Central Bank Communication and Inflation*

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Abstract

The anchoring of inflation expectations in stochastic environments is known to be achieved only under special circumstances, e.g. when the analysis is restricted to locally bounded equilibria near a non-stochastic steady state, or when the "right" mix of monetary and fiscal policy is pursued. We abstract from such considerations and demonstrate that the communication of information by central banks about the future state of the economy can anchor inflation expectations as effectively, steering the path of inflation towards its target, with this regardless of how noisy communication may be. Furthermore, we show how direct communication, i.e. the transmission of signals about future values of variables of interest, can be recast in terms of forward guidance—an indirect form of communication.

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1 Introduction

Long-run price stability is a natural goal for central banks (Reis, 2013). While it still remains unclear whether central banks should favor a price-path target over an inflation target, everyone recognizes the need for central banks to ensure the stability of inflation expectations—it is on them that this paper focuses.

Conventional policies like Taylor-type rules, however, are known to be inherently destabilizing. Here, in an environment in which central banks set rates according to a Taylor-type rule, we characterize the effect of information transmission, or, more broadly, central bank communication, on the stochastic path of inflation. We demonstrate that central bank communication is an essential tool for monetary policy that can help anchor the public's, otherwise arbitrary, long-run inflation expectations.

Broadly defined, central bank communication involves the provision of information by central banks to the public about current matters of monetary policy, the economic outlook, even the outlook for its own policy decisions—the latter is known as forward guidance. By central bank communication, or forward guidance about the future course of monetary policy, we refer to the so called "Delphic" communication/forward guidance in the parlance of Campbell et al. (2012), which implies no commitment for a central bank and serves solely as a means to transmit a central bank's superior information to the market participants. We also distinguish between forward guidance and direct communication, which in our terminology is the transmision of information about the future state of the economy to the market participants. A first contribution is that we show the existence of a direct mapping between forward guidance and direct communication: direct communication can be recast as forward guidance and vice versa.

Turning to the paper's main point, it is well-known that, in stochastic environments, conventional monetary policies fail to anchor inflation expectations. For instance, Bloise et al. (2005) and Nakajima and Polemarchakis (2005) demonstrate that arbitrary interest-rate rule policies can only pin down expected inflation, *not* its distribution. Cochrane (2011) considers the standard New-Keynesian framework and argues that explosive inflation paths, which the Taylor principle is meant to rule out, remain still in line with the

model's optimality conditions. Benhabib et al. (2001) allow for the zero-lower-bound constraint and demonstrate the possibility of self-fulfilling fluctuations even in deterministic environments.[1] The contribution of this paper is to demonstrate that central bank communication can anchor inflation expectations effectively, steering the stochastic path of inflation towards its target.

More to the point, we assume that a monetary authority follows an interest-rate rule, whose systematic component is common knowledge—assuming less transparency or lack thereof would only strengthen our conclusion. We let the monetary authority communicate noisy information about the future values of fundamental (e.g., productivity) or non-fundamental (e.g., monetary policy) shocks, whose, say, zero-mean priors are also common knowledge. Communication may be direct, when the central bank communicates its noisy forecast of a variable of interest such as productivity directly, or indirect, when it communicates an endogenous signal like the inflation forecast—it makes no difference, as we show.

The key equation to consider is the Euler equation, which connects the current-period consumption to the expected consumption in the following period and the expected real interest rate. The Euler equation implies that the expected inflation next period depends on the current values of shocks and their expected values next period. When agents have no information besides their zero-mean priors about the shocks that lie further than the next period into the future (i.e. from two periods ahead onwards), their respective expectations are zero mean, i.e. they are "unshaped" in our terminology, hence in line with the Euler equation. As a result, come tomorrow, the *then* agents' expectations of the shocks in the period right afterwards can affect the *tomorrow's realized* inflation rate in an arbitrary way. When, on the other hand, agents have information about the shocks that lie further into the future, say two periods ahead, their expectations thereof are not zero (conditional) mean any longer, i.e. they become "shaped"—even minimally so when information is very noisy. But, when this becomes the case, they cease being consistent with the Euler equation, hence they cannot matter. By shaping the agents' expectations, then, a

[1] A frequently-suggested remedy is non-Ricardian fiscal policy, which has been suggested, among others, by Sargent and Wallace (1975), Woodford (1994), Nakajima and Polemarchakis (2005), and Cochrane (2011). Here, we completely abstract from fiscal-policy considerations.

monetary authority is able to render them completely irrelevant, a seemingly paradoxical, in our view, result.

By simply providing more information, however, a central bank only "postpones" this endemic indeterminacy. If, say, a central bank were to provide information about the shocks two periods ahead instead of just one, the above argument would repeat and, then, it would be the agents' expectations about the shocks three periods ahead that would affect the following period's inflation rate arbitrarily. This, in turn, would call for the provision of even more information, say up to three periods ahead, and so forth.

Therefore, we conclude that a central bank can phase out the *inherent* inflation indeterminacy by providing information that spans the agents' entire planning horizon; if the latter is infinite, then it is "asymptotic communication" that is needed. Interestingly, the effectiveness of central bank communication in pinning inflation down does not hinge on how noisy it may be. In other words, noisy communication is as effective in determining future inflation as the full resolution of uncertainty.

Finally, we show that casting direct communication about variables of interest, such as productivity or preference shocks, as an indirect form of communication, like the inflation forecasts, requires a central bank to convey its forecasts, not only of mean inflation, but also of the inflation variance for the entire planning horizon of the agents.

The paper is organized as follows. Section 2 presents the main setup. Section 3 analyzes central bank communication in the presence of only productivity shocks. Section 4 extends the argument to the case of more shocks. Section 5 analyzes monetary policy rules with a nonsystematic component. Section 6 extends the argument to incomplete information and real indeterminacy. Section 7 concludes.

1.1 Related literature.

To be added.

2 The Economy

Consider a frictionless, monetary (cashless) representative-agent economy, along the lines of Woodford (2003). Time is discrete and can be either finite or infinite. The equilibrium conditions in log-linear form are given by

$$y_t = a_t \tag{1}$$

$$E_t\left[y_{t+1}\right] - y_t = i_t - E_t\left[\pi_{t+1}\right], \tag{2}$$

where y denotes output, a productivity, i the nominal interest rate, π denotes inflation (lowercase variables refer to logs).[2]

A monetary authority sets the nominal interest rate according to an interest-rate rule that is known to the agents. For simplicity, let the monetary authority initially target expected inflation with a zero-inflation target:

$$i_t = \phi\, E_t\left[\pi_{t+1}\right], \tag{3}$$

where ϕ can take any nonnegative value.

Eqs. (2)-(3) imply the Euler equation can be rewritten as

$$E_t\left[\pi_{t+1}\right] = \frac{1}{\phi - 1}\left(E_t\left[a_{t+1}\right] - a_t\right). \tag{E}$$

It is evident that eq. (E) only pins down the expected inflation, not its distribution. This remark is in line with the more general result in Nakajima and Polemarchakis (2005) and will serve as our starting point.

This result, it is useful to remark, is a mere outcome of uncertainty. In a perfect-foresight economy, we would simply drop the expectations operators and the (then well-defined) inflation would be given by

$$\pi^{pf}_{t+1} = \frac{1}{\phi - 1}\left(a_{t+1} - a_t\right). \tag{4}$$

[2] Appendix A.1 offers details.

2.1 Shock processes and signals

We assume that the current log-productivity, a_t , follows a white noise process:

$$a_t \;=\; \epsilon_t \,, \tag{5}$$

where ϵ is i.i.d. with $\epsilon \sim N(0, \sigma_\epsilon^2)$.

In addition, we assume that the agents and the monetary authority (may) observe, at date t, noisy public signals about the future productivity innovations

$$s_{t+\tau} \;=\; a_{t+\tau} \;+\; e_{t+\tau} \,, \tag{6}$$

where $s_{t+\tau}$ refers to a noisy signal about the productivity innovation in period $t+\tau$, with $\tau \geq 1$, and e is an i.i.d. *noise* shock with $e \sim N(0, \sigma_e^2)$ and σ_e finite. The two shocks ϵ and e are mutually independent, and so are they with the policy shock v .

Both the agents and the monetary authority are Bayesian hence, in the presence of a (bounded) noisy signal about the productivity innovation in $t+\tau$, their expectations of it are

$$E_t\left[a_{t+\tau}\right] \;=\; \mu\, s_{t+\tau}, \tag{7}$$

where $\mu \;=\; (1 \;+\; \sigma_e^2/\sigma_\epsilon^2)^{-1}$ reflects the precision of the signal.

Finally, the state of the economy in period t in the presence of noisy information about the productivity innovations up to τ periods ahead is given by

$$\Omega_t \;=\; \{(\ldots,\, a_{t-1}\,,\, a_t)\,,(\ldots,\, s_{t+\tau})\}\,, \quad \text{where } \tau \;\geq\; 1\,. \tag{8}$$

2.2 Linear Rational Expectations Equilibria

We focus on linear, time-invariant, rational expectations equilibria.[3] To this end, we consider conjectures of inflation of the following form:

$$\pi_{t+1} \;= \theta_0\, a_t \;+\; \theta_1\, a_{t+1} \;+\; \theta_2\, E_{t+1}\left[a_{t+2}\right] \;+\; \theta_3\, E_{t+1}\left[a_{t+3}\right] \;+\; \cdots\,, \tag{C1}$$

where θ's are coefficients that correspond to the productivity shock realizations or expectations thereof, and are, if possible, determined in equilibrium. More specifically, the θ's

[3] In addition, as is well known, since the shocks are Gaussian, the Kalman filter is the optimal filter when agents are rational.

need to be such that the expected inflation generated by conjecture (C1) is in agreement with (E)—the Euler equation.

To motivate the importance of information transmission and help, thereby, illustrate our main point, we analyze first the benchmark case of "no information", that is, the case in which agents receive no signals about future productivity. Instead, their information set only contains current and past realizations of productivity—and we maintain that agents know the prior distribution of productivity shocks.

We demonstrate that, in this benchmark case, monetary policy alone cannot determine the distribution of inflation. To this end, take the expectation of (C1) as of date t and match coefficients with the Euler equation (E) (bearing in mind that we have suppressed the v's). We can only pin down θ_0,

$$\theta_0 \;=\; -\frac{1}{\phi - 1}\,. \tag{9}$$

This is because

$$E_t\left[a_{t+1}\right] \;=\; E_t\left[a_{t+2}\right] \;=\; \ldots \;=\; 0\,.$$

The coefficient θ_1 cannot be determined from the equilibrium conditions, hence we get a family of solutions of the following form:

$$\pi_{t+1} \;=\; -\,\frac{1}{\phi - 1}\,a_t \;+\; \theta_1\,a_{t+1}\,. \tag{10}$$

3 Central Bank Communication

3.1 Direct communication

Suppose instead that a monetary authority communicates each period noisy information about productivity one-period ahead, i.e. suppose that it communicates s_{t+1} each period (see (6)).

Like before, this implies that $E_t[a_{t+2}] \;=\; E_t[a_{t+3}] \;=\; \ldots \;=\; 0$. Yet, unlike before, expectations of the productivity innovation one-period ahead are now non-zero: $E_t[a_{t+1}] \;\neq\; 0$ (see also (7)).

Together with (E) and (C1), this pins down a unique value, not only for θ_0 (like before, $\theta_0 \;=\; 1/(\phi - 1)$), but also for θ_1: $\theta_1 \;=\; 1/(\phi - 1)$. Like before, however, θ_2, $\theta_3 \ldots$, remain

indeterminate. As a result, come period $t+1$, the inflation rate is given by

$$\pi_{t+1} \;=\; \frac{1}{\phi-1}\,(a_{t+1} \;-\; a_t) \;+\; \theta_2\, E_{t+1}[a_{t+2}]\,, \tag{11}$$

where $E_{t+1}[a_{t+2}] \;=\; \mu\, s_{t+2} \neq 0$.

Suppose, more generally this time, that a monetary authority communicates (noisy) information about the productivity innovations up to $\bar{\tau}$ periods ahead and the economy has a long enough (say, infinite) horizon. Then $E_t[a_{t+1}]\,, \ldots\,, E_t[a_{t+\bar{\tau}}] \;\neq\; 0$, whereas $E_t[a_{t+\bar{\tau}+1}] \;=\; E_t[a_{t+\bar{\tau}+2}] \;=\; \cdots \;=\; 0$. As a result, $\theta_1 \;=1/(\phi-1)$, $\theta_2 \;=\ldots\; \theta_{\bar{\tau}} \;=\; 0$, whereas $\theta_{\bar{\tau}+1}\,,\; \theta_{\bar{\tau}+2}\;\ldots$ remain indeterminate. In this case, the inflation rate in $t+1$ writes as

$$\pi_{t+1} \;=\; \frac{1}{\phi-1}\,(a_{t+1} \;-\; a_t) \;+\; \theta_{\bar{\tau}+1}\, E_{t+1}[a_{t+\bar{\tau}+1}]\,, \tag{12}$$

where $E_{t+1}[a_{t+\bar{\tau}+1}] \;=\; \mu\, s_{t+\bar{\tau}+1} \neq 0$.[4]

It becomes clear from this discussion that, when an economy's horizon is long enough, a sequence of finite signals about productivity innovations that lie further into the future does not determine the stochastic path of inflation: it is always the last and latest signal's role that is indeterminate ($s_{t+\bar{\tau}+1}$ in (12)). However, "more" information is indeed valuable: it renders the agents' expectations about productivity in all the periods before the last one (i.e. the periods about which there is a signal) irrelevant. Determinacy of the stochastic path of inflation calls, then, for a countably infinite sequence of signals, which we call "asymptotic communication".

More to the point, all the signals—however noisy they may be—except the one about the following period ("s_{t+1}"), which the Euler equation draws on, and the last one to arrive (in our example, $s_{t+\bar{\tau}+1}$) are not part of an equilibrium but, interestingly, the prevailing class of equilibria would have been different without them. To see this, maintain the assumption that agents have (finitely noisy) information about productivity $\bar{\tau}$ periods ahead, and note that the Euler equation (E) effectively requires that

$$\theta_\tau\, E_t\,[a_{t+\tau}] \;=\; 0 \quad \text{for all } \tau \;>\; 1\,. \tag{13}$$

When agents have information in period t about productivity until period $t+\tau$, where in our example $\tau \;\in\; (1, \bar{\tau}]$, then their expectations of it are *not* zero conditional mean (*i.e.*,

[4]See Appendix A.2 for details.

they are "shaped"), hence the corresponding θ's in (13) have to be equal to zero. For period productivities, however, about which there is no information (in our example, periods $\bar{\tau}+1, \bar{\tau}+2, \ldots$), the respective agents' expectations are zero conditional mean (*"unshaped"*), hence the corresponding θ's are left indetermined. In turn, this implies that the first piece of information about these period productivities that arrives later than period t (in our example, $s_{t+\bar{\tau}+1}$, which arrives in $t+1$) will affect inflation in an entirely arbitrary way.

Proposition 1 summarizes the above discussion:

Proposition 1. *(i) By shaping the agents' expectations about future productivity, a monetary authority renders them irrelevant; (ii) When a monetary authority communicates arbitrarily, yet finitely, noisy information about all the future periods, it lowers the degree of indeterminacy, driving the economy closer to the stochastic counterpart of the perfect-foresight inflation path given by (4); (iii) An infinite-horizon economy calls for asymptotic noisy communication (equivalently, a countably infinite sequence of signals).*

Let us make the following remarks. First, it is implicit in the above that what is essential for a monetary authority is to communicate, not the exact value of ϕ it selects in its interest-rate rule (3), which is unimportant—at least from a "global" point of view.

Second, when it comes to communication, what matters is the existence of a series of signals, not the exact value they take nor their variance—as long as it is finite—both of which are entirely irrelevant.

Third, it follows from Proposition 1 that if a central bank were to communicate noisy information about all the future realizations of all possible shocks, we would obtain a unique linear equilibrium that is the stochastic counterpart of the perfect-foresight one (4) (one would just need to replace ϵ_{t+1} with $E_t[\epsilon_{t+1}]$). In other words, a deterministic economy is sufficient for inflation determinacy, not necessary: noisy information can lead to the same outcome.

Fourth, our results do not hinge on the shocks' prior being zero. We only require that agents have a posterior about each shock that is (generically) different from its prior (see also Appendices A.3.1 and A.3.2).

Fifth, note that part (iii) of Proposition 1 favors, effectively, a "bang-bang" release of

information, not a gradual one: it requires a monetary authority to communicate in period 0 a countably infinite sequence of signals about the future, and it is this (extreme) policy that may help pin a unique path of inflation down.

Finally, our results are robust to different specifications of the interest-rate rule (Appendix A.4 discusses current inflation targeting) and stochastic processes for productivity (see Appendix A.3.3).

3.2 Indirect communication: Inflation forecasts or forward guidance

In practice, however, central banks communicate their inflation forecasts more often than their productivity forecasts, which, as we show in the following section, is at least in part because inflation depends on more variables than just productivity. It is straightforward to see that there is a unique inflation forecast path corresponding to the productivity forecast path here. To see this point, suppose that a central bank has noisy information in period t about productivity in $t+1$ and communicates its forecast about expected inflation for periods $t+1$, $t+2$,... , which are given by:

$$\pi_{t,t+1} = \frac{1}{\phi - 1} \left(E_t^m \left[a_{t+1} \right] - a_t \right) \tag{14}$$

$$\pi_{t,t+2} = -\frac{1}{\phi - 1} E_t^m \left[a_{t+1} \right] \tag{15}$$

$$\pi_{t,t+3} = \pi_{t,t+4} = \ldots = 0 \,, \tag{16}$$

where $\pi_{t,t+\tau}$ denotes the monetary authority's forecast as of t about expected inflation in $t+\tau$, and $E_t^m [\cdot]$ denotes the monetary authority's expectations as of t about a variable of interest. Assuming, with no loss of generality, that agents have no private information, it follows that they will adopt the monetary authority's inflation-forecast path.

For the inflation-forecast path (14)-(16), in turn, to map uniquely to the productivity forecast path, i.e. the sequence of signals with generic element $s_{t+\tau}$ where $\tau \geq 1$ (see (6)), we need to make the additional assumptions that, first, the central bank's inflation forecasts reflect concrete information in the sense that the central bank updates its beliefs

according to signals about productivity it receives and does not communicate arbitrary information and second, the central bank does not hide information. Loosely speaking, these two assumptions reflect a degree of transparency of the central bank with regards to the rules of the game it will play with the private agents. The transparency requirement of the central bank is common knowledge here.

Agents, then, use eq. (16) to infer (generically) the horizon over which a central bank has information. Given this and assuming they know the current productivity, agents can use either (14) or (15) to elicit the central bank's information about productivity in $t+1$. It is straightforward to extend and apply this argument to cases in which a central bank has information over longer horizons, which include the infinite-horizon case.

Finally, from the interest rate rule (3), it follows that announcements about inflation forecasts, as defined above, are equivalent to announcements about expected interest rates; the effects of direct communication can be implemented with forward guidance.

4 More Shocks

Suppose there is more than one fundamental shock. We show that communication requires the announcement of the entire inflation-forecast path - expected mean and information about the variance - rather than announcements only about expected inflation or expected interest rates.

Consider, for instance, two fundamental shocks: a productivity (supply) shock, ϵ, that is distributed like before, and a preference (demand) shock, η, that is i.i.d. with $\eta \sim N(0, \sigma_\eta^2)$. Moreover, we maintain the assumption that the bank receives only one signal about future shocks; and it is common knowledge that the bank receives one signal. However, the signal may reveal information about one of the two shocks, as before, or it may be a composite signal and hence, reveal information about both shocks. The shocks

are such that the Euler equation writes[5]

$$E_t\left[y_{t+1}\right] \,-\, y_t \,=\, E_t\left[\eta_{t+1}\right] \,-\, \eta_t \,+\, i_t \,-\, E_t\left[\pi_{t+1}\right]. \tag{17}$$

In equilibrium, $y_t \,=\, a_t$, so we can rewrite (17) as

$$E_t\left[\pi_{t+1}\right] \,=\, \frac{1}{\phi-1}\left[E_t\left[a_{t+1}\right] \,-\, E_t\left[\eta_{t+1}\right] \,-\, \left(a_t \,-\, \eta_t\right)\right]. \tag{E2}$$

The state space of the economy, then, modifies as

$$\Omega_t \,=\, \left\{\left(\ldots,\, a_{t-1}\,,\, a_t\right),\left(\ldots,\, \eta_{t-1}\,,\, \eta_t\right),\left(\ldots,\, s_{t+\tau}\right)\right\}, \quad \text{where } \tau \,\geq\, 0\,;$$

and our conjecture about the inflation path adjusts accordingly:

$$\pi_{t+1} \,=\, \theta_0\, a_t \,+\, \theta_1\, a_{t+1} \,+\, \theta_2\, E_t\left[a_{t+2}\right] \,+\, \ldots \,+\, \varpi_0\, \eta_t \,+\, \varpi_1\, \eta_{t+1} \,+\, \varpi_2\, E_t\left[\eta_{t+2}\right] \,+\, \ldots. \tag{C1'}$$

Next, suppose with no loss of generality that a central bank possesses noisy private information about the values of the shocks only up to *one* period ahead. As in Section 3, taking expectations as of t in (C1′) and matching coefficients with the Euler equation, (E2), yields

$$\pi_{t+1} \,=\, \frac{1}{\phi-1}\left(a_{t+1} \,-\, a_t\right) \,+\, \theta_2\, E_{t+1}\left[a_{t+2}\right] \,-\, \frac{1}{\phi-1}\left(\eta_{t+1} \,-\, \eta_t\right) \,+\, \varpi_2\, E_{t+1}\left[\eta_{t+2}\right], \tag{18}$$

where θ_2 and ϖ_2 are indeterminate (and the unreported terms involve coefficients multiplying zero conditional mean expectations about future values of the shocks). The argument repeats as in Section 3 and, as a result, the determinacy of the stochastic path of inflation requires asymptotic communication.

4.1 Inflation forecasts

In light of the above, suppose that a central bank followed the same communication policy as in Section 3.2, that is, suppose it only announced its mean inflation expectations. To

[5] Eq. (17) corresponds to the nonlinear Euler

$$Q_t \,=\, \beta\, E_t\left[e^{\eta_{t+1}-\eta_t}\,\frac{1}{\Pi_{t+1}}\,\frac{C_t}{C_{t+1}}\right].$$

simplify exposition, we maintain the assumption that the central bank possesses information about the shocks only up to one period ahead—and relax it afterwards. If, then, the central bank announces

$$\pi_{t,t+1} \;=\; \frac{1}{\phi-1}\,[E_t^m[a_{t+1} \,-\, \eta_{t+1}] \,-\, (a_t \,-\, \eta_t)], \tag{19}$$

$$\pi_{t,t+2} \;=\; -\,\frac{1}{\phi-1}\,E_t^m[a_{t+1} \,-\, \eta_{t+1}], \tag{20}$$

$$\pi_{t,t+3} \;=\; \pi_{t,t+4} \;=\; \ldots \;=\; 0\,, \tag{21}$$

the agents' inference would be problematic. This is so for two reasons: First, agents cannot infer whether the central bank has information about one or both shocks; second, agents cannot infer the length of the central bank's information horizon.

Instead, suppose the central bank makes the following announcement:

$$\pi_{t,t+1} \;=\; \frac{1}{\phi-1}\,[E_t^m[a_{t+1} \,-\, \eta_{t+1}] \,-\, (a_t \,-\, \eta_t)], \tag{22}$$

$$\pi_{t,t+2} \;=\; -\,\frac{1}{\phi-1}\,E_t^m[a_{t+1} \,-\, \eta_{t+1}], \tag{23}$$

$$\pi_{t,t+3} \;=\; \pi_{t,t+4} \;=\; \ldots \;=\; 0, \tag{24}$$

$$Var_t^m\left[\pi_{t+1}\right] \;\geq\; \left(\frac{1}{\phi-1}\right)^2 Var_t[a_{t+1} \,-\, \eta_{t+1}], \tag{25}$$

$$Var_t^m\left[\pi_{t+2}\right] \;\geq\; \left(\frac{1}{\phi-1}\right)^2 \left[Var_t[a_{t+1} \,-\, \eta_{t+1}] \,+\, (\sigma_\epsilon^2 + \sigma_\eta^2)\right], \tag{26}$$

$$Var_t^m\left[\pi_{t+3}\right], Var_t^m\left[\pi_{t+4}\right], \ldots \;\geq\; 2\left(\frac{1}{\phi-1}\right)^2 (\sigma_\epsilon^2 + \sigma_\eta^2)\,, \tag{27}$$

where $Var_t^m[\cdot]$ denotes the central bank's conditional variance as of t about a variable of interest.

The proposed communication strategy involves a central bank communicating the minimum (conditional) variance of future inflation (see eqs. (25)-(27)). A clarification of the argument underlying (25)-(27) is in order. The central bank, using (18) and information about shocks one period ahead, computes the conditional inflation variance $Var_t^m[\pi_{t+1}]$ as follows:

$$Var_t^m[\pi_{t+1}] = \underbrace{\left(\frac{1}{\phi-1}\right)^2 Var_t[a_{t+1} - \eta_{t+1}]}_{\text{current information term}} + \underbrace{\theta_2^2 Var_t[E_{t+1}[a_{t+2}]] + \varpi_2^2 Var_t[E_{t+1}[\eta_{t+2}]]}_{\text{arbitrary "future" information term}}, \tag{28}$$

and one can derive (26)-(27) similarly. The first term on the RHS of (28) reflects the information the central bank *currently* possesses about a_{t+1} and η_{t+1}. The second term reflects the fact that the central bank knows that it *will* receive (noisy) information in period $t+1$ about $t+2$ shocks; that is, it reflects the central bank's knowledge about the arrival of information at some point in the future. The conditional variance as of t of terms like $E_{t+1}[a_{t+3}]$, $E_{t+1}[\eta_{t+3}]$,... , however, is zero because the central bank knows as of t that, come period $t+1$, it will only receive (noisy) information about a_{t+2} and η_{t+2} and not about shocks that lie further in the future, which will, therefore, have no effect on inflation in $t+1$ (see, again, (18)). Crucially, the central bank will not announce anything about the second term since, and according to our transparency criterion, it does not have concrete information at t about shocks at $t+2$; only that it will receive information at $t+1$ about $t+2$.

Agents have to extract information from the announcement of the central bank, that is, from (22)-(27). There are four possible cases: (i) the CB has no "new" information; (ii) the CB communicates a composite noisy signal about $a_{t+1} - \eta_{t+1}$, (iii) a noisy signal only about a_{t+1}; (iv) a noisy signal only about η_{t+1}.

The first case can be ruled out upon observing (22)-(24). The agents compare their prior beliefs about expected inflation with the announcement. If the two are different, then there exists "new" information. Crucially, however, agents cannot extract further information upon observing (22)-(24). It is here that additional information in the form of announcements about the minimum conditional variance plays a key role. In particular, it reveals the information of the central bank or, equivalently, the central bank announces additional information in order to communicate its own private information.

Upon observing (27), and combined with the transparency restriction, agents arrive to the conclusion that the central bank does not have information, as of period t, about $t+2$ onwards—the lower bound in (27) depends only on the prior variances, which means that the central bank does not have additional information at t. As a result, agents have to use (25) and (26) to infer whether the bank has information about only one of the two shocks or about both.

Suppose that agents initially "guess" that the lower bounds in (25) and (26) are consistent with noisy information only about a_{t+1} or η_{t+1}. For this guess to be correct, it must be that the lower bound in (25) is equal to $1/(1-\phi)^2$ times the conditional variance of either a_{t+1} or η_{t+1}. However, these calculations are inconsistent with the announced lower bounds and, as a result, agents infer that there is information about both shocks. Effectively, following a process of elimination—eliminating cases (i), (iii), (iv)—agents arrive to the conclusion that a central bank announces information about both shocks.

Agents understand that, if a central bank were to communicate information directly, it would have announced a composite noisy signal about $a_{t+1} - \eta_{t+1}$, that is, $s_{t,t+1} = a_{t+1} - \eta_{t+1} + e_{t+1}$. Thus, agents have to extract the composite signal $s_{t,t+1}$, and from that, extract a conditional distribution for each shock. Since it is common knowledge that the central bank is Bayesian, it follows, then, that

$$E_t^m[a_{t+1} - \eta_{t+1}] = \delta_e \, s_{t,t+1}, \tag{29}$$

$$Var_t[a_{t+1} - \eta_{t+1}] = \left(\frac{1}{\sigma_e^2} + \frac{1}{\sigma_\epsilon^2 + \sigma_\eta^2}\right)^{-1}, \tag{30}$$

where $\delta_e = \left(\frac{1}{\sigma_e^2}\right) / \left(\frac{1}{\sigma_e^2} + \frac{1}{\sigma_\epsilon^2+\sigma_\eta^2}\right)$. Expression (30) is consistent with (25) and (26). Moreover, combining (22), (23) and (29), agents extract the value of $s_{t,t+1}$. Finally, it follows that agents can use $s_{t,t+1}$ to update their beliefs about the conditional distribution of each shock, that is,

$$a_{t+1}|s_{t,t+1} \sim N\left(\delta_\epsilon \, s_{t,t+1}, \left(\frac{1}{\sigma_\eta^2 + \sigma_e^2} + \frac{1}{\sigma_\epsilon^2}\right)^{-1}\right),$$

$$\eta_{t+1}|s_{t,t+1} \sim N\left(-\,\delta_\eta\, s_{t,t+1}, \left(\frac{1}{\sigma_\epsilon^2+\sigma_e^2} + \frac{1}{\sigma_\eta^2}\right)^{-1}\right),$$

where $\delta_\epsilon = \left(\frac{1}{\sigma_\eta^2+\sigma_e^2}\right) / \left(\frac{1}{\sigma_\eta^2+\sigma_e^2} + \frac{1}{\sigma_\epsilon^2}\right)$ and $\delta_\eta = \left(\frac{1}{\sigma_\epsilon^2+\sigma_e^2}\right) / \left(\frac{1}{\sigma_\epsilon^2+\sigma_e^2} + \frac{1}{\sigma_\eta^2}\right)$ are positive coefficients (one can also confirm that $\delta_e = \delta_\epsilon + \delta_\eta$).

The following proposition summarizes the results of this section and Section 3.2.

Proposition 2. *(i) In the presence of one shock, direct communication can be recast as the announcement of a central bank's mean inflation forecast or mean interest-rate forecast; (ii) For this to be the case in the presence of more than one shock, a central bank has to communicate additional information about the variance of future inflation or interest rates.*

A final remark is in order. We have simplified the signal extraction problem by assuming that, when a central bank has information about more than one shock, it communicates a composite signal rather than individual ones about each shock, which is common knowledge—note that signals of the form $s = a - \eta + e$ follow naturally from the structure of (18). Were we to modify the set-up so as to let a central bank's announcements map, not only to a composite signal, but also to distinct ones, agents would still be able to infer whether a central bank's information comes in the form of a composite signal or distinct ones.

5 Rules with a Non-Systematic Component

In practice, however, interest-rate rules also have a non-systematic component. Assume, then, that a monetary authority follows an interest-rate rule given by

$$i_t = \phi\, E_t\left[\pi_{t+1}\right] + v_t\,, \tag{31}$$

where v is an i.i.d. monetary policy shock with $v \sim N(0, \sigma_v^2)$.

To simplify exposition, we focus only on productivity shocks. Expected inflation is given by (E′):

$$E_t\left[\pi_{t+1}\right] = \frac{1}{\phi-1}\left(E_t\left[a_{t+1}\right] - a_t - v_t\right). \tag{E′}$$

We maintain the assumptions that agents have no private information about the future state of the economy, hence they have to rely exclusively on central bank communication, and that they know the interest-rate rule (3)—as well as that productivity is white noise. Moreover, the conjecture about inflation modifies

$$\pi_{t+1} = \theta_0\, a_t + \theta_1\, a_{t+1} + \theta_2\, E_{t+1}\,[a_{t+2}] + \theta_3\, E_{t+1}\,[a_{t+3}] + \cdots + \kappa_0 v_t + \kappa_1 v_{t+1} + \kappa_2 E_{t+1}[v_{t+2}] + \cdots$$

Suppose the central bank announces its forecasts about inflation and also, announces its own forecasts of the policy shock $v_{t,t+\tau} \equiv E_t^m[v_{t+\tau}]$ - as in McKay et al. (2016). Evidently, the central bank announces forecasts about expected interest rates. Similarly to section 3.2 or 4.1, agents extract information from the announcement and update beliefs.

One could extend this argument as in Section 2.2 to allow for the presence of information about future shocks. In the general case, in which there is information about shocks τ periods ahead, the following family of solutions is compatible with (E′):

$$\pi_{t+1} = \frac{1}{\phi - 1}\,(a_{t+1} - a_t - v_t) + \theta_{\bar{\tau}+1}\, E_{t+1}[a_{t+\bar{\tau}+1}] + \kappa_{\bar{\tau}+1}\, E_{t+1}[v_{t+\bar{\tau}+1}]\,, \tag{32}$$

where $E_{t+1}[a_{t+\bar{\tau}+1}]\,, E_{t+1}[v_{t+\bar{\tau}+1}] \neq 0$. Observe, again, that (32) extends (12).

6 Economies with Frictions

Not surprisingly, the nominal indeterminacy of the previous section turns real if the economy ceases to be frictionless. In this section, we discuss the case of asymmetric information to illustrate this point.

Suppose we draw a distinction between producers and consumers and introduce information asymmetries between them as in Rousakis (2013).[6] Below, we present two examples.

We start with the "no-communication" case presented in the previous section. Assume, then, that productivity has a major, commonly-known component that may be stochastic or not, and a minor, idiosyncratic, consumer-specific one that is stochastic. The entire

[6] As we make clear in Appendix A.5, the asymmetry of information is due to the producers and the consumers deciding sequentially. Here, we let the producers decide before the consumers, however our central message would be intact were the events to unfold in the reverse order.

analysis in the previous section would apply to the commonly-known productivity component, so let us suppress it in order to focus on the consumer-specific one. In addition, we will maintain the assumption that productivity follows a white noise process and we will also assume away the policy shock as well as, temporarily, all forms of central bank communication. Remember that Ω denotes the state/history of the economy (see (8)), so the producers' information set is $\mathcal{I}_t^p = \Omega_{t,t} \setminus \{a_t\}$, while the consumers' overlaps with that of the monetary authority $\mathcal{I}_t^c = \mathcal{I}_t^m = \Omega_{t,t}$.

We consider the following equilibrium conditions (see Appendix A.5 for details):

$$(1+\zeta)\, y_t = E_t^p\,[a_t] + E_t^p\,[\pi_t] - \pi_t + \zeta\, a_t \tag{33}$$

$$E_t^c\,[y_{t+1}] - y_t = (\phi - 1)\, E_t^c\,[\pi_{t+1}], \tag{34}$$

where (33) is the intratemporal labor-market condition, (34) is the Euler equation, and $E^j\,[\cdot]$ with $j = \{p\,,\, c\}$ denotes the expectations of the producer (p) and the consumer (c) conditional on their information sets.

Using the intuition from (10), we conjecture that

$$y_t = \xi\, a_t \tag{C2}$$

$$\pi_t = \vartheta_1\, a_{t-1} + \vartheta_2\, a_t\,. \tag{C3}$$

Substituting (C2)-(C3) into (33)-(34) and matching coefficients yields the following class of equilibria:

$$y_t = \frac{\zeta - \vartheta_2}{1+\zeta}\, a_t \tag{35}$$

$$\pi_t = \vartheta_2\, a_t - \frac{\zeta - \vartheta_2}{(\phi-1)(1+\zeta)}\, a_{t-1}\,. \tag{36}$$

where, note, ϑ_2 is left undetermined.

The next case we consider is the "one-signal" case. Suppose, then, that both producers and consumers know the current productivity shock a_t, yet the monetary authority

transmits a noisy signal about the next-period productivity innovation, s_{t+1}, after the producers have solved their decision problem and before consumers get to decide. The producers' information set then is $\mathcal{I}_t^p = \Omega_{t,t+1} \setminus \{s_{t+1}\}$, while the consumers' one overlaps with that of the monetary authority $\mathcal{I}_t^c = \mathcal{I}_t^m = \Omega_{t,t+1}$.

The equilibrium conditions (Appendix A.5 offers details) become:

$$(1+\zeta)\, y_t = (1+\zeta)\, a_t + E_t^p\left[\pi_t\right] - \pi_t \tag{37}$$

$$E_t^c\left[y_{t+1}\right] - y_t = (\phi - 1)\, E_t^c\left[\pi_{t+1}\right]. \tag{38}$$

Using the intuition from (11), we conjecture that

$$y_t = \xi_1\, a_t + \xi_2\, E_t^c\left[a_{t+1}\right] \tag{C4}$$

$$\pi_t = \vartheta_1\, a_{t-1} + \vartheta_2\, a_t + \vartheta_3\, E_t^c\left[a_{t+1}\right]. \tag{C5}$$

Substituting (C4)-(C5) into (37)-(38) and matching coefficients yields the following class of equilibria:

$$y_t = a_t - \frac{\theta_3}{1+\zeta}\, E_t^c\left[a_{t+1}\right] \tag{39}$$

$$\pi_t = \frac{1}{\phi - 1}\left(a_t - a_{t-1}\right) + \frac{\vartheta_3}{(1+\zeta)(\phi-1)}\, a_t + \vartheta_3\, E_t\left[a_{t+1}\right]. \tag{40}$$

where, again, ϑ_3 is left undetermined.

Proposition 3 summarizes the above discussion:

Proposition 3. *Asymmetric information simply renders the nominal indeterminacy of the previous section real, hence the role of central bank communication is unchanged.*

7 Conclusion

This paper revisits the primary purpose of central bank communication, which is to anchor inflation expectations. We make two points. First, we show that a central bank can

phase out the *inherent* inflation indeterminacy by providing information about the future state of the economy. Interestingly, assuming any "commitment" considerations away, the effectiveness of central bank communication does not depend on how noisy it may be, which, in turn, implies that noisy communication is as effective in pinning future inflation down as the full resolution of uncertainty. Second, we show how direct communication about variables of interest such as productivity can be recast as an indirect form of communication, like forward guidance.

Revisiting this question in the presence of commitment, i.e. studying the "Odyssean" central bank communication in the parlance of Campbell et al. (2012), can be an interesting direction for future work.

A Appendix

A.1 Section 2

The frictionless economy in Section 2 corresponds to agents having preferences given by

$$E_{-1} \sum_{t=0}^{\infty} \beta^t \left(\log C_t \, - \, \frac{1}{1+\zeta} \, N_t^{1+\zeta} \right) , \tag{41}$$

where C_t denotes consumption in period t, N_t denotes employment in t, $\beta \in (0,1)$ denotes the agents' discount factor, and $\zeta > 0$ denotes the inverse "Frisch" labor elasticity.

Agents face a sequence of budget constraints given by

$$P_t \, C_t \, + \, Q_t \, B_{t+1} \, = \, B_t \, + \, W_t \, N_t \, + \, \tilde{\Pi}_t \, , \tag{42}$$

where P_t denotes the consumption-good price in period t, B_{t+1} denotes holdings of a nominal riskless bond (in zero net supply) purchased in t and maturing in $t+1$, Q_t denotes the nominal bond price, W_t denotes the nominal wage, and $\tilde{\Pi}_t$ denotes the firm's profits.

A perfectly competitive firm operates a linear technology given by

$$Y_t \, = \, A_t \, N_t \, , \tag{43}$$

where Y_t denotes production and A_t denotes productivity. Profits, $\tilde{\Pi}_t$, are given by

$$\tilde{\Pi}_t \, = \, P_t \, Y_t \, - \, W_t \, N_t \, . \tag{44}$$

A monetary authority sets the inverse nominal bond price (nominal interest rate) targeting expected inflation

$$\frac{1}{Q_t} = \Phi\left(E_t\left[\frac{\Pi_{t+1}}{\bar{\Pi}}\right]\right), \tag{45}$$

where Π_t denotes the gross inflation rate in period t, defined as $\Pi_t \equiv P_t/P_{t-1}$, and $\bar{\Pi}$ the inflation target (set equal to one in the main text).

In a rational expectations equilibrium under the interest-rate rule given by (45), agents solve their problems, which we specify below, and markets clear at the stated prices, that is $Y_t = C_t$, $N_t^d = N_t^s$, and $B_{t+1} = 0$ for all t with $B_0 = 0$.

Given $B_0 = 0$ agents decide how much to consume, save in the nominal bond, and work to maximize their expected utility (41) subject to the sequence of budget constraints (42) and a standard no-Ponzi-scheme constraint. Their optimality conditions are

$$N_t^{\zeta} = \frac{W_t}{P_t C_t} \tag{46}$$

$$Q_t = \beta E_t\left[\frac{1}{\Pi_{t+1}}\frac{C_t}{C_{t+1}}\right], \tag{47}$$

with Q_t set according to (45). Equation (46) is the usual intratemporal labor supply condition and (47) is the Euler equation.

Firms maximize their profits and, thanks to their linear technology, they accommodate any labor supplied at a wage given by

$$W_t = P_t A_t. \tag{48}$$

We restrict attention to linear rational expectations equilibria and, to this end, we consider an interest-rate rule given by

$$i_t = -\log\beta + \phi E_t[\pi_{t+1} - \overline{\pi}], \tag{49}$$

where we define $i_t \equiv -\log Q_t$ and let π_t denote log-inflation, defined as $\pi_t \equiv p_t - p_{t-1}$, and $\overline{\pi}$ the inflation target. We use ϕ, remember, to parametrize the monetary authority's aversion to expected inflation, and we let it take any non-negative value.

The optimality conditions (46)-(48) in log-linear form are

$$\zeta n_t = w_t - p_t - c_t \tag{50}$$

$$c_t \;=\; const \;-\; i_t \;+\; E_t\left[c_{t+1} \;+\; \pi_{t+1}\right] \tag{51}$$

$$w_t \;=\; a_t \;+\; p_t\,. \tag{52}$$

It is straightforward to see how one gets from (46) and (48) to (50) and (52), respectively. The next section, A.2, discusses in detail, among other things, how one gets from (47) to (51).

Finally, eqs. (50)-(52) lead to eqs. (1) and (2) in the main text.

A.2 The equilibria without approximations

In the above we take a log-linear approximation of the Euler equation. This is with no loss of generality because higher-order (here, only second-order) terms appear as constants, which we have suppressed. To "unsuppress" the constant terms, we will take the same steps as in the Appendix of Rousakis (2013).

Define $x_t \equiv \log(X_t)$. We can rewrite (47) as:

$$e^{\log Q_t} \;=\; e^{\log\beta}\, E_t\,\left[e^{-\,\pi_{t+1}\,+\,a_t\,-\,a_{t+1}}\right], \tag{53}$$

where we have used the equilibrium condition $y_t \;=\; c_t \;=\; a_t$. Using (49) (remember $i_t \;\equiv\; -\,\log Q_t$) and assuming a white noise productivity process, $a_t \;=\; \epsilon_t$, (53) simplifies to

$$e^{-\,\phi\, E_t\,[\pi_{t+1}\,-\,\overline{\pi}]} \;=\; E_t\,\left[e^{-\,\pi_{t+1}\,+\,\epsilon_t\,-\,\epsilon_{t+1}}\right]. \tag{54}$$

We allow noisy information about productivity up to τ periods ahead, where $\tau \;\geq\; 0$, given by (6). Next, conjecture that

$$\pi_{t+1} \;=\; \theta_{-1} \;+\; \theta_0\,\epsilon_t \;+\; \theta_1\,\epsilon_{t+1} \;+\; \theta_2\,\epsilon_{t+2} \;+\; \ldots \;+\; \theta_{\tau+1}\,E_t\,[\epsilon_{t+\tau+1}]. \tag{AC}$$

Using (AC), the LHS in (54) becomes:

$$e^{-\,\phi\, E_t\,[\pi_{t+1}\,-\,\overline{\pi}]} \;=\; e^{-\,\phi\, E_t\,[\theta_{-1}+\theta_0\,\epsilon_t+\theta_1\,\epsilon_{t+1}+\theta_2\,\epsilon_{t+2}+\cdots+\theta_\tau\,E_t\,[\epsilon_{t+\tau}]\,-\,\overline{\pi}]} \;=\; e^{-\,\phi\,\{\theta_{-1}+\theta_0\,\epsilon_t+E_t\,[\theta_1\,\epsilon_{t+1}+\theta_2\,\epsilon_{t+2}+\cdots+\theta_\tau\,E_t\,[\epsilon_{t+\tau}]]\,-\,\overline{\pi}]} \tag{55}$$

where we have used the fact that $E_t\left[\epsilon_{t+\tau+1}\right] = 0$ because, by assumption, agents have information up to τ periods ahead.

The RHS in (54) becomes:

$$E_t\left[e^{-\pi_{t+1}+\epsilon_t-\epsilon_{t+1}}\right] = e^{\epsilon_t} E_t\left[e^{-(\pi_{t+1}+\epsilon_{t+1})}\right] = e^{\epsilon_t - E_t[\pi_{t+1}+\epsilon_t] + \frac{1}{2} Var_t[\pi_{t+1}+\epsilon_{t+1}]}, \tag{56}$$

where the last equality in (56) follows from the fact that the sum $\pi_{t+1} + \epsilon_{t+1}$ is normally distributed (hence $e^{\pi_{t+1}+\epsilon_{t+1}}$ is log-normally distributed), while we use $Var_t[\cdot]$ to denote the variance of a variable conditional on the *period-t* information set. Using (AC), eq. (56) becomes:

$$E_t\left[e^{-\pi_{t+1}+\epsilon_t-\epsilon_{t+1}}\right] = e^{\epsilon_t - \{\theta_{-1}+\theta_0\,\epsilon_t + (1+\theta_1)\,E_t[\epsilon_{t+1}] + \ldots + \theta_\tau\,E_t[\epsilon_{t+\tau}]\} + \frac{1}{2}\{[(1+\theta_1)^2+\theta_2^2+\cdots+\theta_\tau^2]\,(\sigma_e^{-2}+\sigma_\epsilon^{-2})^{-1} + \theta_{\tau+1}^2\,\sigma_\epsilon^2\}}, \tag{57}$$

where we use the fact that

$$\epsilon_{t+1} \sim N\left(\mu\, s_{t+1}, (\sigma_e^{-2} + \sigma_\epsilon^{-2})^{-1}\right)$$

$$\epsilon_{t+2} \sim N\left(\mu\, s_{t+2}, (\sigma_e^{-2} + \sigma_\epsilon^{-2})^{-1}\right)$$

$$\vdots$$

$$\epsilon_{t+\tau} \sim N\left(\mu\, s_{t+\tau}, (\sigma_e^{-2} + \sigma_\epsilon^{-2})^{-1}\right)$$

$$\epsilon_{t+\tau+1} \sim N\left(0, \sigma_\epsilon^2\right)$$

and $\mu = \frac{(\sigma_e)^{-2}}{(\sigma_e)^{-2}+(\sigma_\epsilon)^{-2}}$.

Matching coefficients in (55) and (57) yields:

$$-\phi\,(\theta_{-1} - \overline{\pi}) = -\theta_{-1} + \frac{1}{2}\{[(1+\theta_1)^2 + \theta_2^2 + \cdots + \theta_\tau^2]\,(\sigma_e^{-2} + \sigma_\epsilon^{-2})^{-1} + \theta_{\tau+1}^2\,\sigma_\epsilon^2\} \tag{58}$$

$$-\phi\,\theta_0 = 1 - \theta_0 \tag{59}$$

$$-\phi\,\theta_1 \;=\; -\,(1 \,+\, \theta_1) \tag{60}$$

$$\theta_2 \;=\; \ldots \;=\; \theta_\tau \;=\; 0\,. \tag{61}$$

We can rearrange (58-(61) as:

$$\theta_{-1} \;=\; -\,\frac{1}{2\,(\phi-1)}\left\{\left(\frac{\phi}{\phi-1}\right)^2 (\sigma_e^{-2} \,+\, \sigma_\epsilon^{-2})^{-1} \,+\, \theta_{\tau+1}^2\,\sigma_\epsilon^2\right\} \,+\, \frac{\phi}{\phi-1}\,\overline{\pi} \tag{62}$$

$$\theta_0 \;=\; -\,\frac{1}{\phi-1} \tag{63}$$

$$\theta_1 \;=\; \frac{1}{\phi-1} \tag{64}$$

$$\theta_2 \;=\; \ldots \;=\; \theta_\tau \;=\; 0\,. \tag{65}$$

Observe that the solution is the same as the one in the main text and, crucially, $\theta_{t+\tau+1}$ is indeterminate. The only difference compared to the main text is the presence of the now unsuppressed constant term, θ_{-1}, which captures the slope of the Euler equation. Observe that the constant term depends on $\theta_{t+\tau+1}$, hence the constant term is indeterminate too. One may, therefore, be tempted to argue that, unlike the point made in Nakajima and Polemarchakis (2005), it is also mean inflation that remains indeterminate; this is not correct. What we have shown is that the mean of log-inflation is indeterminate, which should come as no surprise given that the mean of a log-normally distributed variable depends on its variance.

A.3 Treatment of more general productivity processes

A.3.1 Non-zero mean processes

In this section, we maintain the assumption that productivity a_t evolves as a random walk, yet we assume its prior distribution is (generically) non-zero mean, i.e. we assume that

$$a \;=\; \epsilon \sim N(\mu, \sigma_\epsilon^2),$$

where μ can take any finite value.

Modify conjectures (C1) and (AC) in the following way:

$$\pi_{t+1} = \tilde{\theta}_{-1} + \theta_0 (\epsilon_t - \mu) + \theta_1 (\epsilon_{t+1} - \mu) + \theta_2 (E_{t+1} [\epsilon_{t+2}] - \mu) + \theta_3 (E_{t+1} [\epsilon_{t+3}] - \mu) + \cdots .$$

It is straighforward to see that the constant term $\tilde{\theta}_{-1}$ is a modified version of (58), such that it also includes the μ's, and the paper's results remain intact.

We conclude that what matters is that the shocks' posterior distribution is different from their prior—which the presence of information achieves.

A.3.2 Posteriors as priors

One may wonder whether, while ruling out certain classes of equilibria, central bank communication opens the door to other ones.

To answer this question, consider, for instance, the case with a signal only about the following period's productivity. We have shown that, in this case, the class of equilibria given by (11) obtains, where θ_2 is indeterminate. If a central bank, however, also communicated s_{t+2} in period t, then θ_2 woud be equal to zero, but the indeterminate θ_3 would get activated and, as a result, the "new" class of equilibria would be given by

$$\pi_{t+1} = \frac{1}{\phi - 1} (\epsilon_{t+1} - \epsilon_t) + \theta_3 E_{t+1}[\epsilon_{t+3}] , \tag{66}$$

where $E_{t+1}[\epsilon_{t+3}] = \mu s_{t+3}$.

Consider, next, the following modified version of (C1):

$$\pi_{t+1} = \theta_0 \epsilon_t + \theta_1 \epsilon_{t+1} + \theta_2 E_{t+1} [(\epsilon_{t+2} - E [\epsilon_{t+2} | s_{t+2}]) | s_{t+2}] + \theta_3 E_{t+1} \epsilon_{t+3} + \cdots , \tag{CMOD}$$

where $E [\epsilon_{t+2} | s_{t+2}]$ denotes the agents' posterior of ϵ_{t+2} conditional on the signal s_{t+2}. Is θ_2 still determined (and equal to zero) in the presence of s_{t+2}?

Taking expectations in (CMOD) as of t, observe that $E_t [(\epsilon_{t+2} - E [\epsilon_{t+2} | s_{t+2}]) | s_{t+2}] = 0$. (CMOD), then, is compatible with the Euler (E), hence θ_2 is (once again) indeterminate. Come period $t+1$, though, we also have that $E_{t+1} [(\epsilon_{t+2} - E [\epsilon_{t+2} | s_{t+2}]) | s_{t+2}] = 0$. That is, the indeterminate θ_2 multiplies a zero term and, therefore, is irrelevant. We obtain, then, the same class of equilibria as before.

The above analysis would also apply if (CMOD) were to include any term like $E_{t+1}\left[(\epsilon_{t+j} - E\left[\epsilon_{t+j} \mid s_{t+j}\right]) \mid s_{t+j}\right]$ with $j > 2$. If, on the other hand, (CMOD) were to include a term like $\epsilon_{t+1} - E\left[\epsilon_{t+1} \mid s_{t+1}\right]$ instead of ϵ_{t+1}, it would not be in line with the Euler equation (E).

A.3.3 More general productivity processes

The paper's results are robust to different specifications of productivity. A sufficient condition is that the current (log-)productivity, a_t, is a linear function of past productivities and the current productivity innovation ϵ_t:

$$a_t = F\left(a_{t-1}, a_{t-2}, \ldots, a_0\right) + \epsilon_t, \tag{67}$$

where, like in the main text, ϵ is i.i.d. with $\epsilon \sim N(0, \sigma_\epsilon^2)$ and, as already said, $F(\cdot)$ is linear in a's.

For instance, suppose productivity follows an AR(1) process so that

$$a_t = \gamma\, a_{t-1} + \epsilon_t,$$

where $\gamma < 1$.

To uncover different inflation paths, we will take steps similar to the ones in the main text. We conjecture that

$$\pi_{t+1} = \varrho\left(a_{t+1} - a_t\right) + \varrho_2(E_{t+1}[a_{t+2}] - \gamma\, a_{t+1}) + \varrho_3(E_{t+1}[a_{t+3}] - \gamma\, E_{t+1}[a_{t+2}]) + \ldots. \tag{Step C.}$$

(Step C.) can be written equivalently as

$$\pi_{t+1} = \varrho\left(a_{t+1} - a_t\right) + \varrho_2\, E_{t+1}[\epsilon_{t+2}] + \varrho_3\, E_{t+1}[\epsilon_{t+3}] + \ldots. \tag{Gen. C.}$$

It is straightforward to see that the analysis in the main text applies here entirely. Likewise, for any productivity process (67), we can always come up with a step conjecture like (Step C.) that boils down to a conjecture like (Gen. C.), which resembles (C2) in the main text.

A.4 Current inflation targeting

Suppose instead that a central bank sets the nominal interest rate based on the following rule:

$$i_t = \phi(\pi_t - \bar{\pi}), \tag{68}$$

where $\phi > 0$ and the inflation target $\bar{\pi}$ is any finite (positive) number—say, zero.

The Euler equation, then, becomes:

$$E_t[\pi_{t+1}] = \phi\,\pi_t - (E_t[a_{t+1}] - a_t)\,. \tag{69}$$

Let, like in the main text, productivity be white noise, $a_t = \epsilon_t$, and conjecture that

$$\pi_{t+1} = \phi\,\pi_t + \theta_0\,\epsilon_t + \theta_1\,\epsilon_{t+1} + \theta_2\,E_{t+1}[\epsilon_{t+2}] + \theta_3\,E_{t+1}[\epsilon_{t+3}] + \cdots\,. \tag{AC2}$$

Observe that the only difference between (C1) and (AC2) is the additional presence of the first term, the current inflation, on the RHS of (AC2)—which, of course, is known when taking expectations of π_{t+1} as of t. Hence, it is straightforward to see that the (entire) analysis in the main text applies in the case of current-inflation targeting too.

A.5 Asymmetric information

The optimality conditions (50)-(52) in log-linear form (and after suppressing constants) are

$$w_t = E_t^p[a_t] + E_t^p[p_t] \tag{70}$$

$$\zeta n_t = w_t - p_t - c_t \tag{71}$$

$$c_t = \log Q_t + E_t^c[c_{t+1} + \pi_{t+1}], \tag{72}$$

where $i_t \equiv -\log Q_t = \phi\,E_t^m[\pi_{t+1}]$.

Note that the nominal wage, by virtue of the linear technology (43) reflects only the producers' expectations. Assume, then, it is posted before consumers decide on labor so that producers cannot extract the consumers' information by observing their actions.

Adding and subtracting p_{t-1} where necessary and using market clearing ($y_t = c_t$) implies that (70)-(72) become (33)-(34) or (37)-(38) depending on producers' information.

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