TGIF: Trade, Growth, and the Industrial Framework

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Abstract

We study Ricardian trade and economic growth in a framework that avoids counterfactual market-size effects and delivers several new results, some contradicting the previous literature. The channel through which trade affects growth is restricted by construction to comparative advantage. Trade, growth, and the economy's industrial structure interact, each affecting the other two. Trade affects capital exporting and importing countries differently, temporarily but not permanently raising the growth rate of the former and permanently raising the growth rate of the latter. Trade induces shut-down of some industries and an international reallocation of R&D activity. The model can explain the patterns of capital production and R&D activity that we see in the data. Trade leads to "effective technology equalization," with the capital-importing economy behaving as if it adopts its trading partner's technology even though no technology transfer actually occurs. It thus offers a new interpretation of data that supposedly shows technology transfer. Trade induces partial and possibly complete factor price equalization despite being Ricardian with permanently different technologies across countries. The model delivers closed-form solutions for steady states and transition dynamics, allowing sharp results on trade's welfare effects. Trade unambiguously raises the social welfare of capital-exporting countries and has an ambiguous effect on the welfare of capital-importing countries. The change in either country's welfare can be very large.

Keywords: International trade, endogenous growth, embodied technical progress, endogenous industrial structure, social welfare

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1 Introduction

In this paper we use endogenous growth theory to examine the interaction between Ricardian trade and economic growth. The focus is trade per se, not technology transfer facilitated by trade. The novelty of the work is the analytical framework used, which is fundamentally different from nearly all other models of trade and growth because of its treatment of the economy's industrial structure and the resulting implications for the way research and development (R&D) is performed. The analysis delivers several new results on both trade and growth in a framework that is consistent with the relevant IO facts.

Previous work on trade and growth falls into two groups. The larger group in fact does not study trade itself but rather technology transfer that may be facilitated by trade, as Feenstra (1996) noted some time ago. Examples in the older literature are Barro and Sala-i Martin (1997) and Connolly (2003), and very recent examples are Alvarez, Buera, and Robert E. Lucas (2013), Waugh, Tonetti, and Perla (2013), and Stokey (2015). In that body of work, trade affects growth if and only if technology transfer occurs in some form. Technology transfer (or diffusion) really does happen, and empirical work suggests it is significant both economically and statistically in explaining economic growth. However, our interest here is the interaction of economic growth and trade *per se*. The first group of studies thus is complementary to our work but not directly pertinent to it. The second group comprises studies that examine trade itself but do so with either 1st-generation growth models, such as Grossman and Helpman (1990), Redding (1999), Eaton and Kortum (1999), and Baldwin and Robert-Nicoud (2008) or semi-endogenous models, such as Dinopoulos and Segerstrom (1999) and Eaton and Kortum (2001a). All such models deliver counterfactual predictions, leading to empirical rejection of the framework. The underlying problem is two-fold: (1) important aspects of the industrial structure are exogenous and (2) fixed operating costs are omitted. The "second-generation" endogenous growth models developed by Peretto (1996), Peretto (1998), Peretto and Connolly (2007), Dinopoulos and Thompson (1998), and Howitt (1999) build on the earlier models but eliminate the problems and as a result also eliminate the counterfactual implications. The 2nd-generation framework has fared very well in formal tests, e.g., Madsen (2008). Despite their success as theories of growth, the 2nd-generation models have not been used to study the interplay of trade and growth.[1] In this paper, we use the 2nd-generation framework to study how Ricardian trade and endogenous economic growth interact.

A crucial element of our analytical framework is the industrial structure of the economy, which determines how trade and growth interact. Our model's industrial structure is built to be consistent with several facts from the industrial organization literature about what kinds of firms do R&D and what kind of R&D they do. The restrictions imposed by the IO facts play a pivotal role in determining how trade and growth affect each other. Furthermore, the industrial structure is endogenous, so trade and growth not only affect each other but also determine the evolution of the industrial structure. Trade, growth, and the industrial structure thus are simultaneously determined. The model advances our understanding of the interrelation between macroeconomics and microeconomics.[2]

[1] Indeed, we know of only three published articles that apply the 2nd-generation framework to any aspect of openness. Howitt (2000) and Peretto (2003) study technology transfer. Dinopoulos and Syropoulos (2004) study trade's effect on wages under restrictions that either prevent trade from having any effect on growth rates or that confine the effect to a redistribution of growth rates among countries in a zero-sum way.

[2] The type of model pioneered by Melitz (2003) also provides insights into the interaction of trade and IO, but so far it has not been possible to extend it to obtain robust results on growth. Baldwin and Robert-Nicoud (2008), for example, combine Melitz's approach with a 1st-generation growth model. The resulting growth rate suffers from the same market-size effects as all other 1st-generation growth models, rendering it unacceptable empirically. Combining Melitz's approach with a 2nd-generation

The trade part of our model has elements of each of the "big three" models of trade - Ricardian trade arising from cross-country differences in technology, Hecksher-Ohlin trade arising from cross-country differences in factor quantities, and Krugman trade arising from love of variety. In our model, technology, factor quantities, and sets of goods produced all have endogenous elements. By spending resources on various kinds of investment, countries change their technology, their stocks of factors of production, and the sets of goods they produce. So although the trade part of the model looks familiar to standard models, it also is different in important ways.

Trade affects economic growth in a systematic way. Depending on the relative production efficiencies of firms in different countries and the relative R&D efficiencies of those same firms, trade may raise or lower growth and may raise or lower welfare. A country that exports a capital good in return for a consumption good gets a temporary increase in its growth rate but no permanent change. In contrast, the country that imports a capital good gets a permanent change in its long-run growth rate. In general, the change may be positive or negative, but under realistic restrictions it will be positive. Thus what matters for trade's effect on growth is the good that a country imports. The good that it exports is irrelevant, a result in stark contrast to much trade policy that is concerned with the goods being exported (e.g., Chapter 5, Economic Report of the President, 2012) but that is consistent with evidence (DeLong and Summers (1991), DeLong and Summers (1992), DeLong and Summers (1993).; Lee (1995); Kliesen and Tatom (2013)). Trade always produces partial factor price equalization and may produce complete equalization, even though our framework violates all the restrictions necessary for factor price equalization in classical trade theory. The analysis thus extends our understanding of the economic forces underlying factor price equalization. Finally, trade always increases the capital-exporting country's welfare but may reduce the capital-importing country's welfare even if it raises that country's growth rate. The economic mechanisms that lead to these results arise from the industrial structure in the model, especially the specification of who does R&D and what kind of R&D they do, and how that structure changes endogenously in the face of trade and growth.

We present no formal tests of the theory derived herein. Nevertheless, the model does deliver testable implications, some of which we discuss.

2 Underlying Facts and Modeling Assumptions

As with any model, ours requires restrictions to be be tractable. Our choices of which restrictions to apply are driven by the IO and trade literature.

2.1 Factors of Production and Technology

In classic trade models, the factors of production are endowed, and technology is given. In reality, many factors of production are themselves produced, *viz.*, physical capital and intermediate goods, and technology changes through R&D that changes the types of available factors of production, the quantities in use, and the way they are used. Endogenous growth theory's very purpose is to explain changes in the stocks of factors of production and in the technology to use them. A complete theory of international trade must include the insights of endogenous growth theory.

model is desirable, but so far has proven impossible because of a curse of dimensionality.

2.2 R&D

Long-run economic growth arises from technical progress (Klenow and RodrÌguez-Clare (1997); Hall and Jones (1999)) that in turn is driven by R&D (Griliches (1998); Meliciani (2000); Zachariadis (2003), Zachariadis (2004); Griffith, Redding, and Reenen (2004); Guellec and de la Potterie (2004). We thus need an R&D-based growth model. In the first generation R&D-based growth models, entry was either absent (quality-ladder models) or did not play its proper role in determining the number of firms (Romer (1986), Romer (1990); Grossman and Helpman (1993); Aghion and Howitt (1992)). Those limitations lead to counterfactual aggregate predictions, the best-known being the aggregate scale effect. The aggregate scale effect is the prediction that the economy's growth rate is positively related to the economy's scale, usually measured by the labor force. The aggregate scale effect has been solidly rejected by formal tests (Backus, Kehoe, and Kehoe (1992); Jones (1995b); Gong, Greiner, and Semmler (2004)). Second-generation growth models eliminate the scale effect as well as other counterfactual predictions of the first-generation models by making both the number of firms and firms' market size endogenous (Peretto (1998); Dinopoulos and Thompson (1998); Howitt (1999)). Our model is of the second-generation type.[3]

There are three types of R&D. One type ("process R&D") reduces the cost of producing goods, another improves the quality of existing goods, and a third invents new varieties of goods. Most R&D of all types is done by incumbent firms. In the US, about 70% of R&D is private, and about 93% of that is done by incumbent firms. The situation is similar in other countries (Dosi, Freeman, Nelson, Silverberg, and Soete (1988); Dosi, Freeman, Nelson, Silverberg, and Soete (1988); **?**; OECD (2003, Chapter 4); Appendix Table 4-3 of the NSF's *Scientific and Engineering Indicators 2010*; Broda and Weinstein (2010)). In the US, about 40% of incumbents' R&D is devoted to improving the quality of their existing products, and another 20% aims at reducing production costs. The remaining 40% is devoted to developing new products, most of which simply replace existing goods made by the same firm and so really are another form of quality improvement (Mansfield (1968); Pavitt (1984) ; Scherer (1982); Bernard, Redding, and Schott (2010); Broda and Weinstein (2010)). Overall, nearly 80% of incumbents' R&D is for quality improvement, and 20% is for cost reduction. A small amount of private R&D (about 7% in the US) is done by outsider firms. Almost all outsider R&D is for developing new varieties of goods. Very little is for developing better quality versions of goods already produced by existing firms. The main exception occurs in newly-created industries, where new firms bring to market improved versions of the newly-invented product and displace the firms that produced the old versions (Fontana, Nuvolari, Shimizu, and Vezzulli (2012)). That kind of technical progress, however, is responsible for only a small fraction of total progress (OECD (2003)). In mature industries, firm extinctions are uncommon events and mostly result from gradual elimination of firms with high production cost or low quality products (Klepper and Simons (1997)).

The foregoing facts tell us that technical progress is almost entirely "late Shumpeterian" rather than "early Shumpeterian." Early Shumpeterian progress is the well-known "creative destruction," according to which rival firms continually try to displace each other by developing better versions of existing products. Late Shumpeterian progress is not so well known. It appeared in Schumpeter's later work (1942) and sub-

[3]The semi-endogenous growth model (Jones (1995a)) also eliminates the scale effect, but it does so by restricting the efficiency of the R&D production function while leaving intact the partial exogeneity of the economy's market structure. We want a fully-endogenous market structure, so a semi-endogenous growth model is not suitable for our purposes. Also, fully endogenous models fare better than semi-endogenous models in formal tests. See Laincz and Peretto (2006), Ha and Howitt (2007), and Madsen (2008) for some examples of tests of fully-endogenous vs. semi-endogenous models.

sequently has been dubbed "creative accumulation" (Breschi, Malerba, and Orsenigo (2000)). In it, existing firms improve their products and their production technologies, and new firms develop new varieties of goods. As we have just seen, creative accumulation encompasses nearly all R&D. We therefore construct our model as one of creative accumulation rather than creative destruction.[4] Because so little R&D is of the creative destruction type, we simplify by omitting it altogether and restrict R&D to creative accumulation: quality improvement and cost reduction by incumbent firms and variety expansion by newcomers. In principle, we could include creative destruction without affecting the qualitative results (Dinopoulos and Thompson (1998); Howitt (1999)). However, the mathematics would be much more difficult, and the transition dynamics would be intractable. Our qualitative results require only that quality-improving or cost-reducing R&D on a substantial fraction of existing products is done solely by incumbents, which is true in reality. Quality improvement and cost reduction are isomorphic in our model, so we simplify further by considering only quality improvement, which constitute most R&D anyway.

Conducting our analysis of international trade in the framework of a 2nd-generation endogenous growth model distinguishes it from virtually all the previous literature on trade and growth, which has used neoclassical (exogenous), 1st-generation endogenous, or semi-endogenous growth models. Our analytical framework delivers many new results, some of which contradict the previous literature.

2.3 Fixed Operating Costs

Fixed operating costs often are substantial. For example, they constitute 43% of the total operating cost of the Ohio Turnpike (Hendrickson (2000)), 43% of the cost of corn production and 53% of the cost of wheat production in the US (USDA 2014), 59% of the cost of generating electricity at a nuclear power plant (Brinckerhoff (2011)), and 67% of the cost of running a commercial airline (Rodrigue (2013)). Although fixed operating costs often are given scanty treatment (e.g., Mas-Colell, Whinston, and Green (1995)), they are important in endogenous growth models because they prevent variety expansion from being a permanent source of growth in income per capita (Peretto and Connolly (2007)). In light of their magnitude and theoretical importance, we include them in our analysis. They have important effects on our results.

2.4 Trade

In classic trade models, only final goods are traded. In reality, physical capital and intermediates also are traded and in fact constitute the bulk of trade. About one-quarter of international trade is in consumer goods and three-quarters is in factors of production, either physical capital or intermediate goods (Miroudot, Lanz, and Ragoussis (2009)). In this paper we allow trade in both types of goods. For tractability, we restrict attention to two countries and two tradable goods, one consumption good and one capital good. Extending the analysis to many countries and many goods, as in the static trade models of Eaton and Kortum (2002) and Shiozawa (2007), poses a severe curse of dimensionality and renders the model unsolvable. We can solve our model because the number of active state variables never exceeds three. Once we add either more goods or more countries, analytical results become impossible.[5] For the same reason, we cannot entertain the rich

[4] See Fontana, Nuvolari, Shimizu, and Vezzulli (2012) for a brief history of the terms "creative accumulation" and "creative destruction." For an introduction to "creative accumulation" models, see Smulders and van de Klundert (1995)and Peretto (1996), Peretto (1998), Peretto (2007).

[5] Eaton and Kortum (2001a) extend their work to include growth uses a semi-endogenous growth model, where the curse of dimensionality is not as severe. However, as noted above, semi-endogenous growth models have a counterfactual exogenous market structure and also have not done well in empirical tests. There still does not exist a satisfactory model that combines

market structure that Melitz (2003) uses to examine which firms enter the international market. We forego some richness on the trade side to gain insight on the growth side. However, omitting Melitz-type firm heterogeneity costs little in studying the relation between trade and growth. Baldwin and Robert-Nicoud (2008) embed Melitz-type heterogeneity in a 1st-generation growth model. The growth rate still has the scale effect, so Melitz-type heterogeneity does nothing to cure the IO structural problems of the 1st-generation models. Also, the pattern of trade in Melitz (2003) and Baldwin and Robert-Nicoud (2008) is not determined by comparative advantage but rather by exogenous endowment of types of goods for each country, i.e., by an Armington (1969) assumption.

To isolate comparative advantage from other motives for trade, we rule out direct technology transfer by which one country is able to adopt the production technology of another country, and we also rule out foreign direct investment, by which one country can implant its technology in another country's economy. We do not allow migration of labor.

Even though technical progress is intangible, it is indirectly tradable when it is embodied in goods in the form of quality. Quality improvements embodied in an intermediate or capital good affect the productivity of the buyer rather than the seller (Mansfield (1968); Griliches (1986)). Trading goods with embodied progress transfers that progress from the country that created it to the country that buys the enhanced good. The result is that trade leads to effective technology transfer even if no actual transfer of technological processes (such as by imitation or direct foreign investment) occurs. Empirical studies have found that embodied technical progress is important in explaining growth (Gort, Greenwood, and Rupert (1999); Meliciani (2000)), so we include it in our model. It turns out to be a major determinant of how trade and growth interact. Indeed, some of our most important results arise from embodied technical progress that derives from the R&D of incumbent firms.

2.5 Shut-down

Firms and even whole industries often shut down in the face of foreign competition.[6] Eaton and Kortum (2001b) show that capital production is highly concentrated among very few countries, implying that trade in capital has shut down production of capital in most countries

3 Autarky

We begin with autarky to explain the intuition of the underlying framework. Our model is a standard 2nd-generation knowledge-based R&D growth model. The discussion in this section lays the groundwork for the following section on the interaction of Ricardian trade and endogenous growth, where the contribution of our paper lies.

3.1 Final Goods

The final goods industry is made up by a large number of perfectly competitive firms that produce a homogenous final output Y. As is standard in the literature, we condense the final goods firms into a

Eaton and Kortum's multi-country, multi-good approach with an acceptable model of endogenous growth.

[6]For example, the sewing machine and commercial shipbuilding industries in the United States.

representative firm. The production technology is

$$Y_t = A_y K_t^{\beta} L_{yt}^{1-\beta} \quad 0 < \beta < 1 \tag{1}$$

where K is capital, L_y is the amount of labor devoted to final output production, A_y is a constant, and β is capital's income share.

Labor moves freely across all firms in all sectors, so wages are the same everywhere. We set the price of final output as the numeraire, so $P_y = 1$ at all times. The final goods firm takes the price of capital as well as the wage as given and chooses how much capital and labor to hire to maximize profit. We obtain the standard demands:

$$wL_y = (1-\beta)\, Y \tag{2}$$

and

$$p_k K = \beta Y \tag{3}$$

3.2 Capital Production

Capital is produced by perfectly competitive firms with the Dixit-Stiglitz CES technology

$$K = A_k \left[\int_0^N \left(Z_i^{\theta} X_i \right)^{\alpha} di \right]^{\frac{1}{\alpha}} \quad 0 < \theta, \alpha \tag{4}$$

where N is the (endogenous) number of goods, X_i is the firm's purchase of intermediate good i, and Z_i is the product quality attached to good i, and A_k is a constant.[7] The capital-producing firm's profit is

$$\pi_k = p_k A_k \left[\int_0^N \left(Z_i X_i \right)^{\alpha} di \right]^{\frac{1}{\alpha}} - \int_0^N p_i X_i$$

Zero profit and the firm's first-order condition imply the demand

$$X_i = \frac{p_k K p_i^{\frac{-1}{1-\alpha}} Z_i^{\frac{\theta\alpha}{1-\alpha}}}{\int_0^N p_i^{\frac{-\alpha}{1-\alpha}} Z_i^{\frac{\theta\alpha}{1-\alpha}} di} \tag{5}$$

Holding all else constant, demand is increasing in own-product quality and in the size of the total market $p_k K$. The price p_i is determined in the intermediate goods sector, which is imperfectly competitive. Anticipating the results below, the intermediate goods market is characterized by symmetric equilibrium with all the p_i the same, $p_i = p$. In that case, demand for X_i is

$$X_i = \frac{\beta Y}{Np} \tag{6}$$

[7] We could extend the analysis to include labor as a factor of production in the capital sector by using the production function $K = A_k \left[\int_0^N \left(Z_i^{\theta} X_i \right)^{\alpha} di \right]^{\frac{\varphi}{\alpha}} L_k^{\varphi}$. However, the analysis becomes much more tedious without changing any of the results. Therefore we set $\varphi = 1$.

3.3 Intermediates and R&D

The intermediate goods industry is the heart of the model because it is there that all innovation takes place. There are two kinds of innovation: vertical (quality improvement) and horizontal (variety expansion). In the vertical dimension, incumbent firms engage in costly R&D to improve the quality (Z_i) of their own products to increase demand and hence profit. In the horizontal dimension, entrepreneurs enter with new products when profit opportunities exist.

3.3.1 Incumbents

The intermediate goods industry comprises a continuum of monopolistically competitive firms, each of which produces a single and unique intermediate good X_i and undertakes R&D to improve the quality of that good. Higher quality increases the demand for the good. Firms pay a constant unit marginal cost of production and also a fixed operating cost ϕ, so the production technology is

$$l_{xi} = X_i + \phi \tag{7}$$

where l_{xi} is the firm's production labor. The production function for quality-improving R&D is standard in the 2nd-generation literature:

$$\dot{Z}_i = \eta Z l_{zi} \tag{8}$$

where $Z = (1/N)\int_0^N Z_i di$ is the industry-average level of quality and l_{zi} is the labor the firm devotes to R&D.[8]

Firm profit is

$$\pi_{xi} = X_i\left(p_i - w\right) - w\phi - w l_{zi} \tag{9}$$

Firms maximize their lifetime value

$$V_i = \int_0^\infty e^{-\int_0^t r(s)ds}\pi_{it}dt \tag{10}$$

with respect to their price and R&D outlay subject to (8), (9), and (5). Standard dynamic optimization (see Appendix) yields the price and rate of return

$$p_i = \frac{w}{\alpha} \equiv p \tag{11}$$

$$r_{zi} = \eta\theta Z\left(\frac{X_i}{Z_i}\right) + \frac{\dot{w}}{w} - \frac{\dot{Z}}{Z} \tag{12}$$

Assuming identical firms implies symmetric equilibrium, and (12) reduces to

$$r_z = \left(\frac{\alpha\beta\eta\theta}{1-\beta}\right)\frac{L_y}{N} + \frac{\dot{w}}{w} - \frac{\dot{Z}}{Z} \tag{13}$$

Note that the rate of return to quality improvement r_z depends on L/N , not L alone. It is the *firm's* mar-

[8] Using average Z instead of own-firm Z_i is a way of capturing cross-firm knowledge spillovers, which are known from the empirical literature to be of appreciable magnitude. Because of the symmetry of the solution, the results would be little different if we had the change in Z_i depend on Z_i instead of Z.

ket size, not the aggregate market size, that determines the incentive to innovate. That result is exactly the opposite of what emerges from the 1st-generation growth literature and is a hallmark of the 2nd-generation framework. The difference arises from the IO structure assumed for R&D. The 1st-generation literature assumes that quality-improving R&D is done by an independent R&D sector, whereas the 2nd-generation framework assumes that quality-improving R&D is done by the same firms that produce the goods being improved. As we saw earlier, most quality-improving R&D is done by incumbents, not outsiders, i.e., not by an independent R&D sector. The 1st-generation assumption is what gives rise to the counterfactual market size effects of the early literature, such as the aggregate scale effect and the over-sensitivity of the growth rate to tax rates. In the 1st-generation literature, an increase in L raises the return to quality-improving R&D, elicits more R&D per firm (*whose number is fixed* in 1st-generation quality-improvement growth models), and thus raises the growth rate. In the 2nd-generation framework, an increase in L raises the return to quality-improving R&D, *which elicits entry* until the return to quality-improving R&D is back at its original value. The change in the economy's scale has only a transitional effect on growth. There is no scale effect in the long run, consistent with the evidence. See Peretto (1998), Peretto (1999), Dinopoulos and Segerstrom (1999), and Dinopoulos and Sener (2007) for detailed discussions of how the two generations of growth models differ. The important thing to understand here is that the same element of the 2nd-generation framework that kills the counterfactual aggregate market size effects also underlies several of the most striking differences between our results and those of the earlier literature on how trade and growth interact. Those differences are explained below.

3.3.2 Entrants

New firms enter by paying a sunk cost of βX to set up operations. We assume they enter at the average quality level and size, a simplifying assumption that preserves symmetry of equilibrium at all times.[9] To prevent infinite entry, which is impossible, the market guarantees that

$$V_i \leq w\psi X \tag{14}$$

Entry is positive when (14) holds with equality.

Differentiating (10) with respect to time gives the standard asset pricing rule

$$r_n = \frac{\pi}{V} + \frac{\dot{V}}{V}$$

When entry is positive, we can substitute for π and write the foregoing expression as

$$r_n = \frac{X(p-w) - w\phi - wl_z}{w\psi X} + \frac{\dot{L}_y}{L_y} + \frac{\dot{w}}{w} - \frac{\dot{N}}{N} \tag{15}$$

which is the rate of return to entry.

[9] We could allow initial quality to be random if we impose sufficient parameter restrictions to ensure Nash stability, i.e., to insure that firms whose quality gets ahead of other firms have an economic incentive to slow their R&D enough that the lagging firms eventually catch up. The equilibrium in that case is one in which all firms approach the same quality (in the absence of further random shocks) and thus also approach the average quality. Unfortunately it is impossible to solve for the model's transition path if we allow firms to have transitory differences in quality. We avoid the problem by assuming entry at the industry average quality level, which in a symmetric equilibrium is the value that all existing firms have. See Peretto (1996, 1999) for more discussion.

Eqs. (13) and (15) give the rates of return for the two kinds of R&D: quality-improvement and variety expansion, respectively.

3.4 Households

There are L households. Each buys consumption, supplies one unit of labor inelastically, and buys assets in perfectly competitive markets. For simplicity we assume that there is no population growth.[10] The utility function of the representative household is

$$U(t) = \int_t^{\infty} \log(c_s)\, e^{-\rho s} ds \tag{16}$$

where c is consumption per capita and ρ is the rate of time preference. The representative household's lifetime budget constraint is

$$\dot{a}_t = r_t a_t + w_t L_t - C_t \tag{17}$$

where $a = \int_0^N V_i di$ is the asset (the market value the fraction of all firms that the household owns) and C is aggregate consumption. The intertemporal consumption plan that maximizes discounted utility (16) is given by the consumption Euler equation

$$\frac{\dot{C}}{C} = r - \rho \tag{18}$$

3.5 General Equilibrium and Dynamics

As with all models of this type, we consider a symmetric Nash equilibrium in open loop strategies, which implies a one-shot game. At the opening of play, firms commit to a time path, and the paths of economic structure and economic growth are simultaneously determined. Feedback strategies would be more realistic but are unsolvable with imperfect competition in a dynamic general equilibrium context. See Peretto (1996, 1999) and their precursor, Dasgupta and Stiglitz (1980), for a full discussion of the game-theoretic foundations of our model.

To solve the model, we find the prices and quantities of the final good Y, the intermediate goods X_i, consumption C, the labor allocations L_Y, l_{xi}, and l_{zi}, the wage w, the number of firms N, and the rates of return r_n, r_z, and r. Those allow us to solve for the growth rates of the variables we are interested in. The solution is obtained in the usual way, so the details are relegated to the Appendix with only the results discussed here.

Lemma 1. *The amount of labor devoted to producing final output Y is constant at the value*

$$L_y = \left[\frac{(1-\beta)}{1-\rho\psi\beta\alpha}\right] L \tag{19}$$

The wage is

$$w_t = A_y \left(\frac{\alpha\beta A_k}{1-\beta}\right)^{\beta} Z_i^{\beta\theta} N_t^{\beta(1-\alpha)/\alpha} \tag{20}$$

[10] The model can be extended to include population growth, but the only difference is that on the balanced growth path the number of varieties would grow at the rate of population growth instead of being constant. See Peretto (2007).

Let a circumflex over a variable denote that variable's growth rate: $\hat{x} \equiv (1/x)(dx/dt)$. Using the constancy of L_y, we can show that $\widehat{w} = r - \rho$, from which we can obtain the growth rates for Z and N.

Proposition 1. *The growth rate of product quality and the equilibrium number of firms are*

$$\widehat{Z} = \begin{cases} \left(\frac{\alpha\beta\eta\theta}{1-\alpha\beta\rho\psi}\right)\frac{L}{N} - \rho & : N < \bar{N} \\ 0 & : N > \bar{N} \end{cases} \tag{21}$$

$$\widehat{N} = \begin{cases} \frac{1-\alpha(1+\theta+\rho\psi)}{\alpha\psi} - \left[\left(\frac{\alpha\beta\psi}{1-\alpha\beta\rho\psi}\right)\frac{L}{N}\right]^{-1}\left(\phi - \frac{\rho}{\eta}\right) & : N < \bar{N}_z \\ \frac{1-\alpha-\alpha\rho\psi}{\alpha\psi} - \left[\left(\frac{\alpha\beta\psi}{1-\alpha\beta\rho\psi}\right)\frac{L}{N}\right]^{-1}\phi & : N > \bar{N}_z \\ 0 & : N > \bar{N}_n \end{cases} \tag{22}$$

where

$$\bar{N}_z = \left(\frac{\alpha\beta\eta\theta}{1-\alpha\beta\rho\psi}\right)\rho^{-1}L \tag{23}$$

and

$$\bar{N}_n = \begin{cases} \left(\frac{\beta\eta}{1-\alpha\beta\rho\psi}\right)\frac{1-\alpha(1+\theta+\rho\psi)}{\eta\phi-\rho}L & : N < \bar{N}_z \\ \left(\frac{\beta}{1-\alpha\beta\rho\psi}\right)\frac{1-\alpha(1+\rho\psi)}{\phi}L & : N > \bar{N}_z \end{cases} \tag{24}$$

The branches of the growth rate equation for Z correspond to the two possible growth regimes. In the first regime, variety expansion shuts down but quality improvement continues forever at a constant rate. The reason variety expansion stops is that the number of firms eventually rises to the point where the marginal benefit of adding another variety is too small to justify paying the fixed operating cost. Note that it is the fixed operating cost ϕ, not the sunk cost, that caps the number of firms. The sunk cost caps the growth rate of the number of firms. Growth in quality continues because higher quality does not add to any fixed operating cost. See Peretto and Connolly (2007) for a detailed discussion of the roles of the two kinds of costs in an endogenous growth model.[11] In the second regime, quality improvement shuts down before variety expansion does because the number of firms becomes so large that the return to quality-improving R&D is driven down too low to justify diverting resources to quality improvement. Variety expansion continues for a while and then it, too, shuts down for the same reason as in the first regime, and all growth in the economy stops. The real world exhibits positive growth on average, so for most of the remaining discussion we restrict attention to the first regime with positive steady-state growth. We return briefly to the second regime and an interesting implication of it near the end of the paper.

When the first regime prevails, we have

Proposition 2. *Under the parameter restrictions*

$$\eta\phi > \rho\left(\frac{1-\alpha\left(1+\theta+\psi\rho\right)}{\alpha\theta}\right) + \rho \tag{25}$$

$$1 > \alpha\beta\rho\psi\,,\ \alpha\left(1+\theta+\rho\psi\right) \tag{26}$$

[11] The differing effects of sunk and fixed costs were exploited by Krugman (1980), who imposed a fixed operating cost to keep the number of varieties finite, and Grossman and Helpman (1993, Chapter 3), who imposed a sunk entry cost to keep the rate of growth finite.

the steady-state number of firms and growth rate of quality are

$$N_{ss} = \left(\frac{\beta\eta}{1-\alpha\beta\rho\psi}\right)\left[\frac{1-\alpha\left(1+\theta+\psi\rho\right)}{\eta\phi-\rho}\right]L \tag{27}$$

$$\widehat{Z}_{ss} = \frac{\alpha\theta\left(\eta\phi-\rho\right)}{1-\alpha\left(1+\theta+\rho\psi\right)} - \rho \tag{28}$$

Along the transition path, both quality improvement and variety expansion drive growth of the final good Y. Eventually variety expansion stops, and only quality improvement continues. In particular,

Proposition 3. *The growth rate of output is*

$$\begin{aligned}\widehat{Y}_t &= \beta\left[\theta^2\left(\frac{\alpha\beta\eta}{1-\alpha\beta\rho\psi}\right)\frac{L}{N_t} - \theta\rho\right. \\ &\quad \left. + \left(\frac{1-\alpha}{\alpha}\right)\left\{\frac{1-\alpha-\alpha\theta-\alpha\rho\psi}{\alpha\psi} - \left(\frac{\eta\phi-\rho}{\psi}\right)\left[\left(\frac{\alpha\beta\eta}{1-\alpha\beta\rho\psi}\right)\frac{L}{N_t}\right]^{-1}\right\}\right]\end{aligned} \tag{29}$$

In the steady state the foregoing expression simplifies to

$$\widehat{Y}_{ss} = \beta\theta\widehat{Z}_{ss} = \beta\theta\left[\frac{\alpha\theta\left(\eta\phi-\rho\right)}{1-\alpha\left(1+\theta+\rho\psi\right)} - \rho\right] \tag{30}$$

Equations (28) and (30) show the importance of the industrial structure in determining the economy's growth rate. It is unsurprising that the growth rate depends positively on the R&D total factor productivity (TFP) parameter η or the power θ of quality Z in augmenting the associated intermediate good. Why, however, does growth depend *positively* on the sunk cost and fixed operating cost parameters ψ and ϕ? One might expect higher costs of setting up and running the firm to reduce the growth rate. The explanation is that higher sunk and operating costs reduce the equilibrium number of firms, which in turn raises the remaining firms' market size L/N. A bigger market size increases the return to quality-improving R&D and so also increases the growth rate. That kind of interaction between general equilibrium, the economy's IO structure, and the growth rate is absent from the 1st-generation growth models because those models fix either the number of firms or the market size of firms. The more accurate IO structure of the 2nd-generation framework is responsible for eliminating the counterfactual market-size effects that plagued its 1st-generation predecessor. As we shall see, it also underlies most of the results we obtain on the interaction of trade and growth.

3.6 Transition Dynamics and Welfare

The transition values of N_t and Z_t are given by

$$N_t = \frac{N_{ss}}{1+e^{-Mt}\left(\frac{N_{ss}-N_0}{N_0}\right)} \tag{31}$$

and

$$Z_t = Z_0 e^{\widehat{Z}_{ss}t + \left(1-e^{-Mt}\right)\left[\frac{\alpha\beta\eta\psi}{(\eta\phi-\rho)(1-\alpha\beta\rho\psi)}\right]\left(\widehat{Z}_{ss}+\rho\right)\left(\frac{N_{ss}-N_0}{N_{ss}}\right)\left(\frac{L}{N_0}\right)} \tag{32}$$

where $M = \left[\frac{1-\alpha(1+\theta+\rho\psi)}{\alpha\psi}\right]$, and N_0 and Z_0 denote the initial values of the number of firms and product quality.

Substituting (31) and (32) into (1) and combining the result with (16) gives the representative household's utility:

$$\begin{aligned} U &= \rho^{-1}\log\left[\frac{(1-\beta)\,A_y L}{1-\alpha\beta\rho\psi}\right]\left(\frac{A_k\beta\alpha}{1-\beta}\right)^{\beta} Z_0^{\beta\theta} N_0^{\beta\left(\frac{1-\alpha}{\alpha}\right)} \\ &\quad +\beta\theta\left[\rho^{-2}\widehat{Z}_{ss}+\left(\frac{\alpha\psi}{\rho\left[1-\alpha\left(1+\theta\right)\right]}\right)\left(\widehat{Z}_{ss}+\rho\right)\left(\frac{N_{ss}-N_0}{N_0}\right)\right] \\ &\quad +v\left(N_{ss},N_0\right) \end{aligned} \tag{33}$$

where the function $v\left(N_{ss},\,N_0\right)=\int_0^{\infty}e^{-\rho t}\log\left(N_t/N_0\right)dt=\int_0^{\infty}e^{-\rho t}\log\left(\frac{N_{ss}}{N_0+e^{-Mt}(N_{ss}-N_0)}\right)dt$ is the dynamic contribution of variety expansion to welfare.

Unsurprisingly, utility is increasing in the initial value of product quality and the steady-state growth rate. The initial number of firms, however, has an ambiguous affect on welfare. On the one hand, a higher value of N_0 raises utility because the household values variety. On the other hand, a higher value of N_0 reduces utility because it reduces transitional quality improvement. Which effect dominates depends on parameters and so is an empirical issue.

4 The Open Economy

We now consider free and costless trade. There are two countries, Home and Foreign. They have the same utility functions. Their production technologies have identical functional forms but differ in most of their parameter values. Specifically, the following parameters differ across the countries: TFP for final goods (A_{jy}), TFP for capital goods (A_{jk}), TFP for R&D (η_j), entry cost (ψ_j), and fixed cost (ϕ_j).

To keep our analysis simple and to isolate the effects of Ricardian trade, we do not allow foreign investment or direct technology transfer. Furthermore, we restrict trade to final output Y and capital K. This simplifying assumption keeps down the dimensionality of the problem.[12] In addition, this assumption yields a familiar structure; classical (two county, two good) models of trade in which trade may induce specialization. Shutdown has important implications for market size and growth, as we show.

4.1 Comparative Advantage and the Pattern of Trade

The law of one price implies that the prices of the identical final goods are the same in the two countries and so equal 1 because the price of Y is the numeraire. Final goods firms can buy domestic and foreign capital. We assume domestically-produced and foreign-produced capital are perfect substitutes for one another in order to stay as true as possible to the classical notion of comparative advantage. The final output production function of Home now is

$$Y_h=\left(K_{hh}+K_{hf}^{I}\right)^{\beta}L_{hy}^{1-\beta}$$

[12] We can get shutdown by trading intermediates themselves, but the analysis is much more tedious.

where the first and second subscripts respectively denote the buyer and seller of the capital good. Foreign's production function is similar.

Given the perfect substitutability of capital, both countries buy capital only from the country with the lowest price. Without loss of generality we assume

$$p_{hk} < p_{fk} \tag{34}$$

Substituting for the two prices and rearranging terms, we can rewrite (34) as

$$\left(\frac{A_{hy}}{A_{fy}}\right)^{\frac{1}{1-\beta}} \left(\frac{A_{fk}}{A_{hk}}\right) < \left(\frac{N_h}{N_f}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{Z_h}{Z_f}\right)^{\theta} \tag{35}$$

The left-hand side comprises the usual standard comparative advantage parameters. The right-hand side comprises the endogenous variables that reflect the two kinds of R&D, *viz.*, the numbers of varieties and the levels of quality in the two countries.

4.2 General Equilibrium in the World Economy

The analysis under trade is similar to that under autarky, so we relegate all derivations to the Appendix and simply state results here.

4.2.1 The Patterns of Production and Trade

As in autarky, employment in the final output sector jumps to its steady state value.

Lemma 2. *Home's allocation of labor to final output is given by*

$$L_{hy} = \max\left[\frac{(1-\beta)\,L_h - \beta\left[1-\alpha\rho\psi_h\right]\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f}{1-\alpha\beta\rho\psi_h}, 0\right] \tag{36}$$

The first term inside the brackets on the right side of (36) may be positive or negative. If it is positive, Home produces both final goods and capital, and Foreign specializes in final goods, setting$L_{fy} = L_f$. If the first term is negative, the situation is more complicated. Home is in a corner because employment cannot be negative, so Home sets L_{hy} to zero and produces only intermediate goods and capital. Foreign produces both capital and final goods because Home is too small (as measured by its productivity-adjusted labor force) to meet all of Foreign's capital demand. That cross-country configuration of production turns out to be analytically intractable, and it also is unrealistic. As Eaton and Kortum (2001b) have shown, the countries that produce capital are the larger economies (again in the sense of the productivity-adjusted labor force), and they produce final goods as well as capital. The smaller economies do not produce capital. We therefore assume that the parameter values of the world economy are such that the right side of (36) is positive.[13]

[13]The reason the first case is analytically tractable and the second is not is that in the first case the wage is determined by the technologies for final output in the two economies whereas when one country shuts down final output production the wage is determined by a much more complex system. As part of the complexity, the wage and the innovation rate feed on each other, leading to a system with four state variables. A system with more than three state variables is usually intractable, as it is here.

4.2.2 Rate of Innovation and Steady-State Growth

Under trade, only Home does R&D. Its rates of return to vertical and horizontal innovation are

$$r_{zi} \equiv r_{hzi} = \eta_h \theta Z_h \left(\frac{X_{hi}}{Z_{hi}} \right) + \frac{\dot{w_h}}{w_h} - \frac{\dot{Z}_h}{Z_h}$$

and

$$r_n \equiv r_{hn} = \frac{\pi}{\psi_h w_h X} + \frac{\dot{w_h}}{w_h} - \frac{\dot{N}_h}{N_h}$$

where $X_{hi} = p_{hk} K_h p_i^{\frac{-1}{1-\alpha}} Z_{hi}^{\frac{\alpha\theta}{1-\alpha}} / \int_0^{N_h} p_{hi}^{\frac{-\alpha}{1-\alpha}} Z_{hi}^{\frac{\alpha\theta}{1-\alpha}} di$.[14] Solving the model as usual leads to

Proposition 4. *The dynamic equations for quality and the number of varieties are*

$$\widehat{Z}_h^T = \begin{cases} \frac{\alpha\beta\eta_h\theta}{N_h} \left[\frac{L_h + \left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f}{1-\alpha\beta\rho\psi_h} \right] - \rho & : N < \bar{N_z^T} \\ 0 & : N > \bar{N_z^T} \end{cases} \tag{37}$$

and

$$\widehat{N}_h^T = \begin{cases} \frac{1-\alpha(1+\theta+\rho\psi_h)}{\alpha\psi_h} - \left(\left[\frac{\alpha\beta\psi_h}{(1-\alpha\beta\rho\psi_h)N_h} \right] \left[L_h + \left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f \right] \right)^{-1} \left(\phi_h - \frac{\rho}{\eta} \right) & : N_h < \bar{N_{hz}^T} \\ \frac{1-\alpha-\alpha\rho\psi_h}{\alpha\psi_h} - \phi_h \left(\left[\frac{\alpha\beta\psi_h}{(1-\alpha\beta\rho\psi_h)N_h} \right] \left[L_h + \left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f \right] \right)^{-1} & : N_h > \bar{N_{hz}^T} \\ 0 & : N_h > \bar{N_{hn}^T} \end{cases} \tag{38}$$

where the superscript T indicates the trade regime and the cut-off values of N are

$$\bar{N_{hz}^T} = \frac{\alpha\beta\eta_h\theta}{\rho} \left[\frac{L_h + \left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f}{1-\alpha\beta\rho\psi_h} \right]$$

and

$$\bar{N_{hn}^T} = \begin{cases} \frac{\beta\eta_h[1-\alpha(1+\theta+\rho\psi_h)]}{\eta_h\phi_h - \rho} \left[\frac{L_h + \left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f}{1-\alpha\beta\rho\psi_h} \right] & : N_h < \bar{N_{hz}^T} \\ \frac{\beta(1-\alpha-\alpha\rho\psi_h)}{\phi_h} \left[\frac{L_h + \left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f}{1-\alpha\beta\rho\psi_h} \right] & : N_h > \bar{N_{hz}^T} \end{cases}$$

The easiest way to compare the dynamics with and without trade is to suppose that the world starts autarkic and unexpectedly opens to trade. Everyone re-optimizes at that point taking the current values of all values as the initial conditions. Comparing the trade and autarky dynamics (Propositions 4 and 1, respectively) shows that the growth rates of both quality and the numbers of varieties initially are higher under trade than under autarky. When trade opens, the capital sector in Foreign shuts down, leaving the existing Home firms with the entire world market rather than just its original Home customers. The increase in the firms' market raises the rates of return to both kinds of R&D and increases R&D, innovation, and economic growth. However, the rise in growth rates is only temporary, as the next result implies.

[14] The term $\dot{L}/L$ is absent from the equation for r_n because by (36) it is zero at all times.

Proposition 5. *Under the identical parameter restrictions listed in section 2* (25)*, a unique steady-state with positive growth exists where,*

$$N_{hss}^{T} = \frac{\beta\eta_h\left[1-\alpha\left(1+\theta+\rho\psi_h\right)\right]}{\eta_h\phi_h-\rho}\left[\frac{L_h+\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}L_f}{1-\alpha\beta\rho\psi_h}\right] \tag{39}$$

and

$$\widehat{Z}_{hss}^{T} = \frac{\alpha\theta\left(\eta_h\phi_h-\rho\right)}{1-\alpha\left(1+\theta+\rho\psi_h\right)}-\rho \tag{40}$$

$$\widehat{Y}_{hss}^{T} = \widehat{Y}_{fss}^{T} = \beta\theta\widehat{Z}_{hss} = \beta\theta\left[\frac{\alpha\theta\left(\eta\phi-\rho\right)}{1-\alpha\left(1+\theta+\rho\psi\right)}-\rho\right] \tag{41}$$

Equation (38) is a logistic differential equation, and equation (43) is a linear differential equation. Both have well-known solutions:

Corollary 1. *The number of firms and product quality are given by*

$$N_{ht}^{T} = \frac{N_{ss}^{T}}{1+e^{-M_h t}\left(\frac{N_{ss}^{T}-N_{h0}}{N_{h0}}\right)} \tag{42}$$

and

$$Z_{ht}^{T} = Z_{h0}e^{\widehat{Z}_{hss}t+\left(1-e^{-M_h t}\right)\frac{\alpha\psi\left(\widehat{Z}_{hss}+\rho\right)}{1-\alpha\left(1+\theta+\rho\psi_h\right)}\left(\frac{N_{hss}^{T}-N_{h0}}{N_{h0}}\right)} \tag{43}$$

where $M_h^T = \frac{1-\alpha(1+\theta+\rho\psi_h)}{\alpha\psi_h}$

which are the values at any point along the transition path.

Proposition 5 shows that Home's steady-state growth rate is the same under trade as under autarky even though trade increases the size of the market that Home faces, a result that stands in stark contrast to the second proposition of Grossman and Helpman (1990):

> "*An equiproportionate, once-and-for-all increase in the effective labor forces of both countries accelerates long-run growth.*"

That proposition is the aggregate scale effect, it is common to all 1st-generation endogenous growth models, and it has been solidly rejected by the evidence.[15] Our model does not have an aggregate scale effect in the steady state, and understanding the reason gives considerable insight into how the model works. Two elements of the model together eliminate the scale effect: (1) our model includes fixed operating costs, and (2) it includes both variety expansion and quality improvement. First-generation growth models omit fixed operating costs and have either variety expansion or quality improvement, never both. Why do these two differences give us such a different prediction on long-run growth? Variety-expansion versions of 1st-generation models have the scale effect because all firms face the whole economy as their market at all times and so have a fixed market size. Quality-improvement versions have it because the number of firms is fixed so that any expansion of the total market necessarily also permanently expands each firm's market. In our model, fixed operating cost puts an upper bound on the number of firms and so eliminates perpetual growth through variety expansion, thus preventing variety expansion from introducing the scale

[15] E.g., Backus, Kehoe, and Kehoe (1992), Jones (1995b), and Gong, Greiner, and Semmler (2004).

effect. Nevertheless, even though variety expansion is bounded, it prevents the quality-improvement side of R&D from introducing the scale effect. It does that by endogenizing the number of quality-improving firms and thereby endogenizing the size of those firms' market. In our model, an increase in the aggregate market size has a *temporary* scale effect by increasing the size of the market facing *existing* firms. That raises the profit of existing firms and so induces entry of new firms. Entry continues until all economic profit has been eliminated, which happens when firms' market size has been returned to its original value. At that point entry stops, and growth through variety expansion stops with it. Quality-improvement continues forever, but it does it at the same rate as before the increase in the aggregate market. So in our model, opening the world to trade increases existing firms' market size, but entry ultimately eliminates that effect.[16]

Our analysis provides new insight into another issue discussed by Grossman and Helpman (1993, Chapter 6) and also present in Rivera-Batiz and Romer (1991) and Jones (1995a). One way trade can change growth rates in 1st-generation models is by inducing a reallocation of labor into or out of the R&D sector. As Grossman and Helpman (1993, Chapter 6) remark,

> "When trade causes resources to be released from manufacturing sectors, which then find their way into research labs, the rate of innovation rises. But when the sectors that expand in response to trading opportunities compete with the research labs for factor inputs, international integration may retard growth." [Grossman and Helpman (1993, Chapter 6)]

In our model, trade raises Home's R&D labor allocation permanently, just as in Grossman and Helpman (1993, Chapter 6):

Proposition 6. *The steady-state fraction of the labor force employed in quality-improving R&D under autarky is*

$$\left(\frac{L_{zh}}{L_h}\right)_{ss}^{A} = \beta\left\{\alpha\theta - \rho\left[\frac{1-\alpha\left(1+\theta+\rho\psi_h\right)}{\eta_h\phi_h-\rho}\right]\right\}\left(\frac{L_h}{1-\alpha\beta\rho\psi_h}\right)$$

whereas under trade it is

$$\left(\frac{L_{zh}}{L_h}\right)_{ss}^{T} = \beta\left\{\alpha\theta - \rho\left[\frac{1-\alpha\left(1+\theta+\rho\psi_h\right)}{\eta_h\phi_h-\rho}\right]\right\}\left[\frac{L_h+\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}L_f}{1-\alpha\beta\rho\psi_h}\right]$$

which is larger than the autarkic value.

Despite the permanent change in labor devoted to quality-improving R&D, which is the only source of long-run growth in our model, the growth rate changes only temporarily. Steady-state growth is the same as under autarky, as we saw in Proposition 5. The reason R&D resources can rise but the growth rate remain unchanged is interesting and shows the importance of the underlying IO structure. In our framework, the driver of growth is *firm* level R&D, not aggregate R&D. Trade increases Home's aggregate R&D, but it also increases the number of firms, leaving R&D per firm unchanged in the long run and therefore leaving

[16] Mathematically, profit is linear but not affine in pure variety expansion models and is affine in models with fixed cost. Linearity together with the linearity of the R&D technology makes the rate of return positively related to the economy's size and independent of the number of firms. Intuitively, doubling the number of firms causes profit to fall by half, but R&D expense also falls by half, leaving the rate of return to variety expansion independent of the number of firms. Doubling the aggregate market size, such as by doubling the population, thus raises the growth rate permanently. Fixed cost changes matters drastically by making profit affine in market size. Increasing the number of firms then reduces profit by more than the entry cost. Because of that, variety expansion peters out and is not a source of long-run growth. See Peretto and Connolly (2007) for a thorough discussion.

the steady-state growth rate unchanged. Laincz and Peretto (2006) show that the evidence supports this mechanism. For the same reason, our results differ markedly from Rivera-Batiz and Xie (1993), who find that trade reduces growth rate of larger country and raises rate for smaller country. Their result is just the market-size effect again but with countries of different sizes and so different magnitudes of the market-size effects.

Foreign's growth rate under trade may be higher or lower than its autarkic rate. Trade is determined by current comparative advantage, and the country with the current advantage need not also be the country that is more efficient at R&D. It could be, for example, that Foreign has a higher R&D TFP parameter η, implying that it is intrinsically better at quality-improving R&D, but also have a sufficiently low current value of quality Z so that it currently comparatively disadvantaged in the production of capital. Although possible, that situation does not seem typical of reality, where the countries that do not produce capital (i.e., that specialize in final goods) also are those that are relatively weak in R&D. We therefore assume henceforth that Home, the country that produces capital under trade, is also the country that is stronger at R&D:

Assumption 1. $\eta_h > \eta_f$

In that case, trade always raises Foreign's steady-state growth rate because trade allows Foreign to abandon its relatively inefficient R&D and rely instead on Home's R&D for improving the quality of the capital that it uses, which it buys from Home.[17]

An implication of our results is that trade's effect on a country's steady-state growth rate depends on the kind of good that the country imports. Importing a capital good can raise long-run growth, whereas importing a consumption good cannot. The good that is exported is irrelevant. That result is consistent with a long line of evidence (DeLong and Summers (1991); DeLong and Summers (1992), DeLong and Summers (1993); Lee (1995); Kliesen and Tatom (2013)., and the economics behind it is straightforward. Growth is driven by improvements in production, either by expanding the number of varieties of intermediate goods or by improving their quality. Importing a consumption good does nothing to enhance either of those sources of growth, whereas importing an intermediate or capital good does because of the improved technology embodied in capital.[18]

4.2.3 Factor Price Equalization

Our trade framework is Ricardian, depending on technology differences to drive trade. We have ruled out technology transfer and direct foreign investment, so countries do not adopt each other's technology by any means and their technologies are permanently different. Furthermore, one country stops producing one of the traded goods. In classic trade theory, those features would prevent factor price equalization. Nevertheless, in our model trade can lead to factor price equalization. Understanding the reason for that result provides

[17] As in Lucas (1988) and Redding (1999) "dynamic inefficiency" is possible. In our case, this happens when $\eta_h < \eta_f$. However, the empirical evidence presented by Eaton and Kortum (2001b) suggests that this case is highly unlikely to emerge.

[18] Yenokyan, Seater, and Arabshahi (2014) arrive at the same conclusion using a two-sector model without R&D. Our analysis thus extends theirs to the fully endogenous, R&D-based framework. Also, Ji and Seater (2012) obtain a related result in a model in which countries are only allowed to trade intermediate goods: trade's effect on a country's growth rate depends on the R&D efficiency of the trading partner making the imported intermediate good and not at all on the country's own R&D efficiency in improving the quality of the intermediate that it exports.

insight into the economics underlying factor price equalization, even in other kinds of trade models.

Lemma 3. *The wage rates under trade are*

$$w_h = A_{hy}\left(\frac{\alpha\beta A_{hk}Z_h^{\theta}}{1-\beta}\right)^{\beta} N_h^{\beta(1-\alpha)/\alpha}$$

$$w_f = A_{fy}\left(\frac{\alpha\beta A_{hk}Z_h^{\theta}}{1-\beta}\right)^{\beta} N_h^{\beta(1-\alpha)/\alpha}$$

The ratio of the two wages is

$$w_h/w_f = \left(\frac{A_{hy}}{A_{fy}}\right)^{\frac{1}{1-\beta}}, \tag{44}$$

Home's wage is the same as before trade. Foreign's wage now depends on Home's values of A_k, Z_i, and N instead of its own values because Foreign now relies exclusively on capital produced by Home. Foreign wage is the same as if Foreign had replaced its original technology for producing capital with Home's technology, even though it has not done that. In fact, Foreign now does not produce capital at all but instead imports it from Home and so relies on Home's technology for capital production. Trade has resulted in an *effective* transfer of Home's technology to Foreign through the technology (both number of varieties and levels of quality) embodied in the capital it sells to Foreign.

The only difference between the two countries' wages is the final goods TFP parameters A_{hy} and A_{fy}, which trade does not affect. We thus have an interesting result:

Proposition 7. *Trade leads to partial factor price equalization. The disparity in cross-country wages that remains after trade opens depends on the ratio of the TFP parameters A_{hy} and A_{fy} in the two countries' final goods industries as shown in equation* (44)*. Equalization is complete if $A_{hy} = A_{fy}$.*

Notice that under autarky the wages generally are not equal even if $A_{hy} = A_{fy}$ because in wages depend on other parameters that generally differ in the two countries: typically, $A_{hk} \neq A_{fk}$, $N_h \neq N_f$, and $Z_{hi} \neq Z_{fi}$. Thus factor price equalization is non-trivial even when $A_{hy} = A_{fy}$.

Our result here complements and extends the result in Yenokyan, Seater, and Arabshahi (2014). They show that in a two-sector, two-capital growth model trade in the two types of capital equalizes factor prices in the case of complete specialization, that is, where each country specializes in one of the types of capital and trades it to the other country. That is exactly the opposite of the conditions necessary for factor price equalization in the Hecksher-Ohlin framework, which requires that neither country specialize. We now have added a third case where factor price equalization emerges when one country specializes and the other does not. The common thread that ties all the results together is that factor price equalization occurs if and only if trade effectively equalizes technology.[19] That is the deep economic principle underlying factor price equalization.

[19] In classical Hecksher-Ohlin models technology is identical in the two countries but factor endowments differ, leading to different marginal products of the factors in autarky. Trade allows the marginal products to be equalized and so in that sense equalizes "realized" technology as measured by marginal products.

4.3 Empirical Observations and Testable Implications

We perform no tests of our theory here. We do note, however, that the model is consistent with several facts. For example, For example, Eaton and Kortum (2001b) document the following:

1. The bulk of capital is produced by just a few countries.
2. The world's R&D is primarily performed by the countries producing the capital goods.
3. The countries producing the capital and R&D are the historically more advanced countries.

Our model is consistent with those facts and indeed can be regarded as an explanation of them.

Our theory has several testable implications, some of which explain briefly.

As we have seen, opening to trade temporarily raises the growth rate of the country that exports capital but has no effect on that country's steady-state growth rate. In contrast, trade permanently raises the growth rate of the country importing capital and exporting a consumption good. The time series implications for the two countries thus are very different.

Lemma 2 shows that the magnitude of the labor reallocation induced by trade depends on the ratio of the two countries' TFP values in the final output sector. The more productive Foreign is relative to Home, the more labor Home will reallocate away from the final goods sector. That implication is testable if one has estimates of TFP values at the time that trade opens between two countries. The period around 1980 saw a large increase in international openness, so data around that time may be appropriate for carrying out the test.

Proposition 6 shows that the fraction L_z/L of the labor force allocated to quality-improving R&D depends positively on the TFP parameters β and η. Those parameters are measurable from national income account data on factor shares and patents per unit of R&D resources. Proposition 6 also shows thatL_z/L depends positively on both fixed operating cost ϕ and entry cost ψ. Good data on those costs are difficult to obtain, but in principle they can be measured, so this implication of the theory will be testable once the requisite data have been assembled.

Proposition 5 shows that the steady state number of varieties N and the steady-state growth rate of final output depend on the same parameters as L_z/L and in the same way. The steady-state growth rate of quality, however, does not depend on β.

4.4 Welfare

Opening to trade changes both Home and Foreign welfare. By substituting the appropriate values for variables and parameters into the utility function (33) for Home and Foreign before and after trade, we obtain the change in each country's welfare brought about by opening to trade.

Proposition 8. *The changes in Home and Foreign welfare caused by opening to trade are*

$$\begin{aligned} U_{h\triangle} &= \rho^{-1} log\left[1+\alpha\beta\rho\psi A_{hy}^{-1}\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}\frac{L_f}{L_h}\right] \\ &\quad +\beta\theta\left[\left(\frac{\alpha\psi}{\rho\left[1-\alpha\left(1+\theta\right)\right]}\right)\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}\left(\widehat{Z}_{hss}+\rho\right)\frac{L_f}{L_h}\right] \\ &\quad +\left[v\left(N_{hss}^T,N_{h0}\right)-v\left(N_{hss}^A,N_{h0}\right)\right] \\ &> 0 \end{aligned} \tag{45}$$

$$\begin{aligned} U_{f\triangle} &= \rho^{-1} log\left[(1-\psi\alpha\beta\rho)^{-1}\left(\frac{A_{hk}}{A_{fk}}\right)^{\beta}\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{\beta}{1-\beta}}\left(\frac{N_{h0}}{N_{f0}}\right)^{\beta\left(\frac{1-\alpha}{\alpha}\right)}\left(\frac{Z_{h0}}{Z_{f0}}\right)^{\beta\theta}\right] \\ &\quad +\beta\left[\rho^{-2}\theta\left(\widehat{Z}_{hss}-\widehat{Z}_{fss}\right)+\left(\frac{\alpha\psi}{\rho\left[1-\alpha\left(1+\theta\right)\right]}\right)\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}\left(\widehat{Z}_{hss}+\rho\right)\frac{L_f}{L_h}\right] \\ &\quad +\beta\left(\frac{1-\alpha}{\alpha}\right)v\left(N_{hss}^T,N_{h0}\right) \\ &\gtreqless 0 \end{aligned} \tag{46}$$

The first term in Home's welfare change captures the standard static effect of trade on Home consumption and is unambiguously positive. The second and third terms capture the welfare effects from transitional growth arising from quality improvement and variety expansion, respectively. As we have seen above, Home experiences a temporary increase in its growth rate because of the temporary scale effect conferred upon Home by the opening of trade, so the second term is positive. The third term is

$$\begin{aligned} v\left(N_{hss}^T,N_{h0}\right)-v\left(N_{hss}^A,N_{h0}\right) &= \int_0^\infty e^{-\rho t}\log\left(\frac{N_{ht}^T}{N_{ht}^A}\right)dt \\ &= \int_0^\infty e^{-\rho t}\log\left\{\frac{\left(\frac{N_{hss}^T}{N_{h0}+e^{-M_h t}\left(N_{hss}^T-N_{h0}\right)}\right)}{\left(\frac{N_{hss}^A}{N_{h0}+e^{-M_h t}\left(N_{hss}^A-N_{h0}\right)}\right)}\right\}dt \end{aligned}$$

which is positive because Home's number of firms following trade is higher than under autarky.

Foreign's change in welfare is more complicated but perhaps more interesting. The first term in Foreign's welfare change again is the static gain from trade, but its sign is ambiguous because it is a combination of two effects that have opposite signs. The first effect is the standard static gain from trade and is positive. The second effect reflects the loss of monopoly profit that arises from the reallocation of the imperfectly competitive production of intermediate goods from Foreign to Home, an unambiguously negative effect.[20] The sign of the first term in equation (46) thus is ambiguous. The second term in equation (46) reflects the dynamic contributions of quality improvement to Foreign's welfare, and it also comprises two effects. The first effect arises from the change in Foreign's steady-state growth rate. That effect is positive under our assumption that Home is better than Foreign at R&D. Trade allows Foreign to rely on Home's superior R&D for quality improvement, which raises Foreign's steady-state growth rate. The second effect is

[20] There is also an effect from the reallocation of labor from intermediate good production to final output production (increasing Y_f). That reallocation has no effect on consumption and welfare because labor is paid the same in the two sectors .

the contribution of transitional growth to welfare, which is unambiguously positive. Thus the second term in equation(46) is positive. Finally the third term in equation(46) is the dynamic contribution of variety expansion to welfare, which also is unambiguously positive. The sign of equation (46) is ambiguous because the sign of the first term is ambiguous.

An interesting aspect of Proposition 8 is that the welfare change may be much larger for Foreign than for Home. In equation (46), the change in the steady-state growth rate is multiplied by ρ^{-2}, a very big number, so even a small change in Foreign's growth rate has a large effect on Foreign's welfare. Consequently, there is some reason to expect that the positive effects in equation (46) are dominant and that trade raises Foreign's welfare. Trade does not affect Home's steady-state growth rate, so equation (45) does not contain a term inρ^{-2}. We therefore conclude that the country importing the capital goods (and hence the embodied technology) may well have the most to gain from trade, perhaps by a considerable margin.

Finally, notice that the change in both Home and Foreign welfare depends on relative but not absolute population sizes. Thus there is no scale effect on welfare, in sharp contrast to Li (1996) and Rivera-Batiz and Xie (1993). The reason again is the IO structure of the model and the endogeneity of the number of firms.

5 Trade and Take-offs

Recent work by Peretto (2013) shows that population growth can cause innovation and growth in income per person to start in economies that initially do not innovate and grow. In this section we show that trade is another possible (and perhaps plausible) explanation for an economy to turn on economic growth.

Recall from Proposition 1 that it is possible for an economy to be stuck in the no-growth regime. From (23) and (24) we obtain the condition for there to be no growth:

$$N_{h0} > \min\left\{\left(\frac{1}{1-\psi_h\alpha\beta\rho}\right)\left(\frac{\alpha\eta_h\beta\theta}{\rho}\right)L_h, \left(\frac{1}{1-\rho\psi_h\beta\alpha}\right)\left[\frac{\beta\left(1-\alpha-\alpha\psi_h\rho\right)}{\phi_h}\right]L_h\right\} \tag{47}$$

If the population size too small relative to the initial number of firms, the economy will not innovate and grow. That may be a realistic situation in very underdeveloped economies, where a large fraction of households are independent producers (e.g., farmers). Can trade cause such an economy to start growing? Yes, it can.

Under trade, the condition for no growth is

$$N_{h0} > \min\left\{\left(\frac{\alpha\eta_h\beta\theta}{\rho}\right)\left[\frac{L_h + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{1-\psi_h\alpha\beta\rho}\right], \left(\frac{\beta\left(1-\alpha-\alpha\psi_h\rho\right)}{\phi_h}\right)\left[\frac{L_h + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{1-\psi_h\alpha\beta\rho}\right]\right\} \tag{48}$$

Comparing (48) and (47) shows that trade can cause "take-offs". The intuition is that the specialization induced by comparative advantage increases the market that Home firms face. That increase can be enough to induce R&D in both variety expansion and quality improvement. If trade causes both R&D activities to

start (N_{h0} is larger than both terms in (48)), the dynamics and the conditions for stability of the positive growth regime is given in section 3 and the economy can grow forever after.

Causality running from trade to a growth take-off is consistent with the timing Industrial Revolution, which was preceded by an expansion of international trade following the demise of mercantilism and its various restrictions on trade over the late 18th and early 19th centuries.

6 Conclusion

Our theory has delivered several new results on the effects of trade on both growth and welfare. It is instructive to summarize the reasons for the novelty of the results.

Classical trade theory treats technology as given, and it treats factors of production as both endowed and non-tradable. All of those restrictions are false, and relaxing them has profound implications for the effects of trade. Trading factors of production that are produced and that embody technical progress has fundamentally different effects than trading consumption goods. Because factors of production can carry within them embodied technical progress, trading them works to equalize technology across countries even when no technology transfer or diffusion ever takes place. It also allows trade to enhance growth because the technology embodied in the traded factors of production can augment the non-traded non-reproducible factors such as land and labor. As has been well-known since **?**, augmenting the non-reproducible factors of production is the key to sustained economic growth. Those kinds of effects are totally alien to classical trade theory.

The specification of the economy's industrial structure, especially the part concerning who does R&D and the kind of R&D they do, has important implications for the evolution of the economy and trade's effect on it. Early endogenous growth theory incorporated restrictions on the industrial structure and the nature of R&D that are inconsistent with the facts. Correcting the model to align it with the IO facts alters the general equilibrium behavior and produces many results that differ from those obtained in the earlier literature. The key elements are to recognize that firms enter and exit in response to the profit opportunities facing them and that firms must pay fixed costs to operate. Those two facts work together to alter the dynamic behavior of the fully endogenous growth model in decisive ways. The counterfactual market-size effects of the 1st-generation literature are gone. Many results on both the transition dynamics and the steady-state behavior of the economy are changed. The 2nd-generation framework delivers many testable implications that distinguish it from the earlier frameworks for analyzing growth, and so far it has done well in the tests performed. Our analysis has shown that extending the model to include Ricardian trade delivers many more testable implications. It will be interesting to see how those implications fare when confronted with the data in formal tests.

Appendix

Dynamic Optimization of Intermediate Goods Firms in (3.3) The current value Hamiltonian is obtained by substituting (8) and (5) into (10)

$$H_i = \frac{p_k K p_i^{\frac{-1}{1-\alpha}} Z_i^{\frac{\theta\alpha}{1-\alpha}}}{\int_0^N p_i^{\frac{-\alpha}{1-\alpha}} Z_i^{\frac{\theta\alpha}{1-\alpha}} di} (p_i - w) - w\phi - wl_{zi} + \lambda_i \eta Z l_{zi},$$

where p_i and l_{zi} are the control variables, Z is the state variable, and λ_i is the co-state variable. The first-order conditions include

$$\frac{\partial H_i}{\partial p_i} = 0 \Leftrightarrow p_i = \frac{w}{\alpha} \equiv p, \tag{49}$$

$$\frac{\partial H_i}{\partial l_z} = 0 \Leftrightarrow \lambda_i = \frac{w}{\eta Z}, \tag{50}$$

$$\frac{\partial H_i}{\partial Z_i} = \frac{\theta\alpha}{1-\alpha} \frac{p_k K p_i^{\frac{-1}{1-\alpha}} Z_i^{\frac{\theta\alpha}{1-\alpha}-1}}{\int_0^N p_i^{\frac{-\alpha}{1-\alpha}} Z_i^{\frac{\theta\alpha}{1-\alpha}} di} \left(\frac{w}{\alpha} - w\right) = r_t \lambda_i - \dot{\lambda}_i. \tag{51}$$

Substituting (50) and (49) into (51) yields (12).

Proof of Lemma 1. Lemma 1 can be proved by using resource allocation (2), household budget constraint $wL + N\pi = C + wL_n$, returns to entry (15), Euler equation (18), (6) and the fact that $C = Y$.

First, combining the fact $C = Y$ such that $\hat{C} = \hat{Y}$, (2), and (15), we get

$$\widehat{L_y} + \rho = \frac{\pi}{w\psi A_x X} + \widehat{X}. \tag{52}$$

Combining (6) and (2), we have

$$\widehat{X} = \widehat{L_y} - \widehat{N}. \tag{53}$$

Substituting (6), (2), (53) and the household budget constraint $wL + N\pi = C + wL_n$ into (52) leads to

$$\begin{aligned} \rho &= \frac{\pi}{w\psi A_x X} - \frac{wL + N\pi - C}{Nw\psi A_x X} \\ &= \frac{C - wL}{Nw\psi A_x X} \\ &= \frac{C - wL}{Nw\psi \left(\frac{\alpha\beta}{(1-\beta)}\right) \frac{L_y}{N}} \\ &= \frac{\frac{wL_y}{1-\beta} - wL}{\psi \left(\frac{\alpha\beta}{(1-\beta)}\right) L_y}, \end{aligned} \tag{54}$$

which can be solved to get (19). Furthermore, combining (2), (4), (6) and (19) leads to (20). ■

Proof of Proposition 1. Equation (2) leads to $\hat{Y} = \hat{w} + \hat{L_y}$. Given that $\hat{L_y} = 0$ from Lemma 1, (18) and $Y = C$, we get

$$\hat{C} = \hat{Y} = \hat{w} = r - \rho. \tag{55}$$

Combining (55) with (13) leads to the growth rate of quality under autarky (21). Substituting (11), (8), (55), and (21) into (15) we get (22). ■

Proof of Proposition 2. On the steady state, entry stops, and the number of firm is a finite number. Solving $\hat{N} = 0$ in (22) leads to (27). Plugging (27) into (21), we get (28). ■

Proof of Proposition 3. Combining (55) and (20) leads to

$$\hat{Y} = \hat{w} = \beta\theta\hat{Z} + \frac{\beta(1-\alpha)}{\alpha}\hat{N}. \tag{56}$$

Substituting (21) and (22) into (56) leads to (29). Substituting the steady state values (28) and (27) into (56) leads to (30). ■

Proof of Lemma 2. Similar to the case under autarky, to prove Lemma 2, we need the demand of X_{hi}, resource allocation condition, Euler equation, return to entry, household budget constraint, and in addition, the trade balance condition.

Under trade, home produces both final goods and intermediates while foreign specializes in final goods. The aggregate market of intermediate goods in home is

$$X_{hi} = \frac{p_{hk}K_h p_i^{\frac{-1}{1-\alpha}} Z_{hi}^{\frac{\alpha\theta}{1-\alpha}}}{\int_0^{N_h} p_{hi}^{\frac{-\alpha}{1-\alpha}} Z_{hi}^{\frac{\alpha\theta}{1-\alpha}} di} = \frac{\alpha L_{hy}}{N_h}\left(\frac{\beta}{1-\beta}\right)\frac{Y_h + Y_f}{Y_h} = \left(\frac{\alpha\beta}{1-\beta}\right)\left(\frac{L_{hy} + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{N_h}\right). \tag{57}$$

The return to entry after trade is similar to the case under autarky: $r_h = \frac{\pi_h}{V_h} + \frac{\dot{V}_h}{V_h}$, and the free entry condition requires that $V_h = w_h\psi_h X_h$. Combining the return to entry, free entry condition, Euler equation and (57) yields

$$\widehat{C_h} = \frac{\pi_h}{w_h\psi_h X_h} + \widehat{w}_h + \frac{\widehat{L_{hy}}L_{hy}}{L_{hy} + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}} - \widehat{N} - \rho. \tag{58}$$

Trade balance requires that

$$C_h = Y_h + \beta Y_f = \frac{w_h L_{hy} + \beta w_f L_f}{1-\beta} = w_h\left(\frac{L_{hy} + \beta\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f}{1-\beta}\right), \tag{59}$$

which implies

$$\widehat{C_h} = \widehat{w_h} + \left(\frac{L_{hy}}{L_{hy} + \beta\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} L_f}\right)\widehat{L_{hy}}. \tag{60}$$

Combining (58), (60) and the household budget constraint $w_h L_h + N_h \pi_h = C_h + w_h L_{hn}$ yields

$$\left(\frac{L_{hy}}{L_{hy} + \beta \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}} L_f} \right) \widehat{L_{hy}} = \frac{\widehat{L_{hy}} L_{hy}}{L_{hy} + L_f \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}}} + \frac{C_h - w_h L_h}{N w \psi X} - \rho \tag{61}$$

Defining $c_h = C_h / w_h$, the factor income shares imply that

$$L_{hy} = (1-\beta) c_h - \beta \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}} L_f. \tag{62}$$

Straightforward differentiation of (62) yields

$$\widehat{L_{hy}} = \frac{c_h \widehat{c_h}}{(1-\beta) c_h - \beta \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}} L_f}. \tag{63}$$

Substituting (63) and (62) into (61) yields

$$\widehat{c_h} = \frac{c_h \left[1 - \psi \alpha \beta \rho \right] - L_h - L_f \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}} \psi \alpha \beta \rho}{(1-\beta)^{-1} \psi \alpha \beta L_f \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}}}$$

This is an unstable differential equation, so to satisfy transversality c_h must jump to its steady state value

$$c_h = \frac{L_h + L_f \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}} \psi \alpha \beta \rho}{1 - \psi \alpha \beta \rho}$$

This combined with (62) yields

$$L_{hy} = L_h (1-\beta) - L_f \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}} \beta \left[1 - \psi \alpha \rho \right].$$

■

Proof of Proposition 4. Similar to the case under Autarky in Proposition 2, except that under trade, the aggregate market size which the home intermediate goods firms face is the international market: $L_h + \left(\frac{A_{fy}}{A_{hy}} \right)^{\frac{1}{1-\beta}} L_f$. ■

Proof of Proposition 5. In the steady state, entry stops. Setting $\hat{N}_h = 0$ in (38) yields the steady state number of firms in (39). Substituting (39) into (37) yields (40). ■

Proof of Corollary 1. We focus on the case with both dimensions of innovations. Given the steady state value in (39), the first equation of (38) can be written as

$$
\begin{aligned}
\dot{N} &= N\left[\frac{1-\alpha-\alpha\psi\rho-\alpha\theta}{\alpha\psi} - N_h\left(\psi\alpha\beta\left(\frac{L_h+L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{1-\psi\alpha\beta\rho}\right)\right)^{-1}\left(\phi-\frac{\rho}{\eta}\right)\right] \\
&= N\left(\psi\alpha\beta\left(\frac{L_h+L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{1-\psi\alpha\beta\rho}\right)\right)^{-1}\left(\phi-\frac{\rho}{\eta}\right)\left[N_{ss}^T-N_h\right].
\end{aligned}
$$

Define

$$
M_h \equiv \frac{\dot{N}}{N\left[N_{ss}-N_h\right]} = N_{hss}^T\left(\phi-\frac{\rho}{\eta}\right)\left(\frac{\psi\alpha\beta\left(L_h+L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}\right)}{1-\psi\alpha\beta\rho}\right)^{-1},
$$

Then

$$
\begin{aligned}
M_h &= \int_0^T\left(\frac{\dot{N}_h}{N_h}+\frac{\dot{N}_h}{N_{ss}-N_h}\right) \\
&= \ln\left(\frac{\frac{N_{ht}}{N_{h0}}}{\frac{N_{ht}-N_{hss}}{N_{h0}-N_{hss}}}\right) \\
\Longrightarrow \frac{N_{ht}}{N_{h0}} &= \frac{N_{ht}-N_{hss}}{N_{h0}-N_{hss}}e^{M_{ht}} \\
\Longrightarrow N_{ht} &= \frac{N_{hss}^T}{1+e^{-M_h t}\left(\frac{N_{hss}^T-N_{h0}}{N_{h0}}\right)},
\end{aligned}
$$

where $M_h^T = N_{hss}^T\left(\phi-\frac{\rho}{\eta}\right)\left(\frac{\psi\alpha\beta\left(L_h+L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}\right)}{1-\psi\alpha\beta\rho}\right)^{-1}$.

Substituting (40) and (42) into (37) yields

$$\begin{aligned}
\widehat{Z_{ht}} &= \theta\alpha\beta\eta N_h^{-1}\left(\frac{L_h + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{1-\psi\alpha\beta\rho}\right) - \rho \\
&= \theta\alpha\beta\eta\left(\frac{1+e^{-M_h t}\left(\frac{N_{hss}^T - N_{h0}}{N_{h0}}\right)}{N_{hss}^T}\right)\left(\frac{L_h + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{1-\psi\alpha\beta\rho}\right) - \rho \\
\Longrightarrow \ln\left(\frac{Z_{ht}}{Z_{h0}}\right) &= \int\left(\theta\alpha\beta\eta\left(\frac{1+e^{-M_h t}\left(\frac{N_{hss}^T - N_{h0}}{N_{h0}}\right)}{N_{hss}^T}\right)\left(\frac{L_h + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{1-\psi\alpha\beta\rho}\right) - \rho\right) \\
&= T\left(\left(\frac{\theta\alpha\beta\eta}{N_{hss}^T}\right)\left(\frac{L_h + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}}{1-\psi\alpha\beta\rho}\right) - \rho\right) + \frac{1}{M_h}\left(1-e^{-M_h t}\right)\left(\frac{\theta\alpha\beta\eta}{N_{hss}^T}\right)\left(\frac{N_{hss}^T - N_{h0}}{N_{h0}}\right)\left(\frac{L_h + L_f\left(\frac{A_{fy}}{A_{hy}}\right.}{1-\psi\alpha\beta}\right. \\
&= \widehat{Z}_{hss}t + \frac{\psi\alpha}{1-\alpha\left(1+\theta+\psi\rho\right)}\left(1-e^{-M_h t}\right)\left(\frac{N_{hss}^T - N_{h0}}{N_{h0}}\right)\left(\widehat{Z}_{hss}+\rho\right) \\
\Longrightarrow Z_{ht}^T &= \exp^{\widehat{Z}_{hss}t + \left(\frac{N_{hss}^T - N_{h0}}{N_{h0}}\right)\frac{\psi\alpha\left(\widehat{Z}_{hss}+\rho\right)}{1-\alpha(1+\theta+\psi\rho)}\left(1-e^{-M_h t}\right)}
\end{aligned}$$

where $M_h^T = \frac{1-\alpha(1+\theta+\psi\rho)}{\psi\alpha}$. ■

Proof of Proposition 6. Equation (8) indicates

$$L_Z = N\frac{1}{\eta}\hat{Z}. \tag{64}$$

Substituting (27) and (40) into (64) yields $\left(\frac{L_{zh}}{L_h}\right)_{ss}^A$. Substituting (39) and (40) into (64) yields $\left(\frac{L_{zh}}{L_h}\right)_{ss}^T$. ■

Proof of Lemma 3. Zero profits in final output sector in Home and Foreign implies

$$A_{hy}\left(\frac{K_{hh}}{L_{hy}}\right)^{\beta} = p_{hk}\frac{K_{hh}}{L_{hy}} + w_h, \tag{65}$$

$$A_{fy}\left(\frac{K_{fh}}{L_f}\right)^{\beta} = p_{hk}\frac{K_{fh}}{L_f} + w_f. \tag{66}$$

The Cobb-Douglass technology implies,

$$\frac{K_{fh}}{L_f} = \frac{w_f K_{hh}}{w_h L_{hy}} \tag{67}$$

Substituting (67) into (65) and (66) yields,

$$w_h/w_f = \left(\frac{A_{hy}}{A_{fy}}\right)^{\frac{1}{1-\beta}} \tag{68}$$

and

$$\frac{K_{fh}}{L_f} = \left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} \frac{K_{hh}}{L_{hy}}. \tag{69}$$

Using the fact that $K_h = K_{fh} + K_{hh}$ yields

$$K_h = K_{hh}\left(\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}} \frac{L_f}{L_{hy}} + 1\right) \tag{70}$$

Also the production technology (4) and intermediate demand (57) yield

$$K_h = A_{hk} N_h^{\left(\frac{1-\alpha}{\alpha}\right)} Z_h^{\theta} \left(\frac{\alpha\beta}{1-\beta}\right)\left(L_{hy} + L_f\left(\frac{A_{fy}}{A_{hy}}\right)^{\frac{1}{1-\beta}}\right) \tag{71}$$

Setting (71) equal to (70) yields

$$K_{hh} = A_{hk} N_h^{\left(\frac{1-\alpha}{\alpha}\right)} Z_h^{\theta} \left(\frac{\alpha\beta}{1-\beta}\right) L_{hy}. \tag{72}$$

The production technology combined with (72) yields

$$Y_h = A_{hy}\left(A_{hk} N_h^{\left(\frac{1-\alpha}{\alpha}\right)} Z_h^{\theta} \left(\frac{\alpha\beta}{1-\beta}\right)\right)^{\beta} L_{hy} \tag{73}$$

The factor share yields the wage rate. The derivation of Foreign's wage follows trivially.■

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