Cross-Industry Growth Differences with Asymmetric Industries and Endogenous Market Structure

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Abstract

We develop a two-industry R&D growth model with an endogenous market structure to evaluate the impact of industrial fundamentals on cross-industry differences of TFP growth and R&D intensity. Endogenous market structure in our model allows the firm's market size to respond to the firm's entry and exit which complements the models with an exogenous market structure in the previous literature. We find that surprisingly, an industry with a relatively high R&D productivity or appropriability exhibits "relatively" *low* in-house innovation growth and R&D intensity during transition. Moreoever, we examines the effects of R&D subsidies and patent breadth policies on industry differences by implementing both asymmetric and symmetric policy rules. We find that only asymmetric R&D subsidies have impacts on TFP growth and R&D intensity differences.

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1 Introduction

Why do industries differ in their rates of TFP growth? To explore this question, we consider a second generation R&D based growth model with an endogenous market structure pioneered by Peretto (1996, 1998), Young (1998), Howitt (1999), and Segerstrom (2000).[1] To our knowledge, this is the first paper that analyzes the fundamentals that cause cross-industry differences in growth and research intensity in a second-generation fully endogenous growth model.

The previous IO literature suggests that differences in R&D intensity (the ratio between R&D expenditure and sales) underlie differences in TFP growth. See Griliches and Lichtenberg (1984), and Dosi (1988). However, as studied by Jones (1995) and Klenow (1996), the causal relation between the R&D intensity and TFP growth is unclear. Further recent research shows that both R&D intensity and TFP growth differences should result from the differences in deeper industry fundamentals. The line of this research, such as Klenow (1996), Giordani and Zamparelli (2008), and Ngai and Samaniego (2009), explores the fundamentals: *technology opportunity* (the efficiency of research), *market size*, and *appropriability*,[2] in an R&D-based growth framework with an *exogenous* market structure to

In contrast to the previous literature, our model which features an *endogenous* market structure shows two important characteristics that render a feedback effects of market structure on the cross-industry differences in growth and R&D intensity. First, the market structure, measured by the firm's market size and market concentration, endogenously responds to the change of these fundamentals. Specifically, these fundamental factors affect the entry and exit of firms, and therefore determine the equilibrium number of firms and firms' market size. Second, a firm's R&D incentive depends on the firm's market size but not on the aggregate market size.[3] The firm's market size in turn shapes the firm's R&D expenditure and hence the industry TFP growth rate. The feedback effect of market structure on TFP growth and R&D intensity differences substantially differentiates our model's prediction from the previous literature. Moreover, our model characterizes both the steady state and transitional dynamics which the previous literature did not emphasize.[4]

The steady state TFP growth is not related to the industry market size. This finding contrasts with Klenow (1996) who uses the model with a scale effect. Our model with a scale-invariant prediction is consistent with Ngai and Samaniego (2009). However, the underlying mechanisms are different. Ngai and Samaniego (2009) use a semi-endogenous growth model which eliminates the scale effect by assuming a diminishing return to knowledge spillover. Instead, we eliminate the scale effect by an endogenous market structure where entry and exit causes the TFP growth to depend on the firm's market size, but not the aggregate market size.

Our model provides novel and different predictions of cross-industry differences based on the transitional dynamics. Surprisingly, an industry with a relatively high R&D productivity or

[1] The growth literature generally assumes single product firms and monopolistic competition. Minniti (2006) constructs a model with multi-product firms and oligapolistic competition and shows similar aggregate growth results.

[2] Appropriability is the extent to which a firm benefits from its own knowledge.

[3] which is consistent with the IO facts. See Kamien and Schwartz (1982), Baldwin and Scott (1987), Dosi (1988), Tirole (1988, Ch. 10), Cohen and Levin (1989), and Scherer and Ross (1990, Ch. 17); more recently, Cohen and Klepper (1996a, 1996b), Adams and Jaffe (1996).

[4] Klenow (1996) uses the Romer model, Giordani and Zamparelli (2008) use the quality-ladder model, Ngai and Samaniego (2009) use a semi-endogenous model, all of which do not have transition dynamics, or the transition dynamics are too complicated to illustrate, so they only provide the balanced growth path results.

appropriability exhibits a relatively *low* in-house innovation growth and R&D intensity following a change in R&D productivity (appropriability). This result contrasts with the predictions in the previous literature based on an exogenous market structure. See Klenow (1996), Giordani and Zamparelli (2008), Ngai and Samaniego (2009). The endogeneity of the market structure provides an additional channel to affect the relative TFP growth and R&D intensity. A higher R&D productivity or appropriability also induces the entry and exit of firms which endogenously changes the relative firm market size and R&D incentive. An industry with higher R&D productivity induces more entry to split the market, ending up with smaller firm market size. By contrast, an industry with lower R&D productivity induces the exit of more firms which leads to a higher market concentration and higher market size. This market size effect in turn reduces the R&D incentive for an industry with higher R&D productivity but raises the R&D incentive for its counterpart. As a result, a relatively high R&D productivity (or appropriability) leads to a *relatively* low TFP growth. In addition, our model also predicts that the TFP growth differences and R&D intensity ratio are both independent of the industry market size. Intuitively, the endogeneity of the market structure implies that in response to a profit opportunity, the entry or exit of firms immediately absorbs the change in the industry-level market size, which leaves the firm market size and thus the TFP growth rate unchanged. This result again contrasts with the findings in Klenow (1996) but is consistent with the findings in Ngai and Samaniego (2009) based on the same intuition as already referred.

One important implication of our model is the impact of R&D policies on industry differences. Countries have different R&D incentives across industries.[5] For example, the US and China recently introduced a more generous credit for R&D in energy. See OECD (2011). We examine the effects of R&D subsidies (R&D tax reduction) and patent breadth policies on industry differences by implementing both asymmetric and symmetric policy rules. When the government imposes asymmetric R&D subsidies on two industries, we find that increasing relative R&D subsidies is similar to increasing relative R&D productivity, which will reduce the *relative* TFP growth and R&D intensity at the instant that the policy is implemented. When the government imposes an asymmetric patent breadth on two industries, we find that changing the relative patent breadth is similar to changing the relative unit cost, which does not affect cross-industry differences in TFP growth and R&D intensity. Moreover, imposing a symmetric R&D subsidy or symmetric patent breadth on two industries does not affect the relative TFP growth and R&D intensity. Our research complements Giordani and Zamparelli (2008) who examine profit tax policies, and Chu (2011), Chu and Furukawa (2011), Iwaisako and Futagami (2003), Chu et al. (forthcoming), and Cozzi and Galli (2014), who study the effect of patent policies on the aggregate economy.

The remainders of the paper proceeds as follows. Section 2 constructs an asymmetric growth model with an endogenous market structure based on Peretto (Oct. 1999 and 2007). Section 3 highlights the role of technological opportunity, market size and unit costs in determining industry research intensity, and productivity growth differences. Section 4 concludes.

[5] Today, 26 out of the 34 OECD member countries offer R&D tax incentives to business. Amongst non-OECD countries, Brazil, China, India, the Russian Federation, Singapore and South Africa also provide tax incentives for R&D. Different countries might have different targeting across industries.

2 A Multi-industry Endogenous Growth Model

In this section, we first set up the model, and then discuss how industry characteristics affect (1) the balanced growth rate of the whole economy; and (2) the differences between industry productivity growth and research intensity. Two industries are compared without loss of generality.

There are three productive sectors in the model: final goods, processed goods and intermediate goods. One representive firm produces the final goods Y with two types of processed goods, X_1 and X_2. Final goods can be used for consumption, to produce intermediate goods G_{ij}, and to improve the quality of intermediate goods Z_{ij}. Intermediate goods G_{ij} and the quality inside them are used to produce processed goods, X_i.

Models with this type usually have two sectors, one for final goods and one for intermediate goods. In this model, we have two different classes of intermediate goods, which also means two heterogeneous industries. Adding a third sector (the processed good sector) facilitates the discussion.

2.1 Final Goods Sector

One representive firm produces a single homogeneous final good Y using two non-durable processed goods X_1 and X_2 as inputs. The final goods can be consumed, used to produce intermediate goods, and invested in R&D that improves the quality of existing intermediate goods. The final goods sector is perfectly competitive with Cobb-Douglas production:

$$Y = X_1^{\epsilon} X_2^{1-\epsilon}. \tag{1}$$

We take the final good as the numeraire, so $P_Y = 1$. The usual maximization problem leads to the indirect demand functions:

$$P_{X_1} = \epsilon \left(X_2/X_1\right)^{1-\epsilon}, \tag{2}$$

$$P_{X_2} = \epsilon \left(X_1/X_2\right)^{\epsilon}, \tag{3}$$

where P_{X_1} and P_{X_2} are the prices of X_1 and X_2.

The competitive final-good producer pays compensation ϵY and $(1-\epsilon)Y$ to the processed-good 1 and processed-good 2. So we obtain

$$\epsilon Y = P_{X_1} X_1 \quad and \quad (1-\epsilon)Y = P_{X_2} X_2. \tag{4}$$

2.2 Processed Goods Sector

The processed goods sector is also perfectly competitive. This sector comprises two industries, each producing a single homogeneous good. The representative firms in the two industries use non-durable intermediate goods and labor to produce their respective processed goods. Borrowed from Peretto (2007), their production functions are,

$$X_1 = \int_0^{N_1} G_{1j}^{\lambda} \left(Z_{1j}^{\delta_1} Z_1^{\gamma_1} Z_2^{1-(\delta_1+\gamma_1)} l_{1j} \right)^{1-\lambda} dj, \quad 0 < \lambda, \gamma, \delta < 1 \tag{5}$$

$$X_2 = \int_0^{N_2} G_{2j}^{\lambda} \left(Z_{2j}^{\delta_2} Z_2^{\gamma_2} Z_1^{1-(\delta_2+\gamma_2)} l_{2j} \right)^{1-\lambda} dj, \quad 0 < \lambda, \gamma, \delta < 1 \tag{6}$$

where G_{ij}, $i = 1, 2$ are intermediate goods, Z_{ij} is the quality of good G_{ij}, $Z_i \equiv (1/N_i) \int_0^{N_i} Z_{ij} dj$ is the average quality of class-i intermediate goods, l_{ij} is the number of workers working with intermediate good G_{ij}, and N_i is the number of varieties of intermediate goods used in each industry. The quality Z_{ij} of intermediate good G_{ij} is embodied in the good itself, but augments the workers l_{ij} who use that good.

Define L_1 and L_2 as the total amount of workers allocated to industry 1 and 2, respectively. So, $L_1 \equiv \int_0^{N_1} l_{1j} dj$ and $L_2 \equiv \int_0^{N_2} l_{2j} dj$. Note that each intermediate good G_{ij} is used by only a fraction of the industry's labor force, l_{ij}, and not the whole industry's labor force, L_i. This is one of the key differences between this model and existing growth models. The latter assumes that G_{ij} is used by L_i.

There are two classes of intermediate goods, $\{G_{1j}\}_{j=0}^{N_1}$ and $\{G_{2j}\}_{j=0}^{N_2}$, with one class providing inputs for the X_1 industry and the other class providing inputs for the X_2 industry. Each intermediate good is in one and only one class, so the sets of intermediate goods used by the X_1 and X_2 industries are disjointed, and generally have different numbers of elements (i.e., in general $N_1 \neq N_2$). Each intermediate good G_{ij} has its own quality Z_{ij}, determined by the R&D that has been done by the firm that produces G_{ij}. we discuss the industrial structure and R&D of the intermediate goods sector in the next section. Labor productivity depends on the quality of the intermediate good it works with. To allow for knowledge spillovers, we let labor productivity in industry X_1 also depend on both the average quality $Z_1 = (1/N_1) \int_0^{N_1} Z_{1j} dj$ of the $\{G_{1j}\}$ goods used in industry X_1, and $Z_2 = (1/N_2) \int_0^{N_2} Z_{2j} dj$ of the $\{G_{2j}\}$ goods used in industry X_2. Industry X_2's situation is symmetric. The importance of knowledge spillovers is governed by the magnitude of the parameters δ_i and γ_i. It shows that technology spillovers occur at both the intra-industry (γ_i) and inter-industry levels ($1 - \delta_i - \gamma_i$). See evidence in Bernstein and Nadiri (1988) and Nadiri (1993).

The profit maximization problem leads to demand functions for intermediate goods and labor:

$$G_{1j} = \left(\frac{\lambda P_{X_1}}{P_{G_{1j}}}\right)^{\frac{1}{1-\lambda}} Z_{1j}^{\delta_1} Z_1^{\gamma_1} Z_2^{1-(\delta_1+\gamma_1)} l_{1j}, \tag{7}$$

$$G_{2j} = \left(\frac{\lambda P_{X_2}}{P_{G_{2j}}}\right)^{\frac{1}{1-\lambda}} Z_{2j}^{\delta_2} Z_2^{\gamma_2} Z_1^{1-(\delta_2+\gamma_2)} l_{2j}, \tag{8}$$

$$l_{1j} = (P_{X_1} \frac{1-\lambda}{w})^{\frac{1}{\lambda}} G_{1j} (Z_{1j}^{\delta_1} Z_1^{\gamma_1} Z_2^{1-(\delta_1+\gamma_1)})^{\frac{1-\lambda}{\lambda}}, \tag{9}$$

$$l_{2j} = (P_{X_2} \frac{1-\lambda}{w})^{\frac{1}{\lambda}} G_{2j} (Z_{2j}^{\delta_2} Z_2^{\gamma_2} Z_1^{1-(\delta_2+\gamma_2)})^{\frac{1-\lambda}{\lambda}}, \tag{10}$$

where $P_{G_{ij}}$ is the price of G_{ij}, and w is the wage rate. From the demand functions, we see that an increase in the qualities $Z_{ij}'s$ and the spillovers Z_1, Z_2 cause the increases in the demand for intermediate goods and labor. This is the reason why intermediate good firms engage in research to increase their qualities – in order to obtain a higher demand.

The processed goods industries are competitive with Cobb-Douglas production. By combining the resource allocation in the final good sector, (4), we can see that the intermediate goods firms in class 1 (i.e., those in the set $\{G_{1j}\}$) together receive a total payment of $\lambda P_{X1} X_1 = \lambda \epsilon Y$, and the workers in the processed goods industry 1 receive total compensation of $wL_1 = (1-\lambda) P_{X1} X_1 = (1-\lambda) \epsilon Y$. Similarly, the payments to intermediates and workers employed in the processed goods industry 2 are $\lambda P_{X2} X_2 = \lambda (1-\epsilon) Y$ and $wL_2 = (1-\lambda) P_{X2} X_2 =$

$(1-\lambda)(1-\epsilon)Y$. Based on these outcomes, we can obtain the labor allocation across industries as

$$\frac{L_1}{L_2} = \frac{wL_1}{wL_2} = \frac{\epsilon}{1-\epsilon}. \tag{11}$$

Total compensation paid to intermediate good producers and labor is $\int_0^{N_1} G_{1j} P_{G1j}\, dj + \int_0^{N_2} G_{2j} P_{G2j}\, dj = \lambda Y$ and $w(L_1 + L_2) = (1-\lambda)Y$. Quality Z_{ij} does not get paid for directly from the final goods sector. The return to Z_i is generated indirectly by increasing the demand for the intermediate good G_{ij} in which it is embodied, as shown in equations (7) and (8).

2.3 Intermediate Goods Sector

The intermediate good sector is the core of the model. There are two dimensions of technology change in this sector – the vertical (quality improvement) dimension and the horizontal (variety) dimension. In the vertical dimension, incumbents engage in innovation to improve the quality of their own products in order to obtain a larger individual market size – and thus, a higher profit. In the horizontal dimension, assuming Bertrand Competition, if entrepreneurs observe an incipient profit, they enter the market with a new variety, and share the market with existing firms. Thus, the market structure is endogenous. Fixed operating costs cause the horizontal expansion to eventually stop.[6] So, the key to long-run growth is the quality improvement.

The intermediate goods sector, like the processed goods sector, is comprised of two heterogeneous industries. In each industry, all firms face identical production, R&D production and demand functions. However, those differ across industries. We prove that in the same industry all firms make the same decisions on prices and the investments in R&D, and thus all firms within the same industry have the same individual market size and are symmetric. However, firms in different industries are heterogeneous. To simplify the analysis, we assume that entry and exit involve zero costs.[7] Thus, the number of firms is free to jump to its equilibrium level, as in Peretto (1996, Oct 1999). We construct an equilibrium where, at time t, firms commit to time-path strategies, while simultaneously, free entry and exit determine the number of firms in the market. A transitional dynamics of quality accumulation is shown.

We construct this equilibrium in three steps. First, we focus on the determination of the price and investment in R&D of the firms that are already active in the market (the incumbents). Next, we focus on the endogenous market structure, which is the free entry and exit decisions and the determination of the number of firms in the market. Finally, we prove that the incumbents make symmetric decisions within the same industry, but asymmetric decisions across industries.

2.3.1 Incumbents

Each intermediate goods industry is comprised of a continuum of monopolistically competitive incumbents, each of which produces a single intermediate good G_{ij} and also undertakes R&D to

[6] See Peretto and Connolly (2007) for more discussion.

[7] This is the assumption in Peretto (1996) and (Oct 1999). For the discussion of costly entry in a similar framework but with symmetric industries, see Peretto (2007).

improve the quality Z_{ij} of the good it produces. An increase in quality boots the demand for the goods, as shown above, and thus raises profit.

Production, technologies, R&D technologies, and costs are the same for all firms within a given industry but differ across the industries. Thus, the industrial structure is symmetrical within each intermediate goods industry, but is asymmetrical across two industries. All firms in industry i have a linear technology that converts A_i units of the final good into one unit of intermediate good G_{ij}:

$$A_i G_{ij} = Y_{ij}, \tag{12}$$

where Y_{ij} is the amount of the final good used by firm j in industry i. Similarly, the R&D production functions are the same within an industry but differ across them:

$$\dot{Z}_{ij} = \alpha_i R_{ij}, \tag{13}$$

where R_{ij} is the amount of the final good Y expended on R&D. The firm obtains the resources for R from retained earnings.

Firms face a fixed operating cost ϕ_{ij} that depends on the average quality of the firm's own industry Z_i, and of the other industry Z_k. There are two channels of influence. First, the operating cost depends positively on own industry quality. A more sophisticated industry is more complex and requires more sophisticated inputs, so the demand for operating cost inputs is increasing in industry quality. Based on the reasonable assumption, commonly made in the literature, that the cost of producing those inputs rises with their sophistication, higher industry quality then implies a higher price for the factors that are used to run the firms' operations. We borrow a page from the adjustment cost literature and assume that fixed operating costs are convex in the level of industry sophistication. Second, operating costs are reduced by advances in knowledge, which in our model is captured by quality. We suppose that all knowledge is useful in reducing operating costs; that is, knowledge spillovers from both an intermediate goods firm's own industry and also from the other industry lower operating costs. Thus, both the own industry knowledge Z_i and the other industry's knowledge Z_j help reduce costs. The general form of the operating cost function is thus $\phi_{ij} = \Phi_{ij}(Z_i; Z_i, Z_k)$ with $\Phi_1' > 0$, $\Phi_{11}' > 0$, $\Phi_2' < 0$, and $\Phi_3' < 0$. To keep the analysis tractable, we assume that all firms in a given industry have the same cost function, which takes the analytically convenient form

$$\Phi_{ij}(Z_i; Z_i, Z_k) = \theta_i \frac{Z_i^3}{Z_i Z_k} = \theta_i \frac{Z_i^2}{Z_k}. \tag{14}$$

The cubic term in the numerator captures the convexity of cost in complexity,[8] and the two terms in the denominator capture the effect of knowledge in reducing costs. The dependence of cost on only industry averages and not the firm's own quality level is not restrictive because, as we show later, firms within a given industry behave symmetrically so that each firm's quality equals the average quality of the industry.

The intermediate goods firm's *gross profit* is revenue substracted by production costs:

$$F_{ij} = P_{G_{ij}} G_{ij} - A_i G_{ij} - \phi_i. \tag{15}$$

[8] We choose a cubic functional form for the tractability of the model.

The firm retains some amount R_{ij} of its gross profit for investment purposes and distributes the rest to its owners. Gross profit net of retained earnings is the net profit:

$$\Pi_{ij} = F_{ij} - R_{ij}. \tag{16}$$

The present discounted value $V_{ij}(t)$ of this net profit is

$$V_{ij}(t) = \int_t^{\infty} \Pi_{ij}(\tau)\, e^{-\int_t^{\tau} r(s)ds}\, d\tau. \tag{17}$$

The firm chooses the paths of its product price $P_{G_{ij}}$ and its R&D expenditure R_{ij} to maximize (17) subject to the demand function (7), the R&D production function (13), and the average qualities, Z_1 and Z_2, which the firm takes as given. The solutions of the maximization problem for the prices are mark-ups over variable cost

$$P_{Gi} \equiv P_{G_{ij}} = \frac{A_i}{\lambda}, \tag{18}$$

where i denotes industry 1 or 2.

Finally, Lemma 1 shows that the return to R&D positively depends on the market size of individual firm l_{ij} where $i = 1, 2$.

Lemma 1 *The returns to in-house R&D are given by*

$$r_{1j} = \delta_1 \alpha_1 A_1 \frac{1-\lambda}{\lambda} \left(\frac{\lambda P_{X_1}}{A_1/\lambda}\right)^{\frac{1}{1-\lambda}} \left(\frac{Z_{1j}}{Z_2}\right)^{(\delta_1+\gamma_1)-1} l_{1j}, \tag{19}$$

$$r_{2j} = \delta_2 \alpha_2 A_2 \frac{1-\lambda}{\lambda} \left(\frac{\lambda P_{X_2}}{A_2/\lambda}\right)^{\frac{1}{1-\lambda}} \left(\frac{Z_{2j}}{Z_1}\right)^{(\delta_2+\gamma_2)-1} l_{2j}. \tag{20}$$

Proof. See Appendix 1. ■

Given the spillover from the other industry, the return to R&D is diminishing in its own quality level, since $\delta_i + \gamma_i$ is between 0 and 1. Thus, a balanced growth requires that the qualities from both industries grow together.[9] The return to R&D has a convenient property that,

$$r_{ij} = \delta_i \alpha_i \frac{(P_{ij} - A_i) G_{ij}}{Z_{ij}}. \tag{21}$$

We keep this convenient property in mind, for it provides clear intuition for cross-industry differences in section (2.7) and (2.8).

[9] Without inter-industry spillovers, industries grow at different rates on the BGP. However, this does not alter the main results of this paper. See the appendix for the details.

2.3.2 Entrants

The value of the firm V_{ij} $(i = 1, 2)$ is defined by equation (17). To determine the entry and exit of the firm, this value V_{ij} has to be compared with the cost of entry and exit. We assume that entry and exit are costless. For simplicity, we refer only to entry. Whenever the net present value of a new firm V_{ij} differs from the zero entry cost, new firms jump in or out to restore equality between the value of the firm and the entry cost. We thus at all times have

$$V_{ij} = 0. \tag{22}$$

Differentiating Eq.(17) with respect to time gives the firm's rate of return on equity (i.e., entry):

$$r_{ij}^{E} = \frac{\Pi_{ij}}{V_{ij}} + \frac{\dot{V}_{ij}}{V_{ij}}. \tag{23}$$

This is a usual perfect-foresight, no-arbitrage condition for the equilibrium of the captial market. It requires two conditions. First, the returns to R&D for all firms in both industries should be the same, otherwise all investment goes to the firms with higher R&D returns. We will revisit this condition again in general equilibrium. Second, the return to firm ownership should be equal to the rate of return on a riskless loan of size V_{ij}. The return to firm ownership is given by the ratio between profit ($\prod_{ij}$) and the firm's stock market value (V_{ij}), plus the capital gain (loss) from the stock appreciation (depreciation). As a result, we also have $\dot{V}_{ij} = 0$. Multiplying both sides of (23) by V_i and imposing $V_{ij} = 0$ and $\dot{V}_{ij} = 0$ implies the *Zero (net) Profit Condition.*

$$\Pi_{ij} = 0, \tag{24}$$

as in Peretto (Oct 1999). Based on the zero profit condition (24) and (16), incumbents account for all the retained gross profits for R&D. The level of R&D expenditure can be expressed as

$$R_{ij} = F_{ij} = P_{G_{ij}} G_{ij} - A_i G_{ij} - \phi_i. \tag{25}$$

Furthermore, the growth rates of qualities can be written as the quality-adjusted gross profit $\frac{F_{ij}}{Z_{ij}}$, times the R&D productivity α_i

$$g_{ij} = \frac{\dot{Z}_{ij}}{Z_{ij}} = \frac{\alpha_i R_{ij}}{Z_{ij}} = \frac{\alpha_i F_{ij}}{Z_{ij}} = \frac{\alpha_i [(P_{ij} - A_i) G_{ij} - \phi_i]}{Z_{ij}}. \tag{26}$$

Now we assume that (1) at the initial time all firms in industry i have the same level of quality $Z_{ij} = Z_i$; and (2) new firms enter with the average quality level Z_i of the industry, following Peretto (Oct. 1999).[10] These two assumptions plus a zero-cost entry/exit condition lead directly

[10] We should clarify one point regarding the stability of the equilibrium. Consider the general situation in which firms have different quality levels. A firm with below-average quality might be tempted to set R&D expenditures to zero and get a free ride to the average level of quality simply by leaving the market and then immediately re-entering with the average quality. If that strategy were profitable, the market equilibrium would be one in which no firms engage in R&D. However, the strategy is not profitable. Once an incumbent entrepreneur leaves the market, he loses all claims to the niche he just vacated. That is the meaning of exit, after all. If he wants to re-enter the market, even in the instant after he leaves, he must join the pool of other potential entrants vying for the vacated niche. There is an uncountable number of potential entrants, so the probability that the former incumbent will reclaim the vacated niche is zero. The strategy of exit and immediate re-entry therefore has an expected value of zero, rendering it unprofitable. Thus an incumbent with below-average quality will not leave the market and then try to re-enter, and the equilibrium is stable with respect to such possible behavior. For a complete discussion of market equilibrium and its stability in these types of R&D models, see Peretto (1996, 1999, 2007) and the references cited therein.

to an equilibrium that is symmetric within each industry, with all firms in an industry always making the same decisions on pricing, R&D expenditures, and market size.

2.4 Households

The economy is populated by a representative household that supplies labor inelastically in a perfectly competitive market and purchases assets (corporate equity). We assume for simplicity that there is no population growth. The utility function of the representative household is

$$U(t) = \int_t^{\infty} log(C)\, e^{-\rho t}, \tag{27}$$

where C is consumption per capita and ρ is the rate of time preference. The household budget constraint is given by

$$a_t = r_t a_t + w_t L - C_t. \tag{28}$$

$a_t = N_t V_t$ is the real value of assets owned by the household, and r_t is the real interest rate. The household has a labor endowment of L units and supplies them inelastically to earn a real wage rate w_t. The intertemporal consumption plan that maximizes discounted utility (27) is given by the consumption Euler equation, which, as usual, can be written as

$$r = \rho + \frac{\dot{C}}{C}. \tag{29}$$

2.5 General Equilibrium

The equilibrium is a time path of allocations $\{a_t, C_t, Y_t, X_t(i), R_t(i)\}$ and prices $\{r_t, w_t, p_t(i), V_t(i)\}$ such that the following conditions are satisfied:

- the household maximizes utility taking $\{r_t, w_t\}$ as given;
- competitive final goods firms in industry i maximize profits taking $\{p_t(i), w_t\}$ as given;
- incumbents in industry i choose $\{p_t(i), R_t(i)\}$ to maximize $\{V_t(i)\}$ taking $\{r_t\}$ as given;
- entrants in industry i make entry decisions taking $\{V_t(i)\}$ as given;
- the value of all existing monopolistic firms adds up to the value of the household's assets such that $a_t = \sum_{i=1}^{2} N_{it} V_{it}$;
- labor demand equals labor supply, $L = L_1 + L_2 = \epsilon L + (1-\epsilon)L$;
- the no-arbitrage condition in the credit market requires that $r_1 = r_2 = r$, and
- the market-clearing condition of final goods holds.

The market-clearing condition of final goods is

$$Y_t = C_t + \sum_{i=1}^{i=2} N_{it}(X_{it} + \phi_i + R_{it}). \tag{30}$$

Combining the demand functions of intermediate goods (7) and (8) in the final goods production (1) to eliminate G_i, we obtain

$$Y = \kappa Z_1^{\Gamma} Z_2^{1-\Gamma} L, \tag{31}$$

where κ and Γ are constants.[11] The growth rate of Y is a weighted average of the growth rates of the Z_i:

$$\frac{\dot{Y}}{Y} = \Gamma g_1 + (1-\Gamma)\, g_2. \tag{32}$$

The growth rates g_1 and g_2 are given by equation (26) with $i = 1, 2$, and are functions of the model parameters, the current values of the state variables Z_1 and Z_2, and of N_1 and N_2.

We now analyze the dynamics of the economy. In Lemma 2, we show that the consumption-output ratio C_t/Y_t jumps to a unique and stable steady-state value. This equilibrium property simplifies the analysis of transition dynamics.

Lemma 2 *The consumption-output ratio jumps to a unique and stable steady-state value:*

$$(C/Y)^* = 1 - \lambda. \tag{33}$$

Proof. See the household budget constraint (28), zero profit condition (24), and $wL = (1-\lambda)Y$. ■

Equations (29), (32) and (33) imply that,

$$\Gamma \frac{\dot{Z}_1}{Z_1} + (1-\Gamma)\frac{\dot{Z}_2}{Z_2} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = r - \rho. \tag{34}$$

The model permits a full characterization of the economy's transition dynamics. Under the restriction $\delta_1 \geq \delta_2$, the dynamic system is stable and all endogenous variables of interest can be expressed as a function of Z_1/Z_2 only, as shown in Lemma 3.

Lemma 3 *The dynamic system of all endogenous variables is expressed as follows:*

$$\begin{aligned}
(\frac{\dot{Z}_1}{Z_1})/(\frac{Z_2}{Z_2}) &= \frac{\dot{Z}_1}{Z_1} - \frac{\dot{Z}_2}{Z_2} \\
&= \frac{\alpha_1[(P_1 - A_1)G_1 - \phi_1]}{Z_1} - \frac{\alpha_2[(P_2 - A_2)G_2 - \phi_2]}{Z_2} && (35)\\
&= \alpha_1[r_1/\alpha_1\delta_1 - \frac{\phi_1}{Z_1}] - \alpha_2[r_2/\alpha_2\delta_2 - \frac{\phi_2}{Z_2}] && (36)\\
&= \frac{\delta_1(1-\delta_2)\alpha_1\theta_1\frac{Z_1}{Z_2} - \delta_2(1+\delta_1)\alpha_2\theta_2\frac{Z_2}{Z_1} + (\delta_1 - \delta_2)\rho}{\delta_1\delta_2 - \delta_2\Gamma - \delta_1(1-\Gamma)}, && (37)
\end{aligned}$$

$$(\frac{N_1}{N_2}) = \frac{\alpha_1\delta_1}{\alpha_2\delta_2}\frac{\epsilon}{1-\epsilon}(\frac{Z_1}{Z_2})^{-1}, \tag{38}$$

[11] See the appendix for the details.

$$\frac{l_1}{l_2} = \frac{\epsilon L/N_1}{(1-\epsilon)L/N_2} = (\frac{\alpha_1\delta_1}{\alpha_2\delta_2})^{-1}\frac{Z_1}{Z_2}. \tag{39}$$

We define industry research intensity as R&D/Sales, which is $RND_i \equiv \frac{R_i}{P_iG_i}$ as in Klenow (1996) and

$$\frac{RND_1}{RND_2} = \frac{\delta_1\frac{\dot{Z}_1}{Z_1}}{\delta_2\frac{\dot{Z}_2}{Z_2}}. \tag{40}$$

Proof. See the Appendix. ■

On the BGP, the growth rates of Z_1 and Z_2 are equal, and the ratio Z_1/Z_2 is constant. Then the following growth rates, including the growth rates of both industries, are equal:

$$g^\star = \frac{\dot{Z}_1}{Z_1} = \frac{\dot{Z}_2}{Z_2} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{X}_1}{X_1} = \frac{\dot{X}_2}{X_2} = \frac{\dot{G}_1}{G_1} = \frac{\dot{G}_2}{G_2} = \frac{\dot{w}}{w}. \tag{41}$$

Lemma 4 provides the results under the BGP:

Lemma 4 *The growth rate of the economy and the numbers of firms in both industries on the BGP are provided by:*

$$N_1^* = \frac{\alpha_1\delta_1\frac{1-\lambda}{\lambda}[\lambda^2\epsilon]^{\frac{1}{1-\lambda}}(\frac{\epsilon}{1-\epsilon})^{\frac{\epsilon-1}{1-\lambda}}}{\frac{\delta_1}{1-\delta_1}\alpha_1\phi_1(\frac{Z_1}{Z_2})^\star - \frac{\delta_1}{1-\delta_1}\rho}[(\frac{Z_1}{Z_2})^\star]^{\Gamma-1}A_1^{\frac{-\lambda\epsilon}{1-\lambda}}A_2^{\frac{-\lambda(1-\epsilon)}{1-\lambda}}(\epsilon L), \tag{42}$$

$$N_2^\star = \frac{\alpha_2\delta_2\frac{1-\lambda}{\lambda}[\lambda^2(1-\epsilon)]^{\frac{1}{1-\lambda}}(\frac{\epsilon}{1-\epsilon})^{\frac{\epsilon}{1-\lambda}}}{\frac{\delta_2}{1-\delta_2}\alpha_2\phi_2(\frac{Z_2}{Z_1})^\star - \frac{\delta_2}{1-\delta_2}\rho}[(\frac{Z_1}{Z_2})^\star]^{\Gamma}A_2^{-\frac{\lambda\epsilon}{1-\lambda}}A_1^{-\frac{\lambda(1-\epsilon)}{1-\lambda}}(1-\epsilon)L, \tag{43}$$

$$g_1^* = g_2^* = \frac{\delta_1}{1-\delta_1}\alpha_1\phi_1(\frac{Z_1}{Z_2})^\star - \frac{1}{1-\delta_1}\rho, \tag{44}$$

$$\left(\frac{Z_1}{Z_2}\right)^\star = \frac{(\frac{\delta_1}{1-\delta_1} - \frac{\delta_2}{1-\delta_2})\rho + \sqrt{[(\frac{\delta_2}{1-\delta_2} - \frac{\delta_1}{1-\delta_1})\rho]^2 + 4\frac{\delta_1}{1-\delta_1}\frac{\delta_2}{1-\delta_2}\alpha_1\phi_1\alpha_2\phi_2}}{2\frac{\delta_1}{1-\delta_1}\alpha_1\phi_1}. \tag{45}$$

Proof. See the Appendix. ■

2.6 Comparative Statics Analysis

Equation (44) shows the following results: first, the steady state growth $g^\star$ depends positively on both industry-specific R&D productivities α_i and α_j. Intuitively, an increase in α_i increases $g^\star$ because a higher industry-specific R&D productivity directly increases the firm's return to R&D so that it increases the firm's R&D expenditure and hence contributes to both the industry TFP growth rate and thus the steady state growth rate.

Second, the steady state growth positively depends on its industry-specific fixed operating cost parameters θ_i and θ_j. At the first glance, it seems quite counter-intuitive. A higher θ_i

decreases the profit for incumbents and should thus decrease the return to in-house R&D and reduce TFP growth. Nonetheless, the endogeneity of market structure implies that the number of firms N_i also decreases with the firm's profit which in turn drives up the individual market size of firms L_i/N_i. A larger firm market size instead increases the return to R&D and TFP growth. In our model, the latter effect dominates the former, leading to our result that the growth rate on the BGP is positively related to fixed operating costs. This implication had been ignored in the previous literature until Peretto (2007), who formally introuced fixed operating costs into the analysis and obtained similar results.

In addition, the steady growth rate is unrelated to the unit costs of production, $A_i's$. A change in unit costs leads to two opposite effects. On the one hand, a decrease in unit costs directly causes an increase in the return to R&D and hence the TFP growth rate. On the other hand, a decrease in unit costs leads to a higher incipient profit which induces an immediate entry to share the firm market size L_i/N_i and hence decreases the return to R&D and TFP growth. These two effects cancel each other out, so the growth rate is not affected by unit costs.

Finally, the growth rate is independent of the economic scale L. The endogenous market structure in our model indicates that in response to the firm's profit-seeking behavior, a firm's entry or exit leads the number of firms $N_i^\star$ to increase or decrease (proportionally) with aggregate market size L and thus leaves the firm sizes in both industry 1, $\epsilon L/N_1^\star$ and industry 2, $(1-\epsilon)L/N_2^\star$ unchanged. Given that the firm's R&D incentive depends on the firm size rather than the aggregate size, the balanced growth rate of innovation is unchanged as well. Thus, the growth rate (44) does not contain L (the scale effect).

Proposition 1 *The R&D productivity α_i has a positive impact on the balanced growth rate $g^\star$ and the number of firms $N_i^\star$. The fixed operating cost parameter θ_i has a positive impact on $g^\star$ but a negative impact on $N_i^\star$. The unit cost A_i has no impact on $g^\star$ while it has a negative impact on $N_i^\star$. The economic scale L has no impact on $g^\star$ while it has a positive impact on $N_i^\star$.*

Proof. See Lemma 4. $\frac{\partial g^\star}{\partial \alpha_i} > 0$, $\frac{\partial g^\star}{\partial \theta_i} > 0$, $\frac{\partial g^\star}{\partial A_i} = 0$, $\frac{\partial g^\star}{\partial L} = 0$, $\frac{\partial N_i^\star}{\partial \alpha_i} > 0$, $\frac{\partial N_i^\star}{\partial \theta_i} < 0$, $\frac{\partial N_i^\star}{\partial A_i} < 0$, $\frac{\partial N_i^\star}{\partial L} > 0$. ■

3 Growth Differences Across Industries

This section examines the determinants of the differences in research intensity and TFP growth across industries. The model's predictions are substantially different from those in the previous literature because of the endogeneity of market structure and the subsequent dynamics that are absent in other models. In the folowing subsections, we examine how a change in R&D productivity leads to the dynamic changes in market structure (relative number of firms and firm size), relative research intensity and TFP growth rates across industries.

3.1 R&D productivity

The first striking finding emerges in the relation between R&D productivity α_i and relative innovation growth (relative R&D intensity) as shown in (37) and (40). It shows that, during the transitional dynamics, the industry with relatively high R&D productivity has a relatively

Insert Figure 1 Here

Figure 1: The effects of an increase in α_1

low in-house innovation growth and R&D intensity.[12] The key is again the endogeneity of the market structure.

To emphasize the endogenous market structure of the model, we first presume that the market structure is exogenous. In this case, since the individual firm market sizes l_1 and l_2 are exogenous and does not change in response to the R&D productivity α_i, the quality-adjusted gross profit flows $\frac{[(P_1-A_1)G_1-\phi_1]}{Z_1}$ and $\frac{[(P_2-A_2)G_2-\phi_2]}{Z_2}$ in (35) do not change either. An increase of R&D productivity α_1 relative to α_2 directly contributes to a relatively higher R&D intensity and thus higher growth rate of Z_1 than that of Z_2 as shown in (35) and (40). Klenow (1996), Giordani and Zamparelli (2008) and Ngai and Samaniego (2009) predict the similar results due to the exogenous market structure in their models. On the contrary, our model shows that the relative firm size l_1/l_2 is endogenous and changes in response to firm entry. At the instant of the productivity change, given the predetermined quality levels Z_1 and Z_2, a higher α_1 increases the relative profitability for the firms in industry 1, leading to an increase in N_1/N_2, i.e., firms enter industry 1 and exit industry 2, thus leading to a fall in l_1/l_2. This fragmentation (concentration) of the firm market size in industry 1 (industry 2) instead reduces the quality-adjusted gross profit $\frac{[(P_1-A_1)G_1-\phi_1]}{Z_1}$ in industry 1 but increases $\frac{[(P_2-A_2)G_2-\phi_2]}{Z_2}$ in the other industry. A relatively low firm gross profits in industry 1 implies relatively small amounts of resources devoted to in-house R&D in industry 1 and therefore contributes to relatively low R&D intensity and a lower TFP growth rate.

In the following dynamics, given that the growth rate g_1 is lower than g_2, the relative quality level Z_1/Z_2 decreases over time. This result in turn reduces the gap between g_1 and g_2 (and thus the differences between RND_1 and RND_2 as well). The economic rationale behind this is that since g_2 is greater than g_1, inter-industry knowledge spillover from industry 2 to industry 1 becomes stronger than the opposite (from industry 1 to industry 2), which benefits the firms in industry 1 more than those in industry 2, raising the relative quality-adjusted gross profit $\frac{[(P_1-A_1)G_1-\phi_1]}{Z_1}/\frac{[(P_2-A_2)G_2-\phi_2]}{Z_2}$ at each point in time during the dynamics. As a result, more R&D resources shift back from industry 2 to industry 1 and therefore, the TFP growth and thus R&D intensity in industry 1 gradually converges to that of industry 2.

The dynamic response of the market structure to R&D productivity can be shown in (38) and (39). The instant of increasing R&D productivity α_1 gives rise to an immediate increase (decrease) in N_1/N_2 (l_1/l_2) as described above. After that, due to gradually stronger inter-industry knowledge spillovers in a dynamic context, the decrease of Z_1/Z_2 leads to a continuous increase (decrease) in N_1/N_2 (l_1/l_2).

In sum, the endogenous change in l_1/l_2 to R&D productivity α_1, in the short run, changes the relative TFP growth g_1/g_2 in the opposite direction as predicted in the previous models with exogenous market structure. See Figure 1 for a summary.

3.2 Appropriability

In this subsection, we investigate the change in "appropriability" which is captured by δ_i. An increase in δ_1 increases the appropriability of private knowledge and reduces the knowledge

[12] Note that higher industry R&D productivity, even though it reduces the relative industry TFP growth (the ratio of the industry TFP growth to the other industry TFP growth), still increases its industry TFP growth in terms of the absolute level.

Insert Figure 2 Here

Figure 2: The effects of an increase in δ_1

spillover in industry 1.

Interestingly, as exhibited in (37) and (40), the industry with relatively high appropriability also exhibits relatively low in-house innovation growth and R&D intensity[13] during the transitional dynamics. The underlying economic rationale is similar to the previous analyse of R&D productivity.

An increase in appropriability δ_1 relative to δ_2, if the market structure were exogenous (l_i is fixed), would increase the rate of return r_1 relative to r_2 as compared with (19) and (20). With an endogenous market structure in our model, however, at the instant that δ_1 changes, a higher return r_1 relative to r_2 will lead to an immediate increase in N_1 but a decrease in N_2 to equalize the rates of return. This in fact will decrease the firm's (quality adjusted) profit flow $\frac{r_1}{\alpha_1\delta_1} - \frac{\phi_1}{Z_1}$ in industry 1 but raise its counterpart $\frac{r_2}{\alpha_2\delta_2} - \frac{\phi_2}{Z_2}$ in (36), resulting in a relatively low amount of firm in-house R&D expenditure and thus relatively low R&D intensity and TFP growth rate. This finding also contradicts the predictions shown in Klenow (1996) and Ngai and Samaniego (2009) where their models feature an exogenous market structure.

Over time, since the growth rate g_1 is lower than g_2, the relative quality level Z_1/Z_2 will start to decrease. This result in turn reduces the gap between g_1 and g_2 and thus reduces the difference between RND_1 and RND_2. In the long run, g_1 converges to g_2 and RND_1 converges to RND_2. The results are summarized in Figure 2.

3.3 Industry market size

Equation (37) shows that the dynamics of cross-industry TFP growth differences and research intensity ratio do not depend on industry-level market sizes L_1 and L_2. The reason is that changing industry-level market size does not affect the firm market size l_i. The endogenous entry immediately absorbs the change, see (42) and (43). Therefore, industry market size differences do not affect the cross-industry quality growth differences and research intensity. This result also contrasts with the finding in Klenow (1996) who uses a two-sector variant of the variety-expansion model but is similar to the finding in Ngai and Samaniego (2009) using a semi-endogenous growth framework.

Besides, the ratio of the number of firms and firm market size do not depend on industry market size although the absolute levels of N_1 and N_2 do correspond to the change in industry market size.

The above results are summarized in Proposition 2.

Proposition 2. In response to a higher R&D productivity α_1 and appropriability δ_1, the firms enter industry 1 but exit industry 2, which makes the relative number of firms N_1/N_2 jump up (the relative firm market size l_1/l_2 jumps down). The relative TFP growth g_1/g_2 and R&D intensity ratio RND_1/RND_2 become less than one. Over time, N_1/N_2 (l_1/l_2) goes further up

[13] Note that higher industry R&D productivity, even though it reduces the relative industry TFP growth (the ratio of its industry TFP growth to the other industry TFP growth), still increases its industry TFP growth in terms of the absolute level.

(down) to a higher (lower) steady state value, g_1/g_2, and RND_1/RND_2 gradually converges back to 1. In addition, industry market size differences do not affect the cross-industry quality growth differences and research intensity ratio at both the steady state and transitional dynamics.

Proof. See Lemma 3. $\frac{\partial(Z_1/Z_2)}{\partial\alpha_1} < 0$, $\frac{\partial(N_1/N_2)}{\partial\alpha_1} > 0$, $\frac{\partial(l_1/l_2)}{\partial\alpha_1} < 0$, $\frac{\partial(RND_1/RND_2)}{\partial\alpha_1} < 0$; $\frac{\partial(Z_1/Z_2)}{\partial\delta_1} < 0$, $\frac{\partial(N_1/N_2)}{\partial\delta_1} > 0$, $\frac{\partial(l_1/l_2)}{\partial\delta_1} < 0$, $\frac{\partial(RND_1/RND_2)}{\partial\delta_1} < 0$; $\frac{\partial(Z_1/Z_2)}{\partial L_i} = \frac{\partial(RND_1/RND_2)}{\partial L_i} = 0$. ■

This model predicts differently on relative research intensity and across-industry TFP growth differences in comparsion with the previous literature. The endogenous market structure plays a key role as what we emphasized in the previous sections.

4 The Implications of R&D Policies

Now we will evaluate two government R&D policies: the R&D subsidy and patent breadth protection across different industries. We examine both the symmetric rule (in which the government implements the policy to the same extent across different industries) and the asymmetric rule (in which the government implements policy to different extents across different industries) of R&D policies.

4.1 Asymmetric rule

Suppose the government imposes an R&D subsidy s_1 in industry 1 but not in industry 2. The profit of a firm in industry 1 now becomes

$$\pi_{1j} = G(P_{G1j} - A_1) - \phi_1 - (1 - s_1)R_{1j}.$$

Then return to R&D in industry 1 becomes

$$r_{1j} = \delta_1 \frac{\alpha_1}{1-s} \frac{(P_{1j} - A_1)G_{1j}}{Z_{1j}}. \tag{46}$$

The effect of a subsidy on R&D is similar to the effect of an increase in R&D productivity, hich is shown by the comparision between (21) and (46). Specifically, an instant increase in R&D subsidy s_1 on industry 1 (which is isomorphic to an increase in α_1) increases the relative profitability for the firms in industry 1, raises N_1/N_2 to equalize the innovation returns in both industries and thus leads to a fall in l_1/l_2. This immediate fall in the relative firm market size reduces the relative (quality-adjusted) gross profit, decreases in-house R&D expenditure per firm in industry 1 relative to industry 2 and therefore contributes to a relatively low R&D intensity and TFP growth rate.

Second, suppose the government imposes patent breadth protection on industry 1 but not on industry 2. We find that this asymmetric policy has no effect on cross TFP growth differences. To be more specific, to analyze the effects of patent breadth, we impose an upper bound $\mu > 1$ on the markup, following Li (2001), Goh and Olivier (2002), Chu (2011), Chu and Furukawa (2011), Iwaisako and Futagami (2013), and Chu et al. (2014). Therefore, the equilibrium price becomes $P_{G_1} = \min\{\mu A_1, A_1/\lambda\}$. Assume that $\mu < 1/\lambda$. In this case, a larger patent breadth μ leads to a higher markup in industry 1.[14] An increase in the markup (a higher μ), on the one hand,

[14] This implication is consistent with Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

directly causes an increase in the return to R&D. On the other hand, it causes a higher incipient profit and induces firm entry, which reduces firm size L_i/N_i and hence reduces the return to R&D. These two effects cancel each other out. The R&D intensity and thus TFP growth rates are not affected by the patent breadth policy.

4.2 Symmetric rule

Finally, a symmetric policy for both industries has no impact on relative TFP growth and relative research intensity. The symmetric subsidies on R&D are equivalent to an increase in the rates of returns to R&D across industries by the same percentage. See (46). That is equivelant to an increase in α_i to $\frac{\alpha_i}{1-s}$. Lemma 3 shows that a change in $\alpha_i's$ by the same percentage does not change anything. The symmetric patent breadth protection has no impact on the industry differences based on the similar intuition as for the asymmetric rule.

5 Conclusion

In this study, we have analyzed the effects of changing industry fundamentals, namely, technology opportunities, market size, and appropriability, on TFP growth and R&D intensity differences across industries in a Schumpeterian growth model with endogenous market structure.

Our model predicts that the industry with relatively high R&D productivity or appropriability exhibits a relatively *low* TFP growth and R&D intensity during transition, resulting in a lower relative TFP level in the steady state. The key mechanism is the endogenous market structure which leads to novel findings in contrast to the previous studies. In addition, the industry market size has no impact on the TFP growth and R&D intensity differences. This result is, however, consistent with that of the semi-endogenous growth model but differs in terms of the underlying mechanism. Furthermore, when we analyze the asymmetric and symmetric R&D and patent policies across industries, we find that only asymmetric R&D subsidies have impacts on TFP growth and R&D intensity differences. An interesting extension is to study the impacts of both R&D and patent policies on social welfare.

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Proof of Lemma 1 The firm's current-value Hamiltonian is

$$CVH_{ij} = G_{ij}(P_{Gij} - A_i) - \phi_i - R_{ij} + q_{ij}(\alpha_i R_{ij})$$

where i denotes the industry, and q_{ij} is the co-state variable. By taking the first-order derivative subject to $P_{G_{ij}}$, the solutions for the prices are mark-ups over variable cost:

$$P_{G1} \equiv P_{G_{1j}} = \frac{A_1}{\lambda} \tag{47}$$

$$P_{G2} \equiv P_{G_{2j}} = \frac{A_2}{\lambda} \tag{48}$$

The Hamiltonian is linear in R&D expenditure, and so the solution for investment expenditure R_{1j} is bang-bang:

$$R_{ij} \begin{cases} = \infty & \text{if } 1/\alpha > q_{ij} \\ > 0 & \text{if } 1/\alpha = q_{ij} \\ = 0 & \text{if } 1/\alpha < q_{ij} \end{cases}$$

We rule out the first possibility of $R_{ij} = \infty$ because it is inconsistent with the market equilibrium. We also rule out the other corner solution, $1/\alpha < q_{ij}$, because it implies no economic growth, and we am interested here in the case where perpetual growth occurs. We thus have the interior solution

$$\frac{1}{\alpha_i} = q_{ij} \tag{49}$$

The left side of eq. (49) is the same for all j, so that all firms in industry i choose the same level of R&D, which we denote as R_i.

The Maximum Principle gives the necessary condition for the evolution of the co-state variable q_1, which we can rearrange as

$$r_{ij} = \frac{\partial F_{ij}}{\partial Z_{ij}} \frac{1}{q_{ij}} + \frac{\dot{q_{ij}}}{q_{ij}} \tag{50}$$

This equation defines the rate of return to R&D (i.e., on quality innovation), with r_{ij} as the percentage marginal revenue from R&D plus the capital gain (the percentage change in the shadow price). Because $1/\alpha_i = q_{ij}$, we also have $\dot{q}_{ij}/q_{ij} = 0$. As with intermediate goods prices, the expressions for the rates of return differ across the two industries. The rate of return for industry 1 in (19) is obtained by substituting (7), (15), (47), and (49) into (50). Following the same steps as in industry 1, we obtain the rate of return to R&D in industry 2. ■

Proof of Lemma 3 Combining (34), (21) and (26) leads to

$$\begin{aligned} r &= \rho + g = \Gamma g_1 + (1-\Gamma) g_2 + \rho \\ &= \Gamma(\frac{r_1}{\delta_1} - \alpha_1 \theta_1 \frac{Z_1}{Z_2}) + (1-\Gamma)(\frac{r_2}{\delta_2} - \alpha_2 \theta_2 \frac{Z_2}{Z_1}) + \rho. \end{aligned}$$

Imposing $r = r_1 = r_2$ leads to

$$r = \frac{-\Gamma \alpha_1 \theta_1 \frac{Z_1}{Z_2} - (1-\Gamma)\alpha_2 \theta_2 \frac{Z_2}{Z_1} + \rho}{1 - \Gamma/\delta_1 - (1-\Gamma)/\delta_2}.$$

Substituting the above equation into $\frac{\dot{Z}_1}{Z_1} - \frac{\dot{Z}_2}{Z_2}$ leads to

$$
\begin{aligned}
(\frac{\dot{Z}_1}{Z_1})/(\frac{Z_2}{Z_2}) &= \frac{\dot{Z}_1}{Z_1} - \frac{\dot{Z}_2}{Z_2} \\
&= \frac{\alpha_1[(P_1 - A_1)G_1 - \phi_1]}{Z_1} - \frac{\alpha_2[(P_2 - A_2)G_2 - \phi_2]}{Z_2} \\
&= \alpha_1[r_1/\alpha_1\delta_1 - \frac{\phi_1}{Z_1}] - \alpha_2[r_2/\alpha_2\delta_2 - \frac{\phi_2}{Z_2}] \\
&= (\frac{1}{\delta_1} - \frac{1}{\delta_2})r - \alpha_1\theta_1\frac{Z_1}{Z_2} + \alpha_2\theta_2\frac{Z_2}{Z_1} \\
&= (\frac{1}{\delta_1} - \frac{1}{\delta_2})\frac{-\Gamma\alpha_1\theta_1\frac{Z_1}{Z_2} - (1-\Gamma)\alpha_2\theta_2\frac{Z_2}{Z_1} + \rho}{1 - \Gamma/\delta_1 - (1-\Gamma)/\delta_2} - \alpha_1\theta_1\frac{Z_1}{Z_2} + \alpha_2\theta_2\frac{Z_2}{Z_1} \\
&= \frac{\delta_1(1-\delta_2)}{\delta_1\delta_2 - \delta_2\Gamma - \delta_1(1-\Gamma)}\alpha_1\theta_1\frac{Z_1}{Z_2} + \frac{-\delta_2(1+\delta_1)\alpha_2\theta_2\frac{Z_2}{Z_1}}{\delta_1\delta_2 - \delta_2\Gamma - \delta_1(1-\Gamma)} + \frac{\delta_1 - \delta_2}{\delta_1\delta_2 - \delta_2\Gamma - \delta_1(1-\Gamma)}\rho
\end{aligned}
$$

By the no-arbitrage condition $r_1 = r_2$, the dynamics of the relative number of firms N_1/N_2 can be expressed as (38), through which we can also obtain the dynamics of the firm size ratio as (39). Combining $r_1 = r_2$ leads to (40). ■

Proof of Lemma 4 We obtain the quality ratio (45) on the BGP by noting that $g_1 = g_2 = g^*$, $r_1 = r_2 \equiv r$, and, from the Euler equation, $r = g^* + \rho$. From those relations, we obtain a quadratic form with two roots:

$$
\begin{aligned}
\left(\frac{Z_1}{Z_2}\right)^\star &= \frac{(\frac{\delta_1}{1-\delta_1} - \frac{\delta_2}{1-\delta_2})\rho + \sqrt{[(\frac{\delta_2}{1-\delta_2} - \frac{\delta_1}{1-\delta_1})\rho]^2 + 4\frac{\delta_1}{1-\delta_1}\frac{\delta_2}{1-\delta_2}\alpha_1\phi_1\alpha_2\phi_2}}{2\frac{\delta_1}{1-\delta_1}\alpha_1\phi_1} > 0, \\
\left(\frac{Z_1}{Z_2}\right)^\star &= \frac{(\frac{\delta_1}{1-\delta_1} - \frac{\delta_2}{1-\delta_2})\rho - \sqrt{[(\frac{\delta_2}{1-\delta_2} - \frac{\delta_1}{1-\delta_1})\rho]^2 + 4\frac{\delta_1}{1-\delta_1}\frac{\delta_2}{1-\delta_2}\alpha_1\phi_1\alpha_2\phi_2}}{2\frac{\delta_1}{1-\delta_1}\alpha_1\phi_1} < 0.
\end{aligned}
$$

We discard the negative solution which is economically implausible. We derive the number of firms $N_i^\star$ by combining (19), (26), (29) and (35). ■