Rethinking Directed Technical Change with Endogenous Market Structure

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Abstract

I consider directed technical change in an economy where market structure is endogenous. Endogeneity of market structure leads to both theoretical and empirical implications that are substantially different from those in the existing literature and that in some cases are rather surprising. There are two dimensions of directed technical change: directed firm entry (new firms enter the industry with higher returns) and directed in-house research and development (R&D is higher in the industry with higher returns.). Directed firm entry responds to the industry market size effect and the price effect as in the existing literature. In sharp contrast to the existing literature, directed R&D depends on firm rather than industry market size. Furthermore, the firm’s market size is endogenous, and its response to economic conditions affect several results on the behavior of directed technical change. The endogeneity of firm size has generally been ignored in the previous literature. Directed technical change alters the relative demands for factors of production and leads to a change in relative factor returns. Directed firm entry changes relative factor returns through a social return to variety (an externality), and directed R&D changes relative factor returns through changes in relative factor productivities. Empirically, the second channel is the main force shaping relative factor productivities and hence relative factor returns. The model also includes fixed operating cost, which turns out to be important for the direction of R&D and for the existence of balanced growth path (BGP) for the economy. The model provides a complete solution for the economy’s transition dynamics as well as its balanced growth path.

Keywords: Directed Technical Change, Endogenous Market Structure, Relative Factor Returns
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1 Introduction

The direction of technical change plays an important role for many problems in labor economics, environmental economics, and economic development. Technical change often is deliberately aimed at augmenting a specific factor of production rather than being a general, Hicks-neutral improvement in productivity. There is a substantial theoretical literature on the determinants of the direction of technical change, in which the direction and the rate of technical change are the outcomes of the behavior of profit-seeking firms. See Acemoglu (1998, 2002), Kiley (1999), Nahuis & Smulders (2002), Smulders & Nooij (2003). An important feature of that literature is that the firm’s incentive to do R&D depends on the size of the firm’s market, which is presumed to be the whole industry’s market and exogenous. However, the assumption that each firm’s market size is exogenous and equal to the entire industry size does not correspond with the realities of industrial organization (Cohen and Klepper, 1996a, 1996b; Adams and Jaffe, 1996). The firm’s market is endogenous and typically is only a fraction of the industry’s market. In this paper, I construct a model of directed technical change that highlights the endogeneity of market structure. Allowing market structure to be endogenous leads to substantially different implications and a more complete story of the directed technical change than what is found in previous literature.

Microeconomic evidence establishes that market structure, including the size and number of firms, is endogenous, with the individual firm’s size changing in response to market and technology conditions and being endogenously regulated by entry and exit of other firms (Laincz and Peretto, 2006; Pagano and Schivardi, 2003). The endogeneity of market structure has important implications for economic growth and directed technical change. It is the individual firm’s market size that determines the firm’s incentive to do in-house innovation. The larger the firm’s market size, the greater its profit and so the greater the return to R&D. Industry size is irrelevant. See Schumpeter (1950). The existing literature on directed technical change implicitly assumes firm market size equals industry size. The underlying reason is that technology is assumed to be nonrival and available for use by all workers simultaneously. As a result, an increase in industry market size raises firm market size. In reality, workers are specialized and work with only a few types of machines. Technology embodied in a machine (i.e. the machine’s quality) augments only the workers who work with it. In this sense, technology is non-rival within a group of workers, and the market size of the machine is the number of workers who use that machine rather than the aggregate labor force. Knowledge embodied in one kind of machine could contribute to general purpose knowledge as a spillover and augment all workers as an externality. However, firms cannot appropriate the knowledge that spills into the public domain, so profit seeking firms do not take such spillovers into account for their R&D decisions. For example, in a pharmaceutical factory, different workers use different machines to formulate the compounds that go into the pills and capsules, to make the pills and capsules, to put the pills in the bottles, to do quality control checks on the pills, to pay the employees, and so on. In fact, the machines used to make pills are different from those used to make capsules, and the machines are different for different pills. Workers trained on one kind of machine typically are not trained on another. Therefore, the market for the machine which makes pills is the number of workers who use that machine, not all workers. If we assume that one firm produces one type of machine, then returns to the innovation

Footnote 1: Following the endogenous technological change literature, I assume that each intermediate goods firm has a single plant.
on that particular type of machine depend on individual firm market size (how many workers use that kind of machine), which is endogenous and is not equal industry market size (the total number of workers in an industry). When market structure is endogenous, a higher industry market size does not lead to a larger firm size. Instead, an increase in the size of the aggregate market is matched by an increase in firm entry, which keeps the market size of the individual firm constant. This endogenous market structure eliminates not only the scale effect\(^2\) but also, and more important, the underlying reason for it. See Peretto (1998). Similar logic applies to directed technical change. An increase in industry market size induces new firms to enter, and that endogenous element of market structure leads to unchanged incentives for incumbents’ R&D. A change in industry market size therefore does not affect the direction of in-house R&D. This property has a key implication for the direction of technical change and hence for relative factor returns that are different from the results under an exogenous market structure: a change in relative factor endowment does not affect relative factor returns through in-house R&D.

I use a variant of the growth models pioneered by Peretto (Dec. 1998, 2007) and Howitt (1999) to include endogenous market structure in the model. The model is built to be consistent with four major sets of facts. First, economic growth is driven by R&D that is done predominately by incumbent firms (Dosi, 1988; OECD, 2003; NSF, 2010)\(^3\). In-house R&D contributes on average around 75% of total factor productivity (TFP) growth at the industry level (Bartelsman and Doms, 2000; Foster and Krizan, 2000). Second, in-house R&D decisions depend on market concentration and the individual firm’s market size (Cohen and Klepper, 1996a,b; Adams and Jaffe, 1996). Industry market size is irrelevant. Third, firms face fixed as well as variable costs. Fourth, and most important, market structure and hence firm market size are determined endogenously by firm entry and exit (Laincz and Peretto, 2006; Pagano and Schivardi, 2003).

The model contains two dimensions of technology change: variety expansion (horizontal dimension) and in-house quality improvement (vertical dimension). In the horizontal dimension, entrepreneurs create new firms to bring new products to market. New entrants compete with incumbents for market share. The number of firms determines the important elements of market structure: market concentration and firm market size. In the vertical dimension, incumbent firms do in-house R&D to improve the quality of their own products. Returns to in-house R&D are affected by market structure, specifically, firm market size. Technological progress and market structure are jointly determined in the two-dimension growth
to produce a single type of machine (intermediate good). Thus the terms plant, firm, product, product line can be used interchangeably. Clearly, models with firms having multiple plants would be more consistent with empirical analysis but also would not change the main results of the model. See Smulders and van De Klundert (1995) and, more recently, Minniti (2006).

\(^2\) The scale effect is the positive relation between long run growth and aggregate economic scale in much of the growth literature. It is rejected empirically, e.g., Backus, Kehoe, and Kehoe (1992).

\(^3\) Appendix Table 4-3 of the NSF’s Scientific and Engineering Indicators 2010 reports that 73% of R&D done in the US was carried out by incumbent firms in 2008 (last year available). Another 20% was conducted by colleges and universities (13%) and the federal government (7%). Only 7% was conducted by independent research labs. Dosi (1988) shows that the fraction of R&D done by the six leading R&D countries always exceeded 50% and that the value-weighted fraction of R&D done by incumbents was about two-thirds when the article was published. Also, OECD (2003), chapter 4, reports evidence that almost all growth in productivity arises from the activities of existing firms and that new entrants explain very little (approximately none) of the growth of productivity in a sample of countries. The dominance of incumbents in doing R&D dates back to at least the first half of the 20th century (Mowrey and Rosenberg, 1998).
path of the economy. Technological progress is measured by the growth rate of quality improvement. Market structure is measured by the number of firms.

The model is quite different from the one-dimensional variety expansion model built by Romer (1990). First, one cannot embed endogenous market structure in a variety expansion model without killing perpetual growth. Second, when fixed operating costs are present, variety expansion eventually stops and so cannot be a source of long-run growth. Rather, long-run growth is driven by the quality improvement arising from in-house R&D. The model is essentially a smooth version (quality being a continuous variable) of the quality ladder model with an endogenous market structure. Acemoglu (1998), Nahuis & Smulders (2002), Smulders & Nooij (2003) discuss directed technical change where growth is driven by quality improvement, but in their analysis the market structure is exogenous. With endogenous market structure, some of their results survive, and others remain only under certain conditions. We also have some completely new results. The model delivers two sets of related results, one on directed technical change and one on relative factor returns.

Section 2 provides a overview of the results and the differences from related literature. Section 3 lays out the basic model. Section 4 constructs the general equilibrium solution, highlighting the role of endogenous market structure in the determinants of directed technical change. Section 5 discusses relative factor returns. Section 6 discusses the theoretical and empirical implications of the model. Section 7 concludes.

2 Overview

Consider a constant elasticity of substitution (CES) production function with two inputs, $H$ and $L$, that could be any non-reproducible factors, such as land, labor, natural resource, or even skilled and unskilled labor.\footnote{In the standard variety expansion model, each variety is used by all labor. Introducing endogeneity of market structure would require changing the model so that each variety is used by only a group of workers. Then, however, as the number of varieties increases, the market size of each variety converges to zero, returns to new innovation also converge to zero, and growth stops. See detail later and Peretto and Connolly (2007).}

\[
Y = \left[ \gamma (Y_L)^{\frac{1}{\epsilon}} + (1 - \gamma) (Y_H)^{\frac{1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon - 1}}
\]

Define $Y_L = B_L L$ and $Y_H = B_H H$. $B_H$ ($B_L$) is the technology that augments factor $H$ ($L$). $\epsilon \in (0, \infty)$ is the elasticity of substitution between $Y_H$ and $Y_L$ (also between $H$ and $L$, since $Y_H$ ($Y_L$) is a linear function of $H$ ($L$)). When $\epsilon > 1$, $H$ and $L$ are substitutes; when $\epsilon < 1$, $H$ and $L$ are complements.\footnote{Other contributions to the literature, e.g. Kiley (1999) and Acemoglu (2002), use the variety expansion model to discuss directed technical change.}

Assume $Y$ is the numeraire. The relative factor return between $H$ and $L$ is:

When $\epsilon = 1$, then (1) is a Cobb-Douglas function. When $\epsilon = \infty$, the two factors are perfect substitutes; when $\epsilon = 0$, the two factors are perfect complements, and the production function is Leontief.
\[
\frac{MP_H}{MP_L} = \frac{1 - \gamma}{\gamma} \left( \frac{B_H}{B_L} \right)^{\epsilon - 1} \left( \frac{H}{L} \right)^{-\frac{1}{\epsilon}}
\] (2)

There are two elements that determine the relative factor return. The first is relative factor endowment. The relative factor return to \( H \) is decreasing in the relative factor endowment, \( H/L \), because a higher relative factor endowment leads the equilibrium return to decrease. The second element is the technology ratio \( B_H/B_L \). When \( \epsilon > 1 \) (factors are substitutes), an increase in \( B_H/B_L \) increases the relative return to \( H \). In contrast, when \( \epsilon < 1 \) (factors are complements), an increase in \( B_H/B_L \) reduces the relative factor return to \( H \). The intuition is that, with complementarity, an increase in the productivity of \( H \) increases the demand for the other factor \( L \) even more. As a result, the factor return to \( L \) increases by more than that to \( H \).

The existing literature presumes \( H (L) \)-complementary technology is used by all of \( H (L) \). In other words, firm market size equals industry market size. With an increase in \( H \) relative to \( L \), the firm market size and hence the incentives for innovation in the \( H \)-complementary technology increase relative to that of \( L \). As a result, \( B_H/B_L \) increases. \( B_H/B_L \) also depends on the relative price \( P_{YH}/P_{YL} \). If the price of \( Y_H \) is relatively higher, then it is more profitable to do innovation to complement factor \( H \), so \( B_H/B_L \) increases. Acemoglu (1998, 2002) calls these two effects the market size effect and the price effect, respectively. Acemoglu provides two main results. First, irrespective of the elasticity of substitution between factors, an increase in \( H/L \) leads to a change in \( B_H/B_L \) through the market size and price effects, which raises \( w_H/w_L \). Second, if the elasticity of substitution is high enough, as \( H/L \) increases, its positive impact on \( w_H/w_L \) through directed technical change dominates the negative impact, so an increase in relative endowment could increase relative factor returns. However, in this analysis market structure is exogenous. When market structure is endogenous, it interacts with innovation which affects the results. The purpose of the present paper is to revisit the reasons for directed technical change and its economic impacts: what determines \( B_H/B_L \), and how directed technical change alters relative factor returns.

The model considers two dimensions of directed technical change: quality improvement through in-house R&D and firm entry. Both types of technical change affect \( \frac{B_H}{B_L} \). The direction of in-house R&D depends on the cross-industry differences in gross profits per unit of quality, which mainly depends on three elements: R&D productivity, fixed operating cost and individual firm market size but not on industry market size. Surprisingly, an increase in an industry’s R&D productivity reduces the growth rate of that industry’s quality improvement relative to the other industry, which is substantially different from what is predicted by the existing literature. This result arises from an interaction between the endogenous market structure and fixed cost. The industry with a relatively high R&D productivity and/or fixed operating cost has a relatively low quality growth on the transition path and also has a relatively low quality level on the BGP. Therefore, the factor used in that industry gets paid a relatively low (high) return if two factors are substitutes (complements). In contrast to the results in the existing literature\(^9\), a change in relative factor endowment does not affect the direction of quality improvement and hence does not affect relative factor returns through directed in-house R&D. The impact of a change in relative

factor endowment on in-house R&D is absorbed by endogenous market structure through firm entry, a channel ignored by the existing literature.

A change in factor endowment does not affect the direction of in-house R&D, which is different from the literature. Instead, it affects the direction of firm entry through the market size and price effects, which is reminiscent of Acemoglu (1998, 2002). However, directed firm entry does not affect relative factor returns unless there is a social return to variety. In other words, the ratio of numbers of firms does not show up in $\frac{B_K}{B_L}$ in (2) unless there is a social return to variety. As a result, a change in relative factor endowment does not affect relative factor returns unless there is a social return to variety, which is different from the main results in the previous literature.

Embedding directed technical change in the model with endogenous market structure allows us to see under what conditions the relative factor endowment affects the direction of technical change and hence relative factor returns. The ability to address such a question arises from the two-dimensional nature of technical progress in the present model. Traditional one-dimensional models of growth through quality improvement presume each product is used by all workers and ignores both the specialization of labor and the potential entry of new varieties. As a result, in those models a change in relative factor endowment alters the incentives of quality improvement across industries and alters the direction of quality improvement. In contrast, the kind of two-dimensional model used in the present paper shows the impact of endogeneity of market structure on the incentives of quality improvement. A change in relative factor endowment does not affect returns to R&D across industries and hence does not affect the direction of in-house R&D. Traditional one-dimensional models of growth through variety expansion alone must have social returns to variety because otherwise they cannot deliver perpetual growth. In contrast, the kind of two-dimensional model used here can deliver growth without there being any social return to variety. We thus can study how including or excluding social returns to variety affects the results.

The model highlights that the link between relative factor endowment and relative factor returns through directed technical change is social return to variety, which is an externality. Without the externality, a change in factor endowment does not affect relative factor returns. Instead, other variables such as R&D productivity, fixed operating costs are the key that affects the direction of innovation hence relative factor returns.

3 The Structure of The Model

I now develop the framework for analyzing the determinants of directed technical change. First I set up the basic model. In the next section I derive the general equilibrium solution and show the determinants of directed technical change and relative factor returns.

The structure of the model partly follows Acemoglu (1998, 2002), Kiley (1999), and Smulders & Nooij (2003), who develop a framework to analyze the forces that shape the direction of technical change toward
a particular factor.\footnote{The difference between the structure of this model and the other literature is the endogeneity of market structure discussed later.} There are three productive sectors in the model: final goods, processed goods and intermediate goods. One representative firm produces final goods $Y$ with two types of processed goods, $Y_H$ and $Y_L$. $Y_H$ is produced by factor $H$, while $Y_L$ is produced by factor $L$. Final goods can be used for consumption; to produce intermediate goods, $G_{ij}$; and to improve the quality inside the intermediate goods, $Z_{ij}$. Intermediate goods $G_{ij}$ and the quality inside them are used to produce processed goods, $Y_i$. The structure of the model is shown in figure 1.

### 3.1 Final Good Sector

One representative firm produces a single homogeneous final good $Y$ using two non-durable processed goods $Y_H$ and $Y_L$ as inputs. The final good can be consumed, used to produce intermediate goods, and invested in R&D that raises the quality of existing intermediate goods. The final good sector is perfectly competitive with the CES production given in (1). I take the final good as the numeraire, so $P_Y = 1$. The representative firm’s profit is

$$\pi_Y = Y - P_{Y_H}Y_H - P_{Y_L}Y_L$$

from which I obtain the indirect demand functions

$$P_{Y_H} = (1 - \gamma)Y^{\frac{1}{1-\epsilon}}/Y_H^{\frac{1}{1-\epsilon}} \quad (3)$$

$$P_{Y_L} = \gamma Y^{\frac{1}{1-\epsilon}}/Y_L^{\frac{1}{1-\epsilon}} \quad (4)$$

where $P_{Y_H}$ and $P_{Y_L}$ are the prices of $Y_H$ and $Y_L$. The ratio of $P_{Y_L}$ to $P_{Y_H}$ then is

$$\frac{P_{Y_L}}{P_{Y_H}} = \frac{\gamma}{1 - \gamma} \left( \frac{Y_L}{Y_H} \right)^{\frac{1}{1-\epsilon}} \quad (5)$$

and

$$[(1 - \gamma)^\epsilon P_{Y_H}^{1-\epsilon} + \gamma^\epsilon P_{Y_L}^{1-\epsilon}]^{\frac{1}{1-\epsilon}} = 1 \quad (6)$$

### 3.2 Processed Goods Sector

The processed good sector is also perfectly competitive. This sector comprises two industries, each producing a single homogeneous good. The representative firms in the two industries use non-durable intermediate goods and non-reproducible factors to produce their respective processed goods. Following Acemoglu (1998, 2002), Kiley (1999), and Smulders & Nooij (2003), I assume $Y_H$ only uses factor $H$, while $Y_L$ only uses factor $L$.

I denote $i$ as the $i^{th}$ industry ($i = H, L$), and $j$ as the firm $j$ in an industry $i$. I assume that the quality $Z_{ij}$ of intermediate good $G_{ij}$ is embodied in the good but augments the non-reproducible factor.
$s_{ij}$, a specification I have borrowed from Aghion and Howitt (2005) and Peretto (2007). The production function of processed good in industry $i$ is:

$$Y_i = N_i^{(1-\lambda)} \int_0^{N_i} G_i^{\lambda} \left(Z_{ij}^\delta Z_{i}^{1-\delta} s_{ij}\right)^{1-\lambda} dj, \quad 0 < \lambda < 1; \quad 0 \leq \sigma, \delta < 1$$

where $Z_i \equiv (1/N_i) \int_0^{N_i} Z_{ij} dj$ is the average quality of class-$i$ intermediate goods, which is the intra-industry knowledge, and $N_i$ is the amount of varieties of intermediate goods used in industry $i$. The quality $Z_{ij}$ of intermediate good $G_{ij}$ is embodied in the good itself but augments the factors $s_{ij}$ which use that good. The term $N_i^{(1-\lambda)}$ shows a positive social return to variety.\(^{11}\)

Processed goods firms $Y_i$ choose quantities of intermediate goods and factor $s_i$ to maximize their profit:

$$\max_{\{G_{ij}, s_{ij}\}} \pi_Y = P_Y Y_i - \int_0^{N_i} P_{G_{ij}} G_{ij} dj - \int_0^{N_i} w_i s_{ij} dj$$

where $P_{G_{ij}}$ is the price of $G_{ij}$, $w_i$ is the return to factor-$i$, and the firm takes all prices as given. The demand functions for intermediate goods and factors are

$$G_{ij} = N_i^{\sigma} \left(\frac{\lambda P_Y}{P_{G_{ij}}}\right)^{\frac{1-\lambda}{\lambda}} Z_{ij}^\delta Z_{i}^{1-\delta} s_{ij}$$

$$s_{ij} = N_i^{\sigma \frac{1-\lambda}{\lambda}} \left(\frac{1-\lambda}{w_i}\right)^{\frac{1}{\lambda}} G_{ij} (Z_{ij}^\delta Z_{i}^{1-\delta})^{\frac{1-\lambda}{\lambda}}$$

Equation (8) shows that an increase in qualities $Z_{ij}$ and the spillovers $Z_i$ leads to an increase in the demand of intermediate goods. This is the reason why intermediate good firms do research to increase their qualities - in order to get a higher demand for their product. The social return to variety also contributes to intermediate demands with a degree $\sigma$.

Note that each intermediate good $G_{ij}$ is used by only a portion of the total factor of one industry ($s_{ij}$) which is endogenously determined, but not used by all factor in this industry, which is one of the key differences between this model and the previous models. Let $S_i$ be the total use (or, in equilibrium, the supply) of factor input $i$. Then $H = S_H = \int_0^{N_H} s_{Hj} dj$ and $L = S_L = \int_0^{N_L} s_{Lj} dj$.

In the next section, I will show how the incumbents endogenously choose the level of quality $Z_{ij}$ and price $P_{G_{ij}}$, and how the endogenous market structure determines the number of firms $N_i$ and the individual firm market size $s_{ij}$.

\(^{11}\)Social return to variety could be positive. For example, when entrants introduce new products, they develop new production processes. To do this, they might solve some problems that are completely new, or re-organize the existing knowledge by a creative way. When these occur, they contributes to the productivity of the industry. For an excellent discussion of the social return to variety, see Peretto & Smulders (2002).
3.3 Intermediate Good Sector

The intermediate sector is the core of the model. There are two dimensions of technology change in this sector—quality improvement (vertical dimension) and variety expansion (horizontal dimension). In the vertical dimension, the firms that are already active in the market (incumbents) do in-house R&D to improve the quality of its own product \( Z_{ij} \) in order to get a larger demand, thus a higher profit. In the horizontal dimension, entrepreneurs make entry decisions and compete with incumbents for market share. Through firm entry, the number of firms \( N_i \) and the individual firm market size \( s_{ij} \) are endogenously determined.

I proceed in two steps. First, I focus on the determination of the price and investment in R&D of incumbents given the existing market structure. Then I focus on the endogenous market structure which is the free entry and exist decisions.

3.3.1 Incumbents

Each intermediate goods industry comprises a continuum of monopolistically competitive incumbents, each of which produces a single intermediate good \( G_{ij} \) and also undertakes R&D to improve the quality \( Z_{ij} \) of the good it produces. An increase in quality raises the demand for the good and thus raises profit.

Production technologies, R&D technologies, and costs are the same for all firms within a given industry but differ across industries, following the previous literature. The industrial structure thus is one of symmetry within each intermediate goods industry but asymmetry across the two industries. All firms in industry \( i \) have a linear technology that converts \( A_i \) units of the final good into one unit of intermediate good \( G_{ij} \). Similarly, the R&D production functions are the same within an industry but differ across industries

\[
\dot{Z}_{ij} = \alpha_i R_{ij}
\]  

(10)

where \( R_{ij} \) is amount of the final good \( Y \) spent on R&D. The firm obtains the resources for \( R \) from retained earnings.

Firms face a fixed operating cost \( \phi_{ij} \). Following Peretto (2007) and Cozzi & Spinesi (2004), I assume fixed cost is a linearly homogeneous function of the average technology, \( Z_H \) and \( Z_L \). Fixed operating cost is an important element in the determination of market structure and direction of technical change, but is usually ignored by the literature without endogenous market structure. At this stage, I use a general form of fixed operating cost.

\[
\phi_i \equiv \phi_{ij}(Z_H, Z_L)
\]  

(11)

The intermediate goods firm’s gross profit is revenue less production costs:

\[
F_{ij} = P_{G_{ij}} G_{ij} - A_i G_{ij} - \phi_i
\]  

(12)
The firm retains some amount $R_{ij}$ of its profit for investment purposes and distributes the rest to its owners. Net profit is:

$$\Pi_{ij} = F_{ij} - R_{ij}$$  \hspace{1cm} (13)

The present discounted value $V_{ij}(t)$ of net profit is

$$V_{ij}(t) = \int_t^{\infty} \Pi_{ij} e^{-\int_t^\tau r(s) ds} d\tau$$

$$= \int_t^{\infty} \left[ G_{ij} \left( P_{G_{ij}} - A_i \right) - \phi_i - R_{ij} \right] e^{-\int_t^\tau r(s) ds} d\tau$$  \hspace{1cm} (14)

The firm chooses the paths of its product price $P_{G_{ij}}$ and its R&D expenditure $R_{ij}$ to maximize (14) subject to the demand function (8) and the R&D production function (10).

### 3.3.2 Entrants

Entrepreneurs create new firms to compete with incumbents for market share and thereby affect firm market size $s_{ij}$. To determine the entry and exit of the firm, the value of firm $V_{ij}$ defined by (14) has to be compared with the cost of entry and exit. I assume that entry and exit are costless as in Peretto (Oct. 1999). For simplicity, I refer only to entry. Costless entry implies that $N_i$ is a jumping variable. Whenever the net present value of a new firm $V_{ij}$ differs from the entry cost of zero, new firms jump in or out to restore equality between the value of the firm and the entry cost. I thus have at all times

$$V_{ij} = 0$$  \hspace{1cm} (15)

As a result, I also have $\dot{V}_{ij} = 0$.

Differentiating Eq.(14) with respect to time gives the firm’s rate of return to equity (i.e., entry):

$$r_{ij}^E = \frac{\Pi_{ij}}{V_{ij}} + \frac{\dot{V}_{ij}}{V_{ij}}$$  \hspace{1cm} (16)

This is a usual perfect-foresight, no arbitrage condition for the equilibrium of the capital market. Multiplying both sides of (16) by $V_i$ and imposing $V_{ij} = 0$ and $\dot{V}_{ij} = 0$ implies the Zero (net) Profit Condition as in Peretto (Oct 1999).

$$\Pi_{ij} = 0$$  \hspace{1cm} (17)

In addition, the returns to in-house R&D of all firms of both industries must be the same because otherwise all investment goes to the firms (industry) with higher returns. Since entry is costless, new firms always jump into the industry with higher returns to satisfy the no arbitrage condition.

\footnote{I explored an extension of the model with costly entry, but I were unable to obtain closed-form solutions. Costly entry should not change the balanced growth path results in this model. Costly entry leads a slow entry of firms along dynamic adjustment path. With or without costly entry, on the balanced growth path, the number of firms in the industry ultimately is the same and endogenous entry kills the scale effect. See Peretto & Connolly (2007) for more discussion of entry cost. Also see Peretto (2007) for discussion of costly entry in a framework similar to mine.}
I follow Peretto (Oct. 1999) and impose a simplification that avoids technical complications that have no effect on the analysis or results. I assume that (1) at the initial time all firms in industry \( i \) have the same level of quality \( Z_{ij} = Z_i \), and (2) new entrants arrive with the average level of quality in their industry. These assumptions lead directly to an equilibrium that is symmetric within each industry, with all firms in an industry always making the same decisions on pricing, R&D expenditures, and market size. All firms in industry \( i \) choose the same prices and sell the same quantity of goods, all of which have the same quality. See the Appendix for proof. For a complete discussion of market equilibrium and its symmetric stability in these types of R&D models, see Peretto (1996, 1999, 2007) and the references cited therein.

### 3.4 Households

A representative household supplies both production factors (\( H \) and \( L \)) inelastically in a perfectly competitive market and purchases assets (corporate equity). I assume for simplicity that there is no population growth. The utility function is

\[
U(t) = \int_t^\infty \log(C) e^{-\rho t}
\]

where \( C \) is consumption and \( \rho \) is the rate of time preference.

The only assets that the household can accumulate are firms that it owns. The household’s lifetime budget constraint therefore is

\[
0 = \int_0^\infty \left( \int_0^{N_1} \Pi_{1j} dj + \int_0^{N_2} \Pi_{2j} dj + w_L L + w_H H - C \right) e^{-\int_t^\infty r(s) ds} dt
\]

The intertemporal consumption plan that maximizes discounted utility (18) is given by the consumption Euler equation, which as usual can be written as

\[
r = \rho + \frac{\dot{C}}{C}
\]

### 4 General Equilibrium

We must solve for the prices and quantities of the final good (\( Y \)), the processed goods (\( Y_i \)), and the intermediate goods (\( G_{ij} \)). We also must find consumption (\( c \)), the employment levels (\( s_{ij} \)), the investment amounts (\( R_{ij} \)), the return to factor (\( w_{ij} \)), the numbers of firms (\( N_i \)), and the rates of return (\( r_{ij}^{E}, r_{ij}^{R&D}, \) and \( r \)). Those allow us to solve for the growth rates of the variables we are interested in. The solution is obtained in the usual way from a system of simultaneous equations.
4.1 Incumbent Decisions

The optimal values for the prices $P_{G_{ij}}$ come directly from straightforward manipulations of the first-order conditions for the incumbent’s maximization problem in (14), and are the usual mark-ups over variable cost:

$$P_{G_i} \equiv P_{G_{ij}} = \frac{A_i}{\lambda}$$  (20)

Solving the maximization problem, we get that the return to in-house R&D is a combination of revenue less variable cost per unit of quality $\frac{(P_{ij} - A_i)G_{ij}}{Z_{ij}}$, R&D productivity $\alpha_i$ and the exponent $\delta$ of the quality $Z_{ij}$:

$$r_{ij}^{R&D} = \delta \alpha_i \frac{(P_{ij} - A_i)G_{ij}}{Z_{ij}}$$  (21)

Plug (8) and (20) into (21):

$$r_{ij}^{R&D} = \delta \alpha_i \frac{N_i^\sigma A_i}{\lambda} \frac{1 - \lambda}{A_i/\lambda^{1-\lambda}} \frac{1}{\lambda Z_{ij}^{\delta-1} Z_{ij}^{1-\delta}} s_{ij}$$  (22)

Equation (22) shows seven elements that determine the return to in-house R&D: (i) the exponent of the quality $Z_{ij}$ in (7), $\delta$, which is the share of the firm’s own knowledge in the total amount of knowledge that augments factors; (ii) R&D productivity, $\alpha_i$, which is positively related with the return to R&D; (iii) the social return to variety, $N_i^\sigma$, which is a positive externality to in-house R&D returns; (iv) unit cost, $A_i$, which is negatively related with R&D returns; (v) the price of processed good, $P_{Y_i}$, which is positively related with R&D returns; (vi) the knowledge spillover within an industry, $Z_i$, given which the return to R&D is diminishing in its own quality level $Z_{ij}$; (vii) firm market size, $s_{ij}$, which is positively related with the return to in-house R&D.

The internal symmetry of each industry equalizes the rate of returns

$$r_{i}^{R&D} \equiv r_{ij}^{R&D} = \delta \alpha_i N_i^\sigma A_i \frac{1 - \lambda}{\lambda} \frac{\lambda P_{Y_i}}{A_i/\lambda^{1-\lambda}} \frac{1}{\lambda s_i}$$  (23)

where by symmetry $s_i$ is expressed as:

$$s_i \equiv s_{ij} = S_i / N_i$$  (24)

Note that the R&D incentive of the individual firm depends on firm market size $s_{ij}$. Industry market size is irrelevant. Firm market size $s_{ij}$ is endogenously determined by firm entry, as discussed in the following sections. The irrelevance of industry size is one of the main differences between this model and the previous literature, which will lead to substantially different and interesting implications on directed technical change and relative factor returns.
4.2 Entry Decisions

Free entry leads to the zero profit condition shown in (17). Combining (17) with (13), we see that incumbents devote all gross profit \((F_{ij})\) to in-house R&D. The level of R&D expenditure can be written as

\[
R_{ij} = F_{ij} = P_{Gij}G_{ij} - A_iG_{ij} - \phi_i
\]

(25)

The growth rate of quality \(Z_{ij}\) is

\[
\frac{\dot{Z}_{ij}}{Z_{ij}} = \frac{\alpha_iR_{ij}}{Z_{ij}} = \frac{\alpha_iF_{ij}}{Z_{ij}} = \frac{\alpha_i[(P_{ij} - A_i)G_{ij} - \phi_i]}{Z_{ij}}
\]

(26)

which, because of symmetry, can be written as

\[
g_i \equiv \frac{\dot{Z}_i}{Z_i} = \frac{\alpha_iR_i}{Z_i} = \frac{\alpha_iF_i}{Z_i} = \alpha_iN_i^{\sigma}A_i(1 - \frac{1}{\lambda})\left(\frac{P_i}{A_i/\lambda}\right)\frac{1}{\tau} s_i - \alpha_i \phi_i Z_i
\]

(27)

These growth rates depend positively on the gross profit per unit of quality \(F_{ij}/Z_{ij}\) and R&D productivity \(\alpha_i\). Note that the growth rate of quality depends on firm market size \((s_i \equiv S_i/N_i)\) which is endogenously determined but not industry market size \(S_i\). Once again, we see the distinction between this model and the previous literature with exogenous market structure. The previous literature either presumes firm market size equals either industry size or a fixed portion of the industry size (because \(N_i\) is fixed and exogenous), so industry size affects incentives of R&D and hence the growth rate. In contrast, the present model endogenizes firm market size through the firm entry decision. This endogeneity of firm size has some important implications, discussed below. Notice also that the industry with a relatively high fixed cost \(\phi_i\) devotes less resource \(R_i\) to R&D and thus has a relatively low growth rate of quality improvement. Fixed operating cost plays an important role in determining the direction of technical change as shown later, but it generally has been ignored in the previous literature.

Equilibrium in the capital market requires that rates of return satisfy the no-arbitrage condition:

\[
r = r^{R&D}_H = r^{R&D}_L = r^E_H = r^E_L
\]

(28)

Zero entry cost implies that \(V_i = 0\) and hence from (16) that \(\pi_i = 0\) (zero profit condition) given any rate of interest \(r\), so that \(r^E_i = r_i^{R&D}\). Zero entry cost also implies that firms instantaneously enter to make the returns to in-house R&D equal across industries, \(r^{R&D}_H = r^{R&D}_L\). Apply the no-arbitrage condition to (21):

\[
r^{R&D}_H = r^{R&D}_L \iff \delta\alpha_H \frac{(P_H - A_H)G_H}{Z_H} = \delta\alpha_L \frac{(P_L - A_L)G_L}{Z_L}
\]
\[ \Leftrightarrow \alpha_H \frac{(P_H - A_H)G_H}{Z_H} = \alpha_L \frac{(P_L - A_L)G_L}{Z_L} \]  

(29)

If \( r^{R&d}_H > r^{R&d}_L \), then new firms instantaneously enter industry \( H \) and compete with incumbents for market share. A larger number of firms \( N_H \) drives down the firm market size \( s_H \), because \( s_H = H/N_H \). A lower firm market size drives down the return to in-house R&D in industry \( H \) to the point that \( r^{R&d}_H = r^{R&d}_L \). Entry is endogenously directed to the industry with a higher return to in-house R&D.

### 4.3 Directed R&D

This section shows what affects the accumulation of qualities and what affects the direction of in-house R&D. In contrast to the previous literature, the direction of in-house R&D does not depend on the industry endowment ratio \( H/L \). Instead, it depends on firm market size, which is endogenous. How endogenous market structure affects the direction of in-house R&D through firm market size is highlighted in this section.

The zero (net) profit condition indicates that incumbents use all gross profit to do in-house R&D. The growth rate of quality thus depends positively on R&D productivity and gross profit per unit of quality, as discussed in (26), and more specifically in (27). The growth rate of the ratio of the two types of qualities equals the difference of the growth rates:

\[
\frac{\dot{Z}_H}{\dot{Z}_L} = \frac{\dot{Z}_H}{Z_H} - \frac{\dot{Z}_L}{Z_L} = \alpha_H \left[ N_H^\sigma A_H \left( \frac{1 - \lambda}{\lambda} \right) \left( \frac{\lambda P_{Y_H}}{A_H/\lambda} \right)^{1-\lambda} s_H - \phi_H \right] - \alpha_L \left[ N_L^\sigma A_L \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{Y_L}}{A_L/\lambda} \right)^{1-\lambda} s_L - \phi_L \right] \quad (30)
\]

To emphasize the impact of endogenous market structure on the direction of in-house R&D, let us assume the market structure is exogenous for a moment, which means individual firm market size \( s_i \) is exogenous and equal to industry market size (or a fixed portion of industry market) as in the previous literature. Given an exogenous \( s_i \), (30) provides two results. First, the growth rate of \( Z_i \) is increasing in \( \alpha_i \) and decreasing in \( \phi_i \). Second, if relative factor endowment \( H/L \) increases, then relative firm market size in the \( H \)-industry to \( L \)-industry increases and thus the R&D return in the \( H \)-industry relative to that in the \( L \)-industry increases. This is the positive market size effect, which increases the relative growth rate of \( Z_H \). At the same time, a higher \( H \) relative to \( L \) decreases \( P_{Y_H} \) relative to \( P_{Y_L} \) as in (34). This is the negative price effect which leads a lower growth rate of \( Z_H \) relative to the growth rate of \( Z_L \). With an exogenous market structure, as relative factor endowments change, both the market size effect and price effect affect the direction of R&D, which is the same as in the previous literature.

Let us now drop the assumption of exogenous market structure, so that individual firm size \( s_i \) is endogenously determined. In general equilibrium, an endogenous \( s_i \) alters the implications for the direction of in-house R&D from the previous literature. What happens if market structure is endogenous? More specifically, what happen to the direction of in-house R&D if the firm entry decision is endogenous?
Plugging the no arbitrage condition (29) to (30), and using that $\alpha_i N_i A_i \left( \frac{1-\lambda}{\lambda} \right) \left( \frac{A_i}{A_i/\lambda} \right)^{1/\lambda}$, $s_i = r^{R&D}_i / \delta$ from (22), we get the quality growth difference as:

$$\frac{\dot{Z}_H}{\dot{Z}_L} \frac{Z_H}{Z_L} = \frac{\dot{Z}_H}{Z_H} - \frac{\dot{Z}_L}{Z_L}$$

$$= \left[ r^{R&D}_H / \delta - \frac{\alpha_H \phi_H}{Z_H} \right] - \left[ r^{R&D}_L / \delta - \frac{\alpha_L \phi_L}{Z_L} \right]$$

(31)

$$= -\frac{\alpha_H \phi_H}{Z_H} + \frac{\alpha_L \phi_L}{Z_L}$$

(32)

The direction of in-house R&D is determined by the cross-industry differences in the gross profits per unit of quality and R&D productivities. Endogenous firm entry equalizes R&D returns across industries so that $r^{R&D}_H / \delta = r^{R&D}_L / \delta$. Note that the cross-industry differences in industry market size and processed-good price are included in $r^{R&D}_i$. Therefore, the price effect and the industry market size effect are both wiped out by endogenous entry. As a result, a change in relative factor endowment has no impact on the direction of in-house R&D. This result is very different from anything in the existing literature and shows the importance of taking into account the endogeneity of market structure.

A truly surprising implication of endogenous market structure is the relation that emerges between R&D productivity $\alpha_i$ and innovation growth. In general equilibrium, the direction of in-house R&D is determined by the differences in the gross profits per unit of quality and R&D productivities across industries, and more specifically, by the differences on the R&D productivity $\alpha_i$ and fixed operating cost $\phi_i$ per unit of quality. It turns out that the industry with a relatively high R&D productivity has a relatively low in-house innovation growth. The key to understanding this surprising result is the endogeneity of market structure. When $\alpha_H > \alpha_L$, given an exogenous and fixed firm size $s_i$ as in the previous literature, then the growth rate of $Z_H$ is higher than that of $Z_L$. But $s_i$ is in fact endogenous through firm entry. Firms enter the industry with a higher R&D productivities, leading to an increase in $N_H/N_L$ and thus to a fall in $s_H/s_L$. Entry continues until $r_H = r_L$, as required by the no arbitrage condition. At that point, the first part of the expression for in-house innovation growth in (27) is equal across industries. Thus the direction of in-house R&D depends on the cross-industry difference of the second term of (27), which shows how fixed operating costs enter the picture of directed technical change: fixed operating cost subtracts resources from R&D. The higher fixed cost is, the less is the resource devoted to in-house R&D. The magnitude of the subtracting effect that fixed cost has on R&D resource is measured by R&D productivity $\alpha_i$. If $\alpha_i$ is relatively high, then so is the negative impact of fixed cost on R&D resource. Incumbents in the industry with a higher R&D productivity must divert more resource from R&D, leading a lower quality growth. Therefore, in general equilibrium, if $\alpha_H \phi_H > \alpha_L \phi_L$, then $Z_H/Z_H < Z_L/Z_L$. The crucial new element responsible for this result is the equality between $r_H$ and $r_L$ that is brought about by endogenous entry.

In this section, I have shown that the direction of in-house R&D is determined by industry differences in gross profits per unit of quality and R&D productivities. In general equilibrium, a change in relative factor endowment is totally absorbed by directed firm entry and therefore does not affect the direction
of in-house R&D, a result that is substantially different from related literature. Instead, the direction of in-house R&D depends on R&D productivity and fixed operating cost. Surprisingly, the industry with a relatively low R&D productivity and fixed operating cost has a relatively high growth of quality.

4.4 No Arbitrage Condition and Directed Firm Entry

In this section, I show how the no arbitrage condition determines the direction of firm entry. The ratio of numbers of firms across industries is obtained from (29):

$$\frac{N_H}{N_L} = \frac{\alpha_H N_H^\sigma A_H \left( \frac{P_{Y_H}}{A_H} \right)^{\frac{1}{1-\gamma}} H}{\alpha_L N_L^\sigma A_L \left( \frac{P_{Y_L}}{A_L} \right)^{\frac{1}{1-\gamma}} L} = \left( \frac{\alpha_H}{\alpha_L} \right)^{\frac{1}{1-\sigma}} \left( \frac{A_H}{A_L} \right)^{\frac{1}{(1-\lambda)(1-\sigma)}} \left( \frac{P_{Y_H}}{P_{Y_L}} \right)^{\frac{1}{1-\sigma}} \left( \frac{H}{L} \right)^{\frac{1}{1-\sigma}}$$

(33)

where $0 < \sigma, \lambda < 1$.

Combining (8), (7) and (5) and rearranging the equation, we obtain the relative price of two processed goods:

$$\frac{P_{Y_H}}{P_{Y_L}} = \frac{1 - \gamma}{\gamma} \left( \frac{N_H}{N_L} \right)^{\frac{\sigma(1-\lambda)}{1-\gamma}} \left( \frac{A_H}{A_L} \right)^{\frac{1}{(1-\lambda)(1-\sigma)}} \left( \frac{Z_H}{Z_L} \right)^{\frac{1}{1-\sigma}} \left( \frac{H}{L} \right)^{\frac{1}{1-\sigma}}$$

(34)

where $\Psi \equiv \lambda + (1 - \lambda)\epsilon > 0$ is the (derived) elasticity of substitution between two factors $H$ and $L$, and $\epsilon$ is the elasticity of substitution between two processed goods, $Y_H$ and $Y_L$. We have $\Psi > 1$ iff $\epsilon > 1$. That is, the two factors are substitutes if and only if the two processed goods are substitutes. See Appendix for proof. An increase in the endowment ratio ($H/L$) decreases the price ratio.

Plugging (34) back to (33), we get that the ratio of the numbers of firms depends on the ratio of in-house R&D and the ratio of factor endowments. Recall that in the previous section we showed the ratio of in-house R&D does not depend on the ratio of factor endowments.

$$\frac{N_H}{N_L} = \left( \frac{\alpha_H}{\alpha_L} \right)^{\Psi} \left( \frac{A_H}{A_L} \right)^{\frac{\lambda(1-\Psi)}{1-\lambda}} \left( \frac{1 - \gamma}{\gamma} \right)^{\frac{1}{\lambda(1-\sigma)}} \left( \frac{Z_H}{Z_L} \right)^{\frac{1}{1-\sigma}} \left( \frac{H}{L} \right)^{\frac{\phi+1}{\phi^2 - 1}}$$

(35)

where $\Upsilon \equiv \Psi(1 - \sigma) + \sigma > 0$. If the social return to variety is zero, i.e. $\sigma = 0$, then $\Upsilon = \Psi$.

Equation (35) indicates that an increase in relative factor endowment $H/L$ has two opposing effects on entry. On the one hand, an increase in $H/L$ increases relative industry market size and hence the relative return to in-house R&D of the $H$ industry given a fixed number of firms, which induces firms to enter the $H$-industry and so increases $N_H/N_L$. This is the positive industry market size effect. On the other hand, an increase in $H/L$ decreases the relative price $P_{Y_H}/P_{Y_L}$, as shown in (34), and induces firms to enter the $L$-industry. This is the negative price effect. The term $\frac{H}{L}^{\frac{\phi+1}{\phi^2 - 1}}$ reflects the combination of

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13(35) contains an endogenous variable $Z_H/Z_L$, but the quality ratio does not depend on factor endowments nor on relative factor prices as discussed in the previous section.
the industry market size effect and the price effect. If $\Psi > 1$, which means the two factors are substitutes, then as $H/L$ increases, the (positive) industry market size effect dominates, and firms are induced to the $H$-industry. If $\Psi < 1$, which means the two factors are complements, then as $H/L$ increases, the negative price effect dominates, and the new firms are directed to the $L$-industry.

The impact of relative factor endowment on the direction of firm entry is similar to that in Acemoglu (2002) but with two important differences. First, firm entry in the present model is the source of the endogeneity of market structure that determines the market size of each individual firm. By contrast, firm entry in a variety expansion model cannot be the source of endogeneity of market structure because with firm entry the variety expansion model does not have perpetual growth. Second, as we will see later, there are important differences from Acemoglu on how directed entry affects relative factor returns. In addition, the model has two dimensions of technical change, and those interact with each other. Equation (35) indicates that the quality ratio negatively affects the variety ratio through the term $(Z_H/Z_L)^{\tau - 1}$. The quality ratio affects the direction of entry through the price ratio shown in (34). If $Z_H$ increases relative to $Z_L$, given other factors fixed, then the supply of $Y_H$ increases relative to $Y_L$, which decreases $P_{Y_H}/P_{Y_L}$. A lower relative price ($P_{Y_H}/P_{Y_L}$) thus decreases the variety ratio ($N_H/N_L$). Therefore, in-house R&D affects the direction of firm entry through the relative price of processed goods (the price effect).

In this section, I first have shown how a change in relative factor endowment affects the direction of firm entry through the price effect and the industry market size effect. I also have shown how in-house R&D affects the direction of firm entry through the price effect. A summary of all these channels is in figure 2.

4.5 Transition Dynamics and Balanced Growth Path

Following Peretto (2007), and Cozzi & Spinesi (2003), I assume fixed operating cost, $\phi_i$, is a homogeneous degree-1 function, $\phi_i = Z_i^\eta Z_j^{1-\eta}$. Then (32) becomes:

$$\frac{\dot{Z}_H}{Z_L} = -\alpha_H \theta_H (\frac{Z_H}{Z_L})^{\eta - 1} + \alpha_L \theta_L (\frac{Z_L}{Z_H})^{\eta - 1}$$

(36)

I allow $\eta$ to be either higher, equal, or less than 1. The theoretical literature has different assumptions on the relation between fixed cost and the knowledge stock. Peretto (2007) assumes fixed operating cost and average quality are positively related whereas Cozzi and Spinesi (2003) assume the opposite. The empirical literature pays little attention to this issue and does not provide any conclusion on the magnitude of $\eta$, so I discuss three different possibilities: $\eta > 1$, $\eta = 1$ and $\eta < 1$.

If $\eta > 1$, the steady state of (36) is $(\frac{Z_H}{Z_L})^* = (\frac{\alpha_L \theta_L}{\alpha_H \theta_H})^{\eta - 1}$ and is stable. The steady state ratio of $Z_H$ to $Z_L$ is negatively related to $\alpha_H \theta_H$ and positively related to $\alpha_L \theta_L$. As just discussed, an increase in $\alpha_H$

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14 See the Introduction.
15 The quality ratio does not directly affect the direction of entry because it does not directly appear in (33). The quality level of an individual product ($Z_{ij}$) does not directly affect the return to in-house R&D since it is canceled out with the intra-industry knowledge spillover ($Z_i$) shown in (22).
16 The other steady state $- (\frac{\alpha_L \theta_L}{\alpha_L \theta_H})^{\eta - 1}$ is unstable. See the Appendix.
increases the second term in (27) but not the first term, so an increase in $\alpha_H$ instantaneously decreases the growth rate of $Z_H$ while keeping the growth of $Z_L$ unchanged initially. $Z_H$ grows more slowly than $Z_L$ along the entire transition path, so the new steady state ratio for $Z_H/Z_L$ is lower than the initial value. Therefore, the steady state of $Z_H/Z_L$ is negatively related to $\alpha_H$. The steady state value of $\frac{\dot{Z}_H}{\dot{Z}_L}$ is negatively related to $\theta_H$ for a similar reason. If $\eta < 1$, the dynamic system is not stable, so I discard that case. If $\eta = 1$, there is no steady state, and (36) becomes:

$$\frac{\dot{Z}_H}{Z_H} - \frac{\dot{Z}_L}{Z_L} = -\alpha_H\theta_H + \alpha_L\theta_L$$

One type of quality always grows faster than the other, depending on which term on the right dominates.  

Consumption is a constant proportion of final good production, so from the Euler Equation (19) we get:

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = r - \rho$$

(37)

We can write the growth rate of the final good as a weighted average of the growth rates of the two processed goods:

$$\frac{\dot{Y}}{Y} = \Gamma \frac{\dot{Y}_H}{Y_H} + (1 - \Gamma) \frac{\dot{Y}_L}{Y_L}$$

(38)

where $\Gamma \equiv \gamma Y_L^{\frac{1}{\epsilon}} / (\gamma Y_L^{\frac{1}{\epsilon}} + (1 - \gamma) Y_H^{\frac{1}{\epsilon}})$. The weight $\Gamma$ changes along the dynamic adjustment path as the $Y_i$ change. If a BGP exists (the fixed cost parameter $\eta > 1$), then $\frac{\dot{Y}}{Y} = \frac{\dot{Y}_L}{Y_L} = \frac{\dot{Y}_H}{Y_H}$ with a constant $\Gamma$ and the BGP growth rate is:

$$g^* = \frac{\delta}{1 - \delta} \sqrt{(\alpha_H \theta_H)(\alpha_L \theta_L)} - \frac{1}{1 - \delta}\rho$$

(39)

where $g^* = \frac{\dot{Z}_H}{Z_H} = \frac{\dot{Z}_L}{Z_L} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{Y}_H}{Y_H} = \frac{\dot{Y}_L}{Y_L} = \frac{\dot{G}_H}{G_H} = \frac{\dot{G}_L}{G_L} = \frac{w_H}{w_H} = \frac{w_L}{w_L}$.

If a BGP does not exist, then $\Gamma$ is not a constant. Using (8), (7) and (35), we can write the the growth rate of $Y_H/Y_L$ as a function of the growth rates of the qualities:

$$\frac{\dot{Y}_H}{Y_H} - \frac{\dot{Y}_L}{Y_L} = \frac{(\dot{Y}_H)/(Y_H)}{(\dot{Y}_L)/(Y_L)}$$

$$= \varphi_1 \left( \frac{\dot{Z}_H}{Z_H} - \frac{\dot{Z}_L}{Z_L} \right)$$

(40)

where $\varphi_1 \equiv \frac{-1}{\Psi(1-\sigma)+\sigma} + \frac{\epsilon(1-\lambda)}{\Psi(1-\sigma)+\sigma} > 0$ given $\sigma \in [0, 1)$. The difference of the growth rates of processed

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17 In fact, there actually is an asymptotic steady state in that case, in which $Z_H/Z_L$ is either infinite or zero.

18 See Appendix for proof.
goods depends only on the differences of the growth rates of qualities, shown in (36). If the fixed cost parameter $\eta = 1$, according to (40), the growth rate difference between two processed goods is:

$$\frac{\dot{Y}_H}{Y_H} - \frac{\dot{Y}_L}{Y_L} = \theta_1 [-\alpha_H \theta_H + \alpha_L \theta_L]$$

Assume $\alpha_H \theta_H < \alpha_L \theta_L$ without loss of generality. Then $\frac{\dot{Y}_H}{Y_H} > \frac{\dot{Y}_L}{Y_L}$. If $\epsilon > 1$, then $\frac{\dot{Y}_H}{Y_H} \to \frac{\dot{Y}_H}{Y_H} (\Gamma \to 0)$, whereas if $\epsilon < 1$, $\frac{\dot{Y}_H}{Y_H} \to \frac{\dot{Y}_L}{Y_L} (\Gamma \to 1)$. The elasticity of substitution of the two types of processed goods plays an important role here. When $\epsilon > 1$, the processed goods are substitutes, so the processed good that grows faster can substitute for the slower one. Therefore, the growth rate of the final good asymptotically equals the growth rate of the processed good that grows faster. When $\epsilon < 1$, the two types of processed goods are complements. The processed good that grows slower acts as a dragging element in the production of the final good. So the growth rate of the final goods asymptotically equals that of the processed good that grows slower.

5 Relative Factor Returns

This section describes how directed technical change affects relative factor returns, such as the return differences between skilled and unskilled labor. A strand of the literature attributes the change in the relative wage between skilled and unskilled labor to skill-biased technical change. It assumes that technical progress by its nature augments skilled labor. However, new technologies are not complementary to certain factors by nature but by design, as emphasized by Acemoglu (1998, 2002, 2007). Firms choose to invest resources in developing new technologies that augment one or another factor of production according to the relative profitability of each type of investment. The model developed here describes what determines relative factor returns, i.e. relative wages.

Relative factor demand can be obtained in two steps. First, combine (8) into (9) and do some algebra to get factor returns $w_i$. Second, take the ratio of $w_H$ to $w_L$ and combine with (34) to get relative factor demand:

$$\frac{w_H}{w_L} = \frac{N_H}{N_L} \sigma \left( \frac{P_{Y_H}}{P_{Y_L}} \right) \frac{1}{\lambda} \left( \frac{A_L}{A_H} \right) \frac{Z_H}{Z_L} \left( \frac{1 - \gamma}{\gamma} \right)^{\Psi} \left( \frac{A_H}{A_L} \right)^{\frac{1}{\psi-1}} \left( \frac{N_H}{N_L} \right)^{\sigma(\frac{\psi-1}{\psi})} \left( \frac{Z_H}{Z_L} \right)^{\frac{\psi-1}{\psi}} \left( \frac{H}{L} \right)^{\frac{1}{\psi}}$$

(41)

Recall that the (derived) substitution elasticity of the two factors, $\Psi \equiv \epsilon(1 - \lambda) + \lambda > 0$, and $\Psi > 1$ iff $\epsilon > 1$. Relative factor endowment is exogenous, following Acemoglu (1998, 2002) and Smulders & Nooij (2003).

For given value of $Z_H/Z_L$ and $N_H/N_L$, as relative factor endowment $H/L$ increases, the equilibrium relative factor return decreases, which is shown by the term $(H/L)^{\frac{1}{\psi}}$ where $-\frac{1}{\psi} < 0$. The ratios $N_H/N_L$
and \(Z_H/Z_L\) also affect the relative return to factors. If there is an increase in \(H\)-type technology (i.e., \(N_H/N_L\) and/or \(Z_H/Z_L\) increases), then \(w_H/w_L\) increases if the two factors are substitutes (\(\Psi > 1\)) and \(w_H/w_L\) decreases if two factors are complements (\(\Psi < 1\)). The intuition is straightforward. If the two factors are complements, an increase in the productivity of \(H\) will increase the demand for the other factor \(L\) even more and thus will increase the relative return for \(L\). Note that if the social return to variety is zero (\(\sigma = 0\)), the variety ratio has no impact on the relative factor return.

We now look further into what determines the direction of technical change and how directed technical change affects relative factor returns. To see clearly how directed technical change affects the factor return ratio, we plug (35) into (41):

\[
\frac{w_H}{w_L} = \Phi_0 \left( \frac{Z_H}{Z_L} \right) \Phi_1 \left( \frac{H}{L} \right) \Phi_2
\]

where \(\Phi_0 > 0\) is a combination of constants; \(\Phi_1 \equiv -\frac{\Psi-1}{\Psi(1-\sigma)+\sigma} \frac{(\Psi-1)\sigma}{\Psi} + \frac{\Psi-1}{\Psi(1-\sigma)+\sigma} \frac{(\Psi-1)\sigma}{\Psi} - \frac{1}{\Psi}\).

The relative return to factors depends on relative quality, relative factor endowment, and the parameters \(\Psi\) and \(\sigma\) that govern respectively, the elasticity of substitution between the factors and the size of the social return to variety. Let us see the impacts of relative factor endowment on the relative return first. The exponent of the endowment ratio, \(\Phi_2 \equiv -\frac{\Psi-1}{\Psi(1-\sigma)+\sigma} \frac{(\Psi-1)\sigma}{\Psi} - \frac{1}{\Psi}\), indicates two opposite effects of the endowment ratio on relative factor returns. The first component is \(-\frac{\Psi-1}{\Psi(1-\sigma)+\sigma} \frac{(\Psi-1)\sigma}{\Psi}\), which is always positive, and shows how a change in relative factor endowment affects relative factor returns through directed firm entry. A change in \(H/L\) directs new firms to enter through the industry market size effect and the price effect. Entry affects relative factor demand and hence relative factor returns. If \(\Psi > 1\), as \(H/L\) increases, the industry market size effect dominates, leads more firms to enter the \(H\)-industry, and raises \(N_H/N_L\) as shown in (35). A higher \(N_H/N_L\) then increases the relative demand of \(H/L\) because \(H\) and \(L\) are substitutes and therefore increases \(w_H/w_L\). If \(\Psi < 1\), as \(H/L\) increases, the price effect dominates and decreases \(N_H/N_L\). A lower \(N_H/N_L\) still increases the relative demand of \(H/L\) and therefore \(w_H/w_L\) because \(H\) and \(L\) now are complements. No matter what the value of \(\Psi\) is, an increase in \(H/L\) always raises \(w_H/w_L\) through directed firm entry. This impact is similar to the “weak induced-biased hypothesis” in Acemoglu (1998, 2002)\(^{19}\). The second component of \(\Phi_2\) is \(-\frac{1}{\Psi} < 0\). This is the direct impact of a change in relative factor endowment on relative factor returns. An increase in relative factor endowment directly decreases relative factor returns. We thus see that a change in relative factor endowment has two opposite impacts on relative factor returns. Which impact dominates depends on the magnitude of \(\Phi_2\). If \(\Phi_2 > 0 \iff \Psi > 1 + 1/\sigma\), an increase in relative factor endowment increases the relative reward to the factor that has become more abundant. Consequently, an increase in endowment of \(H/L\) raises \(w_H/w_L\), which is similar to the “strong induced-biased hypothesis” in Acemoglu (1998, 2002)\(^{20}\). Figure

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\(^{19}\)Acemoglu (2002), pp. 783 defines “weak induced-biased hypothesis” as, “Irrespective of the elasticity of substitution between factors (as long as it is not equal to 1), an increase in the relative abundance of a factor creates some amount of technical change biased towards that factor”, and thus raises the relative return of that factor.

\(^{20}\)Acemoglu (2002), pp. 783 defines “strong induced-biased hypothesis” as, “if the elasticity of substitution of two factors is sufficiently large, the induced bias in technology can overcome the usual substitution effect and increase the relative reward to the factor that has become more abundant.”
A novel element of this analysis is that the impact of directed firm entry on relative factor returns depends on the social return to variety, which is determined by the parameter sigma. If $\sigma = 0$, there is no social return variety, the impact of directed firm entry on relative factor returns disappears, and both the weak and strong induced-biased hypotheses in Acemoglu (1998, 2002) also disappear. When there is no social return to variety, a change in relative factor endowment has no impact on relative factor returns through directed technical change. The reason is that directed firm entry affects relative demand of factors only through an externality, and when $\sigma = 0$, there is no externality. Embedding directed technical change in a model with endogenous market structure allows us to see under what conditions relative factor endowment affects the direction of technical change and hence relative factor returns. The ability to address such a question arises from the two-dimensional nature of technical progress in the model. Traditional one-dimensional models of growth through quality improvement (e.g. Acemoglu 1998) presume each product is used by all labor and ignore the specialization of labor and the potential entry of new varieties. Each intermediate good in the $H$ industry ($L$ industry) is used by all factors $H$ ($L$). As a result, a change in relative factor endowment alters the incentives for quality improvement across industries and alters the direction of quality improvement. In contrast, the kind of two-dimensional model used here emphasizes the impact of endogeneity of market structure on the incentives for quality improvement. Any changes in relative factor endowment are absorbed by endogenous firm entry, do not affect returns to R&D cross industries, and hence do not affect direction of in-house R&D. Therefore, relative factor endowment does not affect relative factor returns through directed in-house R&D. Traditional one-dimensional models of growth through variety expansion alone must have social returns to variety because otherwise they cannot deliver perpetual growth. In contrast, the kind of two-dimensional model used here can deliver growth without there being any social return to variety. The model thus allows us to see how including or excluding social returns to variety affects the results.

The exponent of the quality ratio, $\Phi_1 = \frac{1}{\Psi(1-\sigma) + \sigma} - \frac{1}{\Psi} + \frac{\Psi - 1}{\Psi}$, shows how a change in the quality ratio affects relative factor returns. The first term $\frac{-1}{\Psi(1-\sigma) + \sigma} + \frac{\Psi - 1}{\Psi}$ shows how a change of $Z_H/Z_L$ affects the direction of firm entry, through which $Z_H/Z_L$ affects $w_H/w_L$. If $Z_H/Z_L$ increases, then the relative supply of the processed good $Y_H/Y_L$ increases which causes a decrease in the relative price $P_{Y_H}/P_{Y_L}$. A lower $P_{Y_H}/P_{Y_L}$ induces firms to enter the $L$-industry and thus decreases $N_H/N_L$ (price effect). A lower variety ratio increases (decreases) relative factor returns if the two factors are substitutes (complements). The second term $\frac{\Psi - 1}{\Psi}$ shows that an increase in the quality ratio (1) directly raises relative factor returns ($\frac{\Psi}{\Psi} = 1$), and (2) decreases the relative price $P_{Y_H}/P_{Y_L}$ and thus decreases relative returns. When two factors are substitutes (complements), the first (second) effect dominates.

The foregoing results can be collected in the following proposition:

**Proposition**  Equation (42) shows that relative factor returns depend on relative factor endowment and relative quality.
1. An increase in relative factor endowment has two impacts on relative factor returns. On the one hand, given \( N_H/N_L \) and \( Z_H/Z_L \), it directly decreases relative factor returns through the usual interaction between supply and demand. On the other hand, a change in relative factor endowment induces new firms to enter, which always increases relative factor returns. If \( \Psi > 1 + 1/\sigma \), then as the relative endowment of factors increases, the positive impact through directed firm entry dominates the negative impact, and an increase in relative factor endowment increases the relative reward to the factor that has become more abundant. See the summary in figure 3.

2. An increase in the quality ratio has a positive impact on relative factor returns if the two factors are substitutes \((\Psi > 1)\), and a negative impact if two factors are complements \((\Psi < 1)\). See figure 4.

3. If the social return to variety is zero, i.e. \( \sigma = 0 \), then directed firm entry has no impact on relative factor returns. Induced firm entry has no effect on relative factor returns at all. In other words, the strength of directed firm entry’s impact on relative factor returns depends on how strong the social return to variety is. If there is no externality, then directed firm entry has no impact on the relative factor return. Instead, the main element that shapes relative factor returns is directed in-house R&D.

The dynamic adjustment path of the relative factor return can be derived from (42) and (36):

\[
\frac{(w_H^*)}{(w_L^*)} = \Phi_1 \left( -\alpha_H \theta_H \left( \frac{Z_H}{Z_L} \right)^{\eta - 1} + \alpha_L \theta_L \left( \frac{Z_L}{Z_H} \right)^{\eta - 1} \right)
\]

where \( \Phi_1 \equiv \left[ \frac{-1}{\Psi(1-\sigma)+\sigma} (\frac{\Psi-1}{\Psi})^\sigma + \left( \frac{\Psi-1}{\Psi} \right) \right] > 0 \) iff \( \Psi > 1 (\Leftrightarrow \epsilon > 1) \); and \( \Phi_1 < 0 \) iff \( \Psi < 1 \).

Assume \( \alpha_H \theta_H < \alpha_L \theta_L \) without loss of generality. Then in-house R&D is directed to industry-\( H \), and \( \frac{\dot{Z}_H}{Z_H} > \frac{\dot{Z}_L}{Z_L} \). Faster growth in \( H \)-type quality increases the productivity of \( H \). As a result, the demand for factor \( H \) increases if \( H \) and \( L \) are substitutes \((\Psi > 1)\) implying that \( \frac{(w_H^*)}{(w_L^*)} > 0 \). If \( H \) and \( L \) are complements \((\Psi < 1)\), then a faster growth in \( H \)-type quality increases the demand for \( L \) by more than the increase in \( H \), implying that \( \frac{(w_H^*)}{(w_L^*)} < 0 \). The transition path of the relative factor return is not affected by relative factor endowment because of instantaneous firm entry\(^{21}\). If the fixed cost parameter \( \eta > 1 \), then there exists a steady state value of relative factor return:

\[
\frac{(w_H^*)}{(w_L^*)} = \Phi_0 \left( \frac{\alpha_L \theta_L}{\alpha_H \theta_H} \right)^{\frac{\Phi_1}{\Phi_2}} \left( \frac{H}{L} \right)^{\Phi_2}
\]

The derivation is the same as for (42).

\(^{21}\)If I were to change the assumption to costly entry, then the firms would enter the market slowly. An increase in the industry market size then would increase individual firm size temporarily because entry is gradual. A change in industry market size then has an impact on in-house R&D during the transition to the BGP. On BGP, however, firm entry totally eliminates the impacts of a change in industry market size, leaves the incentives for in-house R&D unchanged, and does not change the balanced growth rate. As I mentioned above, I cannot get a closed form solution with costly entry. I suspect that my result for the BGP is robust to introducing a positive entry cost. See the discussion in Peretto and Connolly (2007), and Peretto (2007).
6 Implications

In this section, I discuss implications that emerge from the theory developed above.

The general equilibrium model used here has an implication for the impact of energy conservation on technology and economic growth. At the end of 2006, the European Union pledged to cut its annual consumption of primary energy by 20% by 2020. What impacts will such energy conservation have on the technology and economic growth? I follow Smulders & Nooij (2003) and treat aggregate energy usage as exogenous. If we denote energy usage as \( H \) and labor as \( L \), then the model predicts that a reduction in \( H \) has no impact on long run growth, as seen in (39). This result is again because of the endogeneity of market structure. A reduction of energy usage decreases the industry market size of the technology that augments \( H \). Firm exit and firm market size fully respond to the change. Therefore, the return to in-house innovation, which depends on firm market size but not on industry market size, is unchanged, and so the growth rate of the economy also is unchanged. The economy's balanced growth rate depends positively on R&D productivity and the fixed operating cost parameters only but not the size of the industry.

The model suggests several new lines of empirical investigation. First, empirical work needs to pay attention to the difference between industry market size and individual firm market size and to their impacts on the different dimensions of technical change. Industry market size matters only for directed firm entry, not for directed in-house R&D, and the direction of in-house R&D only depends on endogenous firm market size. Acemoglu & Linn (2004) measure the impact of industry market size on the number of varieties in the U.S. pharmaceutical industry. They report that a 1% increase in the industry market size increases the number of varieties by more than 1%. In terms of the theory developed above, this result can be interpreted as a measure of the impact of industry size on directed firm entry. That is an interesting quantity. It is not, however, informative about the other major dimension of directed technical progress, which is directed in-house R&D. The direction of in-house R&D should depend on the gross profit per unit of quality, i.e. R&D productivity, fixed operating cost, and firm market size, but not industry market size. To my knowledge, there is no empirical analysis of these issues.

Second, the model provides a different implication from the existing literature on the impact of directed technical change on relative factor returns. A change in the relative endowment of factors affects relative factor returns through directed firm entry if there is a social return to variety. The empirical literature shows a positive correlation between the relative factor endowment (i.e. skilled/unskilled labor) and the relative factor return (i.e. the relative wage), which could possibly reflect such a social return to variety. Empirical work mentioned in the Introduction shows the in-house R&D is the main force that drives TFP growth. Given that TFP is the productivity of labor, this model indicates that the relative factor return mainly depends on the direction of in-house R&D. The R&D productivity and the fixed operating cost matter for directed in-house innovation, so they also matter for relative factor returns. The industry with a lower R&D productivity and fixed operating cost pays a higher (lower) wage to the factor used in this industry if the two factors are substitutes (complements). Once again, there is to my knowledge no empirical analysis dedicated to this issue.
Third, the elasticity of substitution between factors $\psi$ in the model suggests analyzing in detail whether directed technical change raises the relative return or not. If $H$ and $L$ are substitutes (complements), then a directed technical change towards $H$ increases (decreases) the relative return to $H$.

Last, the model emphasizes the importance of fixed operating costs for the direction of technical change, the relative factor return, and the growth of final output and consumption. The nature of fixed operating costs determines whether the model has a BGP and whether the growth rates of factor returns converge. More specifically, if the fixed cost parameter $\eta > 1$, then the model has an unique BGP, and the relative factor return is a constant on BGP; if $\eta = 1$, then the return of one factor grows faster than the return of the other factor forever, and the final output and consumption converge to the output growth rate of one industry, depending on the elasticity of substitution between processed goods. Current empirical and theoretical work devote little attention to fixed operating costs.

7 Conclusion

This paper has presented a model that incorporates endogenous market structure in an endogenous growth framework and then has used the model to discuss directed technical change. The model is consistent with various IO facts and provides a more complete analysis and substantially different results than the existing literature.

The model considers two dimensions of directed technical change: firm entry and in-house R&D. The direction of firm entry depends on the industry market size effect and the price effect. The direction of in-house R&D depends on cross-industry differences in gross profits per unit of quality and R&D productivities. In general equilibrium, the industry market size and the price effect emphasized in the existing literature affects the direction of firm entry but not the direction of in-house R&D. Surprisingly, in-house R&D is directed to the industry with relatively low R&D productivity and fixed operating cost compared with the other industry. The reason is the interaction between endogenous market structure and fixed cost.

The model also considers the determinants of relative factor returns. A change in relative factor endowment does not affect relative factor returns through directed firm entry unless there is a social return to variety. If there is no social return to variety, then directed firm entry has no impact on relative factor returns, in contrast to the existing literature. The main determinant of relative factor returns is directed R&D, which depends only on cross-industry differences of R&D productivities and fixed costs but not on relative factor endowment. The industry with a relatively high R&D productivity and/or fixed operating cost has a relatively low quality growth on dynamic path, and a relatively low quality level on BGP. Therefore, the factor used in that industry gets paid a relatively low (high) return if the two factors are substitutes (complements). The model emphasizes the importance of the interaction between in-house R&D and endogenous market structure for the behavior of relative factor returns, an issue generally ignored by existing literature.
The model provides implications for the determinants of the wage differences across different types of labor and for the impact of energy conservation policy on economy growth. In contrast to the previous literature, the model predicts that changes in the factor endowment have no impact on the direction of in-house R&D or the balanced growth rate because of the endogeneity of market structure. The model shows that empirical work must take into account the endogeniety of market structure.
References


Figure 1: the Structure of the Model

Figure 2: The Direction of Firm Entry
Figure 3: $H/L$ affects $w_H/w_L$

$Z_H/Z_L \rightarrow w_H/w_L$

Figure 4: $Z_H/Z_L$ affects $w_H/w_L$