

Price Search, Consumption Inequality, and Expenditure Inequality over the Life Cycle*

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Abstract

In this paper, we incorporate price search decision into a life cycle model, and differentiate consumption from expenditure. The consumers with low wealth and bad income shocks search more for cheaper prices and pay less which makes their consumption higher than a model without search option. A plausibly calibrated version of our model predicts that the cross sectional variance of consumption is around 15% smaller than the cross sectional variance of expenditure through out the life cycle. Price search has an alternative productive activity role for the lower income people to increase their consumption levels. We discuss other implications of price search over the life-cycle as well.

Keywords: Consumption inequality, price search, incomplete markets, life-cycle models, partial insurance.

J.E.L. Classification: D10, D91, E21.

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1 Introduction

In this paper, we study the role of price search on the age-inequality profiles of consumption and expenditure. We incorporate price search decision into a quantitative life-cycle model and differentiate consumption from expenditure. A plausibly calibrated version of the model predicts a significant difference in age-inequality profiles of consumption and expenditure throughout the life-cycle.

Our model economy features an incomplete market framework. In general, the incomplete market models ignore the partial insurance role of price search.¹ However empirical literature documented significant dispersion in prices paid for identical goods.² For instance, Aguiar and Hurst (2007) document that richer people pay higher prices for identical goods in the U.S. data. Also, they report that price paid for identical goods change over the life-cycle, which is a result of change in price search due to change in cost of time. Using the U.S. data, Sorensen (2000) documents dispersion in prices paid for the same medicine. Dahlbay and West (1986) report price dispersion in automobile insurance companies in Canadian data. Pratt et al (1979) document price dispersion in several categories of goods. Baye and Morgan (2004) document dispersion in prices for identical goods posted in the internet. These documented facts require a quantitative study on the role of price

¹See Heathcote et al (2009) for a detailed survey on the partial insurance mechanisms in incomplete markets.

²Baye et al (2006) provide a detailed survey on the dispersion in prices paid for identical goods.

search over the life-cycle. Motivated by the reported facts on the dispersion in prices in the empirical literature, this paper focuses on the partial insurance role of price search over the age-inequality profiles of consumption and expenditure. We believe that filling this gap in the literature is important to understand the consumption inequality over the life-cycle.

We solve a life-cycle model, where we allow agents to search for cheaper prices in addition to the consumption/saving decision. As a result of idiosyncratic income shocks, people are ex-post heterogeneous in terms of their income realizations and wealth accumulations. If agents search more for cheaper prices, they pay less and consume more, however they have to enjoy less leisure due to time constraint. Optimality implies the marginal return and marginal cost of price search to be equalized. Marginal return of price search is coming from additional consumption, and it is smaller for individuals who already make high consumption. That implies agents with low wealth and bad income shocks search more and pay less, which we call partial insurance through price search. Our results show that the cross-sectional variance of consumption is roughly 15% smaller than cross-sectional variance of expenditure throughout the life-cycle.

Among many other studies in the quantitative life-cycle literature, this paper is closely related to Guvenen (2007), Storesletten et al (2000), Karahan and Ozkan (2010). Those papers study the role of income processes on the age-inequality profile of consumption. There is a common implicit assumption in those standard models that says price of a consumption good is

unique, therefore consumption is equal to expenditure. However, as we mentioned above, there is a large empirical literature that rejects this assumption. Our paper differs from the standard life-cycle studies in the sense that it differentiates consumption from expenditure. We show that this distinction has a quantitatively significant role on the age-inequality of consumption.

The paper continues as follows. In section 2, we document some important features of the data. We explain the model in section 3, and give the details of the calibration in section 4. In section 5, we report the results, and in section 6 we conclude.

2 Model

We augment an incomplete markets model with a price search technology, which allows individuals to search for cheaper prices and partially insure against bad income shocks. We do it in a life-cycle framework to study age-inequality profiles of consumption and expenditure. The environment is incomplete due to idiosyncratic income shocks. The population consists of continuum of individuals, who work for T periods, and afterwards enjoy retirement until period T^* . Each component of the model is explained in detail below.

2.1 Households

At each period, the individuals have two decisions: one is consumption/saving decision, and the other is leisure/price search decision. The individual can enjoy more consumption by searching for cheaper prices, however he/she has to enjoy less leisure in that case. The individuals maximize life-time expected value of discounted utility:

$$E \sum_{t=0}^{T^*} \beta^t u(c_t^i, l_t^i) \quad (1)$$

where, $u(\cdot)$ is period utility, β is time discount factor, c_t^i and l_t^i are consumption and leisure of individual i at time t .

The individual i has the following time constraint at period t :

$$s_t^i + l_t^i + n_t^i = 1 \quad (2)$$

where, s_t^i and l_t^i are the time spent on price search and leisure for individual i at period t . The variable n_t^i denotes labor supply. It is a constant value, \bar{n} , during employment, and 0 after retirement.

There is an incomplete asset market, where individuals can borrow or save through a risk-free interest bearing asset. The individual i faces the following budget constraint at time t :

$$p(s_t^i)c_t^i + a_{t+1}^i = y_t^i + (1+r)a_t^i \quad (3)$$

where, $p(\cdot)$ is the price of consumption good which depends on the search time of the individual. Consumption and saving at the current period are denoted by c_t^i and a_{t+1}^i , respectively. Current period labor income is denoted with y_t^i , and the labor income process will be explained in detail later on.

2.2 Price Search Technology

We follow Aguiar and Hurst (2007) in price function, because they estimated the parameters of this form in the U.S. data, which we will calibrate accordingly in the benchmark model. It is a log linear form:

$$\log(p) = \theta_0 + \theta \log(s)$$

where θ is the return to search on prices. In the log linear form, doubling search decreases prices by $100 * \theta$ percent. Aguiar and Hurst (2007) estimate the return to search, θ , net of how much and what type of goods purchased by the shopper. They use AC Nielsen data set to estimate the parameters.

2.3 Earning and Pension Processes

We follow the literature in earning process. At each period the individual is assumed to receive a persistent and a transitory income shock. This is a standard model for labor earnings and estimated in several studies.³ The log

³For example: MaCurdy (1982), Storesletten (2000), Guvenen (2005).

earnings read the following process:

$$\log(y_t^i) = \beta_0 + \beta_1 t + z_t^i + \epsilon_t^i, \quad \text{with } \epsilon_t^i \sim (0, \sigma_\epsilon^2)$$

where, β_0 is a scale parameter, β_1 is return to experience, t is the years of experience, z_t^i is persistent income shock and ϵ_t^i is the transitory income shock. The persistent income shocks follow $AR(1)$ process:

$$z_t^i = \rho z_{t-1}^i + \nu_t^i, \quad \text{with } z_0 = 0 \quad \text{and} \quad \nu_t^i \sim N(0, \sigma_\nu^2)$$

We discuss the calibration of the earning process parameters in section 3.

We follow Guvenen (2007) in pension process which mimics the U.S. social security system. After retirement, the pension of each agent is determined by the ratio of his last working period income to the average income at the last working period, $\frac{y_T}{\bar{y}_T}$. The pension function, $\Gamma(\frac{y_T}{\bar{y}_T})$ is as follows:

$$= \gamma \times \begin{cases} 0.9 \frac{y_T}{\bar{y}_T}, & \text{if } \frac{y_T}{\bar{y}_T} < 0.3 \\ 0.27 + 0.32(\frac{y_T}{\bar{y}_T} - 0.3), & \text{if } 0.3 < \frac{y_T}{\bar{y}_T} < 2 \\ 0.81 + 0.15(\frac{y_T}{\bar{y}_T} - 2), & \text{if } 2 < \frac{y_T}{\bar{y}_T} < 4.1 \\ 1.1 & \text{if } 4.1 < \frac{y_T}{\bar{y}_T}. \end{cases}$$

2.4 Utility Function

Utility function is quite standard in the literature and specified as follows:

$$u(c_t, l_t) = \frac{c_t^{(1-\sigma)}}{1-\sigma} + \phi_t \log(l_t).$$

The parameter ϕ_t effects the utility enjoyed from leisure time. It could also be interpreted as the cost of the time the agent spends on price search.

2.5 Recursive Formulations

During the working periods, each individual solves the following optimization problem:

$$V_t^i(a_t^i, z_t^i, \epsilon_t^i) = \max_{c_t^i, s_t^i, a_{t+1}^i} \{u(c_t^i, l_t^i) + \delta E[V_{t+1}^i(a_{t+1}^i, z_{t+1}^i, \epsilon_{t+1}^i) | z_t^i, \epsilon_t^i]\}$$

s.t.

$$p(s_t^i)c_t^i + a_{t+1}^i = y_t^i + (1+r)a_t^i$$

$$s_t^i + l_t^i + n = 1$$

$$a_{t+1}^i \geq \Psi_t^i$$

$$\text{for } t \in \{1, 2, \dots, T\}$$

In the above problem, c_t^i is consumption, s_t^i is the time used for price search, l_t^i is leisure, a_t^i is asset level, a_{t+1}^i is saving, and y_t^i is earnings at

period t . Agents can borrow up to a state dependent borrowing limit Ψ_t^i . The return on savings is denoted with r and time discount factor with β . We have inelastic labor supply n , and the total available time is normalized to 1. Note that the agent can enjoy the same amount of consumption with different expenditure levels. The agent can spend more time to find cheaper prices which will allow her to enjoy a certain amount of consumption with small expenditure levels.

After retirement, individuals receive constant pension which depends on the earnings at the last period of working life. The problem of individual becomes deterministic due to the constant pension after retirement:

$$V_t^i(a_t^i, y^i) = \max_{c_t^i, s_t^i, a_{t+1}^i} \{u(c_t^i, l_t^i) + \delta V_{t+1}^i(a_{t+1}^i, y^i)\}$$

s.t.

$$p(s_t^i)c_t^i + a_{t+1}^i = y^i + (1+r)a_t^i$$

$$s_t^i + l_t^i = 1$$

$$a_{t+1}^i \geq \Psi_t^i$$

$$y^i = \Gamma(y_T^i)$$

$$\text{for } t \in \{T+1, \dots, T^*\} \text{ with } V_{T^*+1}^i = 0$$

The pension of each individual is determined by $\Gamma(\cdot)$ function. Time endowment is larger for retired people since they do not work. We drop the inelastic labor supply n from their time endowment.

3 Calibration

We calibrate the model in two stages. In the first stage, we directly use the values of some parameters which are well established in the related literature. This gives us the opportunity to understand the role of price search extension to the standard models. We calibrate the time period yearly, and each individual starts working at age 20 and retires at 65.⁴ Each individual starts working life with the same asset level at 0. We set $\delta = 0.966$ and $r = 0.04$, which are standard for yearly calibrated models. The value of relative risk aversion parameter, σ is set to 2. We repeated computation with other values, too. The parameters of income process - $\beta_0, \beta_1, \rho, \sigma_\varepsilon^2$ and σ_v^2 - are taken from Guvenen (2005), which provides one of the most recent estimations of income processes.⁵

In the second stage, we calibrate parameters θ_0, ϕ_t to match chosen moments in data. Note that we allow ϕ to change over the life-cycle. We do that in order to match the empirical life-cycle profile of average prices paid. We target the log deviation of average prices from age 25 over the life cycle. We normalize the average price paid in the whole population to 1 by calibrating θ_0 . For a set of parameters we compute the policy functions, and simulate a population of $N = 10000$ individuals. We repeat this process until we match

⁴We assume high school graduates start working at age 18, and college graduates at age 22. We take the average of the two ages, because we don't distinguish the education levels in the model.

⁵Guvenen (2005) estimates two different types of income processes, namely Restricted Income Process and Heterogeneous Income Process. We pick the first one, because it matches model's empirical targets well.

Table 1: Benchmark Model Parameters

Parameter		Value
δ	Time discount factor	0.966
r	Interest rate	0.0416
σ	Relative risk aversion	2
T	Retirement age	65
T^*	Death age	85
θ_0	Scale parameter in prices	0.76
θ_1	Return to price search	-0.1
β_1	Return to experience	0.009
ρ	Persistence of income shocks	0.988
σ_ε^2	Variance of transitory shock	0.061
σ_v^2	Variance of noise	0.015

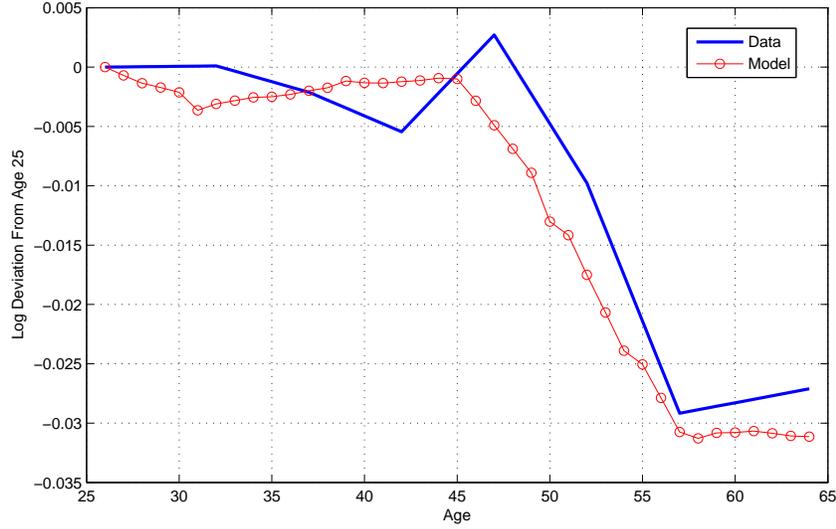
the chosen moments. The benchmark parameters are reported in Table 1. Figure 1 compares the model generated log deviation of average prices to the data.

4 Results

4.1 Age-Inequality Profiles of Consumption and Expenditure

In the earlier studies, consumption was assumed to be equal to expenditure which implied exactly equal age-inequality profiles for consumption and expenditure. In this paper, we differentiate consumption from expenditure by

Figure 1: Life-cycle profile of Prices: Model vs Data



introducing price search into the model. Our model predicts a higher expenditure inequality than consumption inequality throughout the life-cycle. Cross sectional variance of log expenditure starts from 0.09 at age 25 and increases up to 0.35 at age 65. However, variance of log consumption is about 0.07 at age 25 and it is about 0.27 at age 65. The Figure 2 illustrates the age-inequality profiles of consumption and expenditure.

In order to understand the gap between consumption variance and expenditure variance throughout the life-cycle, we decompose expenditure variance:

$$e = p \times c \tag{4}$$

$$var(\log e) = var(\log c) + var(\log p) + 2cov(\log c, \log p) \tag{5}$$

Figure 2: Cross Sectional Variance: Consumption vs Expenditure

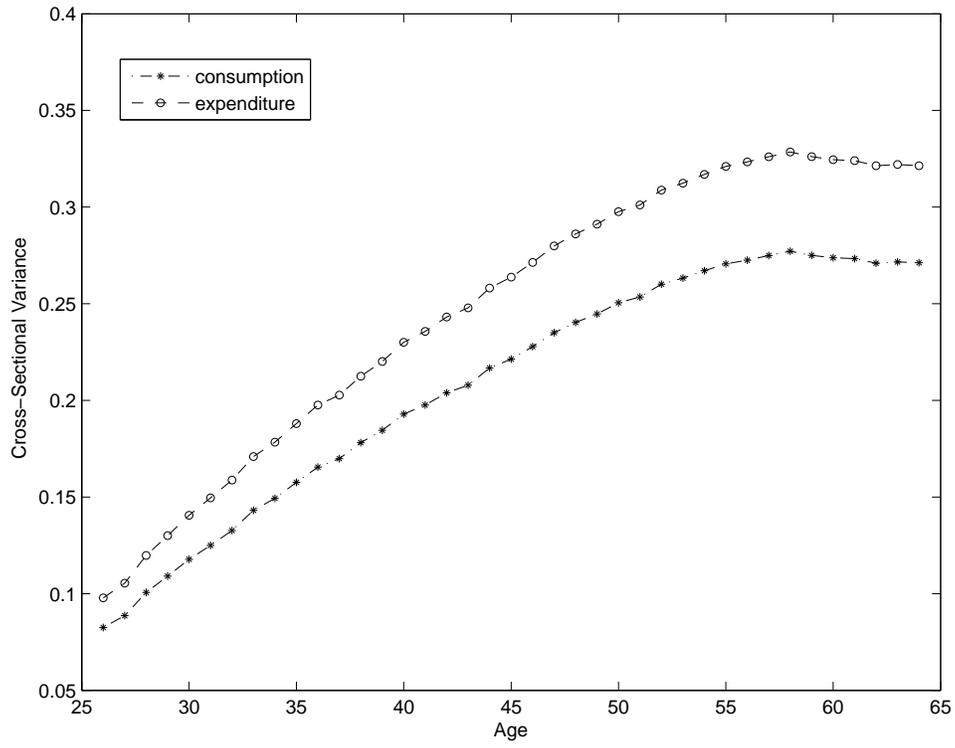
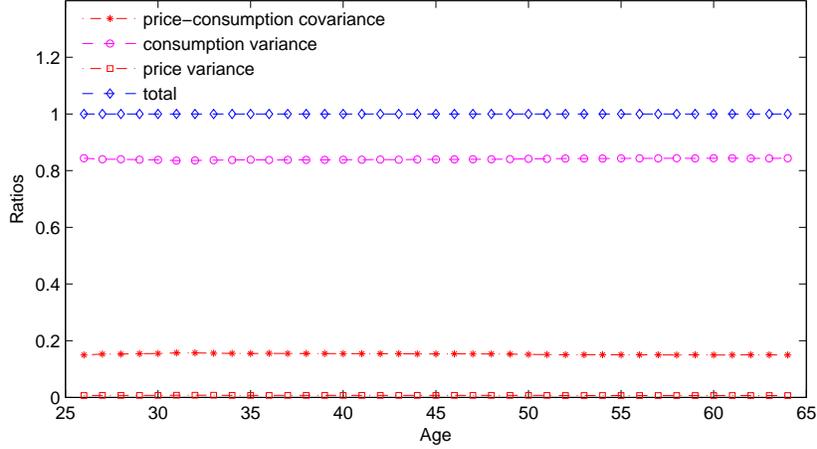


Figure 3: Decomposition of Cross Sectional Expenditure Variance



We calculate each component of $var(\log e)$ from the model's results. Throughout the life-cycle, the model predicts that around 15% of variance in log expenditure comes from covariance between consumption and prices. Around 85% of the expenditure variance comes from consumption variance. Figure 3 summarizes our findings.

We visit the optimality condition for price search to understand the positive covariance between consumption and prices.

$$-\frac{u_1(c_t, l_t)}{p(s)} p'(s_t) c_t = u_2(c_t, l_t) \quad (6)$$

Plugging the utility and price functions into the equation 6, we get the following equation which gives the relationship between search and consumption:

$$\frac{c_t^{1-\sigma}}{s_t} \theta_1 = \frac{\phi_t}{1-s_t}$$

The first order condition for price search implies a diminishing marginal return with consumption. Wealthier people who consume at high levels have less incentive to increase their consumption by sacrificing from leisure. Note that cost of price search is utility from leisure. People with higher income and wealth spend more time with non-search activities instead of searching prices to increase their consumption. People with lower income and wealth spend more time at price search to increase their consumption levels. Price search has a productive activity role for lower income and lower wealth people to increase consumption levels.

4.2 Age-Inequality Profile of Search

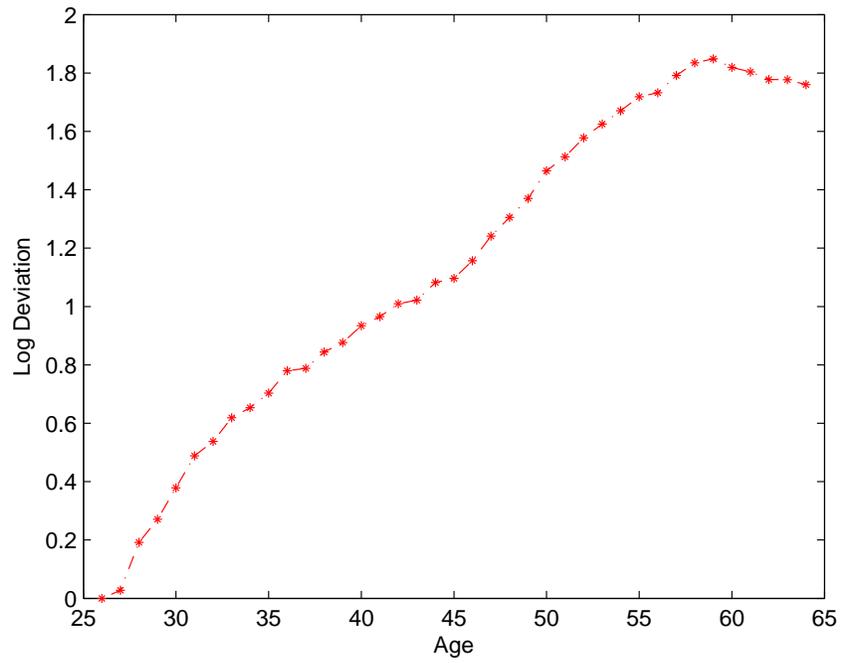
Figure 4 show the age-inequality profile of search over the life cycle. The model predicts an increasing inequality profile for search.

The underlying reason in the increasing profiles of search is the idiosyncratic income shocks over the life-cycle. As people deviate from each other in terms of income and wealth over the life-cycle, they also deviate from each other in terms of time spent for cheaper prices, and that leads to increasing dispersion in search and prices.

4.3 Opportunity Cost of Time

We calculate the marginal cost of time from optimality condition with respect to price search in equation (7), where marginal return of search is equal to

Figure 4: Cross Sectional Variance of Log Search: Model



marginal cost of search. We draw the results in Figure 5, and compare to the opportunity cost of time reported in Aguiar and Hurst (2007). We get roughly the same shape in the life-cycle for levels. Opportunity cost of time is decreasing during the life-cycle. The same figure shows the variance of cost of time over the life-cycle. In order to understand the cost of time over the life-cycle, we revisit the optimality condition with respect to price search, equation (7).

$$\frac{(y_t + (1+r)a_t - a_{t+1})^{(1-\sigma)}}{\theta_0^{1-\sigma}} \theta s_t^{\theta - \theta\sigma - 1} = \frac{\phi_t}{1 - s_t} \quad (7)$$

In equation 7, left hand side is the marginal return on price search and right hand side is the marginal cost of price search, which is the marginal return on leisure time. Note that value of price search is decreasing with higher income and wealth. Therefore, the opportunity cost of time is decreasing over the life-cycle. As a result of increasing wealth and income inequality over the life-cycle, the variance of opportunity cost of time is increasing as well. Figure 5 illustrates the results.

4.4 The Role of Risk Aversion

In this section we study the effect of risk aversion on search behavior and the age-inequality profiles of consumption and expenditure. Figure 6 shows the cross-sectional variance of consumption and expenditure for risk aversion levels 2 and 3. As a result of the increase in risk aversion, the precautionary

Figure 5: Opportunity Cost of Time: Model

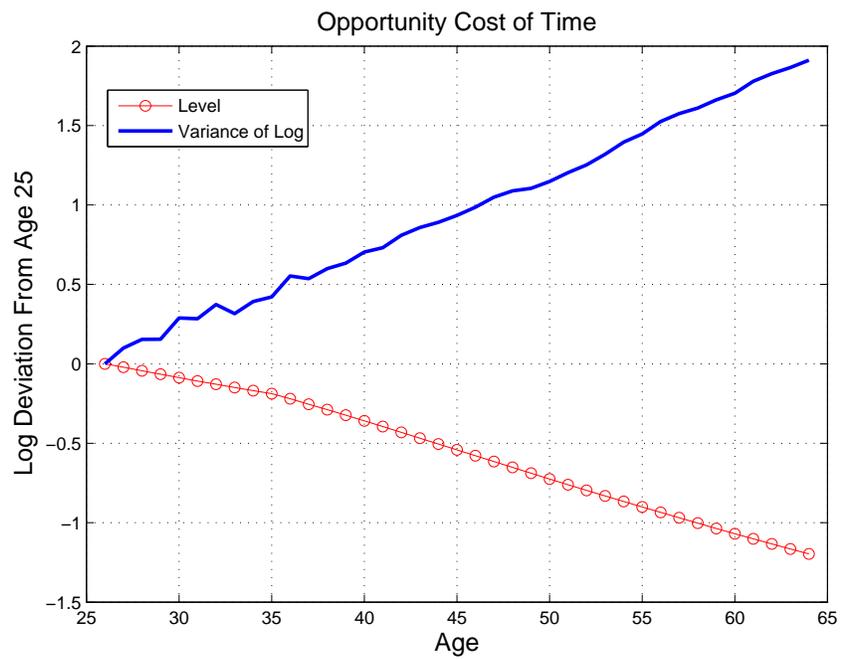
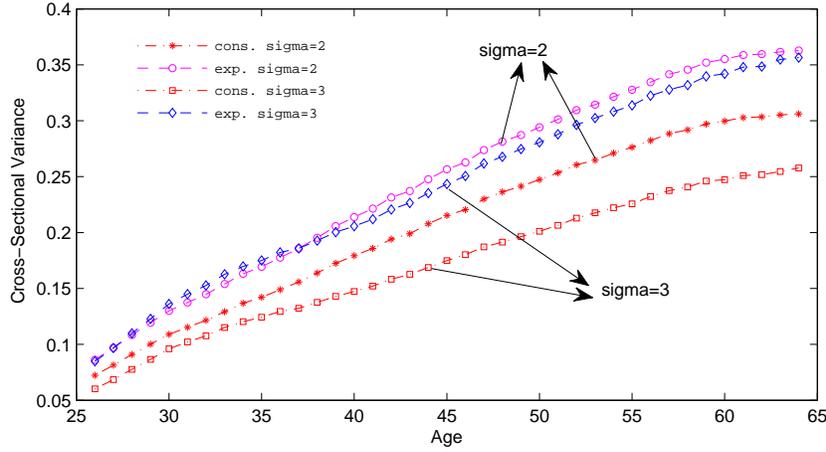


Figure 6: Effect of Risk Aversion: Consumption vs Expenditure



savings increase which makes age-inequality profile of consumption flatter compared to the lower risk aversion case. A flatter age-inequality profile of consumption increases the gap between age inequality profiles of consumption and expenditure. In particular, the gap between the two series increases roughly from 15% to 30% as illustrated in Figure 6.

On the other hand, an increase in risk aversion makes the age-inequality profile of price search flatter. This is a result of a substitution between the two insurance mechanisms. As individuals increase partial insurance through precautionary savings, they decrease the partial insurance through price search. For the same reason, rate of increase in average search gets smaller, when we increase the risk aversion. That can be followed from the optimality condition with respect to price search in equation (7). Figures 8 and 7 illustrate the results.

Figure 7: Effect of Risk Aversion: Average Search

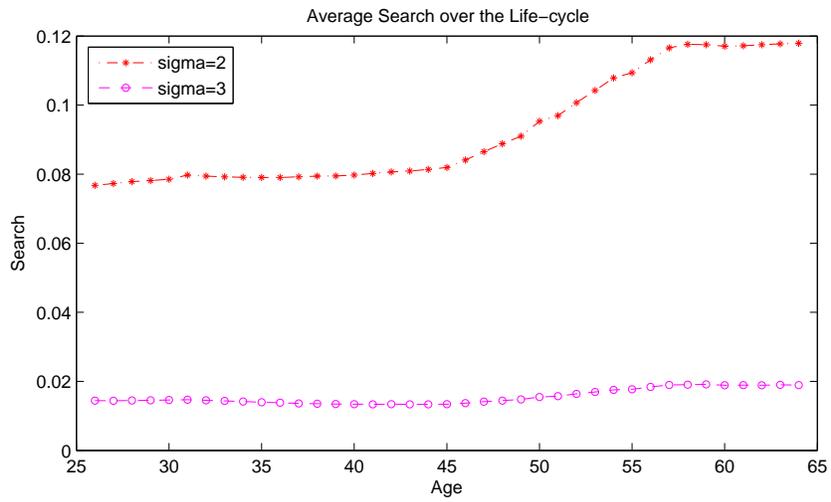
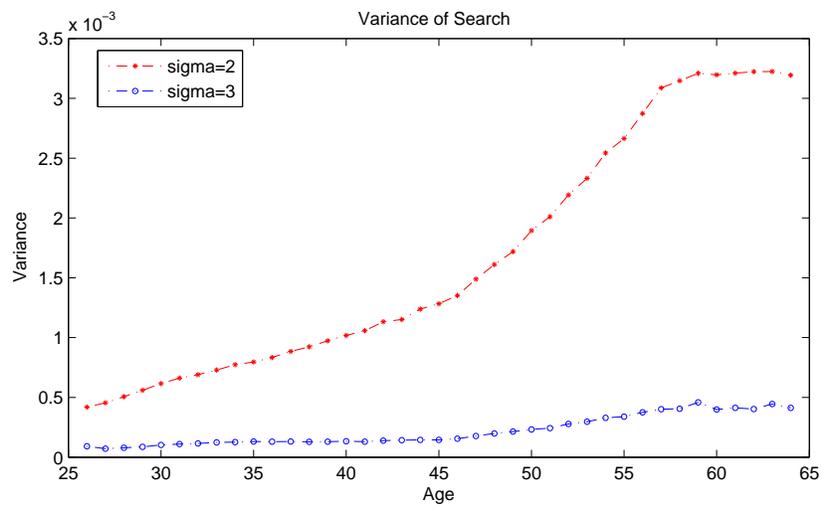


Figure 8: Effect of Risk Aversion: Variance of Search



4.5 The Role of the Search Technology

We solve the model with two values of the parameter θ to see its role in quantitative results. In the benchmark model, we use a value of -0.1 , which is the estimated value in Aguiar and Hurst (2007) for the U.S. data. We also solve the model with a value of -0.2 . This exercise can be considered as implications of a technological innovation in price search, such as internet.

As illustrated in Figure 9, the gap between age-inequality profiles of consumption and expenditure increases as a result of an increase in price search technology. As return to search increases, the increase in the search time of the poor and low income individuals are higher than the increase in the search time of the wealthy and high income individuals. As a consequence, the variance and average of search increases with a higher return to search technology, which is illustrated in Figures 11 and 10.

5 Discussion and Conclusion

In this paper, we study the role of price search on the age-inequality profiles of consumption and expenditure. We solve a life-cycle model to analyze the joint behavior of shopping strategies, individual prices, and expenditures. We introduce price search decision to a life-cycle model, and differentiate consumption from expenditure. The model predicts an increasing age-inequality profile for search, prices, and expenditure. Our quantitative study - using estimated income process and price search functions from literature - predicts

Figure 9: Effect of Price Search Technology: Consumption vs Expenditure

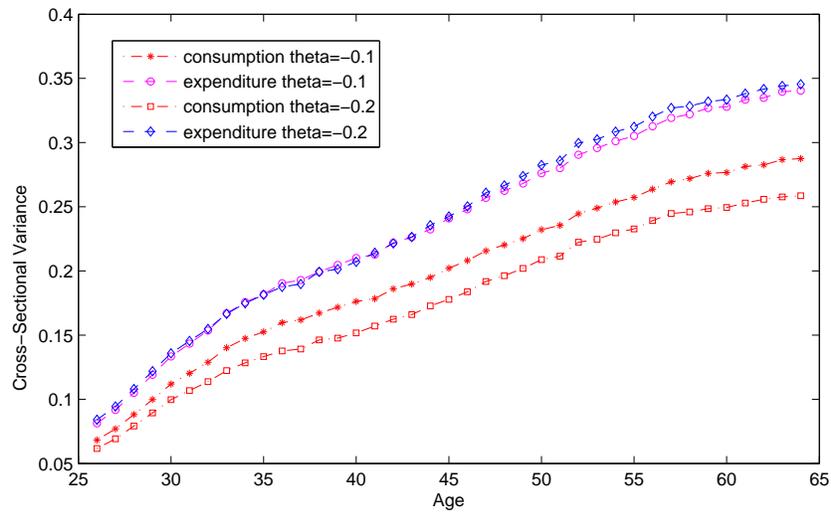


Figure 10: Effect of Search Technology: Average Search

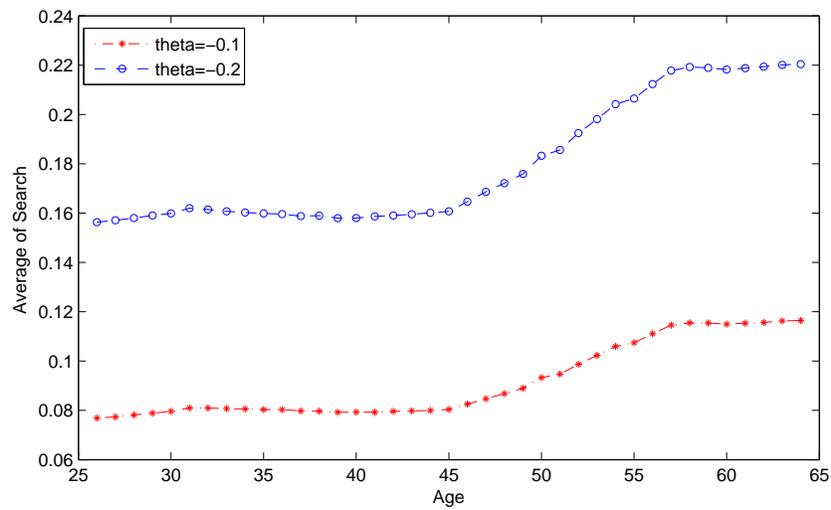
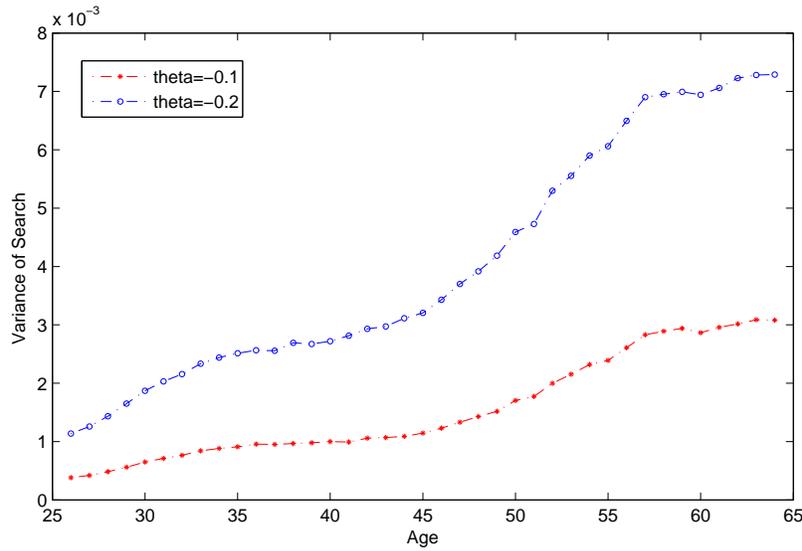


Figure 11: Effect of Search Technology: Variance of Search



that consumption inequality is not equal to expenditure inequality when agents can search for prices. A plausibly calibrated version of our model predicts that cross sectional variance of consumption is around 15% smaller than the cross sectional variance of expenditure through out the life cycle. In the earlier studies ⁶, consumption inequality was implicitly assumed to be the same as expenditure inequality. Also, the life-cycle profile of individual prices and search levels imply a decreasing life-cycle profile for cost of time.

Although we focused on age-inequality profiles, the model can be extended to explain further empirical observations. For instance, Aguiar and Hurst (2009) document different patterns in different expenditure categories. In their study, expenditures on some goods and services have hump-shaped

⁶For example: Aiyagari(1994), Storesletten et al (2000), Krueger and Perri (2006), Guvenen (2007).

profile over the life-cycle, and some others have constantly increasing profiles. Price search could be helpful in explaining the different patterns, because some categories might be more sensitive to price search. The life-cycle search profile may have different implications on the expenditure patterns of different categories due to their different sensitivities. Carroll and Summers (1989) document different expenditure patterns for different education groups. Again, price search together with income processes could be helpful to explain the expenditure patterns. Different price search technologies or time cost profiles for different education or occupation groups could be helpful in explanation to the different expenditure patterns. In this paper we used average cost of time (the coefficient of leisure in the utility function) over the life cycle. It is likely that the variance of the opportunity cost of time changes over the life-cycle with a varying degree for different education and occupation groups. Potentially it will have important implications on inequality in general.

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