quality advantage, v takes a negative value. The values of q_R

[4] Variables with capital letter subscripts are used to denote characteristics of candidates, while variables with small letter subscripts denote characteristics of voters.

[5] The assumption that no two voters have exactly the same bliss point is a mild one given that the probability of two voters having exactly the same position is 0 if all of them are drawn from a continuous distribution function. It considerably simplifies the notation.

[6] This kind of preferences can be called "one and a half dimensional" (Groseclose 2007).

and q_L are drawn from a continuous distribution function before the candidates announce their position. The cumulative distribution of v is given by the function $G(v)$. By assumption, the corresponding probability density function of $g(v)$ has positive support everywhere on the real line. All players, voters as well as candidates, know the basic structure of the game including the policy preferences of the parties as well as the distribution of the bliss points of the voters.[7]

The sequence of moves is the following: First, nature chooses q_R and q_L. Second, candidates announce the policy platform they propose to be implemented after observing the quality difference v. Third, elections in which every voter casts one vote are held. Some of the voters, the so-called informed voters (their number is n_I), can observe the random variable v and the policy platforms offered by the candidates before they make their voting decision. The so-called uninformed voters (their number is n_U) only observe the policy platforms before they cast their votes. Fourth, the candidate who obtains at least $\frac{n+1}{2}$ of the votes in the elections wins and his announced policy platform is implemented. Therefore, $p = p_L$ and $q = q_L$ if candidate L obtains more than half of the votes, and $p = p_R$ and $q = q_R$ if he obtains less than half of the votes.

Abstentions are not allowed. This assumption is made to simplify the notation. It is easily verified that in my model, no voter would ever want to abstain in equilibrium. By assumption, the majority of voters are informed, that is $n_I > n_U$.[8]

In the main part of the paper, the median voter is assumed to be informed. A discussion of the model with an uninformed median voter can be found in Section 3.2. There, it is also shown that the main model is a good approximation of this case for "large" electorates.

For the moment, I assume all voters to be sophisticated in the sense that they are able to understand the Bayesian Nash equilibrium of the voting game and play equilibrium strategies. In Section 4, this assumption is relaxed and boundedly rational voters are introduced into the model.

[7]For the basic model it is sufficient if uninformed voters know the position of the median voter.

[8]This assumption helps avoid implausible additional equilibria with all uninformed voters voting for one party independently of policy positions. The assumption is not necessary for the existence of the type of equilibrium analyzed below.

2.1 Solving the model

I begin my analysis at the last stage of the game and solve the problem of the voters after observing the platforms of the candidates. Then, I solve the problem of the candidates when announcing their policy platforms and show what is the equilibrium policy.

2.2 Informed voters

I consider equilibria where informed voters play the weakly dominating strategy of always voting for the candidate whom they favor.[9] It is possible to determine who is the rightmost informed voter weakly in favor of the candidate with the left policy position. Specifically, the cutoff point is the bliss point b^* that makes a voter indifferent between the two candidates. This point is implicitly defined by (2). Equating the utility of voting for the left candidate and voting for the right candidate gives:

$$\Delta U(b^*, p_L, p_R, q_L, q_R) = U(b^*, p_L, q_L) - U(b^*, p_R, q_R) = -(p_L - b^*)^2 + (p_R - b^*)^2 - v = 0. \tag{3}$$

The cutoff point b^* exists for any v as long as $p_L \neq p_R$ and it is uniquely given by:

$$b^*(p_L, p_R, v) = \frac{p_L + p_R}{2} - \frac{v}{2(p_R - p_L)} \text{ for } p_R \neq p_L. \tag{4}$$

All voters with a bliss point to the left of b^* prefer the candidate with the left position, while all voters with a bliss point to the right of b^* prefer the candidate with the right position.[10] Note that the right candidate could, in principle, be located at the left position (if $p_R < p_L$), although this will never be the case in any plausible equilibrium.[11] The intuition for this formula is straightforward. If $v = 0$, the cutoff

[9] Without this restriction, it is possible to have equilibria where everybody votes left or everybody votes right independently of the candidates' policy positions, so that none of the voters is ever pivotal.

[10] This can be seen from the derivative of the difference in utility from the left candidate's position and the right candidate's position, with respect to a voter's bliss point:

$$d\frac{\Delta U(b, p_L, p_R)}{db} = -2(p_R - p_L) \begin{array}{l} < 0 \text{ if } p_R > p_L \\ > 0 \text{ if } p_R < p_L \end{array}.$$

[11] In theory, a candidate who knows that he will lose against the other candidate's bliss point with any policy position could choose a position further away from his own bliss point than the other candidates bliss point in equilibrium. Throughout the text, I will use the short expression "vote for the left (right) position" instead of the slightly more precise but cumbersome "vote for the candidate with the left (right) policy position".

point is midway between the policy position of the two candidates. A positive v makes the right candidate more attractive and therefore shifts the cutoff point to the left for given policy positions as long as $p_R > p_L$. However, the marginal effect of valence differences on the position of the cutoff point is decreasing in $p_R - p_L$, i.e. the distance in policy. The further the candidates' policy positions are from each other, the more policy matters relative to valence. The reason is that the disutility of distance from the ideal policy point of a voter is quadratic while utility is linear in valence. In the case of large valence differences, it is possible that the cutoff point is to the left of the left policy position or to the right of the right policy position.

The cutoff point between preferred candidates is the same for informed and uninformed voters. The difference between the two types of voters is that uninformed voters do not know where b^* is located since they do not know the valence difference v. However, for informed voters, the voting decision only depends on b^* and therefore $b_I^*(p_L, p_R, v) = b^*(p_L, p_R, v)$, where b_I^* denotes the cutoff point between informed voters who vote for the left position and informed voters who vote for the right position. If an informed voter is located exactly at b_I^* she is indifferent and, by assumption, votes in favor of the candidate with valence advantage.

Now, b^* can be located outside the policy space $[0, 1]$. Whenever this is the case, either all or none of the informed voters vote left or right position, respectively. If $p_L = p_R$ and $v \neq 0$, no value of b solves equation (3) because all informed voters prefer the candidate who has the valence advantage and vote for him. Therefore, $b^* = b_I^* = 0$ if $p_L = p_R$ and $v > 0$, and $b^* = b_I^* = 1$ if $p_L = p_R$ and $v < 0$. If $p_L = p_R$ and $v = 0$, equation (3) holds for arbitrary values of b since all informed voters are indifferent between the candidates independently of their respective bliss points. Without loss of generality, I make the assumption that in this case, all informed voters give their vote to the left candidate L and therefore $b_I^* = 0$.

2.3 Uninformed voters

The problem of an uninformed voter with bliss point b is that she does not know which candidate she favors, because she is not able to observe the valence difference v. Let $F_I(b)$ be the number of informed voters with a bliss point smaller than or equal to b, let $F_I^{-1}(x)$ be the bliss point of the informed voter with the x_{th} lowest bliss point b among the informed voters' bliss points, and let l_U be the number of votes for the left policy position by uninformed voters. I call p_L the left position when $p_L \leq p_R$, and call p_R the left position when $p_L > p_R$. Then I refer to:

$$b_I^d(l_U) = F_I^{-1}(\frac{n+1}{2} - l_U) \tag{5}$$

as the bliss point of the decisive informed voter given l_U.[12]

Lemma 1 *The candidate with the support of the decisive informed voter wins the elections.*

Proof. For a majority, $\frac{n+1}{2}$ votes are necessary and therefore at least $(\frac{n+1}{2} - l_U)$ votes by informed voters in favor of the left position for the candidate with the left position to win, and at least $(\frac{n+1}{2} - (n_U - l_U))$ votes by informed voters in favor of the right position for the candidate with the right position to win. If the decisive informed voter votes for the left position, then $b_I^d(l_U) \leq b^*(p_L, p_R, v)$, and all informed voters with a bliss point $b < b_I^d(l_U)$ vote for the left position. Thus, the left position obtains at least $F_I(b_I^d(l_U)) = \frac{n+1}{2} - l_U$ votes by informed voters. Together with the l_U votes for the left position by uninformed voters, this constitutes a majority. If the decisive informed votes for the right position, then $b_I^d(l_U) \geq b^*(p_L, p_R, v)$, and all informed voters with a bliss point $b > b_I^d(l_U)$ to the right of the decisive voter vote for the right position. Therefore, the left position obtains at most $\frac{n-1}{2}$ votes, and the right position obtains a majority. ■

The candidate preferred by the decisive informed voter wins the elections. Therefore, this voter is decisive in the same sense as the median voter in standard models with full information.

The strategies of the voters can only constitute an equilibrium if none of the voters has an incentive to deviate, given the strategies of the other players and her information. Due to the strategy of the informed voters, this implies that, in equilibrium, none of the uninformed voters would prefer to shift the position of the decisive informed voter by changing her own voting decision. A simple strategy fulfills this condition if it is followed by all uninformed voters. Let $b_U^*(p_L, p_R)$ be the cutoff point

[12] I distinguish between "decisive voters" and "pivotal voters". A voter is pivotal if the winner of the elections wins with one vote and would lose if a pivotal voter changed her vote. All voters who vote for the winner in an election that is decided by one vote are therefore pivotal. If the majority is larger, there are no pivotal voters. A voter is "decisive" if the candidate whom she prefers wins the elections given the preferences of the other voters. In a standard Downsian model, the decisive voter has the median bliss point.

for uninformed voters: i.e. all uninformed voters with $b < b_U^*(p_L, p_R)$ support the left position and all uninformed voters with $b > b_U^*(p_L, p_R)$ support the right position. Specifically, the condition holds with $b_U^*(p_L, p_R) = b_m$ as the cutoff point. All uninformed voters with a bliss point to the left (right) of the informed median voter vote for the the candidate with the left (right) policy position.[13] This cutoff point is independent of the policy platforms that the candidates announce.[14] Moreover, with this cutoff point, the decisive informed voter is the median voter as in standard models without uninformed voters.

Lemma 2 *If the cutoff point for uninformed voters is $b_U^*(p_L, p_R) = b_m$ for all combinations of p_L and p_R, the voting decision of the the informed median voter is decisive for the outcome of the elections.*

Proof.

$$b_I^d(F_U(b_m)) = b_I^d(\frac{n+1}{2} - F_I(b_m)) = F_I^{-1}(F_I(b_m)) = b_m,$$

where the first equality comes from the implicit definition of the median voter's bliss point b_m: $\frac{n+1}{2} = F_I(b_m) + F_U(b_m)$, and the second equality follows directly from the definition of b_I^d given in equation (5). The third equality follows from the fact that, by assumption, an informed voter with bliss point b_m exists. ■

Given Lemmas 1 and 2, it follows:

Lemma 3 *The cutoff point $b_U^*(p_L, p_R) = b_m$ characterizes an equilibrium strategy for uninformed voters given that informed voters play the weakly dominating strategy characterized by the cutoff point $b_I^*(p_L, p_R, v)$. As in standard models with full information, the preferences of the median voter decide the elections.*

Proof. To show the optimality of the strategy of an individual voter, it is sufficient to show that she cannot be better off by changing her strategy, given the strategies of all other voters.

Consider the case of an uninformed voter with her bliss point to the left of b_m. Since such a voter votes for the left policy position, her alternative is voting for

[13] By assumption, there is no uninformed voter with bliss point b_m. Therefore, this cutoff point determines the voting decision of all uninformed voters.

[14] However, it should be kept in mind that because the right candidate could play the left policy position, the candidate whom a voter supports in the elections is not completely independent of the policy positions.

the right position instead. From Lemma 2, we know that if she votes for the left position, the bliss point of the decisive informed voter is b_m. If she instead votes for the right position, the bliss point of the decisive informed voter changes from b_m to $b_I^d(F_U(b_m) - 1) = b_I^d(\frac{n+1}{2} - F_I(b_m) - 1) = F_I^{-1}(F_I(b_m) + 1) > b_m$. This can change the election outcome only if $b^*(p_L, p_R, v)$ takes a value such that $b_m \leq b^*(p_L, p_R, v) \leq F_I^{-1}(F_I(b_m) + 1)$. From Lemma 1, it follows that if the elections outcome changes, then it must be the case that the candidate with the right position wins instead of the candidate with the left position. Because the voter's bliss point is to the left of the median bliss point and, consequently, also to the left of the cutoff point $b^*(p_L, p_R, v)$, this would make her worse off. Therefore, an uninformed voter with bliss point $b < b_m$ is at least as well off voting for the left position as voting for the right position and thus, voting for the left position must be optimal for all possible combinations of p_L and p_R and independently of the candidates' strategies.

An analogous argument can be applied to show that a voter whose bliss point is to the right of b_m can never be better off voting for the left position, given the strategies of the other voters. In the case of $p_L = p_R$, no cutoff point for informed voters exists, but because all of them vote for the candidate with the valence advantage, this candidate wins independently of the decision of the uninformed voters. ■

To provide some intuition for the uninformed voters' strategy, it is helpful to reinterpret the voting strategy of the uninformed as a way of appointing the decisive informed voter. All uninformed voters prefer a decisive informed voter with preferences as close to their own as possible. To achieve this aim, they vote left (right) if their bliss point is to the left (right) of the bliss point of the decisive informed voter. In this way, they attempt to pull the position of the decisive informed voter closer to their own bliss point.

Another interpretation of the result is that uninformed voters ensure that they vote for their favorite (under full information) candidate whenever they are pivotal. They realize that it is not important for whom they vote, as long as their vote does not change the election outcome. If an uninformed voter follows the strategy of only making her decision dependent on her position relative to the median bliss point, she can be certain of never voting against the candidate whose election maximizes her utility when he loses by just one vote. Thus, she can avoid becoming a victim of the swing voter's curse. To see this, imagine that the elections are decided by one vote and assume (without loss of generality) that the right position wins. Because the decisive informed voter is the median voter, this implies that the median voter votes for the right position, but informed voters to the left of the median voter vote for the left position (otherwise the majority would be larger). Because the median voter prefers the right position, all uninformed voters with bliss a point to the right of the median voter who vote for the right candidate indeed have higher utility from a victory of the right candidate than they would in case the left candidate won. This interpretation provides some intuition as to why the position of the decisive informed voter does not change as compared to the standard setup. Whenever they are pivotal, uninformed voters manage to make the same voting decision as if they had full information.

2.4 The candidates

Lemma 3 shows that the candidates' problem is exactly the same as it would be in the full-information setting. The candidates need the support of the decisive informed voter to win, and the decisive informed voter turns out to be the median voter. The candidate with a valence advantage can win by offering the bliss point of the median voter as the policy proposal. However, he can do considerably better by moving as close as possible to his own bliss point without endangering his election victory.[15]

Proposition 1 *Together with the cutoff point $b_U^*(p_L, p_R) = b_m$ for uninformed voters and the weakly dominating strategy of informed voters the following policy platforms of the candidates constitute an equilibrium of the game:*

$$\left.\begin{array}{l} p_R^* = \min(b_R, b_m + v^{0.5}) \\ p_L^* = b_m \end{array}\right\} \text{ if } v \; > \; 0, \qquad (6)$$
$$\left.\begin{array}{l} p_R^* = b_m \\ p_L^* = \max(b_L, b_m - (-v)^{0.5}) \end{array}\right\} \text{ if } v \; \leq \; 0,$$

[15] An interesting aspect of the result is that usual Downsian Competition results do not hold. Due to the additional valence dimension, the winner of the elections does not implement the most preferred policy of the median voter. This is the case despite the fact that Black's theorem applies to the model and the majority's preferences are identical to those of the median voter (Groseclose 2007). The reason for the discrepancy is that (with the exception of $v = 0$) the median policy is not on offer in combination with the high quality candidate because it is not in the interest of the winning candidate to make it available. Black's theorem holds, but the Downsian version of the median voter theorem does not.

and the implemented policy is:

$$p^* = \begin{cases} \min(b_R, b_m + v^{0.5}) & \textit{if } v > 0 \\ \max(b_L, b_m - (-v)^{0.5}) & \textit{if } v \le 0 \end{cases}. \tag{7}$$

Proof. Proof for the case $v > 0$:

From Proposition 1, we know that the candidate with the support of the informed median voter wins the elections. The best reply of the right candidate to a policy p_L by the left candidate is therefore given by the solution to the following constrained maximization problem:

$$p_R^b(p_L, v) = \max_{[0,b_R]} p_R \ s.t. \ -(p_R - b_m)^2 + v \ge -(p_L - b_m)^2.$$

$p_R^b(p_L, v)$ gives the largest $p_R \le b_R$ which makes the median voter at least indifferent between voting for the left candidate and voting for the right candidate and, therefore, leads to an election victory for the right candidate. A unique solution exists and is given by:

$$p_R^b(p_L, v) = \min(b_R, b_m + (v + (p_L - b_m)^2)^{0.5}).$$

$p_R^b(p_L, v) \ge p_L$ for $p_R^b(p_L, v) < b_r$. Therefore, the right candidate could not be better off with a position at which he is defeated by the left candidate and $p_R^b(p_L, v)$ is therefore a best reply to the other players' strategies. $p_R^b(b_m, v) = \min(b_R, b_m + v^{0.5})$, hence right is playing the unique best reply to the left candidate's position $p_L^* = b_m$ in equilibrium. If $-(p_R - b_m)^2 + v \ge -(p_L - b_m)^2$ holds for $p_L = b_m$, it follows that the inequality also holds for all other values of p_L because $p_L = b_m$ maximizes the right-hand side of the inequality. Therefore, the left candidate loses the elections with any reply to p_R, and $p_L = b_m$ (as well as any other p_L) is a best reply to $p_R^b(b_m, v)$. If $p_R^b(b_m, v) < b_R$, the equilibrium response of the left candidate is unique. To see this, consider any other policy platform $p_L \neq b_m$. $p_R^b(p_L, v) > p_R^b(b_m, v)$ for any $p_L \neq b_m$. But against any $p_R > p_R^b(b_m)$, the left candidate can win with $p_L = b_m$. Therefore, p_L cannot be a best reply to $p_R^*(p_L, v)$ for any $p_L \neq b_m$, and $p_L \neq b_m$ cannot be part of the equilibrium. If $p_R^b(b_m, v) = b_R$, then $p_R^b(p_L, v) = b_R$ for all values of p_L, and therefore, $p_R^* = b_R$ in combination with any policy platform p_L constitutes an equilibrium. All values of p_L are best replies to $p_R^* = b_R$, and $p_R^* = b_R$ is a best reply to all values of p_L. For all equilibrium combinations of p_L and p_R, the median voter either prefers the right candidate or is indifferent. Therefore, the median voter votes for the right candidate and Lemma 3 implies that the right candidate wins and policy platform p_R^* is implemented. The implemented policy $p = p_R = p_R^b(b_m)$ is therefore unique for all values of $v > 0$. For the proposition, I assume that the left candidate always chooses $p_L^* = b_m$, even when $p_R^* = b_R$ and the solution is not unique.

An analogous argument can be given for the case $v \leq 0$. ■

2.5 Interpretation as Perfect Bayesian Nash Equilibrium

So far, I have ignored the fact that voters might try to infer the quality difference of the politicians from their chosen policy position. This does not invalidate the above analysis, because the derived strategies of the uninformed voters were (weakly) optimal independently of the players' beliefs about the valence difference v. However, it is interesting that uninformed players can infer the values of v exactly as long as none of the candidates has a valence advantage that is so large that he can choose his bliss point as his policy position and win.

A system of beliefs for the uninformed voters that is consistent with the analysis so far can therefore easily be constructed with the help of the equilibrium outcomes for the candidate positions. All that the uninformed voters have to do to infer the value of v is to observe for what values of v the policy offers by the candidates constitute an equilibrium. As long as the candidates play their equilibrium strategies and neither candidate offers his bliss point, there is only one value of v that is consistent with the policy offers. If one candidate announces his bliss point as the policy platform and the other candidate announces the median bliss point, this combination of policy positions is consistent with an interval of values of v. In this case, Bayes' Theorem is applied to calculate the density of the distribution function of v:

$$v(p_L, p_R) = \begin{cases} (p_R - b_m)^2 & \text{if } p_L = b_m,\ p_R \in [b_m, b_R) \\ -(p_L - b_m)^2 & \text{if } p_R = b_m,\ p_L \in (b_L, b_m], \end{cases}$$

$$g(v|p_L, p_R) = \begin{cases} \left.\begin{array}{ll} \dfrac{g(v)}{1 - G((b_m - b_R)^2)} & \text{for } v \geq (b_m - b_R)^2 \\ 0 & \text{for } v < (b_m - b_R)^2 \end{array}\right\} & \text{if } p_L = b_m,\ p_R = b_R \\ \left.\begin{array}{ll} \dfrac{g(v)}{G(-(b_m - b_L)^2)} & \text{for } v \leq -(b_m - b_L)^2 \\ 0 & \text{for } v > -(b_m - b_R)^2 \end{array}\right\} & \text{if } p_R = b_m,\ p_L = b_L, \end{cases} \tag{8}$$

where $v(p_L, p_R)$ is a value of v that leads to the combination of p_L and p_R in equilibrium. For all other (out of equilibrium) combinations of policy offers, I assume that the uninformed voters do not update their a priori beliefs of the distribution of v, so that in this case $g(v|p_L, p_R) = g(v)$.

Because the voting decision of the uninformed voters does not depend on the beliefs about v, candidates cannot manipulate the voters' beliefs to their own advantage by choosing their policy position. Nonetheless, in equilibrium, all uninformed voters support the candidate they favor, given their beliefs about v whenever their information is sufficient to determine who this candidate is. Only if v is either so high or so low that the winning candidate can offer his bliss point and win nonetheless, an uninformed voter cannot determine the exact value of v. In this case, it can happen that an uninformed voter votes against the candidate whom she would prefer to win if she were fully informed. But since in this case the candidate wins even without her vote, this has no consequences for the outcome of the elections.

Voters do not actually need to be so sophisticated that they posses the beliefs they are required to have in the Perfect Bayesian Nash Equilibrium. As should be clear from the way in which the equilibrium was derived, their strategies are optimal, independently of the strategies of the candidates if all other voters follow the same strategy. Voters could have any conceivable beliefs about the distribution of v without their strategies ceasing to be optimal. The reason is that their vote is only decisive for certain values of v. Therefore, the belief of how likely those values of v are is of no importance for their voting decisions. The requirements for the sophistication of the uninformed voters are considerably lower than in most cases of a Perfect Bayesian Nash Equilibrium and the results are less sensitive to its strong rationality and information requirements.

2.6 Uniqueness of the equilibrium

The equilibrium derived in Sections $2.1 - 2.4$ is not unique. This is not surprising considering that "implausible" equilibria with voter coordination exist already in models with fully informed voters. A trivial example is that of all voters always voting for the same candidate. Since none of the voters is pivotal, this constitutes a Bayesian Nash equilibrium if the winning candidate announces his bliss point as the policy platform (the other candidate can announce an arbitrary policy position).

A more interesting example is that all uninformed voters always vote in favor of the candidate who chooses the policy closest to the median position, while informed voters follow the strategy to vote for their preferred candidate. What uninformed voters do when both candidates choose a position with equal distance to the median voter's bliss point is of no importance, because the higher-quality candidate wins in this case with the votes of all informed voters. Candidates are forced to choose the median bliss point as policy position, even if they have a small valence advantage. If the advantage is sufficiently large to ensure a majority with only informed votes at some more advantageous position, they choose this position. Since, in equilibrium, the higher-quality candidate chooses the median position, or wins at a position closer to his bliss point with a majority consisting only of informed voters, no uninformed voter could increase her utility by deviating. However, it remains unclear how uninformed voters should be able to coordinate on such an equilibrium and why they would want to do so. Every single uninformed voter would be better off if the candidates knew before the elections that she deviated to the strategy of making her voting decision dependent on the median position, because that would for some values of v lead to implemented policies closer to her own bliss point. Moreover, the equilibrium given in Sections $2.1 - 2.4$ seems to be the relevant one, given that I am interested in solving the problem of an uninformed voter who attempts to make the correct voting decision under the condition that she is pivotal, since the other equilibria are only equilibria since no uninformed voter can ever be pivotal.

To provide some more formal justification, I focus on equilibria with a cutoff point $b_U^*(p_L, p_R)$ for uninformed voters. This means that I rule out equilibria in which an uninformed voter with bliss point b_1 votes for the right candidate while an uninformed voter with bliss point $b_2 > b_1$ votes for the left candidate. Such equilibria exist, but they require some coordination by voters that is hard to achieve in large scale elections and there is no intuition that might justify them. Moreover, I assume that the uninformed voter's beliefs are such that every value of v is considered to be possible for any combination of p_L and p_R. Let $G(v^b|p_L, p_R)$ be the cumulative distribution function of the beliefs of an uninformed voter about the value of v conditional on the policy platforms.[16] I assume the corresponding density function $g(v^b|p_L, p_R)$ to be positive for every value of v. This implies that voters assign at least a small probability ε to the possibility that candidates do not play equilibrium strategies, even when they observe equilibrium positions being played. More importantly for my argument, it rules out out-of equilibrium beliefs that assign a zero probability to certain values of v for certain combinations of candidates' policy positions without further justification. The idea is to rule out equilibria that are only justified by restrictions on voters' out out-of equilibrium beliefs. In such equilibria, voters believe that they are never pivotal, which is then, in return, justified by the equilibrium response of the candidates to the voters' strategies. Such equilibria are at odds with the basic idea of my model that voters try to make the right decision conditional on being pivotal. An example of such an equilibrium is the one mentioned above with uninformed voters voting for the candidate who offers the position closest to the median voter's bliss point.[17]

Lemma 4 *If voters' beliefs about the value of v are such that the probability distribution function $g(v^b|p_L, p_R) > 0$ for all combinations of v, p_L and p_R, the Bayesian Nash equilibrium given in Sections $2.1 - 2.4$ is the unique equilibrium with informed voters playing their weakly dominating strategy and uninformed voters' voting determined by a cutoff point $b_U^*(p_L, p_R)$.*

Proof. See the Appendix ■

[16] For the argument, it is not essential that all uninformed voters have the same beliefs. It is only important that they all believe all values of v to be possible.

[17] The standard way of restricting out-of equilibrium beliefs in a game in extensive form is to use the Sequential Equilibrium concept by Kreps and Wilson (1982). Unfortunately, there is no straightforward extension to a setup where players have a continuum of moves to choose from as candidates have in my model.

3 Generalizations

In Section 3.1, I relax the assumption that uninformed voters can observe the policy positions of the candidates. In 3.2, I show what happens if the median voter is uninformed. Finally, in Section 3.3 I show that my results are valid for more general utility functions of voters as well as candidates.

3.1 Voters who are ignorant about policy positions

So far, I have assumed that uninformed voters can observe the policy positions of the candidates. However, even if a voter does not know the policy platforms of the candidates, she can still vote for the candidate with the left (right) bliss point b_L (b_R) if her own bliss point is to the left (right) of the median bliss point. While in equilibrium all voters support the same candidate and candidates choose the same position as in Section 2, updating their beliefs about the valence difference v is impossible for the completely uninformed voters.[18]

Proposition 2 *If candidates and informed voters follow the same strategies as in Section* $2.1-2.4$*, and completely uninformed voters vote for the left (right) candidate if they have a bliss point* $b < b_m$ ($b > b_m$)*, these strategies constitute a Bayesian Nash equilibrium in the game with completely uninformed voters. The implemented policy is the same as in Proposition 1.*

Proof. See the Appendix ■

It is straightforward to verify that the equilibrium policy positions and the implemented policy are also robust to scenarios where some uninformed voters observe the policy positions, while some do not. Some of the voters who do not observe the policy positions could observe the valence term. If such voters ignore the information about the valence term and follow the same strategy as completely uninformed voters in Proposition 2 and all other players follow the given strategies, this once more constitutes an equilibrium.

The fact that voters may not necessarily need to know the exact announced policy positions of candidates to make an optimal voting decision was already pointed out

[18] This is the case despite the fact that the equilibrium strategies of voters are formally not the same. The strategies can, by definition, not be identical because voters have different information on which to base their moves if they observe the policy positions as compared to the case when they do not.

by McKelvey and Ordeshook (1986) in a different setup. It is an important point, since it shows that the information requirements for voters are often much weaker than the standard assumption that each voter has full information.

3.2 The case of an uninformed median voter

If the median voter is not informed, but all other voters follow the strategies derived in Sections 2.2 − 2.3, she is forced to decide between a decisive informed voter with a bliss point to the left or to the right of her own bliss point. Therefore, she does not have the possibility to make a decision that is optimal for all possible beliefs about the true value of v. Nonetheless, if she follows the simple strategy to always vote in favor of the policy position closest to her bliss point, this leads to an election outcome in which she never votes against her preferred candidate. The reason is that the higher-quality candidate who wins the elections adjusts his position to win the elections, even without the support of the uninformed median voter. I assume that the uninformed median voter votes for the left policy position if both candidates have the same distance to her bliss point.

Given the vote of the uninformed median voter, the decisive informed voter has the bliss point:

$$b_I^d = \begin{cases} b_l = F_I^{-1}(\frac{n+1}{2} - F_U(b_m)) & \text{if } |p_L - b_m| \leq |p_R - b_m| \\ b_r = F_I^{-1}(\frac{n+1}{2} - (F_U(b_m) - 1)) & \text{if } |p_L - b_m| > |p_R - b_m| \end{cases} . \tag{9}$$

The informed voter with bliss point b_l is the one with the bliss point closest to the left of the median bliss point, and the informed voter with bliss point b_r is the informed voter closest to the right of the median bliss point.

The position of the decisive voter given in (9) leads to the following strategies of the candidates:

Lemma 5 *The candidates' strategies in the case of an uninformed median voter are:*

$$\left.\begin{array}{c} p_L^* = \max(b_m - \frac{v}{4(b_m - b_l)}, b_l) \\ p_R^* = \min(b_R, b_l + (v + (b_l - p_L^*)^2)^{0.5}) \end{array}\right\} \text{ if } v > 0$$

$$, \tag{10}$$

$$\left.\begin{array}{c} p_R^* = \min(b_m + \frac{-v}{4(b_r - b_m)}, b_r) \\ p_L^* = \max(b_L, b_r - (-v + (b_r - p_R^*)^2)^{0.5}) \end{array}\right\} \text{ if } v \leq 0$$

and implemented policy is:

$$p^* = \begin{cases} p_R^* \text{ if } v > 0 \\ p_L^* \text{ if } v \leq 0 \end{cases}. \tag{11}$$

Proof. See the Appendix ■

It remains to be shown that given these strategies of the candidates, the voters' decision and consistent beliefs are indeed a best reply and we have a Perfect Bayesian Nash equilibrium of the game. $V(p_L, p_R)$ denotes the set of all values of $v(p_L^*, p_R^*)$ that lead to equilibrium policy positions p_L^* and p_R^* in (10). The following is a consistent belief system for uninformed voters, given the equilibrium strategies of the candidates:

$$\begin{array}{ll} v(p_L, p_R) \text{ with probability } 1 & \text{if } v(p_L, p_R) \text{ is the only element in } V(p_L, p_R), \\ g(v|p_L, p_R) = \dfrac{g(v)}{\int_{v \in V(p_L, p_R)} g(v)dv} & \text{if } V(p_L, p_R) \text{ contains more than one element,} \\ g(v|p_L, p_R) = \begin{cases} \dfrac{g(v)}{\int_0^\infty g(v)dv} \text{ for } v > 0 \\ 0 \text{ for } v \leq 0 \end{cases} & \text{if } V(p_L, p_R) = \emptyset \text{ and } |p_L - b_m| > |p_R - b_m|, \\ g(v|p_L, p_R) = \begin{cases} \dfrac{g(v)}{\int_{-\infty}^0 g(v)dv} \text{ for } v \leq 0 \\ 0 \text{ for } v > 0 \end{cases} & \text{if } V(p_L, p_R) = \emptyset \text{ and } |p_L - b_m| \leq |p_R - b_m|, \end{array} \tag{12}$$

where $g(v|p_L, p_R)$ is the probability distribution function of v conditioning on the policy positions.

The beliefs can simply be calculated from the equilibrium values for p_L^* and p_R^* given in Lemma 5. For the out of equilibrium beliefs, there are no restrictions in Perfect Bayesian Nash equilibrium and they are chosen in a way that makes them consistent with the strategies of the uninformed voters.

Proposition 3 *Together with the candidates' strategies given in Lemma 5 and the beliefs in 12, the voters' strategies constitute a Perfect Bayesian Nash equilibrium.*

Proof. See the Appendix ■

The intuition for the strategy of the median voter is simple. She punishes the candidate who deviates further from her ideal point. Since in equilibrium the candidate with the valence advantage adjusts his position in a way that ensures his victory, a situation in which the median voter regrets her vote ex post cannot occur. The candidate with valence advantage chooses his platform in a way that brings him as close as possible to his bliss point without losing the elections. The candidate with valence disadvantage chooses his platform so that he defeats the winning candidate if the latter chooses a platform even closer to his own bliss point.

It is remarkable that an uninformed median voter is actually better off as compared to the game in which she is informed. Her lack of information makes it possible for her to commit to a strategy that would otherwise not have been credible. An informed median voter cannot commit to vote against the candidate who takes a position further away from her bliss point, since that can imply that she has to vote against a candidate whom she prefers, even if that leads to the loss of this candidate.

Let $\varepsilon = \max(b_m - b_l, b_r - b_m)$ be the maximal distance of the median voter to an informed voter on either side. For $\varepsilon \rightarrow 0$, the strategies and the implemented policy given by (10) and (11) converge to the solution with an informed median given in (6) and (7). If informed voters are located "close" to the uninformed median voter, candidates' strategies in the case of the informed median provide a good approximation in the case of an uninformed median. This is likely to be the case if n is large.

3.3 Generalizing the utility functions

In the baseline model discussed so far, the utility functions of the voters as well as of the candidates are chosen to be as simple as possible. This section shows that the results are quite robust to the choice of the utility functions.

3.3.1 The utility function of the voters

First, consider the utility function of the voters given in (2). The proofs in Section 2 are based on there being a single cutoff point between informed voters who prefer the left candidate and informed voters who prefer the right candidate. As a result, all proofs hold without any major modification if there is at most one cutoff point or all informed voters prefer the same candidate. To show that a more general function leads to the same type of equilibria as in Section 2, I only need to show that the assumptions about functional form imply a unique cutoff point.

A more general utility function that depends only on distance and the quality of politicians is:

$$U_i(p, b_i) = u(d_i, q), \tag{2'}$$

with $d_i = |b_i - p|$. If $u(d_i, q) = -d_i^2 + q$, (2') is identical to (2).

Sufficient restrictions on the utility function for having a unique cutoff point are that the following derivatives exist and fulfill the conditions:

$$\begin{array}{ll} (a) & u_d(d,q) \ \leq 0, \\ (b) & u_{dd}(d,q) < 0, \\ (c) & u_q(d,q) \ \geq 0, \\ (d) & u_{qd}(d,q) \leq 0. \end{array}$$

Condition (a) naturally follows from saying that a point b is a bliss point. Condition (b) is somewhat stronger, but nonetheless standard. A voter suffers less from departing from her ideal point b to some alternative policy p' than from departing the same distance $|b - p'|$ away from p' to a policy p'' which has the distance $2|b - p'|$ from b. Without this restriction, for example, a high-quality Democratic candidate could be preferred by Democrats as well as very conservative voters, while moderate Republicans would prefer the low-quality Republican candidate. This would lead to at least two cutoff points. Condition (c) implies that voters never feel worse off with a higher-quality candidate ceteris paribus. It is necessary to ensure that, for example, very conservative voters do not prefer a low-quality Democrat to a high-quality moderate Republican. Condition (d), for example, helps to rule out cases of a very conservative voter preferring a high-quality Democrat to a moderate Republican, if the latter is preferred by moderate Republican voters.

Lemma 6 *Given the conditions on its derivatives, the generalized utility function* $u(d_i, q)$ *leads to at most one cutoff point in* b *for a given combination of* q_L, q_R, p_L *and* p_R.

Proof. See the Appendix ■

3.3.2 The utility function of the candidates

What about the utility function of the candidates given in (1)? Candidates are assumed to care neither about office nor about the quality of the winner. Both assumptions can be relaxed, because there is no uncertainty about the winner in the model. Concerns about victory do not enable the lower-quality candidate to win. On the other hand, even if the lower-quality candidate actually preferred to lose to let a higher quality candidate govern, he would still have an incentive to choose a position that forces the winner to make compromises with respect to his position. Therefore, the equilibria given in Section 2 do not disappear if the utility of the candidates depends on the quality of the winner of the elections or on winning the elections.

4 Swing voters

The equilibrium strategy for uninformed voters derived in Section 2.3 is relatively simple. Nevertheless, it requires a certain level of sophistication of the uninformed voters. Relaxing the sophistication requirements allows the reader, or future empirical researchers, to decide if they are indeed an attribute of a typical electorate. Moreover, modeling less sophisticated voters implies interesting effects on political competition that run counter to the common expectations regarding the effects of a less sophisticated and informed electorate. Specifically, I show that such an electorate actually leads to increased electoral control in the sense of forcing the winning candidate to a policy closer to the bliss point of the median voter. This does not only increase the welfare of the median voter, but that of a majority of all voters.

The methods I use to solve the model are similar to Section 2. First, I solve the problem of the voters and then the problem of the candidates. Finally, I show that the strategies of the sophisticated voters and the candidates together constitute a Perfect Bayesian Nash equilibrium. In Section 5, I analyze the welfare implications of having sophisticated uninformed voters instead of informed voters, as well as of having unsophisticated uninformed voters instead of sophisticated voters. For this analysis, I hold the overall distribution of voters' bliss points constant. In Section 6, I change the basic setup. Instead of a finite number of voters, I assume a continuum. This makes comparative statics possible.

4.1 The voting decision of unsophisticated voters

In this section, I introduce a third class of voters. These unsophisticated uninformed voters or swing voters vote naively without considering the fact that being the pivotal voter reveals information about the quality of politicians. Instead, they calculate their expected welfare given the policy platforms of the candidates and their a priori belief of the distribution of v. Therefore, they have the cutoff point $b^*_{UU}(p_L, p_R) = \frac{p_L+p_R}{2} - \frac{E(v)}{2(p_R-p_L)}$, which reduces to $b^*_{UU}(p_L, p_R) = \frac{p_L+p_R}{2}$ under the assumption that $E(v) = 0$ which I make from now on. All unsophisticated uninformed voters with a bliss point to the left of this cutoff point vote for the left candidate, all unsophisticated uninformed voters with a bliss point to the right of this cutoff point vote for the right candidate. To ensure the existence of an equilibrium, I assume that a voter with bliss point $b^*_{UU}(p_L, p_R)$ votes for the left candidate if $b^*_{UU}(p_L, p_R) \leq b_m$ and for the right candidate if $b^*_{UU}(p_L, p_R) > b_m$.

4.2 The problem of the sophisticated uninformed voters

The decisive informed voter now has the bliss point:

$$b^d_I(p_L, p_R) = F_I^{-1}\left(\frac{n+1}{2} - l_{SU}(p_L, p_R) - l_{UU}(p_L, p_R)\right), \tag{13}$$

where $l_{SU}(p_L, p_R)$ is the number of sophisticated uninformed voters voting in favor of the left policy position, and $l_{UU}(p_L, p_R)$ the number of unsophisticated uninformed voters voting in favor of the left policy position. Using the same arguments that were used to derive the equilibrium in Section 2, it is possible to show that if all sophisticated uninformed voters vote for the left position if their bliss point is smaller than b^d_I, and for the right position if their bliss point is larger than b^d_I, their strategies constitute an optimal reply to the strategies of the other voters for arbitrary combinations of p_L, p_R and v. This gives the second condition:

$$l_{SU}(p_L, p_R) = F_{SU}(b^d_I(p_L, p_R). \tag{14}$$

If conditions (13) and (14) hold, all sophisticated uninformed voters vote optimally independently of the strategies of the candidates. However, just as in the case with only sophisticated uninformed voters when the median is uninformed, it is sometimes impossible for a sophisticated uninformed voter to make her position only dependent on her own position relative to that of the decisive informed voter. The reason is that her own decision changes the decisive informed voter's identity. Consider the following cutoff point between voting left and right for sophisticated uninformed voters:

$$b^*_{SU}(p_L, p_R) = F_S^{-1}\left(\frac{n+1}{2} - l_{UU}(\frac{p_L + p_R}{2})\right), \tag{15}$$

where $F_S(b) = F_I(b) + F_{SU}(b)$ is the cumulative distribution function of sophisticated voters (that is voters who are either informed or sophisticated uninformed), and $F_S^{-1}(x)$ gives the sophisticated voter with the x_{th} smallest bliss point b among sophisticated voters. If the sophisticated voter at $b^*_{SU}(p_L, p_R)$ is informed, she is decisive because if she votes left (right), all informed and sophisticated uninformed voters to the left (right) of her vote left (right). Together with the unsophisticated uninformed voters who vote left (right), this constitutes a majority. Moreover, (14) holds and the voting stage of the game is in equilibrium.

If the sophisticated voter with bliss point $b^*_{SU}(p_L, p_R)$ is uninformed, she faces a situation similar to that of the uninformed median voter in Section 3.2. If she votes for the left candidate, the bliss point of the decisive informed voter is located to the left of her bliss point, and if she votes for the right candidate, the bliss point of the decisive informed voter is located to the right of her bliss point. Therefore, it is not independent of her beliefs of the value of v which decisive informed voter she prefers. I assume that she votes for the candidate whose position is closer to her own bliss point. Similarly to the case of an uninformed median voter, this turns out to be consistent with an equilibrium. The reason is once more that the candidates adjust their positions to the voters' strategies and the candidate with valence advantage wins. If both candidates have the same distance from b^*_{SU}, I assume that a sophisticated uninformed voter with this bliss point votes left if $\frac{p_L+p_R}{2} \geq b_m$ and right if $\frac{p_L+p_R}{2} < b_m$. Therefore, the decisive informed voter has the bliss point:

$$b_I^d(p_L, p_R) = \begin{cases} b^*_{SU}(p_L, p_R) & \text{if } b^*_{SU}(p_L, p_R) \in B_I \\ F_I^{-1}(F_I(b^*_{SU}(p_L, p_R))) & \text{if } b^*_{SU} \notin B_I \text{ and } \frac{p_L+p_R}{2} > b^*_{SU} \\ F_I^{-1}(F_I(b^*_{SU}(p_L, p_R))) & \text{if } b^*_{SU} \notin B_I \text{ and } \frac{p_L+p_R}{2} = b^*_{SU} \text{ and } \frac{p_L+p_R}{2} \geq b_m \\ F_I^{-1}(F_I(b^*_{SU}(p_L, p_R) + 1)) & \text{if } b^*_{SU} \notin B_I \text{ and } \frac{p_L+p_R}{2} = b^*_{SU} \text{ and } \frac{p_L+p_R}{2} < b_m \\ F_I^{-1}(F_I(b^*_{SU}(p_L, p_R) + 1)) & \text{if } b^*_{SU} \notin B_I \text{ and } \frac{p_L+p_R}{2} < b^*_{SU}, \end{cases} \tag{16}$$

where B_I is the set of bliss points of informed voters.

4.3 The problem of the candidates

The candidates face a somewhat more complicated problem than in Section 2 for there is now a trade-off in winning the support of sophisticated versus unsophisticated voters. Let $l_I(p_L, p_R, v)$ once more be the number of votes for the left position by informed voters, $l_{SU}(p_L, p_R)$ the number of votes for the left position by sophisticated uninformed voters, and $l_{UU}(p_L, p_R)$ the number of votes for the left position by unsophisticated uninformed voters. Then, there exists a best reply function for the candidate with valence advantage:

Lemma 7 *If $v > 0$, a best reply p_R^b to any p_L exists and is given by:*

$$p_R^b(p_L, v) = \begin{cases} \max_{[0,b_R]} \; p_R \; s.t. \; l_I(p_L, p_R, v) + l_{SU}(p_L, p_R) + l_{UU}(p_L, p_R) < \frac{n+1}{2} & \text{if } p_L \leq b_R \\ b_R & \text{if } p_L > b_R \end{cases}. \tag{17}$$

If $v \leq 0$, a best reply p_L^b to any $p_R \geq b_L$ exists and is given by:

$$p_L^b(p_R, v) = \begin{cases} \min_{[b_L,1]} \; p_L \; s.t. \; l_I(p_L, p_R, v) + l_{SU}(p_L, p_R) + l_{UU}(p_L, p_R) > \frac{n+1}{2} & \text{if } p_R \geq b_L \\ b_L & \text{if } p_R < b_L \end{cases}. \tag{18}$$

Proof. See the Appendix ■

The equilibrium policy platforms of the candidates are:

Proposition 4 *Equilibrium policy platforms of the candidates are:*

$$\left.\begin{array}{l} p_R^* = \min_{p_L \in [0,1]} \; p_R^b(p_L, v) \\ \\ p_L^* = \arg\min_{p_L \in [0,1]} p_R^b(p_L, v) \end{array}\right\} \text{if } v > 0, \tag{19}$$

$$\left.\begin{array}{l} p_L^* = \max_{p_R \in [0,1]} \; p_L^b(p_R, v) \\ \\ p_R^* = \arg\max_{p_R \in [0,1]} p_L^b(p_R, v) \end{array}\right\} \text{if } v \leq 0, \tag{20}$$

and the implemented policy is:

$$p = \begin{cases} p_R^* \text{ if } v > 0 \\ \\ p_L^* \text{ if } v \leq 0 \end{cases}. \tag{21}$$

Proof. See the Appendix ■

As in Section 2, the candidate with the valence advantage wins the elections. However, without any further assumptions about the distribution of the voters, it is not possible to give a more precise solution than that in Proposition 4.

The intuition for Proposition 4 is straightforward. The candidate with the valence advantage chooses a position that is as close as possible to his own bliss point without being defeated. The lower-quality candidate chooses his position to force the winner as close to the median voter's bliss point as possible.

4.4 Voters' beliefs and equilibrium

Similarly to the case without swing voters in Section 2.5, I now formulate beliefs for the voters that are consistent with the equilibrium:

Let $V(p_L, p_R)$ once more denote the set of all values of v that would lead to equilibrium policy positions p_L^* and p_R^* in (4). The following is a consistent belief system for the uninformed voters, given the equilibrium strategies of the candidates:

$$\begin{array}{ll}
v(p_L, p_R) \text{ with probability } 1 & \text{if } v(p_L, p_R) \text{ is the only element in } V(p_L, p_R), \\
g(v|p_L, p_R) = \dfrac{g(v)}{\int_{v \in V(p_L, p_R)} g(v) dv} & \text{if } V(p_L, p_R) \text{ contains more than one element,} \\
g(v|p_L, p_R) = \begin{cases} \dfrac{g(v)}{\int_0^\infty g(v) dv} \text{ for } v > 0 \\ 0 \text{ for } v \leq 0 \end{cases} & \begin{array}{l} \text{if } V(p_L, p_R) = \emptyset \text{ and} \\ \left| p_L - b_{SU}^*(p_L, p_R) \right| > \left| p_R - b_{SU}^*(p_L, p_R) \right|, \end{array} \\
g(v|p_L, p_R) = \begin{cases} \dfrac{g(v)}{\int_{-\infty}^0 g(v) dv} \text{ for } v \leq 0 \\ 0 \text{ for } v > 0 \end{cases} & \begin{array}{l} \text{if } V(p_L, p_R) = \emptyset \text{ and} \\ \left| p_L - b_{SU}^*(p_L, p_R) \right| \leq \left| p_R - b_{SU}^*(p_L, p_R) \right|. \end{array}
\end{array} \tag{22}$$

It remains to be shown that the voters' and the candidates' strategies, together with these beliefs, indeed constitute a Perfect Bayesian Nash Equilibrium:

Proposition 5 *Taking the nonstrategic decisions by the unsophisticated uninformed voters as given, the candidates' strategies together with the voting decisions by the sophisticated uninformed and the informed voters constitute a Bayesian Nash equilibrium. Together with the beliefs in (22), they constitute a Perfect Bayesian Nash equilibrium.*

Proof. See the Appendix ■

The interpretation of the equilibrium is similar to the equilibria in Section 2. Once more, sophisticated uninformed voters vote in a way that ensures that they vote for the candidate whom they prefer when they are pivotal. Because the candidates adjust their positions accordingly, even a sophisticated uninformed voter with bliss point b^*_{SU} never votes against her favorite candidate when she is pivotal. In fact, she is never pivotal in equilibrium and always votes against the winner.

4.5 Swing voters and equilibrium policy

The solution for the candidates' problem given in Proposition 5 is rather abstract. Nonetheless, I am able to make some welfare statements and analyze how voters' welfare depends on the number of swing voters for a given overall distribution of voters $F(b)$. Let B be the set of all bliss points and B_I, B_{SU} and B_{UU} the sets of the bliss points of the informed, the sophisticated uninformed and the unsophisticated uninformed voters so that $B = B_I \cup B_{SU} \cup B_{UU}$.[19] Then, the following results hold:

Lemma 8 *Taking the overall set of bliss points B as given, having sophisticated uninformed voters (case $'$) instead of informed voters (case $''$) at some bliss points ($B' = B''$, $B'_I \subsetneq B''_I$, $B''_{SU} \subsetneq B'_{SU}$, $B'_{UU} = B''_{UU}$) leads to equilibrium policies as close or closer to the median bliss point for all values of v ($|p^{*\prime}(v) - b_m| \leq |p^{*\prime\prime}(v) - b_m|$).*

Proof. See the Appendix ■

Lemma 9 *Taking the overall set of bliss points B as given, having unsophisticated uninformed voters (case $'$) instead of sophisticated uninformed voters (case $''$) at some bliss points ($B' = B''$, $B'_I = B''_I$, $B'_{SU} \subsetneq B''_{SU}$, $B''_{UU} \subsetneq B'_{UU}$) leads to equilibrium policies as close or closer to the median bliss point for all values of v ($|p^{*\prime}(v) - b_m| \leq |p^{*\prime\prime}(v) - b_m|$).*

Proof. See the Appendix ■

The lemmas show that turning an informed voter into a sophisticated uninformed voter as well as turning a sophisticated uninformed voter into an unsophisticated one, can only lead to policies closer to the median voter's bliss point for a given value of the quality difference v. So far, I have only shown that equilibrium policy can only

[19] Using sets is possible because of the assumption that all voters' bliss points are distinct from each other.

move towards the median bliss point if it changes, not that it actually does change. It is difficult to give general rules for a change of equilibrium policy. However, examples can provide some insight into this. Therefore, I discuss two examples and then provide a further example with a continuum of voters in Section 6.

The example for the effect of having a voter switch from being informed to being sophisticated uninformed was already provided by the generalization of the basic model in Section 2 to the model with an uninformed median voter in Section 3.2. A comparison of (7) and (11) confirms the result of Lemma 8. For values of v that do not allow the high-quality candidate to achieve his bliss point in spite of an uninformed median voter, the implemented policy is closer to the median bliss point if the median voter is uninformed.

To provide an example of uninformed voters becoming unsophisticated informed voters, consider the case of an electorate with only unsophisticated uninformed voters and compare it with the equilibrium in Section 2. The equilibrium with only sophisticated voters is given in Proposition (1). The given equilibrium policies cannot constitute an equilibrium with unsophisticated uninformed voters if the candidate with lower valence now wins given the equilibrium positions. If $0 < v \leq (b_R - b_m)^2$, any unsophisticated uninformed voter in the interval $(b_m, b_m + v^{0.5}/2)$ forces the right candidate to choose a position closer to the median bliss point to win the elections. If he does not adjust, he loses because, in equilibrium, he wins with just one vote and the swing voters in the interval are now voting against him instead of in his favor. If $0 < (-v) \leq (b_L - b_m)^2$, any unsophisticated uninformed voter in the interval $(b_m - (-v)^{0.5}/2, b_m)$ forces the left candidate to choose a position closer to the median bliss point to win. Unsophisticated uninformed voters change the equilibrium policy for a larger interval of values of v, if they are located closer to the median bliss point.[20]

From the last example, it should be clear that if the electorate is large and the number of swing voters is neither very small nor their distribution very different from the distribution of sophisticated voters, the equilibrium will not be the same as the equilibrium without swing voters.

[20]The median position is not necessarily a best reply for the candidate with valence disadvantage. Therefore, the examples in the text give sufficient, but not necessary conditions for a change of equilibrium positions due to the existence of swing voters.

5 Welfare analysis

In this section, I show the welfare impact of having swing voters in the electorate and, as a consequence, policies that are at least weakly closer to the median voter's bliss point.

Take $g(v)$ as given. Let again $p'(v)$ be an equilibrium policy and $p''(v)$ a different one resulting from the same distribution of bliss points $F(b)$, but with some informed voters instead of sophisticated uninformed voters and/or some sophisticated uninformed voters instead of unsophisticated uninformed voters. By Lemma 8 and Lemma 9, we know that the policy $p'(v)$ is at least as close to the median bliss point as the policy $p''(v)$. Thus, $(p''(v) - b_m)^2 \geq (p'(v) - b_m)^2$ for any difference in quality, v. Therefore, the median voter must be (weakly) better off with policy $p'(v)$ for every value of v. Conditioning on v, the majority of voters must be better off with $p'(v)$ instead of $p''(v)$. If $v > 0$ ($v \leq 0$), all voters to the left (right) of the median bliss point are better off with $p'(v)$ since the implemented policy is closer to their bliss point.

In an equilibrium with policy $p(v)$, the expected utility (before nature chooses the quality of candidates) of a voter with bliss point b is :

$$E[U(p,b)] = \int_{-\infty}^{\infty} -(p(v) - b)^2 g(v) + E[\max(q_L, q_R)], \tag{23}$$

where the first term is the utility from implemented policy dependent on the valence difference and the second part is the utility from the quality or valence of the winner of the elections. Since the candidate with a valence advantage always wins, the valence of the winner is the larger of the two valence factors, q_R and q_L.

The difference in ex ante expected utility from the different equilibrium policies $p''(v)$ and $p'(v)$ for a voter with bliss point b is therefore:

$$\Delta E(U,b) = E[U(p'',b)] - E[U(p',b)] = \int_{-\infty}^{\infty} (-p''(v)^2 + p'(v)^2 + 2b(p''(v) - p'(v)))g(v). \tag{24}$$

We know that the difference is weakly negative for the median voter because we know that she is better off with $p'(v)$:

$$\Delta E(U, b_m) = \int_{-\infty}^{\infty} (-p''(v)^2 + p'(v)^2 + 2b_m(p''(v) - p'(v)))g(v) \leq 0 \tag{25}$$

If $\int_{-\infty}^{\infty}(p''(v)-p'(v)))g(v)>0$, all voters with $b<b_m$ are better off with $p'(v)$ in expectation, and if $\int_{-\infty}^{\infty}(p''(v)-p'(v)))g(v)<0$, all voters with $b>b_m$ better off with $p'(v)$ in expectation. Together with the median voter, either group constitutes a majority and therefore, the majority of voters is better off with $p'(v)$.

If the expected value of $p'(v)$ is the same as the expected value of $p''(v)$, all voters are better off without exception. The intuition is simply that in this case, the volatility of policy decreases, which is good for all voters because they are risk averse, while the expected policy remains the same. If $p'(v)$ is not same as $p''(v)$, it is possible to calculate a cutoff point between the voters who are better off and those who are worse off. This is given by:

$$\begin{aligned}\Delta E(U,b_{cut}) &= \int_{-\infty}^{\infty}(-p''(v)^2+p'(v)^2+2b_{cut}(p''(v)-p'(v)))g(v)=0 \qquad (26)\\ \Rightarrow\ b_{cut} &= \frac{\int_{-\infty}^{\infty}(p''(v)^2-p'(v)^2)g(v)}{2\int_{-\infty}^{\infty}(p''(v)-p'(v))g(v)} \text{ for } \int_{-\infty}^{\infty}(p''(v)-p'(v))g(v)\neq 0.\end{aligned}$$

If $\int_{-\infty}^{\infty}(p''(v)-p'(v)))g(v)>0$, all voters with $b<b_{cut}$ ($b>b_{cut}$) are better (worse) off with $p'(v)$ in expectation and if $\int_{-\infty}^{\infty}(p''(v)-p'(v)))g(v)<0$, all voters with $b>b_{cut}$ ($b<b_{cut}$) are better (worse) off with $p'(v)$ in expectation.

A utilitarian (Benthamite) social welfare function that takes all voters equally into account is given by:

$$E[\sum_{i=1}^{n}(U(p,b_i)]=\sum_{i=1}^{n}\int_{-\infty}^{\infty}-(p(v)-b_i)^2g(v)+E[\max(q_L,q_R)] \qquad (27)$$

The difference between the aggregate welfare of policy $p''(v)$ and $p'(v)$ is given by:

$$\Delta E\sum_{i=1}^{n}(U,b_i)=\sum_{i=1}^{n}\int_{-\infty}^{\infty}(-p''(v)^2+p'(v)^2+2b_i(p''(v)-p'(v)))g(v) \qquad (28)$$

From this, it follows that aggregate utility is increased if the average bliss point $b_{av}=\frac{\sum_{i=1}^{n}b_i}{n}$ is on the side of the cutoff point where the change in policy leads to a welfare improvement for voters.[21]

[21] This last result is due to the quadratic disutility in distance and is therefore not robust to changes in the utility function of voters.

6 An example with a continuum of voters

For an example with more specific assumptions about the distribution of voters, I will depart from the basic setup and assume a continuum of voters instead of a finite number. Working with a continuum of voters ensures that the strategies of the candidates become continuous in v and makes it possible to analyze the impact of a marginal change in the number of unsophisticated voters on the welfare of voters. Comparative statics analysis is possible.

As was just shown, the welfare analysis leads to somewhat ambiguous results. The majority of the voters is better off with swing voters, but it is possible to construct examples where the average voter is not. However, under some symmetry assumptions, I show unambiguous ex ante welfare improvements in the specific example I give. Similar welfare improvement can be expected in polities where the symmetry assumptions do not hold exactly, but give a good approximation.

The assumptions about utility functions, candidates and the sequence of moves by the players remain the same as before. Instead of a finite number, there is now a continuum of voters with mass 1. $1 - \alpha - \beta$ of these are informed, α of these are uninformed but sophisticated and β are uninformed but not sophisticated. I assume that $a + \beta < \frac{1}{2}$. This implies that the winner of the elections needs the support of some informed voters and therefore, insures the existence of a decisive informed voter independently of the voting decision of the uninformed voters. Moreover, to simplify the analysis, I assume that the bliss points of all three groups of voters are uniformly distributed on the policy space $[0, 1]$. The expected value of the quality difference v is assumed to be 0 with density $g(v)$ symmetric around 0, that is $g(v) = g(-v)$ for all values of v. This implies that a certain valence advantage is as likely for the right candidate as for the left candidate. In addition, $b_R - 0.5 = 0.5 - b_L$, that is, the distance between both candidates' bliss points and the median position is the same.

Voters are assumed to vote as in Section 4. Informed voters cast their ballot for the candidate they prefer, after observing the policy positions as well as the valence factors. The unsophisticated uninformed voters vote for the candidate whose policy is closer to their bliss point. The sophisticated uninformed voters one more have to find an optimal cutoff point that determines their strategy.

To deal with the assumption of a continuum of voters, I need to make an additional tie-breaking assumption in case both candidates get exactly half of the votes. To ensure the existence of an equilibrium, I assume that in this case the candidate with the valence advantage wins. The intuition from the case with a finite number of voters still applies and therefore the continuum should be interpreted as a convenient approximation of the case of a finite number of voters. Therefore, this assumption seems innocent.

6.1 Solving the model with a continuum of voters

Given the decision of the two kinds of uninformed voters and the assumption about the distribution of voters, the bliss point of the decisive informed voter for $p_L \neq p_R$ is given by:

$$b_I^d(p_L, p_R, l_{SU}(p_L, p_R), l_{UU}(p_L, p_R)) = \frac{0.5 - l_{SU}(p_L, p_R) - \beta\frac{(p_L+p_R)}{2}}{1 - \alpha - \beta}. \tag{29}$$

Once more, the voter with bliss point b_I^d is decisive because when she votes left at least 50% of the voters vote left, and when she votes right, at least 50% of the voters vote right. Therefore, the candidate who has the support of the decisive informed voter wins the elections.

Sophisticated uninformed voters vote for the left (right) policy position if their bliss point is to the left (right) of the decisive informed voter's bliss point. They want to pull the decisive informed voter's bliss point closer to their own. From this follows the second condition:

$$F_{SU}(b_I^d(p_L, p_R, l_U)) = \alpha b_I^d(p_L, p_R, l_{SU}) = l_{SU}(p_L, p_R). \tag{30}$$

Putting (29) and (30) together, I obtain that:

$$b_I^d(p_L, p_R) = \frac{0.5 - \beta\frac{(p_L+p_R)}{2}}{1 - \beta}. \tag{31}$$

The share α of sophisticated uninformed voters drops out of the equation for the decisive informed voter because these voters manage to vote just as informed voters when they are pivotal. This implies that I can analyze the equilibrium policy with only informed voters and unsophisticated uninformed voters and know that the equilibrium policy positions must be identical for all cases with the same share β of unsophisticated uninformed voters in the electorate.

The number of votes for left in the case $\alpha = 0$ is:[22]

$$\beta \frac{(p_L + p_R)}{2} + (1-\beta)(\frac{p_L + p_R}{2} - \frac{v}{2(p_R - p_L)}). \tag{32}$$

What happens if $p_L = p_R$? Several assumptions seem plausible. In the case without any unsophisticated voters, the sophisticated uninformed ones had little reason to deviate from their strategy of making voting dependent only on their position relative to the median position. Here, I assume that sophisticated as well as unsophisticated uninformed voters randomize over their voting decision if $p_L = p_R$. Because there is a continuum of voters, there is nonetheless no uncertainty about the mass of votes for the left and the right candidate. I also assume that if $p_L = p_R$ and $v = 0$, all informed voters vote for the left candidate.

The equilibrium policy platforms of the candidates and the implemented policy are now:

Lemma 10

$$\left.\begin{array}{l} p_R^* = \min((1-\beta)^{0.5}v^{0.5} + 0.5, b_R) \\ p_L^* = \max(p_R^* - (1-\beta)^{0.5}v^{0.5}, 0) \\ p^* = p_R^* \end{array}\right\} \quad \textit{if } v > 0,$$
$$\left.\begin{array}{l} p_L^* = \max(-(1-\beta)^{0.5}(-v)^{0.5} + 0.5, b_L) \\ p_R^* = \min(p_L^* + (1-\beta)^{0.5}(-v)^{0.5}, 1) \\ p^* = p_L^* \end{array}\right\} \quad \textit{if } v \leq 0. \tag{33}$$

Proof. See the Appendix ■

6.2 Electoral control and welfare analysis

The ex-ante expected utility of any voter with bliss point b given an equilibrium policy $p(v)$ is still given by (23):

$$E(U(b)) = \int_{v=-\infty}^{v=\infty} -(p(v) - b)^2 g(v) dv + E(\max(q_R, q_L)),$$

Lemma 11 *The welfare of every voter is increasing in the share of unsophisticated uninformed voters β, independently of her bliss point b. The change in expected utility*

[22] To simplify the notation, I ignore the fact that the share of informed voters voting left cannot be smaller than 0 or larger than 1. This has no influence on any of the results.

of a voter (or candidate) with bliss point b due to a marginal increase in the share of boundedly rational voters is given by the formula:

$$\frac{dE(U(b))}{d\beta} = \int_{v=-\frac{(b_L-0.5)^2}{(1-\beta)}}^{v=\frac{(b_R-0.5)^2}{(1-\beta)}} |v|g(v)dv > 0, \tag{34}$$

which is larger than 0 *as long as the distribution of* v *is not degenerate.*

Proof. See the Appendix ■

The intuition is straightforward. An increase in the number of uninformed unsophisticated voters increases electoral control and forces the politician with the valence advantage to stay closer to the median position to win the elections. This is advantageous for all voters. Even though a voter might profit from an increased likelihood of somewhat more extreme policies close to her bliss point, she always suffers more from the equally increased likelihood of extreme policies on the other side of the median position. The overall effect of a change in β is the same for all voters independently of their bliss points due to the quadratic disutility in policy.

The impact of a change in β increases in:

$$\int_{v=-\frac{(b_L-0.5)^2}{(1-\beta)}}^{v=\frac{(b_R-0.5)^2}{(1-\beta)}} |v|g(v)dv.$$

This term could be called the "adjusted" absolute deviation of the valence difference v. The reason for this "adjusted" absolute deviation being the relevant measure of dispersion is that for large absolute values of v, the winning candidate implements his bliss point. Therefore, an additional dispersion of large absolute values of v does not lead to an additional variance in implemented policy. The advantage of a larger β disappears if v is always 0, that is in a standard model of Downsian Competition without a valence component. Because every single voter is better off with an increase in β, this constitutes a Pareto improvement from an ex-ante perspective.

7 Conclusion

This paper combines elements of the two approaches in political economics that interpret elections as preference aggregation and information aggregation, respectively.

I merge a Downsian model with voter disagreement on policy on a left-right scale, with a model of voter agreement over the quality of political candidates, which is not observable to all voters.

A lack of information on the part of some voters about the quality of politicians is shown to have no consequences at all if every uninformed voter is rational. But if there are some boundedly rational swing voters, a lack of information increases electoral control, in the sense of pulling the implemented policy closer to the preferences of the median voter. This surprising result arises because boundedly rational voters support whoever offers them a policy closer to their own bliss point. They do not consider the fact that their vote is only of importance in a close election with both candidates obtaining exactly half of the votes. This voting strategy works as a commitment device, forcing the winning candidate to moderate his policy position. The larger the group of unsophisticated uninformed voters is, the stronger is the favorable effect.

A remarkable aspect of my findings is that always voting for the candidate of the same party is entirely rational for uninformed voters. In equilibrium, uninformed voters support a candidate whose preferences are located on the same side of the median position as their own, in spite of the fact that they do not know whether this is the candidate who they would support if they were fully informed. This forces candidates to consider the preferences of uninformed voters as much as those of informed voters and stands in stark contrast to the literature that claims that abstentions can be the result of rational choice even when voting is costless (Feddersen and Pesendorfer 1999).

I show that voting patterns that are considered to be evidence for irrational partisan behavior can be the rational response to a lack of information about the quality of candidates. Nevertheless, the belief that partisan voting can lead to less desirable policies is confirmed. This is surprising, because the belief that partisan voting leads to bad policy is usually based on the belief that it is irrational. In my model, partisan voters decrease the welfare of the majority of voters, not because they act irrationally, but because they are rational and therefore cannot commit to "punishing" the candidate who announces a policy position further away from their own preferences. The individual rationality of their decisions leads to a decrease in electoral control, policies further away from the median voter's bliss point and an expected loss in welfare for the majority of voters. Boundedly rational swing voters, on the other hand, turn out to be a blessing, not a curse.

One possibility for testing my model is offered by experiments. Similar models have already been tested experimentally (Palfrey 2009), and it might be the best way of testing if the size of the electorate is likely to have an influence on the relative number of swing voters. This is left for future research.

Appendix A

Proof Section 2

Proof Lemma 4. Suppose that there was a cutoff point $b'_U(p_L, p_R) \neq b_m$ such that $F_U(b'_U(p_L, p_R)) \neq F_U(b_m)$ for any combination of p_L and p_R.[23] Without loss of generality, I assume that an uninformed voter at this bliss point votes left. (To describe a strategy with this voter voting right given p_L and p_R, while the other voters make the same voting decision, it is always possible to choose a cutoff point slightly further to the left.)

If $F_U(b'_U(p_L, p_R)) < F_U(b_m)$, the number of votes for the left policy position by uninformed voters is smaller than in the equilibrium given in Sections $2.1 - 2.4$, and at least one uninformed voter with a bliss point $b < b_m$ votes for the right policy position. The bliss point of the decisive informed voter is given by $b_I^d(l_U) = F_I^{-1}(\frac{N+1}{2} - F_U(b'_U(p_L, p_R)))$, and the bliss point of the decisive informed voter for $F_U(b'_U(p_L, p_R)+1$ votes for the left position by uninformed voters is given by $b_I^{d+1}(l_U) = F_I^{-1}(\frac{N+1}{2} - F_U(b^*_U(p_L, p_R)) - 1)$. The assumption $F_U(b'_U(p_L, p_R)) < F_U(b_m)$ implies that $b_I^d(l_U) > b_I^{d+1}(l_U) \geq b_m$. An uninformed voter with bliss point $b < b_m$ who votes right can only be pivotal if $b_I^{d+1}(l_U) \leq b^*(p_L, p_{R,} v) \leq b_I^d(l_U)$. Because v^b has positive support everywhere, voters believe that the possibility of this happening is positive. An uninformed voter with bliss point $b < b_m \leq b_I^{d+1}(l_U)$ prefers the left position to win when she is pivotal. Therefore, $F_U(b'_U(p_L, p_R)) < F_U(b_m)$ cannot be a cutoff point that is consistent with a Bayesian Nash equilibrium that is consistent with $g(v^b|p_L, p_R) > 0$ for all combinations of v, p_L and p_R.

The case $F_U(b^*_U(p_L, p_R)) > F_U(b_m)$ can be ruled out by an analogous argument. Therefore, only cutoff points of informed voters with $F_U(b^*_U(p_L, p_R)) = F_U(b_m)$ can characterize an equilibrium. From Section $2.1 - 2.4$, we know that in combination with the candidates' policy positions given in Proposition 1 and the weakly dominating

[23] If $F_U(b^*_U(p_L, p_R)) = F_U(b_m)$, both cutoff points lead to the same voting decision and therefore describe the same strategy by uninformed voters. For every distribution of uninformed voters, such alternative cutoff points exist around b_m.

strategies of the informed voters, the strategies characterized by this cutoff point constitute a Bayesian Nash equilibrium for all possible beliefs about $g(v^b|p_L, p_R)$. ■

Appendix B

Proofs Section 3

Proof Proposition 2. Consider the case $v > 0$.

The left candidate obtains $F_U(b_m)$ votes by uninformed voters. Therefore, the left candidate wins if and only if at least $\frac{n+1}{2} - F_U(b_m) = F_I(b_m)$ informed voters vote in his favor. This will be the case if and only if the informed median voter prefers the left candidate. This implies that the candidates face the same problem as if all voters observed the policy positions that is analyzed in Sections $2.1 - 2.4$. The analysis as well as the results of Proposition 1 therefore apply to the candidates' problem.

The informed voters' strategy constitutes a best reply to the other players' strategies, because they play a weakly dominating strategy. The completely uninformed voters who vote for the left candidate play a best reply to the other players' strategies since they are not pivotal. The completely uninformed voters who vote for the right candidate have a bliss point $b > b_m$. From the fact that the informed median voter votes for the right candidate in equilibrium, we know that $b^*(p_L, p_R, v) \leq b_m$. Therefore, all uninformed voters voting for the right candidate have a bliss point $b > b^*(p_L, p_R, v)$ and obtain higher utility with the right candidate. Voting right is therefore optimal for them.

The argument for the case $v \leq 0$ is analogous. ■

Proof Lemma 5. Outline of the proof: First, I show that the informed voter with bliss point b_l is decisive if the median voter votes left for the left policy position, and the informed voter with bliss point b_r is decisive if the median voter votes for the right policy position. I continue by showing that the implemented policy is as given in (11). Then, I show that the strategies are optimal for the candidates. It is straightforward to verify that for all values of v, we have $p_R^* \geq p_L^*$, so that the right candidate adopts the right policy position.

Consider the case when the median voter votes for the left policy position: If the informed voter with bliss point b_l votes for the left policy position, this implies that $b_I^* \geq b_l$. Therefore, all informed voters with a bliss point $b < b_l$ vote for the left policy position. It follows that all voters with a bliss point $b \leq b_m$ vote for the left policy position, and the candidate with the left policy position obtains at least $\frac{n+1}{2}$ votes and wins the elections. If the informed voter with bliss point b_l votes for the right policy position, this implies that $b_I^* \leq b_l$. Therefore, all informed voters with a bliss point to the right of the median voter vote for the right policy position. This implies that all voters with a bliss point $b > b_m$ and the voter with bliss point b_l vote for the right policy position. Therefore, the candidate with the right policy position obtains at least $\frac{n+1}{2}$ votes and wins the elections.

An analogous argument shows that the informed voter with bliss point b_r is decisive if the median voter votes for the right policy position.

Implemented policy:
If $v > 0$, then $p_R^* \leq b_l + (v + (b_l - p_L^*)^2)^{0.5}$ and therefore $\Delta U(b_l, p_L^*, p_R^*) = -(b_l - p_L^*)^2 + (b_l - p_R^*)^2 - v \leq 0$. This implies that the informed voter with bliss point b_l votes for the right candidate and therefore, the right candidate obtains a majority independently of the vote of the uninformed median voter.

If $v \leq 0$, then $p_L^* \geq b_R - (-v + (b_r - p_R^*)^2)^{0.5}$ and therefore, $\Delta U(b_r, p_L^*, p_R^*) = -(b_r - p_L^*)^2 + (b_r - p_R^*)^2 - v \geq 0$. This implies that the informed voter with bliss point b_r votes for the left candidate and therefore, the left candidate obtains a majority independently of the vote of the uninformed median voter.

Candidates' strategies for the case $v > 0$:
If $p_R^*(v) = b_R$, the right candidate cannot do better with any other position. If $p_R^* < b_R$, then $\Delta U(b_l) = -(b_l - p_L^*)^2 + (b_l - p_R^*)^2 - v = 0$. Therefore, the right candidate would not have the support of the informed voter with bliss point b_l for any position $p_R > p_R^*$. If $p_L^* = b_m - \frac{v}{4(b_m - b_l)}$, then $p_R^* = b_l + (v + (b_l - b_m + \frac{v}{4(b_m - b_l)})^2)^{0.5} = b_m + \frac{v}{4(b_m - b_l)}$ and if $p_L^* = b_l$ then $p_R^* = b_l + v^{0.5} \geq 2b_m - b_l$ (where the last inequality is due to the fact that $p_L^* = b_l$ implies that $v \geq 4(b_m - b_l)^2$). In both cases $|p_R^* - b_m| \geq |p_L^* - b_m|$, and therefore for any position $p_R > p_R^*$, the median voter votes for the left candidate. Therefore, for any $p_R > p_R^*$, the decisive informed voter is the voter with bliss point b_L who votes for the left candidate, and the left candidate wins with a position $p_L^* < p_R^*$. Thus, the right candidate cannot obtain any better implemented policy than p_R^*, and his position is a best reply to the strategies of the other players.

The left candidate would only have a better reply than p_L^* to the other players' strategies if he could win with a position $p_L < p_R^*$. Consider first the case with $p_L^* = b_l$. Any position p_L such that $p_L \neq b_l$ and $p_L < p_R^*$ loses because $\Delta U(b_l, b_l, p_R^*) = (b_l - p_R^*)^2 - v \leq 0$ implies that $\Delta U(b_l, p_L, p_R^*) = -(b_l - p_L)^2 + (b_l - p_R^*)^2 - v \leq 0$

for any $p_L < p_R^*$. Second case: $p_L^* = b_m - \frac{v}{4(b_m - b_l)}$. Then $p_R^* \leq b_m + \frac{v}{4(b_m - b_l)}$ and therefore $|p_R^* - b_m| \leq |p_L^* - b_m|$. This implies that if the left candidate chooses a position $p_L < p_L^*$, the median voter votes for the right candidate. The left candidate loses because $\Delta U(b_l, p_L^*, p_R^*) \leq 0$ implies that $\Delta U(b_r, p_L^*, p_R^*) < 0$ which implies that $\Delta U(b_r, p_L, p_R^*) < 0$ for all $p_L < p_L^*$, so that the informed voter with bliss point b_r votes for the right candidate. If the left candidate chooses a position p_L such that $p_R^* > p_L > p_L^*$, then $p_L > b_l$ and from $\Delta U(b_l, p_L^*, p_R^*) \leq 0$ it follows that $\Delta U(b_l, p_L, p_R^*) < 0$ for such a value of p_L. Therefore, with a position $p_L < p_R^*$, the left candidate can neither win the vote of the informed voter with bliss point b_l, nor the vote of the informed voter with bliss point b_r, and thus loses. For $p_R^* < b_R$, the given combination of p_R^* and p_L^* is the only one that can be part of an equilibrium. For any $p_L \neq p_L^*$, the right candidate could choose a position closer to his bliss point and win, so that p_R^* would not be a best reply to any $p_L \neq p_L^*$. However, any $p_R > p_R^*$ can be defeated by p_L^*. If $p_R^* = b_R$, any left reply is a best reply. For the Lemma, I assume that the left candidate chooses the position given in (10).

An analogous argument applies to the case $v \leq 0$. ■

Proof Proposition 3. That the strategies of the candidates are best replies is shown in Lemma 5. Informed voters choose weakly dominating strategies that are best replies to any strategy profile by other players. It remains to be shown that the strategies of the uninformed voters are best replies.

The uninformed voters with bliss points $b > b_m$ who always vote for the right policy position can make a difference by voting left only in elections where they are pivotal. In equilibrium, this only occurs if the informed voter with bliss point b_l votes for the right candidate. In this case, the uninformed voters who vote right have a bliss point $b > b_m > b_l \geq b^*$ and maximize their utility with voting for the right candidate. An analogous argument applies for uninformed voters voting left. The uninformed median voter is never pivotal in equilibrium and thus, her strategy is a best reply.

For any out of equilibrium combinations of p_L and p_R, voters believe that the candidate with a position closer to the median voter has the valence advantage. Take the case $|p_L - b_m| > |p_R - b_m|$ and $p_L < p_R$. All uninformed voters believe that $v > 0$ and that all voters with $b > b_m$ (informed as well as uninformed) vote for the right candidate and therefore constitute a majority for him. Thus, for the uninformed voters with $b > b_m$, voting right is a best reply because they believe that $b^* < b_m$. For the uninformed voters with $b < b_m$, voting left is a best reply because they believe that the right candidate wins independently of their voting decision. A similar argument applies to the other possible cases. ■

Proof Lemma 6. Assume that $p_R \geq p_L$, as is always the case in any equilibrium (the proof is analogous for $p_R < p_L$). Then, there are two possibilities, $q_R \geq q_L$ and $q_R < q_L$. If $q_R \geq q_L$, then either every voter prefers right (and the unique cutoff point is $b^* = 0$), or there is at least one value $b \in [0,1]$ that solves $u(|b - p_L|, q_L) = u(|b - p_R|, q_R)$. The latter follows from the mean value theorem because u is continuous in d (this is implied by the fact that u has a derivative with respect to d), and therefore also in b (because d is a continuous function of b). Let b^* denote the largest b that solves the equation. From $q_R \geq q_L$, it follows that $b^* \leq \frac{p_L + p_R}{2}$, because a higher-quality candidate is always preferred if he is located closer to a voter's bliss point. From this and the fact that b^* is the rightmost bliss point with $u(|b - p_L|, q_L) = u(|b - p_R|, q_R)$, it follows that all voters to the right of b^* prefer right. If $b^* > p_L$, all voters with a bliss point b such that $p_L \leq b < b^*$ must prefer left because their bliss point is closer to p_L and further away from p_R than for the indifferent voter with bliss point b^*. Voters with $b < \min(p_L, b^*)$ must have a preference for left because the two assumptions $u_{dd}(d, q) < 0$ and $u_{qd}(d, q) \leq 0$ ensure that $u_d(p_{L,} q_L) > u_d(p_{R,} q_R)$ everywhere to the left of p_L. Therefore, there can be only one cutoff point and all voters with a bliss point to the left of b^* prefer left.

An analogous argument can be given to show that in the case of $q_R < q_L$, the bliss point is also unique. ■

Appendix C
Proofs Section 4

The proof of Lemma 7 and Proposition 4 requires the following three lemmas:

Lemma A-1 *The candidate with valence advantage wins if he chooses a position as close or closer to the median voter's bliss point than the other candidate.*

Proof. Consider the case $v > 0$ and $p_R \geq p_L$:

If $p_R = p_L$, all informed voters vote in favor of the candidate with valence advantage. Because the informed voters are the majority of voters, the candidate with valence advantage wins the elections.

If $p_R > p_L$, it follows from the assumptions that $|p_R - b_m| \leq |p_L - b_m|$ and $p_R > p_L$

that $b^*_{UU} = \frac{p_R+p_L}{2} < b_m$. Therefore, all unsophisticated uninformed voters with a bliss point at and to the right of the median bliss point vote right. If an unsophisticated uninformed voter is located at the median bliss point, then $l_{UU}(\frac{p_L+p_R}{2}) < F_{UU}(b_m)$ and therefore from equation (15):

$$b^*_{SU}(p_L, p_R) = F_S^{-1}\left(\frac{n+1}{2} - l_{UU}(\frac{p_L+p_R}{2})\right)$$
$$= F_S^{-1}\left(F_S(b_m) + F_{UU}(b_m) - l_{UU}(\frac{p_L+p_R}{2})\right) \geq F_S^{-1}(F_S(b_m)+1) > b_m.$$

If a sophisticated voter has the median bliss point, then $l_{UU}(\frac{p_L+p_R}{2}) \leq F_{UU}(b_m)$ and therefore:

$$b^*_{SU}(p_L, p_R) = F_S^{-1}\left(\frac{n+1}{2} - l_{UU}(\frac{p_L+p_R}{2})\right) =$$
$$F_S^{-1}\left(F_S(b_m) + F_{UU}(b_m) - l_{UU}(\frac{p_L+p_R}{2})\right) \geq F_S^{-1}(F_S(b_m)) = b_m.$$

In both cases, $b^*_{SU}(p_L, p_R) \geq b_m > b^*_{UU}$. Moreover, $v > 0$ together with $p_R > p_L$ implies that $b^*_{UU} > b^*_I$. Because $b^*_{SU}(p_L, p_R) > b^*_{UU}$, a sophisticated uninformed voter with bliss point b^*_{SU} votes right and thus $b^d_I \geq b^*_{SU}(p_L, p_R)$ and $b^d_I > b^*_{UU} > b^*_I$. Thus, the decisive informed voter votes for the right candidate and the right candidate wins.

An analogous argument applies to the cases $v > 0$, $p_R < p_L$ and $v < 0$. ■

Lemma A-2 *As long as $v > 0$ and $p_R > p_L$ the number of votes for left candidate is nondecreasing in p_R.*

Proof. The cutoff points for informed voters and unsophisticated uninformed voters b^*_I and b^*_{UU}, respectively, are increasing in p^*_R. Therefore, the number of informed and unsophisticated uninformed voters voting for the left candidate is nondecreasing in p_R. Remember that the cutoff point for uninformed voters is given by (15):

$$b^*_{SU}(p_L, p_R) = F_S^{-1}\left(\frac{n+1}{2} - l_{UU}(\frac{p_L+p_R}{2})\right),$$

and therefore decreasing in $l_{UU}(\frac{p_L+p_R}{2})$, the number of unsophisticated uninformed voters voting left. Therefore, the number of sophisticated uninformed voters voting right is nondecreasing in p_R. However, the number of sophisticated uninformed voters with a bliss point at or to the left of the cutoff point given by $F_{SU}\left(F_S^{-1}\left(\frac{n+1}{2} - l_{UU}(\frac{p_L+p_R}{2})\right)\right)$, and this can by the definition of $F_S^{-1}(x)$ and $F_{SU}(b)$ not decrease faster than $l_{UU}(\frac{p_L+p_R}{2})$ increases. Moreover, a sophisticated uninformed voter at the cutoff point votes right only if $\frac{p_L+p_R}{2} < b_{SU}^*$. Therefore, the number of all uninformed voters, sophisticated and unsophisticated, voting left is nondecreasing in p_R. Given that neither the number of votes by informed voters nor the number of votes by uninformed voters can decrease, he number of votes for left candidate must be nondecreasing in p_R. ■

Lemma A-3 *The interval of values of $p_R \geq p_L$ for which $l_I(p_L, p_R, v) + l_{UU}(p_L, p_R) + l_{SU}(p_L, p_R) < \frac{n+1}{2}$ holds is closed for all values of p_L.*

Proof. Suppose the statement were to be false and that the interval of values of $p_R \geq p_L$ for which $l_I(p_L, p_R, v) + l_{UU}(p_L, p_R) + l_{SU}(p_L, p_R) < \frac{n+1}{2}$ holds is not closed for some p_L. From Lemma A-2 , we know that if a value $p_R^w > p_L$ wins against p_L, any other p_R such that $p_R > p_L$ and $p_R < p_R^w$ must also win. Therefore, if the interval is not closed, there is a value $\bar{p}_R$ with $l_I(p_L, \bar{p}_R, v) + l_{UU}(p_L, \bar{p}_R) + l_{SU}(p_L, \bar{p}_R) > \frac{n+1}{2}$, but $l_I(p_L, p_R, v) + l_{UU}(p_L, p_R) + l_{SU}(p_L, p_R) < \frac{n+1}{2}$ for all p_R such that $p_L \leq p_R < \bar{p}_R$.

Define $\bar{l}_I = l_I(p_L, \bar{p}_R, v)$. This implies that $b_I^*(\bar{p}_R, p_L, v) > F_I^{-1}(\bar{l}_I)$, where the strict inequality is due to the fact that, by assumption, indifferent informed voters vote for the candidate with valence advantage. Then, by the continuity of b_I^* in p_R (for $p_R > p_L$), there exists a $p_R^I < \bar{p}_R$ for which $b_I^*(p_R, p_L, v) > F_I^{-1}(\bar{l}_I)$, and $l_I(p_L, p_R^I, v) = \bar{l}_I$. Define $\bar{l}_{UU} = l_{UU}(p_L, \bar{p}_R, v)$. This implies that $b_{UU}^*(\bar{p}_R, p_L) > F_U^{-1}(\bar{l}_{UU})$, with $b_{UU}^* = \frac{p_L+\bar{p}_R}{2}$. The strict inequality follows from the assumption that the left candidate wins the elections. Lemma A-1 implies that in this case $|p_L - b_m| < |p_R - b_m|$ and therefore $b_{UU}^*(p_L, p_R) > b_m$. By the assumption stated on page 25, an unsophisticated uninformed voter with bliss point $b_{UU}^*(p_L, p_R)$ votes for the right candidate in this case. By the continuity of b_{UU}^* in p_R, there exists a $p_R^{\bar{l}_{UU}} < \bar{p}_R$ for which $b_{UU}^*(p_R, p_L, v) > F_U^{-1}(\bar{l}_{UU})$. For $p_R \in [p_R^{\bar{l}_{UU}}, \bar{p}_R]$, the cutoff point for sophisticated uninformed voters is $b_{SU}^*(p_L, \bar{p}_R)$.

Define $\bar{l}_{SU} = l_{SU}(p_L, \bar{p}_R, v)$. This implies $b_{SU}^*(\bar{p}_R, p_L) > F_{SU}^{-1}(\bar{l}_{SU})$. The strict inequality follows from the fact that left wins and therefore $\frac{p_L+\bar{p}_R}{2} > b_{SU}^*$. But then there must be some $p_R^{\bar{l}_{SU}}$ such that $p_R^{\bar{l}_U} < p_R^{\bar{l}_{SU}} < \bar{p}_R$ for which $\frac{p_L+p_R^{\bar{l}_{SU}}}{2} > b_{SU}^*$ and a sophisticated uninformed voter with bliss point b_{SU}^* votes right. Therefore, $l_{SU}(p_L, p_R, v) = \bar{l}_{SU}$ for all p_R such that $p_R^{\bar{l}_{SU}} \leq p_R < \bar{p}_R$.

Putting the results together, for any $p_R \geq \max(p_R^{\bar{l}_{SU}}, p_R^{\bar{l}_I})$, $p_R < \bar{p}_R$, we have $l_I(p_L, \bar{p}_R, v) + l_{UU}(p_L, \bar{p}_R) + l_{SU}(p_L, \bar{p}_R) = \bar{l}_I + \bar{l}_{UU} + \bar{l}_{SU} > \frac{n+1}{2}$. This is a contradiction and thus, the interval is closed for all values of p_L. ■

Proof Lemma 7. If $p_L \leq b_R$, we know from Lemma A-1 that there is a $p_R \geq p_L$ for which $l_I(b_m, v) + l_{SU}(p_L, b_m) + l_{UU}(p_L, b_m) < \frac{n+1}{2}$ for all p_L and $v > 0$. From Lemma A-3, we know that the interval of values p_R for which $l_I(b_m, v) + l_{SU}(p_L, b_m) + l_{UU}(p_L, b_m) < \frac{n+1}{2}$ is closed. Moreover, it is bounded. Thus, a solution to the maximization problem (17) exists. Its solution must constitute a best reply because $p_R^b \geq p_L$ and therefore, the right candidate is weakly better off with p_R^b than with choosing a position that loses the elections and therefore leads to policy position p_L being implemented.

If $p_L > b_R$, we know from Lemma A-1 that the right candidate can win the elections with his bliss point as the policy position, so that $p_R = b_R$ is the best reply in this case.

The proof for the case $v \leq 0$ is analogous. ■

For the proof of Proposition 4 I need the following Lemma:

Lemma A-4 *The best reply functions given in Lemma* 7 *are continuous*

Proof. Consider the case $v > 0$:

Let $\varepsilon > 0$. Consider the two policy platforms p_L and $p'_L = p_L + \varepsilon$. First, I show that $p_R^b(p_L) + \varepsilon \geq p_R^b(p'_L)$. If $p_R^b(p_L) = b_R$ this is obvious. If $p_R^b(p_L) < b_R$, right must be losing with any $p_R > p_R^b(p_L)$ given p_L, if not $p_R^b(p_L)$ is not a best reply. Now if $p_R^b(p_L) + \varepsilon < p_R^b(p'_L)$, it follows that $b_I^*(p_L, p_R^b(p_L), v) < b_I^*(p'_L, p_R^b(p'_L), v)$ and $b_U^*(p_L, p_R^b(p_L), v) < b_U^*(p'_L, p_R^b(p'_L), v)$. But right cannot win with $p_R^b(p'_L)$ against p'_L if every $p_R > p_R^b(p_L)$ loses against p_L because there must be $p_R > p_R^b(p_L)$ that gains at least as many votes against p_L as $p_R^b(p'_L)$ against p'_L. This is a contradiction, and therefore $p_R^b(p_L) + \varepsilon \geq p_R^b(p'_L)$. Consider the reply $p_R^b(p_L) - \varepsilon$ to p'_L. Then $b_I^*(p_L, p_R^b(p_L), v) > b_I^*(p'_L, p_R^b(p'_L) - \varepsilon, v)$ and $b_{UU}^*(p_L, p_R^b(p_L), v) = b_{UU}^*(p'_L, p_R^b(p'_L) - \varepsilon, v)$ and therefore right must win. Thus $p_R^b(p'_L) \geq p_R^b(p_L) - \varepsilon$. Taking both results together, we know that if the best reply to p_L is $p_R^b(p_L)$, the best reply to $p_L + \varepsilon$ must be within distance ε of $p_R^b(p_L)$. From this, it follows that if there are two policy platforms p_L and $\tilde{p}_L$ such that $|p_L - \tilde{p}_L| < \frac{\delta}{2}$, than $\left|p_R^b(p_L) - p_R^b(\tilde{p}_L)\right| < \delta$ and therefore $p_R^b(p_L)$ is continuous.

The proof for the case $v \leq 0$ is analogous. ■

Proof Proposition 4. Proof for the case $v > 0$:

First, I prove that p_R^* and p_L^* exist, then that they constitute best responses to each other and the voters' strategies. From Lemma A-4**,** we know that $p_R^b(p_L, v)$ is a continuous function. In addition, $[0, 1]$ is compact. Therefore $p_R^* = \min_{p_L \in [0,1]}$

$p_R^b(p_L, v)$ exists according to Weierstrass' maximum theorem. Consequently, $p_L^* = \arg\min_{p_L \in [0,1]} p_R^b(p_L, v)$ also exists, but is not necessarily unique.

By the definition of p_R^*, there is no p_L that can obtain a majority against it given the strategies of the voters. Therefore, p_L^* must be a best reply by the left candidate. By construction, p_R^* is a best reply to p_L given the strategies of the voters.

The argument for the case $v \leq 0$ is similar. ■

Proof Propositon 5. The informed voters always vote for the candidate they prefer. That the candidates maximize their utility given the strategies of the voters was already shown in Proposition 4. The sophisticated uninformed voters with bliss points $b > b_{SU}^*(p_L, p_R)$ who vote for the right policy position can make a difference by voting left only in elections where they are pivotal. In equilibrium, this only occurs if the informed voter with bliss point $F_I^{-1}(b_{SU}^*(p_L, p_R))$ votes for the right candidate and therefore $F_I^{-1}(b_{SU}^*(p_L, p_R)) \geq b^*$. In this case, the uninformed voters who vote right have a bliss point $b > b_{SU}^*(p_L, p_R) > F_I^{-1}(b_{SU}^*(p_L, p_R)) \geq b^*$ and are better off with the right candidate for whom they vote. An analogous argument applies for uninformed voters with $b < b_{SU}^*(p_L, p_R)$ who vote for the left policy position. The uninformed voter with bliss point b_{SU} is never pivotal in equilibrium and is therefore never worse off following her strategy.

For any out of equilibrium combinations of p_L and p_R, voters believe that the candidate with a position closer to the uninformed voter with bliss point $F_I^{-1}(b_{SU}^*(p_L, p_R)) \geq b^*$ has the valence advantage. Take the case $|p_L - b_m| > |p_R - b_m|$ and $p_L < p_R$. All uninformed voters believe that $v > 0$ and that all voters with $b > F_I^{-1}(b_{SU}^*(p_L, p_R)) \geq b^*$ (informed as well as uninformed) vote for the right candidate and therefore constitute a majority for him. Thus, for the uninformed voters with $b > F_I^{-1}(b_{SU}^*(p_L, p_R))$, voting right is a best reply because they believe that $b^* < F_I^{-1}(b_{SU}^*(p_L, p_R))$. For the uninformed voters with $b < F_I^{-1}(b_{SU}^*(p_L, p_R))$, voting left is a best reply because they believe that the right candidate wins independently of their voting decision. A similar argument applies to the other possible cases. ■

Proof Lemma 8. Consider the case $v > 0$:

If $|b_m - p_R| \leq |b_m - p_L|$, right wins (by Lemma A-1) independent of the details of the distribution. Therefore, I have to check only combinations of p_L and p_R with $|b_m - p_R| > |b_m - p_L|$.

From Proposition 4, we know that the right candidate wins the elections with some position $p_R \geq b_m$. Moreover, the cutoff point for sophisticated uninformed voters $b^*_{SU}(p_L,p_R) = F_S^{-1}\left(\frac{n+1}{2} - l_{UU}(\frac{p_L+p_R}{2})\right)$ depends only on the distribution of sophisticated and unsophisticated uninformed voters and is therefore the same independent of the exact distribution of informed and sophisticated uninformed voters among the sophisticated voters. From $p_R \geq b_m$ and $|b_m - p_R| > |b_m - p_L|$, it follows that $b^*_{UU} > b_m$. Therefore, all unsophisticated uninformed voter at and to the left of the median bliss point vote for the left candidate and if an unsophisticated uninformed voter has the median bliss point, thus $l_{UU}(\frac{p_L+p_R}{2}) \geq F_{UU}(b_m)$ and therefore using equation (15):

$$\begin{aligned} b^*_{SU}(p_L,p_R) &= F_S^{-1}\left(\frac{n+1}{2} - l_{UU}(\frac{p_L+p_R}{2})\right) \\ &= F_S^{-1}\left(F_S(b_m) + F_{UU}(b_m) - l_{UU}(\frac{p_L+p_R}{2})\right) \\ &\leq F_S^{-1}(F_S(b_m)) \leq b_m. \end{aligned}$$

It follows that $b^*_{SU}(p_L,p_R) < b^*_{UU}$, and therefore the voter with bliss point b^*_{SU} votes left if she is uninformed. The decisive informed voter is thus given by $b_I^{d\prime\prime}(p_L,p_R) = F_I^{\prime\prime-1}(F_I^{\prime\prime}(b^*_{SU}(p_L,p_R)))$ respectively $b_I^{d\prime}(p_L,p_R) = F_I^{\prime-1}(F_I^{\prime}(b^*_{SU}(p_L,p_R)))$. From the fact that there are more informed voters in case ($''$) than in case ($'$), it follows that $F_I'(b) \leq F_I''(b)$ which in turn implies that $F_I^{\prime-1}(F_I'(b)) \leq F_I^{\prime\prime-1}(F_I''(b))$ for all b. Therefore, $b_I^{d\prime} \leq b_I^{\prime\prime d}$ and every position p_R that wins given $(B_I', B_{SU}', B_{UU}', p_L)$ wins also given $(B_I'', B_{SU}'', B_{UU}'', p_L)$, but not vice versa. This implies that $|p^{*\prime}(v) - b_m| \leq |p^{*\prime\prime}(v) - b_m|$.

The argument for the case $v \leq 0$ is analogous. ■

Proof Lemma 9. Consider the case $v > 0$:

It follows from Lemma A-1 that before and after the sophisticated uninformed voters switch to being unsophisticated uninformed, the right candidate can win with some position $p_R \geq b_m$.

If $|b_m - p_R| \leq |b_m - p_L|$, the right candidate wins (by Lemma A-1) independent of the details of the distribution. Therefore, I focus on combinations of p_L and p_R with $|b_m - p_R| > |b_m - p_L|$.

The informed voters a make the same voting decision for given p_L, p_R and v for both $(B_I'', B_{SU}'', B_{UU}'')$ and (B_I', B_{SU}', B_{UU}').

From $p_R \geq b_m$ and $|b_m - p_R| > |b_m - p_L|$ follows that $b^*_{UU} = \frac{p_L+p_R}{2} > b_m$. Thus,

$l_{UU}(\frac{p_L+p_R}{2}) \geq F_{UU}(b_m)$ and therefore using equation (15):

$$\begin{aligned} b^*_{SU}(p_L,p_R) &= F_S^{-1}\left(\frac{n+1}{2} - l_{UU}(\frac{p_L+p_R}{2})\right) \\ &= F_S^{-1}\left(F_S(b_m) + F_{UU}(b_m) - l_{UU}(\frac{p_L+p_R}{2})\right) \\ &\leq F_S^{-1}(F_S(b_m)) \leq b_m. \end{aligned}$$

From this is follows that $b^*_{SU}(p_L, b_m) \leq b_m \leq b^*_{UU}$ for (B'_I, B'_{SU}, B'_{UU}) as well as $(B''_I, B''_{SU}, B''_{UU})$.

Therefore, the voters who are sophisticated uninformed in one case and unsophisticated in the other either do not change their voting decision, or vote for the right candidate if they are sophisticated and for the left candidate when they are not sophisticated. This can be partly offset by sophisticated informed voters voting left instead of right. However, the number of votes for the right candidate by uninformed voters cannot be larger given p_L and p_R in case $(')$ compared to case $('')$. This can be seen by comparing

$$b^{*''}_{SU}(p_L,p_R) = F_S^{-1''}\left(\frac{n+1}{2} - l''_{UU}(\frac{p_L+p_R}{2})\right)$$

and

$$b^{*'}_{SU}(p_L,p_R) = F_S^{-1'}\left(\frac{n+1}{2} - l'_{UU}(\frac{p_L+p_R}{2})\right)$$

If a sophisticated voter is turned into an unsophisticated voter, but would also vote left if he had stayed sophisticated, $F'_{SU}(b^{*'}_{SU}(p_L,p_R))$ does not increase compared to $F''_{SU}(b^{*''}_{SU}(p_L,p_R))$. If a sophisticated voter is turned into an unsophisticated voter and votes left instead of right, $F'_{SU}(b^{*'}_{SU}(p_L,p_R))$ can increase compared to $F''_{SU}(b^{*''}_{SU}(p_L,p_R))$, but not by more than $l'_{UU}(\frac{p_L+p_R}{2})$ increases compared to $l''_{UU}(\frac{p_L+p_R}{2})$, so once more the number of uninformed voters voting left cannot increase.

Therefore, the total number of votes for right cannot be larger given p_L and p_R in case $(')$ compared to case $('')$. Every position p_R that wins given $(B'_I, B'_{SU}, B'_{UU}, p_L)$ wins also given $(B''_I, B''_{SU}, B''_{UU}, p_L)$, but not vice versa. This implies that $|p^{*\prime}(v) - b_m| \leq |p^{*\prime\prime}(v) - b_m|$.

The argument for the case $v \leq 0$ is analogous. ■

Appendix D

Proofs Section 6

Proof Lemma 10.

Consider the case $v > 0$:

The left candidate can never be better off offering a policy position $p_L \geq p_R$. For $p_L < p_R$, his vote-maximizing strategy is given by:

$$p_L^{\max}(p_R) = \arg\max_{p_L \in [0, p_R)} \beta \frac{(p_L + p_R)}{2} + (1-\beta)(\frac{p_L + p_R}{2} - \frac{v}{2(p_R - p_L)}). \tag{35}$$

The first-order condition for an interior maximum is $\frac{1}{2} - (1-\beta)\frac{v}{2(p_R - p_L)^2} = 0$. From this, it follows that:

$$p_L^{\max}(p_R) = \max(p_R - (1-\beta)^{0.5} v^{0.5}, 0) \tag{36}$$

With this $p_L^{\max}(p_R)$ it is possible to calculate the optimal strategy for the right candidate. He must take the position that is closest to his bliss point, subject to the constraint that he wins against $p_L^{\max}(p_R)$. Therefore:

$$p_R^*(v) = \arg\max_{p_R \in [0, b_R]} p_R \ s.t. \frac{(p_L^{\max}(p_R, v) + p_R)}{2} - (1-\beta)\frac{v}{2(p_R - p_L)} \leq 0.5. \tag{37}$$

The left-hand side of the constraint is increasing in p_R, while the right-hand side is constant. Therefore, the solution is either b_R as long as $p_L^{\max}(b_R)$ obtains less than 50% of the votes for the left candidate, or the solution to:

$$\frac{2p_R - (1-\beta)^{0.5} v^{0.5}}{2} - (1-\beta)\frac{v}{2((1-\beta)^{0.5} v^{0.5})} = 0.5. \tag{38}$$

The solution to the right candidate's problem is therefore:

$$p_R^*(v) = \max((1-\beta)^{0.5} v^{0.5} + 0.5, b_R). \tag{39}$$

Given that he cannot win the elections, the vote maximizing reply must be an optimal reply for the left candidate:

$$p_L^*(v) = \max(p_R^*(v) - (1-\beta)^{0.5} v^{0.5}, 0). \tag{40}$$

The argument for the case $v \leq 0$ is analogous. ■

Proof Lemma 11. The expected equilibrium utility of a voter is given by equation (23). It can be rewritten to facilitate the calculation of the derivative with respect to the share of unsophisticated voters β:

$$\begin{aligned}
E(U(b)) &= \int_{v=-\infty}^{v=\infty} -(p(v)-b)^2 g(v)dv + E(\max(q_R, q_L)) \qquad (41)\\
&= \int_{v=-\infty}^{v=\infty} (-p(v)^2 + 2bp(v))g(v)dv - b^2 + E(\max(q_R, q_L)\\
&= \int_{v=-\infty}^{v=\infty} -p(v)^2 g(v)dv + b - b^2 + E(\max(q_R, q_L)\\
&= -(1 - G\left(\frac{(b_R-0.5)^2}{(1-\beta)}\right) p_R^{*2} - G\left(-\frac{(b_L-0.5)^2}{(1-\beta)}\right) p_L^{*2}\\
&\quad - \int_{v=-\frac{(b_L-0.5)^2}{(1-\beta)}}^{v=\frac{(b_R-0.5)^2}{(1-\beta)}} p(v)^2 g(v)dv + b - b^2 + E(\max(q_R, q_L)\\
&= -(1 - G\left(\frac{(b_R-0.5)^2}{(1-\beta)}\right) p_R^{*2} - G\left(-\frac{(b_L-0.5)^2}{(1-\beta)}\right) p_L^{*2}\\
&\quad - \int_{v=-\frac{(b_L-0.5)^2}{(1-\beta)}}^{v=\frac{(b_R-0.5)^2}{(1-\beta)}} ((1-\beta)|v| + sign(v)((1-\beta)|v|)^{0.5} + \frac{1}{4})g(v)dv\\
&\quad +b - b^2 + E(\max(q_R, q_L)
\end{aligned}$$

The third equality is due the fact that the expected implemented policy is $\int_{v=-\infty}^{v=\infty} p(v)g(v) = 0.5$, while the fourth equality reflects the fact that equilibrium policy is the bliss point of the voter with valence advantage if $v \geq \frac{(b_R-0.5)^2}{(1-\beta)}$ or $v \leq -\frac{(b_L-0.5)^2}{(1-\beta)}$. The last equality follows from substituting in the equilibrium policies as summarized in Lemma 10.

Taking the derivative of (41) with respect to β by applying Leibniz's rule gives (34):

$$\frac{dE(U(b))}{d\beta} = \int_{v=-\frac{(b_L-0.5)^2}{(1-\beta)}}^{v=\frac{(b_R-0.5)^2}{(1-\beta)}} |v|g(v)dv > 0. \qquad (42)$$

■

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