Mother's Time Allocation, Child Care and Child Cognitive Development

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Abstract

This paper analyzes the effects of maternal time allocation between work, child care and leisure, and non-parental child care on a child’s cognitive development. By using data for the U.S. from the Panel Study of Income Dynamics, I estimate a behavioral model that takes into account the heterogeneity in the mother’s child-care productivity induced by her level of education, and the diversity in the non-parental child care impact given by the different child care types available in the market. The results show that mothers with at least some college education are more effective than their low-educated counterpart in boosting the child’s cognitive skills through their child-care time. Moreover, formal child care is found to be more productive than the informal one, especially during the child’s first years of life. The simulation of policies aimed at regulating the non-parental child care market, so that only high-quality arrangements are available, shows that the effects on the child’s cognitive outcome are larger for the children of low-educated mothers, who benefit more from replacing their mother’s time with the alternative care provider’s time.

JEL Classification: D13, J13, J22, C15

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1. Introduction

There has been a long-standing interest in the social sciences literature in learning about the production of child cognitive achievement. Psychologists and economists agree that one of the most valuable inputs for child development is the time the child spends with the mother (Cunha et al., 2006). The increase in the maternal employment rate and the associated rise in the use of non-parental forms of child care have raised concerns about the impact they might have on child development, in particular through the decline in maternal child-care time. However, the actual effect of maternal employment may also depend on the quality of the alternative form of care, as well as on the quality of the time the child spends with the mother.

This paper analyzes the effects of maternal employment and non-parental child care on children’s cognitive development, by taking into account the mother’s time allocation between child care and leisure, the potential heterogeneity in the mother’s child-care time productivity, and the heterogeneous impact of different types of non-parental child care.

In the United States, the participation of mothers in the labor market has increased from around 50 percent in the 1970s to more than 70 percent at the end of the 1990s (U.S. Census Bureau, 2000). Nevertheless, recent data from the American Time Use Survey show that, while employed mothers work on average five hours per day, the time spent with their child is only half an hour lower than that of non-employed mothers (U.S. Census Bureau, 2013). Moreover, employed mothers are found to spend a substantially lower amount of time in leisure activities, such as socializing, doing sport or watching TV (U.S. Census Bureau, 2013). This suggests that there might not be a one-to-one corresponding relationship between time spent at work and child-care time, and that employed mothers may try to prioritize their child-care time over leisure. In the last decades, also the use of non-parental child care has increased, especially in the formal market: while the informal services refer to the care provided by relatives, friends or baby sitters, the formal sector of the non-parental child care market includes center-based programs (such as day care centers, nurseries, preschools, Head Start and after-school programs) and family day care facilities.\footnote{Bianchi (2000) reports that, from the 1970s, the fraction of 3 to 5 year-old children enrolled in some forms of non-parental child care programs has increased from 7.9 to 51.7 percent for mothers in the labor force, while, in the same period, the percentage of pre-kindergarten age children attending a center-based program has doubled (U.S. Census Bureau, 2012).}

In this paper, I estimate a behavioral model, in which maternal labor supply and time allocation, as well as formal and informal child care, are considered to be the endogenous choices of the mother, while the child cognitive development depends on the time the child spends with the mother, and on the amount of time the child spends in formal and informal child care. The mother’s child-care time can have a heterogeneous productivity, depending on the mother’s level of education, because mothers with different educational levels may engage in different activities during their child-care time, which may have a different effect on the cognitive skills of the child: more precisely, I distinguish between mothers with more than twelve years of education (High educated) and mothers with less than twelve years of education (Low educated). The
estimation of such a model makes it possible to deal with the endogeneity and the simultaneity of all the mother’s choices, and to identify the contributions of both maternal child-care time and non-parental child care for the cognitive development of the child.

There have been several studies assessing the effects of maternal employment or non-parental child care use on the subsequent cognitive development of children, but only Bernal (2008) evaluates the impact of the two simultaneous choices using a structural approach. Bernal (2008) finds that one year of maternal employment and non-parental child care reduces the child’s test scores by 1.8 percent, suggesting a substantial negative effect of both choices. Similarly, other studies adopting a structural approach find negative effects of maternal employment (Mroz et al., 2010, Ermisch and Francesconi, 2013), while Bernal and Keane (2011) show that the negative effect of child care mainly comes from the use of informal child care services. While accounting for the simultaneity of the employment and non-parental child care decisions, these papers do not consider the additional choice that the mother can make regarding her time allocation between time with the child and leisure, and assume that a mother’s time out of work is entirely spent by the mother with the child. However, employed mothers may allocate their time out of work in such a way as to give priority to the time spent with the child (Hoffert and Sandberg, 2001, Bianchi, 2000), so that time out of work may be a poor proxy for a mother’s child-care time. Recent studies have exploited the information on the actual amount of time spent by the mother with the child to assess the effects of maternal time inputs on child development, although not considering the role played by non-parental child care. This paper builds on Del Boca et al. (2014), who model parental time investments in a child’s ability until sixteen years of age, by distinguishing the cases in which each parent is actively engaged in the child’s activities, or just around and not participating. They find that the productivity of a mother’s active time declines over a child’s age, and that a father’s active time with the child becomes more productive as the child reaches adolescence. Differently from Del Boca et al. (2014), this paper focuses on an earlier developmental stage of the child when the main substitute for a mother’s child-care time is non-parental child care rather than the father, and accounts not only for differences in productivity between formal and informal child care services, but also for heterogeneity in the mother’s child-care time productivity induced by the mother’s level of education.

This paper contributes to the existing literature on the effects of parental decisions on children’s cognitive development in three ways. First, the model imposes no restrictions on the relationship between a mother’s labor supply and a mother’s child-care time: it allows a direct estimation of the impact of maternal time on a child’s development, accounting for the fact that the mother not only chooses how many hours to work and how much time to use non-parental child care, but also how much time to devote to the child instead of engaging in leisure activities. To this purpose, this paper exploits the actual measure of maternal time instead of using a proxy, hence allowing all the mother’s choices to be treated as endogenous.

Second, the specification of the model allows the productivity of a mother’s child-care time to vary according to the mother’s level of education, in order to account for the fact that mothers

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2 See Ermisch and Francesconi (2005) for a review of studies assessing the effects of maternal employment on children’s development, and Bernal and Keane (2011) for a review of studies looking at the impact of non-parental child care services in the U.S.
with different educational levels may carry on activities with a heterogeneous effect on the child’s cognitive skills. Indeed, the paper provides evidence that high-educated mothers are more likely than the low educated to engage in language-based activities with their child, such as reading or doing homework, and shows that such heterogeneity matters for the child’s subsequent language and math skills. The introduction of this form of heterogeneity extends the existing studies on the effects of maternal employment and non-parental child care, such as Bernal (2008) and Bernal and Keane (2011), by allowing for a heterogeneous productivity of the actual time spent by the mother with the child. Moreover, it adds to the papers using the information on the actual child-care time of the mother, such as Del Boca et al. (2014), by explicitly taking into account that mothers may differ not only in the amount of time they spend with the child, but also in the type of activities performed together.

Third, this paper provides the first estimates of the elasticity of a child’s ability with respect to both maternal time and non-parental child-care time. To the best of my knowledge, there are no studies that simultaneously evaluate the productivity of both mother and non-parental child care, by also distinguishing between the types of care received by the child. To this purpose, the paper benefits from rich data not only on the amount of time spent by the mother with the child, but also on the non-parental child care arrangements used by the mother for the child at any age of the child. In addition, the model can account for two relevant features of non-parental child care use that have been neglected by the literature so far: the first is the simultaneous use of a formal and an informal arrangements; the second refers to the use of non-parental child care services when the child is beyond kindergarten age.

In the model estimated in this paper, the mother’s utility maximization problem is subject to the mother’s time and budget constraints, as well as the child’s cognitive ability production function. The mother cares about consumption, leisure and the child’s cognitive ability, while child’s ability depends on a mother’s child-care time and the amount of time the child spends in non-parental child care. The productivity of a mother’s child-care time is allowed to vary by a mother’s level of education, while the productivity of non-parental child care varies according to whether the used arrangement is formal or informal. In each period, the mother decides her own labor supply and the investments in the child development process. The empirical specification of the model takes into account that mothers who work and use non-parental child care are systematically different from those who do not. The model allows mothers to allocate their time between labor, time with the child and leisure, depending on their preferences, their productivity in the labor market, and their productivity in the child development process.

The model is estimated using U.S. data from the Panel Study of Income Dynamics (PSID), linked to data from the Child Development Supplement (CDS) and the Time Diary (TD) component. The CDS provides information on all child-care arrangements used from birth until kindergarten and on the arrangement currently used at the time of the survey, if the child is beyond kindergarten age. At every point in time, it is possible to observe what type(s) of child care arrangement the mother is using for the child (whether formal, informal or both), the weekly amount of time each arrangement is used, and the hourly price paid for each arrangement. The Time Diary (TD) component provides unique information on the amount of time the child spends with the mother, while the main PSID surveys give detailed information on
the mother’s work history and household income during the child’s life cycle. The parameters of the model are retrieved using a Method of Simulated Moments estimator, which minimizes the distance between several data statistics and their model counterparts.

The results show a strong heterogeneity in the productivity of a mother’s time with the child by a mother’s level of education, since the productivity of child-care time for mothers with at least some college education almost doubles the one for the low educated. The productivity of non-parental child care also differs according to whether the service is formal or informal: formal child care is found to be more productive than the informal one, especially during the child’s first years of life. These patterns seem consistent with a framework where the educational content of the activities performed either at home or at the non-parental child care facility matters for the subsequent cognitive development of the child. Overall, however, a mother’s time with the child is more productive than any type of non-parental child care, regardless of her level of education; this implies that an increase in a mother’s labor supply induces a reduction in a child’s ability through a decrease in a mother’s child-care time, which may not be compensated for by the increase in non-parental child care use.

The estimated model is used to simulate the effects of policies aimed at subsidizing the mother’s wage or regulating the non-parental child care market. The results confirm that there is not a one-to-one corresponding relationship between a mother’s time out of work and child-care time, and that the mother’s time allocation between child care and leisure is affected by her level of education: in fact, high-educated mothers, who are more productive than the low educated in the child cognitive development process, are less willing to reduce their child-care time in favor of non-parental child care. Policies regulating the non-parental child care market, in such a way that only high-quality arrangements are available, determine a larger effect on the child’s test score for the children of low-educated mothers, who gain more from replacing their mother’s time with the alternative care provider’s time.

While the main results of the paper refer to the cognitive development of a child, I also perform a preliminary analysis on the effects of a mother’s time allocation and non-parental child care on a child’s non-cognitive outcome. The results show that, if the behavioral skills of a child are considered, both the heterogeneity by a mother’s education and the one across non-parental child care types appear to be less relevant; if anything, formal child care is found to be less productive than the informal one when the child is an infant.

The rest of the paper is organized as follows. Section 2 presents key stylized facts in maternal time allocation and non-parental child care use in the U.S. context. Section 3 describes the model that is estimated, while Section 4 introduces the data. Section 5 discusses the empirical method used for the identification of parameters, while Section 6 presents the results and the fit of the model. Section 7 reports the results from the policy simulations, and Section 8 presents the results from the analysis on a child’s non-cognitive outcome. Section 9 concludes.

2. Background

This section reports key stylized facts about the time allocation of mothers and the use of formal and informal child care in the U.S., that represent the basis for the features of the model presented below.
Even though data on mothers’ and children’s time use have become available only very recently, there have been some studies suggesting that mothers differ not only in terms of labor market participation decisions but also in terms of the allocation of leisure and child-care time. For instance, Leibowitz (1974, 1977) points out that more skilled mothers may also have a higher propensity to stay with their child, even if working. More recent studies on mothers’ time use confirm this point, since they do not find significant differences across employment status in the amount of time mothers spend with their child (Hoffert and Sandberg, 2001, Bianchi, 2000). Furthermore, working and non-working mothers may allocate their time out of work differently, so that the actual time that they spend with the child does not correspond to the time they spend out of work. According to data from the American Time Use Survey (ATUS) 2005-2009, the amount of time spent by mothers reading and playing with the child does not vary substantially across employment status: while employed mothers work, on average, five hours per day, they spend with their child only 30 minutes less than their non-employed counterpart; in contrast, employed mothers spend, on average, 2.5 hours per day in activities like socializing, doing sports or watching TV, against the 4 hours per day spent by non-employed mothers (U.S. Census Bureau, 2013).

This pattern is also supported by data on the amount of time spent by children with their mother, which is used in this paper. Figure 1 plots the fitted values from two regressions where the dependent variables are, respectively, maternal child-care time and leisure time, regressed on a child’s age fixed effects and a binary variable indicating whether the mother works in each period. The graph on the left (i.e., maternal child-care time) confirms that employed mothers allocate their time out of work in order to spend a positive amount of time with their child. The graph on the right shows that employed mothers spend a lower amount of time out of work in leisure, while the corresponding level for non-working mothers is considerably higher. Notice that while the difference in maternal time with the child between working and non-working mothers is equal to 8 hours per week, the difference in leisure is equal to 28 hours per week. These graphs suggest that working and non-working mothers allocate their time out of work differently and that the choice of devoting time to the child instead of having leisure should be considered endogenous as the one of labor supply.

The measure of maternal child-care time used in Figure 1-Left represents the amount of weekly time spent by the child with the mother, so it can be considered a measure of quantity of maternal time investments. However, mothers may be heterogeneous in the quality of child-care time, which can be inferred from the types of activities they perform with their child. For instance, Raikes et al. (2006) and Hale et al. (2011) in the psychological literature, as well as Kalb and Van Ours (2014) and Price (2010) in economics, show a positive effect of maternal book-reading and language-based activities on the cognitive and language development of the child; these studies also report that the probability of mothers reading to their children or performing language-based activities is higher for high-educated mothers. Following this line of argument, Table 1 lists the main categories of activities that are included in the maternal time measure used in Figure 1-Left. The same table reports the amount of time spent by the child in each

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3The leisure time is computed as the difference between the total time endowment, assumed to be 112 hours per week, and the sum between working time and time with the child.
Maternal child-care time and leisure by mothers’ employment status.

NOTE. The vertical axis in the graph on the left represents the fitted values of the following regression:

$$\tau_{it} = \eta_0 + \sum_{t=1}^{T} \eta_1 t_{it} + \eta_2 d_{it} + \epsilon_{it}$$

while the vertical axis in the graph on the right represents the fitted values of the following regression:

$$l_{it} = \beta_0 + \sum_{t=1}^{T} \beta_1 t_{it} + \beta_2 d_{it} + \epsilon_{it}$$

$\tau_{it}$ represents (weekly) maternal time with the child and $l_{it}$ represents leisure time, computed as $l = TT - \tau - h$, where $TT = 112$ is the total time endowment and $h$ represents weekly hours of work. $t_{it}$ are child’s age fixed effects (with $t = 1, \ldots, 12$) and $d_{it}$ is a dummy variable equal to 1 if the mother of child $i$ works in period $t$. $\eta_2 = -7.92$ represents the difference in average maternal time (conditional on child’s age) between working and non-working mothers. $\beta_2 = -28.28$ represents the difference in average leisure time (conditional on child’s age) between working and non-working mothers. Source: own elaboration from PSID-CDS data ($N = 572$). For these graphs, the information on a mother’s employment status available for the year 1996 has been used also for the year 1997, in order to match it with the mother’s child-care time and leisure information. See Section 4 and Appendix B for a description of the dataset.

category by distinguishing between mothers with at least some college education (High educated) and mothers without a college education (Low educated). Apart from the residual category that includes any time spent on eating, sleeping or traveling, children spend the majority of their time with the mother in socializing activities, educational activities and leisure. While there are not statistically significant differences by a mother’s level of education in the amount of time spent in educational activities (e.g. doing homework) or in activities related to the personal care of the child, it is interesting to notice that children of high- and low-educated mothers allocate a different amount of time in the leisure categories. Indeed, children of high-educated mothers spend less time than the children of the low-educated ones in activities such as watching

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4Throughout the paper, High educated indicates mothers with at least some college education, i.e., more than 12 years of education, while Low educated indicates mothers with less than 12 years of education.
TV, which may be considered as having a low educational content, and more time in activities like reading, characterized by a higher educational content. This supports the importance of allowing for heterogeneity in maternal time productivity, as induced by a mother’s education, in a model of child cognitive development.

Table 1
Activities performed by the child with the mother, by a mother’s level of education.

<table>
<thead>
<tr>
<th></th>
<th>Low Educated</th>
<th>High educated</th>
<th>T-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household activities</td>
<td>0.76</td>
<td>0.79</td>
<td>−0.21</td>
</tr>
<tr>
<td>Care of other children</td>
<td>0.02</td>
<td>0.02</td>
<td>−0.26</td>
</tr>
<tr>
<td>Activities to obtain goods and services</td>
<td>1.94</td>
<td>1.71</td>
<td>0.76</td>
</tr>
<tr>
<td>Personal care</td>
<td>0.99</td>
<td>1.4</td>
<td>−1.79</td>
</tr>
<tr>
<td>Help and care to others</td>
<td>0.06</td>
<td>0.07</td>
<td>−0.41</td>
</tr>
<tr>
<td>Socializing activities</td>
<td>1.23</td>
<td>1.08</td>
<td>0.54</td>
</tr>
<tr>
<td>Computer-related activities</td>
<td>0.24</td>
<td>0.25</td>
<td>−0.13</td>
</tr>
<tr>
<td>Educational activities</td>
<td>1.84</td>
<td>1.99</td>
<td>−0.45</td>
</tr>
<tr>
<td>Sport and outdoor activities</td>
<td>0.99</td>
<td>0.75</td>
<td>1.17</td>
</tr>
<tr>
<td>Leisure: radio, TV, music</td>
<td>4.88</td>
<td>3.24</td>
<td>3.42***</td>
</tr>
<tr>
<td>Leisure: reading, being read to</td>
<td>0.38</td>
<td>0.67</td>
<td>−2.49***</td>
</tr>
<tr>
<td>Others (Eating, Sleeping, Traveling)</td>
<td>8.34</td>
<td>8.65</td>
<td>−0.39</td>
</tr>
</tbody>
</table>

NOTE. The table reports weekly hours spent by the child with the mother in each category of activities. The category **Household activities** include any activities performed at home, e.g., preparing meals, cleaning, gardening; **Care of other children** refers to child-care activities performed by the child with the mother to other children; **Activities to obtain goods and services** includes any activity performed to obtain a good or a service, such as shopping at the grocery store; **Personal care** refers to the personal care of the child (washing hairs, taking a bath, dressing, etc); **Help and care to others** refers to any activity performed by the child with the mother to help or take care of other adult people; **Socializing activities** includes both the participation in groups or organizations, or the attendance to entertaining events; **Computer-related activities** refers to any activity performed with a personal computer; **Educational activities** include structured learning activities, such as doing homework; **Sport and outdoor activities** includes any sport or outdoor activity performed by the child, when the mother was also present; **Leisure: radio, TV, music** refers to passive leisure time, when the child and the mother listen to the radio or watch TV; **Leisure: reading, being read to** instead refers to leisure reading activities, either active or passive; the residual category **Others** mainly refers to eating, sleeping, and traveling. A mother’s level of education is defined as **High** if she has more than 12 years of education. ***** indicates that the difference between the two samples is statistically significant at the \( p < 0.01 \) level. Source: own elaboration from Time Diary-CDS data.

As previously mentioned, Figure 1-Left shows that the difference in maternal time with the child between working and non-working mothers is much smaller than the difference in leisure. A potential explanation for this pattern can also be related to the use of non-parental child care, that has increased crucially during the last decades, also for non-working mothers. For instance, Bianchi (2000) reports that from the end of the 1960s to the end of the 1990s, the fraction of 3 to 5 year-old children in the US enrolled in some forms of pre-primary educational programs increased from 4.8 to 44 percent for mothers not in the labor force. This figure seems to suggest that non-parental child care can be used, from the mother’s point of view, not only for a custodial purpose, in case the mother works and needs someone to look after the child, but also for an educational purpose, especially before the child begins formal schooling. Descriptive evidence from the data on non-parental child care used for this paper supports the existence of this pattern. Figure 2 reports the fitted values from two regressions where the dependent variables are, respectively, formal and informal child care weekly hours, regressed on a child’s age fixed effects and a binary variable indicating whether the mother works in each period. Formal child care (Figure 2-Left) refers to center-based child care arrangements or family day care services, while the Informal one (Figure 2-Right) refers to the care of relatives or nannies. The Figure shows that non-working mothers also use a positive amount of non-parental child care, especially the formal one, during the child’s first years of life. This implies that children
Formal and informal child-care time by mothers’ employment status.

NOTE. The vertical axis in the graph on the left represents the fitted values of the following regression:

\[ f_{it} = \eta_0 + \sum_{t=1}^{T} \eta_1 t_{it} + \eta_2 d_{it} + \epsilon_{it} \]

while the vertical axis in the graph on the right represents the fitted values of the following regression:

\[ i_{it} = \beta_0 + \sum_{t=1}^{T} \beta_1 t_{it} + \beta_2 d_{it} + \epsilon_{it} \]

where \( i_{it} \) represents (weekly) hours of formal child care, and \( i_{it} \) represents (weekly) hours of informal child care, in each year \( t \); \( t_{it} \) are child’s age fixed effects (with \( t = 1, \ldots, 12 \)), \( d_{it} \) is a dummy variable equal to 1 if the mother of child \( i \) works in period \( t \). \( \eta_2 = 10.55 \) represents the difference in average formal child care use (conditional on child’s age) between working and non-working mothers. \( \beta_2 = 4.68 \) represents the difference in average informal child care use (conditional on child’s age) between working and non-working mothers. Source: own elaboration from PSID-CDS data (\( N = 2021 \)). See Section 4 and Appendix B for a description of the dataset.

Figure 2 also shows that formal child care is more used than the informal one before kindergarten age, which may suggest that mothers value the educational role of the service and choose it as an investment in their child’s human capital, before the child can get an education in formal schooling. This pattern in formal and informal child-care use can also be justified by the evidence that formal arrangements, by providing more structured educational activities than the informal ones, are more likely to have positive effects on the child’s academic achievement (Bernal and Keane, 2011, Loeb et al., 2007). For this reason, it is important to incorporate in a model of non-parental child care decisions and production of human capital a form of heterogeneity in
the productivity of different types of child care.

From Figure 2, other two main features emerge, that should be taken into account in the model specification. First, the use of non-parental child care declines over the child’s age, but it is still positive for working mothers when the child starts attending kindergarten or primary school. Second, at any child’s age, the mother may use for her child either a formal, or informal service or a combination of the two. These patterns seem in line with recent evidence from Carver and Iruka (2006), reporting that non-parental child care is also needed after the child starts attending kindergarten: 40 percent of children between kindergarten age and 8th grade are found to attend at least one after-school arrangement, which might be either center-based or informal, while almost 20 percent in this age range use a combination of the two. In the data used in this paper, I observe that combining a formal and informal arrangements is particularly important also during the child’s first years of life, when this is the case for almost 10 percent of children.

Summing up, the model presented in the next section aims at taking into account the features that I have observed in the data, namely that: (i) a mother’s choice between leisure and child-care time should be considered endogenous, as the ones of labor supply and non-parental child care use; (ii) the effects of a mother’s child-care time on the child’s cognitive development should differ according to the mother’s level of education, since high-educated mothers may be more likely than the low educated to perform education-enhancing activities with their child; (iii) a mother may use non-parental child care even if not working; (iv) the use of different types of child care (i.e. formal versus informal) may respond to different purposes of the service (i.e., educational or custodial) from the mother’s point of view, and may have heterogeneous effects on the child’s cognitive development; (v) at any child’s age a mother may use a combination of formal and informal arrangements.

3. The model

This section describes the model that is estimated: paragraph 3.1 presents the basic structure, while paragraph 3.2 derives the demand functions for all the choice variables; paragraph 3.3 describes the empirical specification.

3.1. Basic structure

The model follows a standard framework from Becker and Tomes (1986), where household preferences are described by a unitary utility function, with child’s ability as an argument, and subject to a production function for child’s ability and budget and time constraints.

The model is dynamic and evolves in discrete time. In each period, the mother decides her own labor supply and time allocation, as well as the amount of non-parental child care to use: in each period the mother can use a positive amount of both formal and informal child care. The choice variables are then: (i) $h_t$, representing hours of work; (ii) $\tau_t$, the time the mother spends with the child, (iii) $i_t$, hours of informal child care, and (iv) $f_t$, hours of formal child care. The timing is defined as follows: $t = 0$ represents the birth of the child and the mother makes
all the decisions at each child’s age $t$ until the child reaches $T$ years of age.\(^5\) The functional form assumptions of the model are based on the theoretical model developed in Del Boca et al. (2014), even though the present model considers a different set of inputs in the child’s cognitive ability production function and uses a different empirical specification.\(^6\)

The mother is the unique decision maker in the household concerning the investment decisions on the child.\(^7\) This assumption implies that the father’s time allocation is exogenous with respect to the mother’s choices and to the child development process.\(^8\) The model applies to intact households, where both the mother and the father are present, and only households with one child are considered.\(^9\)

**The Mother’s Utility Function**

The mother’s utility in each period is a function of her own leisure time ($l_t$), i.e., the time the mother spends alone without working, household consumption ($c_t$), including the father’s and the child’s consumption, and the child’s cognitive ability ($A_t$). I assume a Cobb-Douglas form for preferences and I restrict the preferences parameters to be stable over time:

$$u(l_t, c_t, A_t) = \alpha_1 \ln l_t + \alpha_2 \ln c_t + \alpha_3 \ln A_t$$

where $\sum_{j=1}^{3} \alpha_j = 1$ and $\alpha_j > 0$, $j = 1, 2, 3$.\(^1\)

The mother maximizes her utility subject to the time and budget constraints. The time constraint is defined as:

$$TT = l_t + h_t + \tau_t$$

where $TT$ is the mother’s total time endowment.\(^10\) Notice that, in each period, the mother can choose to spend her leisure time alone ($l_t$) or to devote some time to the child ($\tau_t$): hence, the

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\(^5\) $t = 1$ indicates the first 12 months of the child’s life, $t = 2$ refers to the next 12 months of the child’s life, and so on. $t = T = 13$ represents the terminal period of the model. It may be interpreted as the final period of middle childhood before the child enters adolescence.

\(^6\) While the present paper considers as inputs in the child’s cognitive ability production function the mother’s child-care time, by distinguishing between high- and low-educated mothers, and the amount of time the child spends in formal and informal child care, Del Boca et al. (2014) considers the mother’s and the father’s child-care time (by distinguishing between time spells in which each parent is actively involved in the child’s activities and time spells in which each parent is just present but not engaged in the child’s activities), and the expenditure in goods and services for the child. Concerning the empirical specification of the model, this paper improves on Del Boca et al. (2014) by allowing the mother’s preferences to be correlated with the mother’s unobserved productivity in the labor market: this allows the model to account for the fact that a mother’s skills in the labor market may affect her preferences for her child’s development, and vice versa. This section and Section 3.3 provide further details on the model specification.

\(^7\) To ease the exposition, in the remainder of the paper, the mother will be referred to as feminine and the child as masculine.

\(^8\) The model allows the father to affect child development through his labor income, which influences the mother’s choices concerning work, formal and informal child care and time with the child. In the sample of intact households that I use for the estimation of the model, all fathers work and the average working time does not change across a mother’s employment status.

\(^9\) In the data used to estimate the model, the sample of intact households represents 52.7 percent of the overall sample, while 36.2 percent of families in the sample have only one child. These figures roughly correspond to the official statistics: according to the US Census Bureau data (2012), 68.1 percent of children under 18 live with both parents and 47.8 percent of married women live with their spouse; finally 21.6 percent of married women have only one child. The sample selection may have implications for the estimated parameters; this issue will be further discussed in Section 4.

\(^10\) $TT = 112$ hours per week. All choice variables are defined on a weekly basis.
model allows the mother to further choose between leisure and time with the child when she is not at work.

The budget constraint takes into account household consumption and expenditure for non-parental child care, as well as the total income available in the family (from both parents’ labor supply and non-labor income); it is given by:

\[ c_t = w_t h_t + I_t - p_i i_t - p_f f_t \]  

(3)

where \( w_t \) is a mother’s hourly wage; \( I_t \) represents household earnings (including father’s labor income and household non-labor income); \( i_t \) represents the number of hours that the mother uses informal child care, while \( f_t \) represents the amount of time that the mother uses formal child care; \( p_i \) and \( p_f \) represent the hourly price of informal and formal child care, respectively. The model predicts strictly positive prices, implying that services with a potentially zero price in the market (as it is the case for most of the informal arrangements) are characterized by a shadow price, representing, for instance, the limited availability of informal care or the value of the unpaid care provider’s time in alternative activities (Ribar, 1992, Blau and Currie, 2006). The mother does not make saving decisions, hence household income defined by \( I_t \) can be considered exogenous with respect to all the mother’s choices.

It should be noticed that no type of child care, despite being measured in terms of weekly hours, has been included in a time constraint, or conditioned to the amount of time that the mother works. The current specification, in which the amount of child care is freely determined by the model, has been preferred because in the data I do not observe a clear pattern in the relationship between a mother’s labor supply and non-parental child care use that could justify additional assumptions on the distribution of these variables.\(^{11}\) A possible implication of not specifying the non-parental child care variables in a time constraint is that the model may predict a total number of child-care hours which is larger than the number of hours in a week. However, this does not appear to be the case, since the model almost always predicts a total number of non-parental child care hours which is below the total number of hours in a week.\(^{12}\)

The Child’s Cognitive Ability Production Function

The child’s cognitive ability production function (hereafter CAPF) is defined using a value-added

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11In the data, 49 percent of mothers use child care for an amount of time that is slightly lower than their labor supply, while 51 percent report a total non-parental child care use equal to or larger than the mother’s labor supply. This implies that mothers may use non-parental child care (i) when they are working, because they need someone to look after the child, (ii) for an amount of time that is larger than their labor supply, e.g. if they think the time spent in non-parental child care can be beneficial to their child’s cognitive development, or (iii) for an amount of time lower than their labor supply, e.g. if other forms of care or activities are available for the child (such as the father’s child-care time, schooling time, or other sport/social activities) or if the child can spend time alone. In relation to case (iii), the flexibility of the child care specification adopted has the drawback of not considering other caring inputs that the child may receive, which, however, would be difficult to properly take into account, given the multitude of categories they may belong to, and the fact that they may become relevant for the child’s time allocation at different points in time. However, the implications of omitting two important inputs, such as the time at school and the father’s child-care time, from the model will be further discussed in Section 6 and Appendix E.

12Assuming, as it has been done for the mother, that the total number of hours in a week is 112, the total time spent in non-parental child care simulated by the model is above or equal to such value only for 0.6 percent of the observations.
specification and taking a Cobb-Douglas form:

\[ A_{t+1} = \delta_{0t} \times \tau_t^{\delta_{1t}} \times i_t^{\delta_{2t}} \times f_t^{\delta_{3t}} \times A_t^{\delta_{4t}} \]  

(4)

where \( A_{t+1} \) is the outcome for a child at time \( t + 1 \); \( \tau_t, i_t \) and \( f_t \) are the inputs decided by the mother in each period \( t \), where \( \tau \) represents the amount of time the mother spends with the child, \( i \) the amount of time in informal child care, and \( f \) the amount of time in formal child care; \( A_t \) is the level of child ability at period \( t \). \( \delta_{0t} \) represents a total factor productivity component, which proxies for the role of missing inputs, and varies over time. Since current ability influences the child’s future ability, Equation (4) shows that inputs operate with a lag. Moreover, the structure of the CAPF implies that when deciding the inputs on child development, the mother knows the productivity of each of them and the level of a child’s ability in the previous period.

The main inputs in the child’s cognitive ability production function are the amount of time the mother spends with the child and the time the child attends formal and informal child care. Despite posing some limitations on the substitution pattern across inputs because of the assumed functional form, the model allows the parameters in (4) to vary across the age of the child in order to capture the fact that marginal productivity of inputs varies over the stages of child development (Cunha et al., 2010, Heckman, 2007). Moreover, the elasticity of a child’s cognitive ability with respect to a mother’s child-care time \( \delta_{1t} \) is allowed to vary by a mother’s level of education, in order to account for the fact that mothers with different levels of education may perform activities that may lead to heterogeneous effects on the cognitive development of the child. The distinction between types of child care allows me to account for the heterogeneous productivity across different child care types, which may also induce differences in a mother’ behavior related to their use. The specification of the CAPF also allows the mother to use, in each period, a combination of formal and informal child care, so that the two types of care are not considered as mutually exclusive.

One important issue to discuss in relation to the specification of the CAPF is the absence of other inputs that may be relevant for the child development process. One example is given by other forms of care that the child may receive, such as the father or self care, while another important case is given by the school, where the child spends most of his time after age 5. Section 6 and Appendix E discuss the implications of the omission of these inputs for the estimated productivity parameters. Another input that is missing in the CAPF specified in Equation (4) is the expenditure in goods bought by the mother for the child. Even though, ideally, I would have liked to include in the CAPF both the services (i.e. non-parental child care) and the goods (e.g., clothes, toys, books, etc.) bought by the mother for the child, model tractability and data-related issues prevented me to do so. In fact, the inclusion of such additional input in the CAPF and in the budget constraint would have made the model intractable and prevented me from getting closed-form solutions for these choice variables. In addition, the information on the goods bought by the parents for the child is available in the data only in one point in time, and missing for a large proportion of children, especially at young ages, so that it is not possible to recover any information about the trend of this variable over time, nor about its correlation with the measure of a child’s cognitive ability. In order to account for the effects on a child’s ability induced by such missing inputs, Equation (4) includes a total factor productivity component,
that, like the productivity parameters for the other inputs, varies over time.\textsuperscript{13}

Finally, a mother’s work is not explicitly included in the CAPF, because it may not have a direct impact on child development \textit{per se}. A mother’s employment may indirectly affect child development through a change in her time allocation, together with the use of formal and informal child care. This specification makes it possible to test whether, in each period, maternal time is more productive than any type of non-parental child-care.

\textbf{Maximization Problem}

In each period, the mother maximizes her expected life time utility, optimally choosing her labor supply, the child care inputs and the number of hours to devote to the child. In this decision-making process the mother takes into account the level of ability reached by the child in each period, the wage offer that she receives from the market and the level of income in the household. The child’s cognitive ability represents an endogenous state variable, while the wage offer the mother receives in each period and the household income are exogenous with respect to the maximization problem but differ for each mother in each period. The initial condition of the problem is given by the value of the state variables in the first period.\textsuperscript{14}

The value function for the mother at period \( t \) is given by:

\[
V_t(S_t) = \max_{h_t,\tau_t,i_t,f_t} u(l_t, c_t, A_t) + \beta E_t V_{t+1}(S_{t+1})
\]

\[
s.t. \quad c_t = w_t h_t + I_t - p_i i_t - p_f f_t
\]

\[
TT = l_t + h_t + \tau_t
\]

\[
\ln A_{t+1} = \ln \delta_0 + \delta_1 t \ln \tau_t + \delta_2 \ln i_t + \delta_3 \ln f_t + \delta_4 \ln A_t
\]

where the CAPF has been log-linearized for computational convenience, \( \beta \in [0,1] \) and \( S_t = \{A_t, w_t, I_t\} \) represents the vector of state variables. The timing of the model implies that after childbirth and during the first 12 months of a child’s life the mother observes the initial level of her child’s ability and the level of income in the household and receives a wage offer; then she makes her decisions. Similarly, in the following periods, the mother chooses \( h_t, i_t, f_t \) and \( \tau_t \) after having observed the corresponding level of \( A_t \) and \( I_t \) and after having received the wage offer from the labor market.

It should be noticed that the maximization problem of the mother can be solved analytically only if the wage offer is exogenous with respect to the mother’s past and current labor supply choices. This implies that the offer the mother receives in period \( t \) is not affected by her working decisions in \( (t-1) \) and that it does not reflect any depreciation in the mother’s productivity as a result of her absence from the labor market after childbirth. The exogeneity of wage is necessary to estimate the model with continuous choice variables and closed-form solutions, which is needed to allow for four choices and, in particular, to take into account the additional choice between leisure and time with the child. However, this assumption may have implications

\textsuperscript{13}Notice that the introduction of the total factor productivity, though making it possible to capture the effects of missing inputs on a child’s ability, does not change the mother’s optimal investments decisions.

\textsuperscript{14}The structure of the initial condition for child’s ability and the draws from which the initial values of \( w_t \) and \( I_t \) are taken will be defined in paragraph 3.3.
for the estimated parameters and the fit of the model. In fact, since the definition of the wage process does not take into account the potentially negative effect on wages of leaving the labor market after childbirth, mothers may find it profitable to stay out of the labor market more than they would do in the case of endogenous wages. Thus, the model may overestimate the proportion of mothers not working and underestimate their labor supply, especially during the child’s early years of life.

3.2. Terminal period value function and solutions of the model

The mother makes her decisions (that are relevant for the child development process described by Equation (4)) in the first \( T \) years of the child’s life. After period \( T \), both the mother’s optimization problem and the child’s ability production function change: the mother may continue to optimally choose labor supply and consumption, but she will no longer consider maternal and non-parental child care choices. The terminal level of a child’s cognitive ability is \( A_{T+1} \), i.e., the level of ability reached in \( T + 1 \), that will not be affected by the mother’s subsequent decisions. This level of ability may be interpreted as the starting point for the child’s future development during adolescence, from \( T + 1 \) on.

The period \( T + 1 \) maximization problem for an infinitely-lived household may be written as:

\[
V_{T+1} = \bar{V}_{T+1} + \sum_{\kappa=0}^{+\infty} \beta^\kappa \alpha_3 \ln A_{T+1}
\]

where

\[
\bar{V}_{T+1} = \max_{h_{T+1}} \alpha_1 \ln l_{T+1} + \alpha_2 \ln c_{T+1} + \beta E_{T+1} \bar{V}_{T+2}(l_{T+2}, c_{T+2})
\]

and \( \sum_{\kappa=0}^{+\infty} \beta^\kappa = \rho \) represents the value given by the mother to the child’s ability in the last developmental period.\(^{15}\) Equation (6) represents the terminal period value function and implies that the mother’s maximization problem after period \( T \) becomes stationary and does not depend on the choices made by the mother in the previous periods.

The model is solved by backward induction and yields closed-form solutions for all the choice variables. The solution of the model involves the computation of the value function starting from the terminal period and the corresponding optimal solutions in each period. Following a two-stage process, I first derive the optimal solutions for non-parental child care \((i_t\) and \(f_t\)) and maternal time \((\tau_t)\), conditional on \( h_t \), and then compute the solutions for the mother’s labor supply \( h_t \). Analytical derivations of the results are in Appendix A.

The demands for maternal child-care time and non-parental child care, conditional on the

\(^{15}\)In the estimation, the discount factor is set at \( \beta = 0.95 \). In order to increase the flexibility of the model and to allow the discount factor of the mother to differ in the last period of investments with respect to the previous ones, the parameter \( \rho \) is estimated.
mother’s labor supply, for any period \( t \), are given by:

\[
\tau^*_t = \frac{\beta \delta_{1t} D_{t+1}}{(\alpha_1 + \beta \delta_{1t} D_{t+1})} (TT - h_t) \\
i^*_t = \frac{\beta \delta_{2t} D_{t+1}}{p_i (\alpha_2 + \beta \delta_{2t} D_{t+1} + \beta \delta_{3t} D_{t+1})} (w_i h_t + I_t) \\
f^*_t = \frac{\beta \delta_{3t} D_{t+1}}{p_f (\alpha_2 + \beta \delta_{2t} D_{t+1} + \beta \delta_{3t} D_{t+1})} (w_i h_t + I_t)
\]

where \( D_{t+1} = \frac{\partial V_{t+1}}{\partial h_{t+1}} \) represents the marginal utility the mother gets from the child’s future cognitive ability, in each period. The sequence of marginal utilities from period \( T + 1 \) to period 1 is given by:

\[
D_{T+1} = \rho \alpha_3 \\
D_T = \alpha_3 + \beta \delta_{3T} D_{T+1} \\
D_{T-1} = \alpha_3 + \beta \delta_{4T-1} D_T \\
\vdots \\
D_t = \alpha_3 + \beta \delta_{3t} D_{t+1} \\
\vdots \\
D_2 = \alpha_3 + \beta \delta_{42} D_3 \\
D_1 = \alpha_3 + \beta \delta_{41} D_2
\]

Equations (8) and (9) show that the demand for child care may be driven by necessity of custodial care, i.e., if the mother is working and needs someone to look after the child, or by valuing the educational role of the service. In fact, non-working mothers (for which \( h_t = 0 \)) may demand of non-parental child care if they value the child’s ability and they think child care may represent an input for the child’s development, as long as the household income is strictly positive and sufficiently high.

An implication of the Cobb-Douglas specification used in the mother’s utility function and in the child’s cognitive ability production function is that all inputs should be strictly positive.\(^{16}\) However, I do allow for the possibility of corner solutions for the mother’s labor supply decisions.

The mother’s latent labor supply, conditional on \( \tau^*_t, i^*_t \) and \( f^*_t \), is given by:

\[
h^*_t = \frac{\alpha_2 (TT - \tau^*_t) - \alpha_1 (I_t - p_i i^*_t - p_f f^*_t)}{w_i (\alpha_1 + \alpha_2)}
\]

Substituting (7), (8) and (9) in Equation (11), the latent labor supply becomes:

\[
h^*_t = \frac{TT (\alpha_2 + \beta \delta_{2t} D_{t+1} + \beta \delta_{3t} D_{t+1})}{(\alpha_1 + \beta \delta_{1t} D_{t+1} + \alpha_2 + \beta \delta_{2t} D_{t+1} + \beta \delta_{3t} D_{t+1})} - \frac{I_t (\alpha_1 + \beta \delta_{1t} D_{t+1})}{w_i (\alpha_1 + \beta \delta_{1t} D_{t+1} + \alpha_2 + \beta \delta_{2t} D_{t+1} + \beta \delta_{3t} D_{t+1})}
\]

The actual labor supply in each period is determined according to the following rule:

\(^{16}\)This means that the model always predicts a positive amount of formal and informal child care, regardless of a mother’s working status or household income.
\[ h_t = \begin{cases} h_t^* & \text{if } h_t^* > 0 \\ 0 & \text{if } h_t^* \leq 0 \end{cases} \]

According to Equation (12), the mother’s latent labor supply is negative or zero only if household income is strictly positive and sufficiently high. Notice that a mother’s decision to work also depends on the productivity of the alternative forms of care \( \delta_2t \) and \( \delta_3t \), because if any of them increases, the mother may be more willing to substitute her time with the external child care provider’s time. Substituting (12) into (7), (8) and (9) yields the unconditional demands for time with the child, informal and formal non-parental child care.

### 3.3. Empirical specification of the model

Unobserved and observed heterogeneity enters any stage of the decision-making process of the mother described in the previous paragraphs. Consider first the mother’s utility function, where the parameters, because of the functional form assumptions, should be positive and sum to one. In order to respect these requirements without posing additional constraints to the estimation algorithm, I use a suitable transformation of the original parameters. More precisely, I allow the coefficients in the mother’s utility function to vary according to unobserved taste shifters, representing the utility from consumption \((\gamma_2)\) and the utility from child’s ability \((\gamma_3)\). Thus, the parameters representing the mother’s preference for leisure \((\alpha_1)\), consumption \((\alpha_2)\) and child’s ability \((\alpha_3)\) are defined as:

\[
\alpha_1 = \frac{1}{1 + \exp(\gamma_2k) + \exp(\gamma_3k)}
\]

\[
\alpha_2 = \frac{\exp(\gamma_2k)}{1 + \exp(\gamma_2k) + \exp(\gamma_3k)}
\]

\[
\alpha_3 = \frac{\exp(\gamma_3k)}{1 + \exp(\gamma_2k) + \exp(\gamma_3k)}
\]

where \( \gamma_2 \) and \( \gamma_3 \) follow a discrete distribution with two points of support \((k = h, l)\).

In each period, the mother receives a wage offer and decides whether to enter into the labor market by comparing the value of this offer with her reservation wage. The offer the mother receives is described by the following wage equation:

\[
\ln(w_t) = \mu_t + \epsilon_t
\]

where \( \epsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2_\epsilon) \) is assumed to be uncorrelated over time and represents a transitory shock on wage. The term \( \mu_t \) is the mean of the log wage draws of the mother at time \( t \) and it is defined as follows:

\[
\mu_t = \mu_{mk} + \mu_1 Edu + \mu_2 Age_t + \mu_3 Race + \mu_4 Cohort + \mu_5 MacroArea + \mu_6 Cohort \times MacroArea
\]

where \( Edu \) represents a mother’s years of education; \( Race \) is a dummy variable equal to one if the mother is white; \( Cohort \) indicates the year of birth of the mother, and \( MacroArea \) reports
the geographical area where the mother lives. The interaction term between Cohort and MacroArea captures differences in the wage opportunities for mothers who belong to the same cohorts but live in different geographical areas.

The component \( \mu_{mk} \), where \( k = h, l \), represents the mother’s unobserved skills in the labor market that are allowed to be correlated with the mother’s preferences. The specification of the model assumes that the mother’s unobserved productivity and her preferences for child’s ability follow a bivariate discrete distribution (Heckman and Singer, 1984), with two points of support. This determines four types of mothers, identified by their level of productivity in the labor market and by their level of preference for the child’s ability. The probability that a mother belongs to each type should be estimated.

Concerning the child’s cognitive ability production function, as stated in Section 3.1, the parameters can vary across a child’s age. Moreover, the productivity of a mother’s time with the child is allowed to vary according to the mother’s level of education. They are defined as follows:

\[
\begin{align*}
\delta_{0t} &= \exp(\xi_{0fp} + \xi_{1fp} \times t) \\
\delta_{1t} &= \exp(\xi_{0r} + \xi_{1Edu} \times \text{HighEduMom} + \xi_{1r} \times t) \\
\delta_{2t} &= \exp(\xi_{0i} + \xi_{2i} \times t) \\
\delta_{3t} &= \exp(\xi_{0f} + \xi_{2f} \times t) \\
\delta_{4t} &= \exp(\xi_{0A} + \xi_{4A} \times t)
\end{align*}
\]

where \( t \) indicates the age of the child, and \( \text{HighEduMom} \) is a binary variable equal to 1 if the mother has at least 12 years of education. Allowing the parameters to vary across a child’s age partially compensates for the lack of substitutability implied by the Cobb-Douglas functional form used to define the CAPF. Moreover, it allows me to capture whether the inputs included in the CAPF become less or more productive as the child ages and receives other inputs, such as schooling. The time-varying total factor productivity \( \delta_{0t} \) captures the contributions of such missing inputs on the level of ability of the child in each period.

As for the wage process, the income process is also exogenous with respect to the mother’s

\[\text{MacroArea} \] is a binary indicator equal to 1 if the mother lives in the Northeast region of the U.S., where wages are observed to be higher than in the rest of the country, and 0 otherwise.

It should be noticed that the unobserved skills type of the mother \( \mu_{mk} \) is fixed over time, and thus represents the only form of persistence in the wage process allowed in the model. While adding serial auto-correlation in the wage process would still make the model tractable, the identification of the auto-correlation coefficient would require to use exactly the same variation in the data that is already used to identify the mother’s unobserved type, i.e. the wage correlation over time. Since I believe that the inclusion of an unobserved component for the mother’s productivity in the labor market is crucial to describe the mother’s decision making process, I decided to include only this form of persistence in the wage equation.

The specification of the parameters in the CAPF implies that, while the productivity of a mother’s child-care time varies across mothers depending on their level of education, the productivity of the non-parental child-care inputs is the same for all mothers/children. The reason for not allowing the productivity of formal and informal child care to vary by a mother’s level of education is because I do not observe in the data significant differences in the use of any type of services by a mother’s educational level. Since the estimation of one productivity parameter by a mother’s education is already demanding, I prefer to simplify the model as much as possible concerning the other parameters in the CAPF.
input decisions in each period. The household income is assumed to have a lognormal distribution and to depend on the fathers’ observable characteristics and a shock:

$$\ln(I_t) = \mu_{inc0} + \mu_{inc1}FatherEdu + \mu_{inc2}FatherRace + \mu_{inc3}FatherAge_t + \iota_t$$

where $$\iota_t \overset{iid}{\sim} N(0, \sigma_{inc}^2)$$.

Equations (8) and (9) show that the demands for informal and formal child care also depend on the price of both types of service. Since the data used for the estimation of the model provides information on the hourly price paid by each mother for each arrangement (either formal or informal), I exploit such information in the empirical analysis. The drawback, however, is that the price observed in the data and paid by the parents for non-parental child care is very likely to be endogenous to other decisions that are correlated with the usage of such service. In order to overcome this issue, I specify two child care costs equations (for formal and informal child care), where the main variation for the identification of the parameters is given by exclusion restrictions, i.e., variables that affect the decision to use formal or informal child care only through their effects on the price. For formal child care, such variable is represented by the amount of funding that each U.S. state allocates to kindergarten, which is taken from NIEER (2003) and refers to 3- and 4-year-old children. For informal child care, I instead use an additional variable from the Child Development Supplement, asking the primary caregiver whether other family members live in the same neighborhood, and thus might be available for taking care of the child in an informal setting. The child care cost equations are thus specified as follows:

$$p_i = \exp(\lambda_{i0} + \lambda_{i1}I[family] + \epsilon_i)$$

$$p_f = \exp(\lambda_{f0} + \lambda_{f1}StateFunding + \epsilon_f)$$

where $$p_i$$ and $$p_f$$ represent the cost of informal and formal child care, respectively, and the exponential forms ensure that such costs are positive.

In order to estimate the model and to take into account the dynamic optimization problem faced by the mother, it is necessary to know the starting level of ability, i.e., the child’s cognitive ability the mother observes in the first period before making her investments decisions. The initial ability endowment is assumed to be a function of observed characteristics of the child and the mother at birth. Specifically:

$$A_1 = \exp(\eta_{0} + \eta_1BirthWeight + \eta_2Male + \eta_3MotherAgeBirth + v)$$

where BirthWeight is a dummy variable indicating if a child has a low birth weight (i.e., lower than 2500 grams), Male is a dummy variable indicating whether the child is a male, MotherAgeBirth indicates the age of the mother at birth, and $$\eta_0$$ is a constant. $$v \overset{iid}{\sim} N(0, \sigma_v^2)$$ is a shock representing the variation in initial ability not captured by the observed characteristics.

Recalling the value-added specification of the CAPF, defined in (4), the estimation provides
consistent estimates of the productivity parameters for each input if the following conditions hold: (i) $A_t$ is a sufficient statistic for the inputs history received by the child in the previous periods; (ii) the child’s initial endowment $A_1$ (that the mother observes but the researcher does not) is only reflected in the level of ability in the subsequent period and does not affect a child’s ability in the future periods (Todd and Wolpin, 2003).

Finally, it should be described how the child’s true cognitive ability is related to the measure of that given by the test scores. The score measures used in the empirical analysis are the Letter Word (LW) and the Applied Problems (AP) raw scores, which are simple sums of the number of questions answered correctly by the test-taker. Following the approach based on classical test theory (Novick, 1966), and also adopted by Del Boca et al. (2014), I define the probability that the child answers correctly each item as follows:

$$\pi_{score} = \frac{\exp(ln(A_t + \kappa LW))}{1 + \exp(ln(A_t + \kappa LW))} = \frac{A_t + \kappa LW}{1 + A_t + \kappa LW}$$ (27)

where $A_t$ is the child’s true cognitive ability, and $LW$ is a dummy variable indicating whether the test score is the LW raw score, capturing the differences in the item difficulty between the LW and the AP scores. The final test score is distributed as a Binomial random variable, with parameters $(J_t, \pi_{score})$, where $J_t$ is the maximum number of items in the test.\(^{21}\) This specification properly accounts for measurement error in the test score measure, since a child’s score may not perfectly reflect his true cognitive ability.

4. Data

This paper uses data from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS) and Time Diary (TD) component. The PSID is a longitudinal study that began in 1968 with a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. Starting from 1968, information about each family member was collected, but much greater detail is obtained about the head and the spouse. From 1997, the Child Development Supplement (CDS) has gathered information on children aged 0-12 in PSID families through extensive interviews with their primary caregiver. The CDS has been replicated in 2002 and 2007 for children under 18.

For this analysis, I exploit the child cognitive ability measures and non-parental child care data provided in the Primary Caregiver Interview of the CDS, together with the time use details given in the Time Diary (TD) component of the CDS. To the best of my knowledge, this is the first study linking all the components of the PSID surveys introduced in 1997 and exploiting the rich information on non-parental child care use provided in the CDS.

The CDS asks the primary caregiver about the non-parental child-care arrangements used for the child since childbirth until kindergarten, and at the time of the survey in case the child is beyond kindergarten age. More precisely, the mother can report more than one arrangement used in each period, and is asked to indicate the type of arrangement, as well as the weekly amount of

\(^{21}\)In the empirical application, $J = 57$ for both the LW and the AP scores. The specification allows the LW and the AP to differ through the presence of the $\kappa LW$ component in the probability of answering correctly each item, as well as from a different stochastic process from which the two test scores errors are drawn.
time that is used and its hourly price. For the analysis, I define the formal and informal child care variables by exploiting the information on the formal and informal arrangement used more hours per week, for each age of the child. The formal category includes family day care, Head Start and preschool, while the informal category includes relative’s, non-relative’s or baby-sitter’s care. The same distinction applies when the child reaches school age: formal arrangements include any type of before- or after-school programs or any other forms of center-based arrangements that the child may attend out of the schooling time (e.g. extra-curricular activities, sport, training sessions), while the informal ones include again relatives or nannies.22 Using the 1997, 2002 and 2007 waves, I can recover the complete child-care history (from birth until kindergarten) of the sampled children, as well as information on the formal and informal arrangements that they use at the time of the survey.

In 1997 and 2002, the Child Development Supplement includes another instrument to assess the time use of children: the Time Diary (TD). The TD is a unique feature of the CDS and consists in a chronological report filled out by the child or by the child’s primary caregiver about the child’s activities over a specified 24-hour period.23 Each participating child completed two time diaries: one for a weekday (Monday-Friday) and one for a weekend day (Sunday or Saturday). The TD additionally collects information on the social context of the activity by specifying with whom the child was doing the activity. The variable weekly time with the mother is constructed by multiplying the daily hours the child spends with the mother by 5 for the weekday and by 2 for the weekend day, and summing up the total hours in a week.

The CDS supplement provides several measures of child cognitive skills, based on the Woodcock Johnson Achievement Test Revised (WJ-R) (Woodcock and Johnson, 1989). The outcome measures considered in this study are the Letter Word (LW) and the Applied Problems (AP) test scores, that are applied to all children older than three and prove, respectively, a child’s learning and reading skills, and a child’s skills in analyzing practical problems in mathematics (Hoffert et al., 1997). These measures are available in 1997, 2002 and 2007.

The main PSID surveys are used to gather information about the labor supply of mothers and fathers, and the household non-labor income. PSID interviews have been conducted annually until 1997 and, since then, they have been biennial. Since children in 1997 have different ages, ranging from 0 to 13, and in order to identify the necessary information for all of them at every age, CDS data should be matched with family information from PSID surveys in the years 1985-2007.24 The family information I gather includes each parent’s hours of work, wage and non-labor income in each period.25

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22The CDS questionnaire also allows the mother to report whether the child, once he reaches primary school age, takes care of himself by staying alone. For the definition of non-parental child care at this age, in order to be consistent with the previous time period, I have excluded the time category of self care.

23The primary caregiver completed the time diary for the very young children, while older children and adolescents were expected to complete the time diaries themselves (ISR, 2010a,b).

24For instance, to identify household information for all relevant periods for a child born in 1996 (1 year old in 1997) I need to use PSID surveys from 1997 to 2007; instead, if a child is born in 1986 (aged 11 years in 1997) I need to use PSID surveys from 1987 to 1999. All PSID surveys in the period 1985-2007 have been exploited, and the children included in the final sample are born between 1984 and 1996. See Appendix B, Tables B.1 and B.2.

25Note that all the variables that I use from the main PSID surveys concerning labor and non-labor income of the household members refer to the year before the survey. All monetary variables are deflated into 1997 US$ using the Consumer Price Index (CPI) History for the U.S. See Appendix B for further description of the data.
All relevant variables are constructed for each age of every child, defining age one as the first 12 months of child’s life, age two as the next 12 months, and so on. For the estimation of the model I consider all children without siblings interviewed in CDS I, living in intact households (where both mother and father are present for the entire period), without missing data on personal and parents’ demographic characteristics. The final sample is made up of 417 observations.

Before presenting the descriptive statistics, it should be discussed what biases might be introduced into the analysis by focusing on the subsample of children in intact households without siblings. This sample selection, in fact, implies that all mothers’ investments in child’s ability are unrelated with the decision to marry or to cohabit and with fertility. However, if mothers in intact households have more marriage-oriented attitudes, which also influence their time allocation and fertility, they may be more likely to stay at home and to spend more time with their child instead of working. This may lead to an overestimation of the proportion of mothers not working or to an overestimation of the mothers’ preferences for a child’s ability. Similarly, mothers with only one child may have higher preferences for a child’s ability and this may lead to an overestimation of the mother’s use of the most productive input. On the other hand, women in long-term relationships and with fewer children may also be more desirable in the labor market; in addition, the fact of having only one child means that the mother has experienced only one work interruption as a result of childbirth, thus making the sample disproportionately represented by highly productive mothers, and leading to an overestimation of a mother’s attachment to the labor market. Even though it is difficult to derive a unique direction of the bias induced by the sample selection, the arguments provided above suggest that it may oversample mothers who are more productive either in the labor market or at home with the child.

Table 2 shows the average values of all the variables for the sample. Mothers work, on average, 27 hours per week, and spend with their child 21 hours per week. Formal child care is used on average 10 hours per week, and this value is larger than the amount of time informal child care is used. The mother’s hourly wage is on average 14 US$, while household income amounts to, on average, around 800 US$ per week. In the sample, the average LW score is around 35 out of 57, while the AP score is around 30 out of 52.26

Table 3 provides some descriptive statistics on the mothers’ work, non-parental child care and time with the child choices, by the child’s age. The number of hours mothers work, as well as the proportion of mothers who work, slightly increase over time: mothers work, on average, 24 hours per week when the child is very young, and 29 hours, when the child reaches 11 years of age; when the child is still an infant, 23 percent of mothers do not work, and this percentage decreases to 18 when the child reaches school age. The average number of hours the child is cared for by someone other than the parents decreases as the child ages: formal child care ranges

26Table B.4 in Appendix B compares the characteristics of the subsample used for the analysis (N = 417) with the ones of the entire PSID-CDS sample (N = 3243). It shows that mothers in this subsample are, on average, older and more educated, work more, use more non-parental child care (both formal and informal) and spend less time with their child, than in the entire sample. However, the wage before childbirth of the mothers in the subsample is not statistically different from the one in the entire sample.
## Descriptive statistics on all variables for the entire period.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child’s LW raw score</td>
<td>35.10</td>
<td>14.46</td>
<td>1</td>
<td>57</td>
</tr>
<tr>
<td>Child’s AP raw score</td>
<td>29.62</td>
<td>10.53</td>
<td>1</td>
<td>52</td>
</tr>
<tr>
<td>Mother’s hours of work</td>
<td>27.30</td>
<td>17.53</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Non-working mother</td>
<td>0.19</td>
<td>0.39</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Mother’s time with the child</td>
<td>21.16</td>
<td>17.01</td>
<td>0.17</td>
<td>95.75</td>
</tr>
<tr>
<td>Formal child care</td>
<td>10.26</td>
<td>16.92</td>
<td>0</td>
<td>70</td>
</tr>
<tr>
<td>Informal child care</td>
<td>5.84</td>
<td>13.26</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>Mother’s wage</td>
<td>14.36</td>
<td>10.27</td>
<td>5.01</td>
<td>133.92</td>
</tr>
<tr>
<td>Household income</td>
<td>791.36</td>
<td>644.15</td>
<td>0.09</td>
<td>8834.95</td>
</tr>
<tr>
<td>Price of formal child care</td>
<td>1.08</td>
<td>3.60</td>
<td>0</td>
<td>72</td>
</tr>
<tr>
<td>Price of informal child care</td>
<td>0.28</td>
<td>1.29</td>
<td>0</td>
<td>33.33</td>
</tr>
<tr>
<td>Child’s gender: male</td>
<td>0.51</td>
<td>0.50</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Child’s birth weight</td>
<td>3387.15</td>
<td>614.56</td>
<td>907.18</td>
<td>6917.28</td>
</tr>
<tr>
<td>Mother’s age at child’s birth</td>
<td>28.20</td>
<td>5.10</td>
<td>16</td>
<td>43</td>
</tr>
<tr>
<td>Mother’s education</td>
<td>13.27</td>
<td>2.48</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>Mother’s race: white</td>
<td>0.61</td>
<td>0.49</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**NOTE.** Monetary variables deflated into 1997 US$. Mother’s hours of work, formal and informal child care hours, mother’s time with the child and household income are weekly values. Mother’s wage, and the price of formal and informal child care are hourly values. Child’s birth weight is expressed in grams. Household income includes father’s labor income and household non-labor income. Source: own elaboration from PSID-CDS data.

from 14 hours per week in the first years of life of the child to almost 3 hours per week when the child is 11; informal child care is always less used and ranges between 9 hours per week during infancy and almost 2 hours per week as the child ages. The average number of hours the child spends with the mother also decreases as the child grows up: the mother spends with the child around 30 hours per week when the child is younger than five, while the mother’s child-care time drops by almost ten hours when the child reaches six years of age.

### 5. Estimation

The model parameters are estimated using a Method of Simulated Moments estimator that minimizes the distance between several data statistics and their model counterparts. The full list of statistics used for the estimation is reported in Table 4.

The data generating process implied by the model described in Section 3 allows me to simulate the same statistics for the individuals (mothers and children) in the sample over the child’s life cycle. The simulation is obtained by taking \( N \times R \) random draws from the initial distribution implied by the model, i.e., the shock in the child’s initial ability, the mother’s skills and type preference distributions, and, for each period, the wage and income distributions.\(^\text{27}\)

After having drawn the child’s level of ability, the wage offer and the level of income in the first period, the optimal choices of the mother are obtained by exploiting the optimal solutions

\(^{27}\)\( N = 417 \text{ and } R = 5. \)
Table 3
Descriptive statistics on maternal employment, formal and informal child care and maternal time by child’s age.

<table>
<thead>
<tr>
<th>Child’s Age</th>
<th>1-2</th>
<th>3-5</th>
<th>6-10</th>
<th>11-12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mother’s hours of work</td>
<td>24.75</td>
<td>26.46</td>
<td>28.08</td>
<td>29.75</td>
</tr>
<tr>
<td>(17.67)</td>
<td>(17.41)</td>
<td>(17.28)</td>
<td>(17.75)</td>
<td></td>
</tr>
<tr>
<td>Non-working mother</td>
<td>0.23</td>
<td>0.20</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>(0.42)</td>
<td>(0.40)</td>
<td>(0.38)</td>
<td>(0.39)</td>
<td></td>
</tr>
<tr>
<td>Mother’s time with the child</td>
<td>28.55</td>
<td>29.05</td>
<td>19.31</td>
<td>16.35</td>
</tr>
<tr>
<td>(18.06)</td>
<td>(20.27)</td>
<td>(14.82)</td>
<td>(15.11)</td>
<td></td>
</tr>
<tr>
<td>Formal child care</td>
<td>11.40</td>
<td>14.64</td>
<td>4.25</td>
<td>3.57</td>
</tr>
<tr>
<td>(17.85)</td>
<td>(18.67)</td>
<td>(11.19)</td>
<td>(10.51)</td>
<td></td>
</tr>
<tr>
<td>Informal child care</td>
<td>9.41</td>
<td>6.70</td>
<td>1.76</td>
<td>1.87</td>
</tr>
<tr>
<td>(16.16)</td>
<td>(14.20)</td>
<td>(6.54)</td>
<td>(6.93)</td>
<td></td>
</tr>
</tbody>
</table>

NOTE. This table shows mean values by a child’s age; standard deviations are reported in parenthesis. Source: own elaboration from PSID-CDS data.

derived in Section 3.2. This process is repeated for every period, up to the final one $T$. The simulated data are used to compute the same statistics defined in Table 4. Both actual and simulated statistics are used to construct the objective function to be minimized. The Method of Simulated Moments estimator is then:

$$\hat{\theta} = \arg \min \hat{g}(\theta)'W\hat{g}(\theta)$$

(28)

where

$$\hat{g}(\theta) = \hat{m} - \hat{M}(\theta)$$

(29)

$\hat{m}$ is the vector of statistics defined from the actual data, while $\hat{M}(\theta)$ is the vector of simulated statistics according to the model. Given $S$ number of moments, the weighting matrix is defined as:

$$W = \begin{pmatrix}
\hat{V}[\hat{m}_1]^{-1} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & \hat{V}[\hat{m}_S]^{-1}
\end{pmatrix}$$

where $\hat{V}[\hat{m}]$ is estimated with non-parametric bootstrap. The standard errors are also computed with non-parametric bootstrap, by changing the starting values in each bootstrap iteration. Appendix C provides further details about the estimation.

The estimation requires a unique solution for the minimization of the objective function, which, in practice, depends on the uniqueness of the minimum and on the curvature around it. Even though testing for this would necessitate to account for the multidimensional nature

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28 To test numerically the accuracy of the solutions given by the theoretical model, I also perform a grid search, assuming that the mother’s decision to work was actually discrete. In other words, I compute the value of the demands for formal and informal child care and time with the child, as well as the mother’s inter temporal utility, for different levels of the mother’s labor supply (with the number of hours of work ranging from 0 up to the total time endowment) and I define as optimal choices those that provide the highest utility. The solutions do not differ from the ones provided by the theoretical model, though the computation becomes more time consuming.

29 The estimation is done using the simplex algorithm, which is robust to non-smooth objective functions, by setting a smaller step function than the routine’s default.
Table 4
Statistics of actual and simulated data used for the estimation of the model.

<table>
<thead>
<tr>
<th>Mother’s choices</th>
<th>Mean mother’s hours of work, formal and informal child care and mother’s time with the child by child’s age</th>
<th>Std dev mother’s hours of work, formal and informal child care and mother’s time with the child by child’s age</th>
<th>Proportion of mothers not working by child’s age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test scores</td>
<td>Mean test scores by child’s age</td>
<td>Std deviation test scores by child’s age</td>
<td></td>
</tr>
<tr>
<td>Correlation between mother’s choices and exogenous variables</td>
<td>Corr mother’s wage and mother’s hours of work</td>
<td>Corr household income and mother’s hours of work</td>
<td>Corr mother’s wage and mother’s time with the child</td>
</tr>
<tr>
<td>Correlation between mother’s choices</td>
<td>Corr mother’s hours of work and mother’s time with the child</td>
<td>Corr mother’s hours of work and formal child-care time</td>
<td>Corr mother’s hours of work and informal child-care time</td>
</tr>
<tr>
<td>Productivity of inputs</td>
<td>Coefficient of mother’s time with the child in $t-5$ in a OLS regression on test score in $t$, conditional on a dummy for LW</td>
<td>Coefficient of formal child care in $t-1$ in a OLS regression on test score in $t$, conditional on a dummy for LW</td>
<td>Coefficient of informal child care in $t-1$ in a OLS regression on test score in $t$, conditional on a dummy for LW</td>
</tr>
<tr>
<td>Child’s initial ability and test score specification</td>
<td>Variance of residuals from a child’s test score OLS reg on a dummy for LW and child’s age fixed effects</td>
<td>Average residuals from a child’s test score OLS reg on a dummy for LW and child’s age fixed effects by birth weight, gender and mother’s age at birth</td>
<td>OLS regression of test score on a dummy for LW (coefficient)</td>
</tr>
<tr>
<td>Wage equation and household income</td>
<td>Mean and std deviation of mother’s wage</td>
<td>Average mother’s wage by mother’s level of education, race, age</td>
<td>OLS regression of log wage on a mother’s cohort, area of residence and their interaction (coefficients)</td>
</tr>
<tr>
<td>Price of formal and informal child care</td>
<td>Mean and std deviation of the price of formal child care</td>
<td>Mean and std deviation of the price of informal child care</td>
<td>OLS regression of formal child care price on the number of family members present in the neighborhood</td>
</tr>
<tr>
<td>Mother’s unobserved productivity and preferences</td>
<td>Variance of the residuals from a mother’s wage OLS reg on mother’s education, age, race, cohort, area of residence and their interaction</td>
<td>Variance of the residuals from a mother’s wage OLS reg on child’s age, mother’s wage and household income</td>
<td>Variance of the residuals from a formal child care OLS reg on child’s age, mother’s wage and household income</td>
</tr>
</tbody>
</table>

NOTE. These statistics are computed using PSID-CDS data on children aged 0-12 in 1997 without siblings, and simulated data according to the model defined in Section 3. Mother’s time with the child is measured in 1997 and 2002; child’s test scores are measured in 1997, 2002 and 2007, and refer to both the LW and the AP scores; from 1997 on, mother’s hours of work, mother’s wage and household income are measured every two years and these variables refer to the year before the survey (see Section 4 and Appendix B for a description of the data). Child’s age $t$ ranges from 1 to 13. Ranges $p_y$, with $y = 1997, 2002, 2007$ are defined according to the following ranges of the score distribution: 1st – 25th perc, 25th – 50th perc, 50th – 75th perc, higher than 75th perc.

The identification of the model parameters relies on parametric and functional form assumptions, exclusion restrictions (i.e., variables entering in some parts of the model and not in others), and the choice of the moment conditions (i.e., the moments listed in Table 4 should be
informative of the parameter, in such a way that a slight variation in the parameter results in different values of the moments). While the model assumptions and exclusion restrictions have been presented in Section 3, in what follows I briefly discuss the variation in the data that I exploit for the specification of the moment conditions. Appendix C.2 provides a more formal test, by showing the variation in selected moment conditions, that is induced by the perturbation of the parameters from their estimated values.

The cross-sectional average and standard deviation of choices, together with the correlation between the choices and the exogenous variables (i.e., mother’s wage and household income), are used to recover the mother’s preferences parameters. For the estimation of the parameters in the formal and informal child care cost equations, I use the correlation between each price and its own determinant as a moment.

The productivity parameters in the child’s ability production function are identified by the correlation between a mother’s choices in $t$ and the child’s test scores in $t + 1$. Concerning the contribution of the mother’s level of education to the productivity of her own child-care time, it should be taken into account that a mother’s education also enters in the wage equation. For this reason, I use as a moment the coefficient of a mother’s level of education in a regression of test score, where I also control for the mother’s wage, in order to partial out the effect of education on the mother’s labor market productivity.

In order to identify the mothers’ labor market opportunities, which are proxied by the wage equation, I exploit the variation in wages over the mother’s life cycle (i.e., age profile), over time (i.e., between cohorts), and between geographical areas (i.e., by accounting for the macro-area where a mother lives). The wage offer is also a function of the mother’s unobserved productivity in the labor market, and of a transitory shock. To identify these parameters, I use the residuals from a OLS regression of the mother’s wage on education, race, cohort, area of residence and the interaction between the latter two. While the variance of these residuals captures the variation of both the (time-invariant) mother’s unobserved productivity and the transitory shock, by regressing the residuals in each period onto their lagged value, I get a moment that depends only on the persistence of types.

Finally, the estimation of the model crucially relies on the identification of the initial condition for the child’s level of ability. Unfortunately, while the research question requires to focus also on the first years of life of the child, the data do not provide a measure of cognitive ability before age 4. For this reason, the child’s initial ability has been specified as a function of observable characteristics of the child and the mother at birth (e.g., gender, birth weight, mother’s age at birth), that would help me to proxy the true initial ability, as reported in Section 3.3. In order to estimate the contribution of each characteristics for the child’s development, I could use as moments the correlation between these characteristics and the test scores. The limitation of this strategy is that the moments may not be informative enough of the parameters, especially if

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30Due to the structure of the data, when defining this moment for the productivity parameter of a mother’s time with the child, I use as outcome the test score observed in the next survey, i.e., after 5 years. For the specification of all moments, the test score refers to both the LW and the AP scores.

31Structural papers studying a similar research question, such as Bernal (2008), have adopted a similar strategy. For this aspect, the present paper differentiates itself from Del Boca et al. (2014), who retrieve the initial ability of each child from the first test score observed in the data, from age 4 onward.
derived from test score observations at later ages. In order to partially solve this issue, I define these moments by using only the first test score observed for each child, and by taking the residuals from a OLS regression of such scores on a child’s age fixed effects, in order to partial out any age effects. The variance of these residuals is also used to recover the error variation in the initial ability. I provide evidence in Appendix C.2 that the implemented correction helps the identification of the parameters in the child’s initial ability. However, it should be kept in mind that the issue of not observing an ability measure before age 4 may still affect the fit of the model to the data, as it will be further discussed in Section 6.1.

6. Results

This section presents the main results of the model estimation, by discussing the parameters in the mother’s utility function, the wage and child care cost equations, as well as the child’s cognitive ability production function.\textsuperscript{32}

Panel A of Table 5 reports the preference parameters for leisure ($\alpha_1$), consumption ($\alpha_2$) and a child’s ability ($\alpha_3$) for each one of the four subgroups in the sample, which are defined by the levels of preference for consumption ($\gamma_2$) and a child’s ability ($\gamma_3$), according to the specifications in Equations (13), (14) and (15): Type I corresponds to a low level, while Type II corresponds to a high level. The results show that, for both $\alpha_1$ and $\alpha_2$, the lowest value is 13 percent lower than the highest, and that the largest variation across the four groups is induced by the taste for a child’s ability: in fact, mothers with a low level of taste for a child’s ability (Type I, or $\gamma_3^l$) have higher preferences for both leisure and consumption. Concerning the preferences for a child’s ability $\alpha_3$, mothers with a high taste (Type II, or $\gamma_3^h$) have a preference for a child’s ability, which is 35 percent larger than the level of preferences for Type I mothers ($\gamma_3^l$). Also in this case, the largest difference across the four groups comes from the taste for a child’s ability $\gamma_3$, while there are not significant differences between the Type I and Type II levels of taste for consumption $\gamma_2$. Table 6 shows that 48 percent of mothers in the sample belong to the Type II group for the taste for consumption ($\gamma_2^h$), while the corresponding proportion for the taste for a child’s ability ($\gamma_3^h$) is almost 53 percent.

The model allows the preference parameter for a child’s ability to be correlated with the unobserved skills of the mother in the labor market ($\mu_m$), which are similarly discrete. The estimated values, reported in Panel B of Table 5, show that there are not large differences in the skills level for the two types of mothers, as the skills level of the Low Type is just 4 percent lower than the skills level of the High Type, though only the skill level for the High Type $\mu_h$ is statistically different from zero. According to Table 6, almost 44 percent of mothers in the sample belong to the Low type. Panel C of Table 5 reports the correlation coefficient between the mother’s unobserved skills in the labor market and the preference for her child’s ability, which is negative. This suggests that mothers face a trade-off between working and using non-parental child care on the one hand, and not working and spending time with the child on the other. The final decisions in terms of time allocation and labor supply depends on the estimated

\textsuperscript{32}Appendix D reports the estimated values for the remaining parameters, namely the untransformed parameters in the mother’s utility function and in the cognitive ability production function (Table D.1), and the parameters in the household income function, in the child’s initial ability and in the test score specification (Table D.2).
Table 5
Estimated parameters in the mother’s utility function and the wage equation.

<table>
<thead>
<tr>
<th>Panel A. Utility function</th>
<th>Estimate</th>
<th>Std. Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{1\gamma_{I}}$</td>
<td>Preference for leisure (Type I consumption, Type I child ability)</td>
<td>0.4037</td>
</tr>
<tr>
<td>$\alpha_{1\gamma_{II}}$</td>
<td>Preference for leisure (Type I consumption, Type II child ability)</td>
<td>0.3494</td>
</tr>
<tr>
<td>$\alpha_{2\gamma_{I}}$</td>
<td>Preference for leisure (Type II consumption, Type I child ability)</td>
<td>0.4030</td>
</tr>
<tr>
<td>$\alpha_{2\gamma_{II}}$</td>
<td>Preference for leisure (Type II consumption, Type II child ability)</td>
<td>0.3490</td>
</tr>
<tr>
<td>$\alpha_{3\gamma_{I}}$</td>
<td>Preference for leisure (Type I consumption, Type I child ability)</td>
<td>0.3949</td>
</tr>
<tr>
<td>$\alpha_{3\gamma_{II}}$</td>
<td>Preference for leisure (Type I consumption, Type II child ability)</td>
<td>0.3419</td>
</tr>
<tr>
<td>$\alpha_{4\gamma_{I}}$</td>
<td>Preference for leisure (Type II consumption, Type I child ability)</td>
<td>0.3959</td>
</tr>
<tr>
<td>$\alpha_{4\gamma_{II}}$</td>
<td>Preference for leisure (Type II consumption, Type II child ability)</td>
<td>0.3427</td>
</tr>
<tr>
<td>$\gamma_{1\gamma_{I}}$</td>
<td>Preference for child ability (Type I consumption, Type I child ability)</td>
<td>0.2014</td>
</tr>
<tr>
<td>$\gamma_{1\gamma_{II}}$</td>
<td>Preference for child ability (Type I consumption, Type II child ability)</td>
<td>0.3087</td>
</tr>
<tr>
<td>$\gamma_{2\gamma_{I}}$</td>
<td>Preference for child ability (Type II consumption, Type I child ability)</td>
<td>0.3083</td>
</tr>
<tr>
<td>$\gamma_{2\gamma_{II}}$</td>
<td>Preference for child ability (Type II consumption, Type II child ability)</td>
<td>0.3083</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Weight on future child’s ability in the last period</td>
<td>44.2298</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Wage equation</th>
<th>Estimate</th>
<th>Std. Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{mL}$</td>
<td>Skill level for Low Type mothers</td>
<td>0.1212</td>
</tr>
<tr>
<td>$\mu_{mH}$</td>
<td>Skill level for High Type mothers</td>
<td>0.1256</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>Coefficient of a mother’s years of education</td>
<td>-0.3323</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>Coefficient of a mother’s age</td>
<td>0.2897</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>Coefficient of a mother’s race</td>
<td>0.3283</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>Coefficient of a mother’s cohort</td>
<td>-0.3367</td>
</tr>
<tr>
<td>$\mu_5$</td>
<td>Coefficient of a mother’s macro-area of residence</td>
<td>-0.1283</td>
</tr>
<tr>
<td>$\mu_6$</td>
<td>Coefficient of a mother’s cohort $\times$ macro-area of residence</td>
<td>-0.2356</td>
</tr>
<tr>
<td>$\sigma_{wage}$</td>
<td>Std deviation wage shock</td>
<td>0.4876</td>
</tr>
</tbody>
</table>

| Panel C. Corr between labor market skills and preference for child ability | Estimate |
| Corr($\mu, \alpha_3$) | -0.0925 |

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap iteration. See Appendix C for further details on the estimation.

The productivity parameters for the mother’s child-care time and for the alternative forms of care available in the market, as well as on the out-of-pocket price of non-parental child care.

Table 5 also reports the estimated value for the parameter $\rho$ (bottom of Panel A), indicating the weight that the mother poses on the child’s level of ability reached in the last developmental period, which is estimated to be 44.33 Moreover, Panel B of Table 5 lists the estimated parameters in the wage equation, of which almost all have the expected sign and reasonable magnitude, though not all of them are statistically significant.

Figure 3 reports the time-varying elasticity of a child’s cognitive ability with respect to maternal time with the child, by a mother’s level of education, and non-parental child care, by distinguishing between formal and informal arrangements. According to Figure 3-Left, the productivity of maternal time crucially differs by a mother’s level of education. The elasticity of a child’s cognitive ability with respect to maternal time for high-educated mothers ranges between 1.8 when the child is one year old and 0.1 when the child is 13; the same elasticity for low-educated mothers ranges between 1 and 0.1, meaning that the largest differential appears during the child’s first years of life. When the child is one year old, a ten percent increase in

---

33 As Del Boca et al. (2014) point out, having found a discount factor in the last period larger than the one we could get by fixing it to the value assigned to $\beta$ (i.e., $\beta = 0.95$ so that $\rho = \sum_{n=0}^{\infty} \beta^n = 1/(1-\beta) = 20$) implies that the mother gives a lot of importance to the level of ability that the child reaches in the final period.
Table 6
Estimated proportions of types of mothers.

<table>
<thead>
<tr>
<th>Proportion Type</th>
<th>Estimate</th>
<th>Std. Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I consumption</td>
<td>0.5147</td>
<td>0.0764</td>
</tr>
<tr>
<td>Type II consumption</td>
<td>0.4853</td>
<td>(...)</td>
</tr>
<tr>
<td>Type I child ability &amp; Low Type mothers</td>
<td>0.2197</td>
<td>0.0682</td>
</tr>
<tr>
<td>Type I child ability &amp; High Type mothers</td>
<td>0.2505</td>
<td>0.0935</td>
</tr>
<tr>
<td>Type II child ability &amp; Low Type mothers</td>
<td>0.2278</td>
<td>0.0316</td>
</tr>
<tr>
<td>Type II child ability &amp; High Type mothers</td>
<td>0.3019</td>
<td>0.0571</td>
</tr>
</tbody>
</table>

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap iteration. See Appendix C for further details on the estimation.

the mother’s child care time for high-educated mothers, corresponding to almost 2.5 hours per week, leads to an increase in the level of cognitive ability of the child by 18 percent; at the same age, an increase in a mother’s child-care time for low-educated mothers by 10 percent, which corresponds to almost 3 hours per week, leads to an increase in a child’s cognitive ability by 10 percent. It should be noticed that the productivity of high-educated mothers is larger than that of low-educated, despite the latter spending slightly more time with their child than the former. Thus, this heterogeneity could not be identified by simply looking at the amount of time that each type of mothers spends with the child, but could be instead related to the educational content of the activities that mothers with different levels of education perform with their child, as it has been described in Section 2.

Figure 3
Elasticity of a child’s cognitive ability with respect to mother’s time with the child and non-parental child care.

Figure 3-Right shows that there is no such a large heterogeneity in the productivity of non-parental child care by child care types, even though formal child care is found to be slightly more productive than the informal one during the child’s early years of life. For instance, at one year of age, a ten percent increase in formal child care time, corresponding to slightly more than one hour per week, leads to an increase in the cognitive ability of the child by 7.23 percent, while an increase in informal child care by ten percent, which roughly corresponds to one hour,
leads to an increase in a child’s ability by 5.26 percent. Even though a direct comparison with the existing studies on the topic is not feasible, because they use a binary indicator for a child’s external child care attendance instead of a continuous time measure, this result is in line with the findings in Bernal and Keane (2011) and Loeb et al. (2007) for the United States and Hansen and Hawkes (2009) for the United Kingdom, stating that the attendance of a formal arrangement before kindergarten age improves the child’s language and math competences.

It should be noticed that the two types of care start having the same productivity when the child reaches 5 years of age, and, from age 9 onward, their productivity goes to zero. This pattern could be explained by the different purposes that non-parental child care may have from the mother’s point of view once the child starts attending school. In fact, before the child reaches school age, a mother could choose a positive amount of child care not only if she needs someone looking after the child, but also if she thinks it may represent an input for the child’s subsequent development (which may lead the mother to choose the formal one). Once the child starts going to school, the educational role of non-parental child care, also for the center-based one, becomes less important, and even more structured environments offering before- or after-school programs may decide to prioritize social or athletic activities over the educational ones. Another reason for the declining productivity of both types of non-parental child care can be related to the fact that as the child grows up, he can take care of himself and spend time alone: this implies that the only educational input that he receives is the one provided by the school.

Figure 3 also shows that the elasticity of a child’s cognitive ability with respect to all inputs (i.e., maternal time, formal and informal child care) is higher during the early years and decreases over time, which seems in line with previous studies on human capital accumulation (Heckman, 2008, Carneiro and Heckman, 2003). The estimated parameters shed also light on how the different productivity of inputs affects the mothers’ decision-making process, especially their decision to work. Indeed, their final decision of whether to join the labor force depends on whether the alternative form of care (i.e., formal or informal child care) can compensate for the reduction in the mother’s child-care time induced by their labor supply, and, if not, whether the loss they would incur in terms of a child’s ability could be compensated for by the gains in the labor market. High-educated mothers face a significantly large loss when spending time in the market, since the alternative forms of care available - either formal or informal - cannot fully compensate for the reduction in a child’s ability induced by their lower child-care time.34

An additional determinant of the mother’s decision to use a particular type of child care is represented by its price. Table 7 reports the estimated parameters in the cost equations for informal and formal child care respectively. As previously discussed, the cost determinants in the two equations act as exogenous restrictions for the cost of informal and formal child care; thus, the coefficients reported in Table 7 allows me to test how well these variables predict the cost of each service. The fact that the coefficients of the state funding for center-based child care and the presence of family members in the neighborhood are both statistically significant is reassuring, because it confirms that also in the simulated data these variables represent strong predictors.

34This may explain the recent evidence of highly educated women exiting the labor force to care for their children at higher rates than their less educated counterparts. This trend has been reported and analyzed, for instance, by Juhn and Potter (2006) and Macunovich (2010).
of the child care costs. Indeed, the model also predicts, as it is the case in the actual data, that formal child care is slightly more expensive than the informal one: the average simulated price of formal child care is 2.95\$/per hour, while the average simulated price of informal child care is 2.41\$/per hour.

Table 7
Estimated parameters in the child care cost equations.

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std. Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Informal child care cost</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{0}$: Intercept</td>
<td>0.2695</td>
<td>0.1166</td>
</tr>
<tr>
<td>$\lambda_{1}$: Coefficient of indicator for family members in neighborhood</td>
<td>0.1940</td>
<td>0.0757</td>
</tr>
<tr>
<td>$\sigma_{\text{informal cost}}$: Std deviation informal cost shock</td>
<td>1.0514</td>
<td>0.0145</td>
</tr>
<tr>
<td><strong>Panel B. Formal child care cost</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{f0}$: Intercept</td>
<td>0.3718</td>
<td>0.1063</td>
</tr>
<tr>
<td>$\lambda_{f1}$: Coefficient of state funding for center-based child care</td>
<td>0.1434</td>
<td>0.0428</td>
</tr>
<tr>
<td>$\sigma_{\text{formal cost}}$: Std deviation formal cost shock</td>
<td>1.1095</td>
<td>0.0281</td>
</tr>
</tbody>
</table>

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap iteration. See Appendix C for further details on the estimation.

Finally, Figure 4 reports the time-varying elasticity of a child’s cognitive ability with respect to the level of ability in the previous period and the estimated total factor productivity. It should be noticed that, differently from all the other inputs, the total factor productivity is increasing over time, which seems to suggest that other inputs not explicitly included in the model play a more important role as long as the child ages. As discussed in Section 3.1, the omission of time inputs in the CAPF, such as the time the child spends with the father, or the time the child spends at school, may have implications for the estimated productivity parameters. Appendix E further elaborates on this issue, by performing two sensitivity analyses that account for the time spent at school, and for the father’s child-care time. The results from these exercises seem to suggest that such omission induces an upward bias in the estimates, especially for the productivity of a mother’s time with the child.

Figure 4
Elasticity of a child’s cognitive ability with respect to the level of ability of the child in the previous period, and estimated total factor productivity (TFP).

NOTE. This graph represents the productivity parameters for the level of ability of the child in the previous period ($A_{t-1}$), and the estimated total factor productivity parameter, as a function of child’s age $t = 1, 2, 3, \ldots, 13$. The specification of the parameters is reported in Equations (18) and (22). See Appendix C for further details on the estimation.
6.1. Fit of the model

This section discusses the fit of the model to the data, by presenting the actual and simulated moments for the mother’s choices and the child’s test scores, by a child’s age. Table D.3 in Appendix D reports the fit for the other unconditional moments.

Table 8 shows the fit of the model for the mother’s choice variables, conditional on the age of the child. The overall fit of the model for the mother’s choices is good, though the model overestimates the proportion of mothers not working and the mother’s child-care time at early ages. As discussed in Section 3.1, this could be due to one of the assumptions needed to solve the model, which is the exogeneity of the wage process with respect to the mother’s decisions, implying that the mother does not face any costs associated with her absence from the labor market. Over time, the model well predicts a negative trend in mother’s time with the child, formal and informal child care, and a positive trend in labor supply.

Table 8
Goodness of fit for mother’s choices by child’s age.

<table>
<thead>
<tr>
<th>Child age</th>
<th>1-2</th>
<th>3-5</th>
<th>6-10</th>
<th>11-12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion non-working mothers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual data</td>
<td>0.2291</td>
<td>0.2011</td>
<td>0.1780</td>
<td>0.1830</td>
</tr>
<tr>
<td>Simulated data</td>
<td>0.4393</td>
<td>0.3301</td>
<td>0.1773</td>
<td>0.1156</td>
</tr>
<tr>
<td>Mother’s hours of work</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual data</td>
<td>24.7488</td>
<td>26.4614</td>
<td>28.0840</td>
<td>29.7518</td>
</tr>
<tr>
<td>Simulated data</td>
<td>20.6888</td>
<td>24.2698</td>
<td>36.0740</td>
<td>38.1984</td>
</tr>
<tr>
<td>Mother’s time with the child</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual data</td>
<td>28.5513</td>
<td>29.0493</td>
<td>19.3114</td>
<td>16.3548</td>
</tr>
<tr>
<td>Simulated data</td>
<td>50.5177</td>
<td>29.3575</td>
<td>9.3919</td>
<td>17.8782</td>
</tr>
<tr>
<td>Informal child care</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual data</td>
<td>10.4095</td>
<td>7.7021</td>
<td>2.7569</td>
<td>2.8662</td>
</tr>
<tr>
<td>Simulated data</td>
<td>5.9008</td>
<td>3.3511</td>
<td>0.8769</td>
<td>1.5064</td>
</tr>
<tr>
<td>Formal child care</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual data</td>
<td>12.4029</td>
<td>15.6419</td>
<td>5.2526</td>
<td>4.5742</td>
</tr>
<tr>
<td>Simulated data</td>
<td>8.1524</td>
<td>3.6678</td>
<td>0.9340</td>
<td>1.5729</td>
</tr>
</tbody>
</table>

NOTE. Actual data represent PSID-CDS data on children aged 0-12 in 1997, without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

Figure 5 shows the model fit for the child’s score measure, which appears to be underestimated in the first time periods.\(^{35}\) As discussed in Section 5, the deviation of the simulated test scores from the data, especially for the first years of life of the child, may be related to the difficulty of identifying the initial level of ability at birth.\(^{36}\) However, the model predicts quite well the increasing trend in the raw scores over time.

\(^{35}\)In Figure 5, the child’s test score represents the average between the LW and the AP raw scores.

\(^{36}\)Such weak identification could also be seen from the estimated parameters in the equation specifying the child’s cognitive ability at birth, that are reported in Table D.2-Panel B in Appendix D, since not all of them are statistically different from zero.
7. Policy simulations

I use the estimated model to simulate the effects of policies aimed at increasing the mother’s opportunities in the labor market, or at regulating the child care market. The results of the model estimation suggest that a mother may face two main trade-offs: the first one relates to the mother’s level of education, since more educated mothers can be more productive in the labor market, but are also more productive with the child; the second one refers instead to the type of non-parental child care, because formal child care is slightly more productive for the child’s cognitive development than the informal one, but it is also more expensive. In this context, the effects of policies aimed at empowering mothers with higher wages or regulating the child care market, in order to offer higher-quality services, depend on the relative weights that mothers pose to their preferences for leisure or consumption versus their preferences for the cognitive development of their child, as well as the relative productivity of each input. In order to fully understand the implications of the above-mentioned trade-offs, I report the results of the policy simulations for the entire sample and by distinguishing between high- and low-educated mothers.

7.1. Wage subsidy

Table 9 reports the results of the simulation of a policy that increases, in every period, the wage offer of all mothers by 20 percent. Such policy can resemble interventions aimed at increasing the participation of mothers in the labor market, by lowering labor market taxation or by providing in-work benefits.

The policy induces, on average, an increase in the mother’s labor supply by 4 percent, even though such increase is larger for high-educated mothers than for the low educated. This difference can be due to the fact that the amount of the subsidy is higher for larger wages: as long as more educated mothers are more likely to be in this category, they are also more responsive to the increase. The increase in a mother’s labor supply is associated with a decrease in both child-care time and leisure, with the high-educated mothers reducing child care less and
leisure more than the low-educated ones. This heterogeneity in the time allocation by a mother’s level of education confirms that the additional choice between child care and leisure plays an important role for the effects of policies affecting the mother’s labor supply. The increase in a mother’s wage also leads to a larger use of non-parental child care: while there are not large differences in the increase of formal and informal child care, I observe that low-educated mothers increase more the use of non-parental child care than the high-educated ones. These results seem to suggest that, while high-educated mothers renounces to child-care time to a lower extent, the low-educated ones are more likely to spend the additional money for buying non-parental child care. The final effect on the child’s test score in the last period is very small, and differ according to the mother’s level of education: children of high-educated mothers face a negative change in the final test score, which is induced by the fact that the reduction in their mother’s child-care time cannot be compensated for by the use of (formal or informal) non-parental child care; children of low-educated mothers instead face a slightly positive variation in the final test score, because their mothers’ child-care time is less productive and their reduction in time with the child can be compensated for by the use of non-parental child care.

### 7.2. Regulation of the child care market

The model has been estimated by using data from the U.S., where the non-parental child care market is mainly private and heterogeneous in terms of quality and price. Indeed, the estimated parameters in the child care cost equations and the CAPF indicate that the formal (center-based) child care services are more productive but also more expensive than the informal ones. The policy maker may be interested in regulating such market, by setting rules and regulations that guarantee a more homogeneous quality, and/or by providing such services at a subsidized price. In order to assess the effects of such interventions, I design two types of policies aimed at regulating the non-parental child care market: the first sets the productivity of informal child care to the level of the formal one, according to the estimated values reported in Figure 3 - Left, by leaving the price unaffected, while the second also regulates the price, by setting the hourly cost for both types of services at a subsidized value of 0.5$ per hour. The first policy implies that the policy maker only changes the provision of child care (in terms of quality standards), without influencing the price, and may resemble the case of a child care market where each provider can set its own price, provided that it fulfills the policy requirements. The second policy instead wants to mimic the case of a subsidized and high-quality child care system, where a child care

### Table 9
Effects of a policy subsidizing the mother’s wage, on the entire sample and by a mother’s level of education.

<table>
<thead>
<tr>
<th></th>
<th>All sample</th>
<th>High educated</th>
<th>Low educated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score in last period</td>
<td>−0.0110</td>
<td>−0.0216</td>
<td>0.0013</td>
</tr>
<tr>
<td>Mother’s hours of work</td>
<td>4.0236</td>
<td>5.3113</td>
<td>3.0118</td>
</tr>
<tr>
<td>Mother’s time with the child</td>
<td>−1.6072</td>
<td>−1.5683</td>
<td>−1.6827</td>
</tr>
<tr>
<td>Formal child care</td>
<td>17.1968</td>
<td>13.7376</td>
<td>18.6240</td>
</tr>
<tr>
<td>Informal child care</td>
<td>17.2432</td>
<td>13.9188</td>
<td>18.6981</td>
</tr>
<tr>
<td>Leisure</td>
<td>−1.5629</td>
<td>−1.6428</td>
<td>−1.4732</td>
</tr>
</tbody>
</table>

NOTE. This table shows the percentage changes with respect to the simulated values of the child’s test score in the last period, as well as of the average (over the period) mother’s choices, as induced by the implementation of a policy that increases the mother’s wage by 20 percent.
slot in a center-based facility is available to whoever demands it; this is the context of some Northern European countries, such as Sweden and Norway, where the only type of non-parental child care available is regulated and highly subsidized.

Table 10
Effects of policies regulating the non-parental child care market and subsidizing non-parental child care, on the entire sample and by a mother’s level of education.

<table>
<thead>
<tr>
<th></th>
<th>All sample</th>
<th>High educated</th>
<th>Low educated</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Both types of child care are regulated</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Score in last period</td>
<td>0.0018</td>
<td>0.0000</td>
<td>0.0040</td>
</tr>
<tr>
<td>Mother’s hours of work</td>
<td>0.9683</td>
<td>0.9806</td>
<td>0.9586</td>
</tr>
<tr>
<td>Mother’s time with the child</td>
<td>-0.6789</td>
<td>-0.5329</td>
<td>-0.9622</td>
</tr>
<tr>
<td>Informal child care</td>
<td>18.9312</td>
<td>20.3184</td>
<td>18.3242</td>
</tr>
<tr>
<td>Leisure</td>
<td>-0.2643</td>
<td>-0.1871</td>
<td>-0.3511</td>
</tr>
<tr>
<td><strong>B. Both types of child care are regulated and subsidized</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Score in last period</td>
<td>0.0349</td>
<td>0.0273</td>
<td>0.0439</td>
</tr>
<tr>
<td>Mother’s hours of work</td>
<td>0.9683</td>
<td>0.9806</td>
<td>0.9586</td>
</tr>
<tr>
<td>Mother’s time with the child</td>
<td>-0.6789</td>
<td>-0.5329</td>
<td>-0.9622</td>
</tr>
<tr>
<td>Formal child care</td>
<td>63.1795</td>
<td>63.3538</td>
<td>63.1077</td>
</tr>
<tr>
<td>Informal child care</td>
<td>94.9248</td>
<td>87.2256</td>
<td>98.2942</td>
</tr>
<tr>
<td>Leisure</td>
<td>-0.2643</td>
<td>-0.1871</td>
<td>-0.3511</td>
</tr>
</tbody>
</table>

NOTE. This table shows the percentage changes with respect to the simulated values of the child’s test score in the last period, as well as of the average (over the period) mother’s choices, as induced by the implementation of policies that (i) set the productivity of informal child care to the level of the formal one, according to the estimated values reported in Figure 3 - Left, (Panel A), and (ii) subsidize both types of non-parental child care, by setting their price to 0.5$/h (Panel B).

Table 10 reports the results of the simulation of the two policies. Panel A reports the results of the policy setting the productivity of informal child care to the level of the formal one (Policy A), while Panel B reports the results of a policy that, in addition to the regulation, also subsidizes non-parental child care by setting its price to 0.5$ per hour (Policy B). Policy A implies that, since informal services have a lower price than the formal ones, but are equally productive, they are the most convenient in the market and thus preferred by the mothers: for this reason, both high- and low-educated mothers switch from the use of formal to the use of informal child care. The increase in child care use leads to an increase in labor supply, which also affects the mother’s time allocation between child care and leisure: high-educated mothers decrease more their leisure time and less their child-care time than their low-educated counterpart, because of their higher productivity with the child. The final effect on the child’s test score in the last period is very small (i.e., on average it increases by 0.002 percent), and differs by a mother’s level of education. Children of high-educated mothers do not face any improvements in their final score, which seems to suggest that the potential positive effect of attending a higher-quality child care has been canceled out by the negative effect induced by a lower time with the mother. For children of low-educated mothers, instead, the final effect is positive, though extremely small: this suggests that in their case the positive effect of using a higher-quality child care slightly dominates the negative effect induced by a lower maternal child-care time.

The main difference between Policy B and A is that, in the context of the former, there is no price discrimination between formal and informal child care and mothers consider the two types of services as perfect substitutes. Again, there is an effect induced by the mother’s level of education: high-educated mothers are less willing to substitute their time with the alternative care available in the market, even though such alternative is more convenient and of better
quality than before. Indeed, the results show that with Policy B the use of both formal and informal child care increases, even though high-educated mothers decrease less their child-care time and switch less to informal child care than the low-educated ones. The effect on the child’s score in the last period is larger than for Policy A (i.e., on average, the final score increases by 0.03 percent) and positive for children of both high- and low-educated mothers. However, as a consequence of the differential reaction of mothers according to their level of education, the effect is larger for children of low-educated mothers than for those of the high educated.

8. Effects on a child’s non-cognitive ability

The analysis presented so far has shown that (i) a mother’s education matters for the cognitive development of a child, because it determines a different productivity of the time they spend together, and (ii) formal child care may be more effective in boosting a child’s cognitive development than the informal one, especially during the child’s first years of life. It could be the case that the same findings do not hold with respect to the non-cognitive ability of the child, and that the inputs considered in the analysis (i.e., maternal child-care time, formal and informal child care) play a different role for the social and behavioral aspects of the child development process. In order to shed light on this issue, this paragraph presents the results of a preliminary analysis that looks at the non-cognitive development of the child.

In order to assess the role played by maternal and non-parental child care for the child’s non-cognitive development, I re-estimate the model described in Section 3 by assuming that $A_t$ represents the non-cognitive ability of the child in each period $t$, and by using as a measure of ability an index summarizing the child’s behavioral skills. Two considerations are however in order. First, the aim of this exercise is to retrieve a set of productivity parameters related to the non-cognitive development of the child, that could be compared with the baseline results presented in Section 6. For this reason, the estimation is conducted separately from the baseline analysis.\footnote{Recent studies have established the relationship and the inter-dependency between the cognitive and the social-emotional skills (see, for instance, Heckman (2008), Heckman et al. (2006)). However, the estimation of a version of the model where both the cognitive and non-cognitive ability of the child are included would require too many additional assumptions on the relationship between the two types of ability, that would complicate the estimation tremendously, and would be beyond the scope of the paper.} Second, in order to ease the comparison between the productivity parameters estimated in the model with non-cognitive ability and in the baseline analysis, I keep the same model specification, including the specification of the score measure. This means that also the behavioral problem index used in this exercise is specified as described in Section 3.3: as for the cognitive measures, it represents a sum of items, where each item indicates one issue that may be present or absent from the child’s behavior, and the probability that each item is true is given by Equation (27). The measure of non-cognitive ability that I use for this exercise is a corrected version of the Behavioral Problem Index (BPI), which is available in the CDS data in 1997, 2002 and 2007. The BPI is made up of 30 items, representing questions asked to the primary caregiver about whether a problem (out of 30) is often, sometimes, or never true for the child. Examples of these behaviors are having sudden mood changes, anxiousness, meanness towards others, and obsessiveness. In order to adapt these questions to the structure of the score
specification in the model, I sum up the items in which the caregiver reports that the behavior is not present. This ensures that (i) also this measure represents a sum of positive answers, and that (ii) higher values imply better outcomes, as is the case for the LW and AP scores used in the baseline analysis. The drawback of this specification is that, differently from the cognitive measures, the behavioral one does not necessarily improve over time, because some behaviors might be more likely to happen in certain (later) ages than in others.

Figure 6
Elasticity of a child’s non-cognitive ability with respect to a mother’s time with the child and non-parental child care.

![Graph](image)

NOTE. This graph represents the productivity parameters for maternal child-care time ($\tau_t$) and non-parental child care ($i_t$), as a function of child’s age $t = 1, 2, 3, \ldots, 13$, when the non-cognitive ability of the child is measured by a corrected version of the Behavioral Problem Index (BPI).

Figure 6 reports the estimated elasticities of a child’s non-cognitive ability with respect to a mother’s child-care time and non-parental child care. By comparing them with the estimates reported in Figure 3, it can be seen that both maternal time with the child and non-parental child care are less productive for the non-cognitive development of the child, at any age of the child. Interestingly, Figure 6-Left shows no difference in productivity between high- and low-educated mothers, which seems to suggest that a mother’s education affects the types of activities enhancing the child’s cognitive development, but not those which are more directly related to the non-cognitive development of the child. Figure 6-Right, instead, shows that formal and informal child care plays a different role for the non-cognitive development according to the age of the child: informal child care matters more during the child’s first years of life, while the formal one becomes more important from age 3 onward. This result seems in line with previous studies in psychology and economics assessing the effects of formal and informal child care not only on cognitive but also on behavioral outcomes: for instance, NICHD (1998, 2002) and Loeb et al. (2007) finds that the attendance of formal child care before kindergarten, while having a positive effect on the child’s language and math competences, is detrimental for the child’s behavioral and social skills. This effect might be related to the different types of activities and to the level of attention that children may get in the two types of care. Informal child care services, in fact, often imply a one-to-one relationship between the caring adult and the child, which might be more important when the child is still an infant than later on. Formal
center-based child care services, instead, are often characterized by a lower adult-children ratio, which may help the child socialize with other children and improve his social skills at later ages (Brazelton, 1992). However, more work is needed to further understand the role played by maternal child-care and non-parental child care for the child’s non-cognitive development, by looking at other dimensions of non-cognitive skills and at the interdependency between these skills and the child’s cognitive ability.

9. Concluding remarks

This paper estimates a behavioral model where labor supply, formal and informal child care, and time allocation choices of the mother are considered endogenous. The model takes into account that a mother’s time productivity can be influenced by her level of education, and that non-parental child care may differently affect the cognitive development of the child according to whether it is a formal or informal arrangement. The paper also shows how a mother’s labor market participation decision, as well as its final effect on a child’s cognitive ability, is affected by the relative productivity of maternal child-care time with respect to the non-parental child-care type available in the market.

In line with previous studies on human capital accumulation reporting diverse productivity of investments over time, the results show that the productivity of both maternal child-care time and non-parental child care decreases as the child ages. Moreover, the elasticity of a child’s cognitive ability with respect to maternal child-care time is larger for high-educated mothers than for the low educated. This result may be due to the different educational content of a mother’s time according to her level of education, since more educated mothers may be more likely than the low-educated ones to engage in language-based activities or routines aimed at stimulating the child’s cognitive development. The results also show that formal child care is more productive than the informal one, especially before the child starts attending primary school. Also in this case, the larger productivity of formal child care can be justified by the fact that trained care providers may give more cognitive stimulation to the children than informal providers, and that center-based child care may provide more educational activities than informal care.

The simulation of the effects of policies that increase the mother’s opportunities in the labor market or regulate the non-parental child care market, by setting quality standards, shows that there is a differential effect induced by whether the mother has a college education or not. In fact, while the policies increase the labor supply of all mothers, the high-educated ones have less incentive to decrease their child-care time than their low-educated counterpart because the alternative forms of care in the market cannot fully compensate for the reduction in the child’s cognitive ability induced by a lower maternal child-care time. Indeed, the effect on the child’s test score in the last period is very small and positive only for the policies that increase the productivity of non-parental child care, while the wage-subsidy policy has a negative effect on the test score of children with high-educated mothers.

A preliminary analysis on the non-cognitive development of the child shows that the heterogeneity by a mother’s education and between non-parental child-care types is less relevant for the child’s behavioral skills, and that during the child’s early years of life formal child care can
induce more behavioral problems than the informal one. While this result seems in line with previous findings in the psychology and economics literature, more work is needed to properly understand the implications of these inputs, and to account for the inter-dependency between the child’s cognitive and non-cognitive development.

The analysis leaves space for further research. For instance, little is known about the substitutability or complementarity of a mother’s child-care time and non-parental child care in the production of human capital. While the model assumes that maternal time and non-parental child care are substitute, future research should better understand how the mother’s investment decisions interact in the production function for the child’s cognitive and non-cognitive ability.
Acknowledgments

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References


Appendix A. Analytic solution of the model

In this Appendix I derive analytically the closed-form solutions of the model, for all the choice variables. The process of backward induction involves the solution of the optimization problem in each period, starting from the last one, $T$. Consider first the choice variables $i_t$, $f_t$ and $\tau_t$. The first step is to find the optimal child care and time input decisions at time $T$. The value function of the mother at period $T$ can be written as:

$$V_T = \max_{\tau_T, i_T, f_T} \alpha_1 \ln(T - h_T - \tau_T) + \alpha_2 \ln(w_T h_T + I_T - p_i i_T - p_f f_T) + \alpha_3 \ln(A_T) + E_T \beta \{\bar{V}_{T+1} + \rho \alpha_3 \ln(A_{T+1})\}$$

where the variables $l_T$ and $c_T$ have been already substituted using the time and budget constraints, the CAPF has been log-linearized for computational convenience, and the braces include the terminal period value function, as specified in (6).

The optimal solutions for $\tau^*_c$, $i^*_c$ and $f^*_c$ at period $T$, conditional on $h_T$, are given by the solutions of the following first order conditions (FOCs):

$$\tau^*_c \Rightarrow \frac{\partial \bar{V}_T}{\partial \tau_T} = 0$$
$$i^*_c \Rightarrow \frac{\partial \bar{V}_T}{\partial i_T} = 0$$
$$f^*_c \Rightarrow \frac{\partial \bar{V}_T}{\partial f_T} = 0$$

(A.2)

Because of the value-added specification of the child cognitive ability production function, as defined by (4), child ability in period $T + 1$ is a function of the inputs received by the child at period $T$. Hence, (A.2) can be rearranged, using total differential, in the following way:

$$\tau^*_c \Rightarrow \frac{\partial \bar{V}_T}{\partial \tau_T} + \frac{\partial \bar{V}_{T+1}}{\partial \ln A_{T+1}} \times \frac{\partial \ln A_{T+1}}{\partial \tau_T} = 0$$
$$i^*_c \Rightarrow \frac{\partial \bar{V}_T}{\partial i_T} + \frac{\partial \bar{V}_{T+1}}{\partial \ln A_{T+1}} \times \frac{\partial \ln A_{T+1}}{\partial i_T} = 0$$
$$f^*_c \Rightarrow \frac{\partial \bar{V}_T}{\partial f_T} + \frac{\partial \bar{V}_{T+1}}{\partial \ln A_{T+1}} \times \frac{\partial \ln A_{T+1}}{\partial f_T} = 0$$

(A.3)

where $\bar{V}_T$ is the current utility in period $T$:

$$\bar{V}_T = \alpha_1 \ln(T - h_T - \tau_T) + \alpha_2 \ln(w_T h_T + I_T - p_i i_T - p_f f_T) + \alpha_3 \ln(A_T)$$

The corresponding derivatives are given by the following expressions:
\[ \frac{\partial V_T}{\partial \tau_T} = \frac{-\alpha_1}{TT - h_T - \tau_T} \]  
\[ \frac{\partial V_{T+1}}{\partial \ln A_{T+1}} \times \frac{\partial \ln A_{T+1}}{\partial \tau_T} = (\beta \rho \alpha_3) \left( \frac{\delta_{1T}}{\tau_T} \right) \]  
\[ \frac{\partial V_T}{\partial i_T} = -p_i \alpha_2 \]  
\[ \frac{\partial V_{T+1}}{\partial \ln A_{T+1}} \times \frac{\partial \ln A_{T+1}}{\partial i_T} = (\beta \rho \alpha_3) \left( \frac{\delta_{2T}}{i_T} \right) \]  
\[ \frac{\partial V_T}{\partial f_T} = \frac{-p_f \alpha_2}{w_T h_T + I_T - p_i i_T - p_f f_T} \]  
\[ \frac{\partial V_{T+1}}{\partial \ln A_{T+1}} \times \frac{\partial \ln A_{T+1}}{\partial f_T} = (\beta \rho \alpha_3) \left( \frac{\delta_{3T}}{f_T} \right) \]  

and the FOCs become:

\[ \tau^c_T \Rightarrow -\frac{\alpha_1}{TT - h_T - \tau_T} + (\beta \rho \alpha_3) \left( \frac{\delta_{1T}}{\tau_T} \right) = 0 \]  
\[ i^c_T \Rightarrow -\frac{p_i \alpha_2}{w_T h_T + I_T - p_i i_T - p_f f_T} + (\beta \rho \alpha_3) \left( \frac{\delta_{2T}}{i_T} \right) = 0 \]  
\[ f^c_T \Rightarrow -\frac{p_f \alpha_2}{w_T h_T + I_T - p_i i_T - p_f f_T} + (\beta \rho \alpha_3) \left( \frac{\delta_{3T}}{f_T} \right) = 0 \]

The solutions for the three inputs at period \( T \) are given by:

\[ \tau^c_T = \frac{\beta \delta_{1T} D_{T+1}}{\alpha_1 + \beta \delta_{1T} D_{T+1}} (TT - h_T) \]  
\[ i^c_T = \frac{\beta \delta_{2T} D_{T+1}}{p_i (\alpha_2 + \beta \delta_{2T} D_{T+1} + \beta \delta_{3T} D_{T+1})} (w_T h_T + I_T) \]  
\[ f^c_T = \frac{\beta \delta_{3T} D_{T+1}}{p_f (\alpha_2 + \beta \delta_{2T} D_{T+1} + \beta \delta_{3T} D_{T+1})} (w_T h_T + I_T) \]

where \( D_{T+1} = \frac{\partial V_{T+1}}{\partial \ln A_{T+1}} = \rho \alpha_3 \).

These solutions can be substituted into the value function of the mother at period \( T \), in order to get \( V_T(\tau^c_T, i^c_T, f^c_T) \).

This expression can be used to write down the value function at period \((T - 1)\). Using the same process described for periods \( T \) and computing the corresponding derivatives yield the solutions for period \((T - 1)\). The solutions for all the periods up to period \( t = 1 \) can be retrieved similarly.

At the end, three sequences of optimal choices can be obtained. The sequence of optimal choices for time with the child, conditional on the mother’s labor supply, is given by:
Equation (A.19) is equal to Equation (7) in the text.

The sequences of the optimal informal and formal child care choices, conditional on the mother’s labor supply, are given by:

$$i_T^c = \frac{\beta_2 T D_{T+1}}{p_i(\alpha_2 + \beta_2 T D_{T+1} + \beta_3 T D_{T+1})} (w_{T} h_{T} + I_T)$$  \hspace{1cm} (A.22)

$$i_{T-1}^c = \frac{\beta_2 T_{-1} D_{T}}{p_i(\alpha_2 + \beta_2 T_{-1} D_{T} + \beta_3 T_{-1} D_{T})} (w_{T-1} h_{T-1} + I_{T-1})$$  \hspace{1cm} (A.23)

$$i_{T-2}^c = \frac{\beta_2 T_{-2} D_{T-1}}{p_i(\alpha_2 + \beta_2 T_{-2} D_{T-1} + \beta_3 T_{-2} D_{T-1})} (w_{T-2} h_{T-2} + I_{T-2})$$  \hspace{1cm} (A.24)

$$i_T^f = \frac{\beta_1 T D_{T+1}}{p_i(\alpha_1 + \beta_1 T D_{T+1})} (TT - h_T)$$  \hspace{1cm} (A.16)

$$i_{T-1}^f = \frac{\beta_1 T_{-1} D_{T}}{p_i(\alpha_1 + \beta_1 T_{-1} D_{T})} (TT - h_{T-1})$$  \hspace{1cm} (A.17)

$$i_{T-2}^f = \frac{\beta_1 T_{-2} D_{T-1}}{p_i(\alpha_1 + \beta_1 T_{-2} D_{T-1})} (TT - h_{T-2})$$  \hspace{1cm} (A.18)

$$\vdots$$

$$i_t^c = \frac{\beta_1 t D_{t+1}}{p_i(\alpha_1 + \beta_1 t D_{t+1})} (TT - h_t)$$  \hspace{1cm} (A.19)

$$\vdots$$

$$i_2^c = \frac{\beta_1 2 D_3}{p_i(\alpha_2 + \beta_1 2 D_3 + \beta_3 2 D_3)} (TT - h_2)$$  \hspace{1cm} (A.20)

$$i_1^c = \frac{\beta_1 1 D_2}{p_i(\alpha_1 + \beta_1 1 D_2)} (TT - h_1)$$  \hspace{1cm} (A.21)

$$i_T^c = \frac{\beta_2 T D_{T+1}}{p_i(\alpha_2 + \beta_2 T D_{T+1} + \beta_3 T D_{T+1})} (w_T h_T + I_T)$$  \hspace{1cm} (A.22)

$$i_{T-1}^c = \frac{\beta_2 T_{-1} D_{T}}{p_i(\alpha_2 + \beta_2 T_{-1} D_{T} + \beta_3 T_{-1} D_{T})} (w_{T-1} h_{T-1} + I_{T-1})$$  \hspace{1cm} (A.23)

$$i_{T-2}^c = \frac{\beta_2 T_{-2} D_{T-1}}{p_i(\alpha_2 + \beta_2 T_{-2} D_{T-1} + \beta_3 T_{-2} D_{T-1})} (w_{T-2} h_{T-2} + I_{T-2})$$  \hspace{1cm} (A.24)

$$i_T^f = \frac{\beta_1 T D_{T+1}}{p_i(\alpha_1 + \beta_1 T D_{T+1})} (TT - h_T)$$  \hspace{1cm} (A.16)

$$i_{T-1}^f = \frac{\beta_1 T_{-1} D_{T}}{p_i(\alpha_1 + \beta_1 T_{-1} D_{T})} (TT - h_{T-1})$$  \hspace{1cm} (A.17)

$$i_{T-2}^f = \frac{\beta_1 T_{-2} D_{T-1}}{p_i(\alpha_1 + \beta_1 T_{-2} D_{T-1})} (TT - h_{T-2})$$  \hspace{1cm} (A.18)

$$\vdots$$

$$i_t^c = \frac{\beta_1 t D_{t+1}}{p_i(\alpha_1 + \beta_1 t D_{t+1})} (TT - h_t)$$  \hspace{1cm} (A.19)

$$\vdots$$

$$i_2^c = \frac{\beta_1 2 D_3}{p_i(\alpha_2 + \beta_1 2 D_3 + \beta_3 2 D_3)} (TT - h_2)$$  \hspace{1cm} (A.20)

$$i_1^c = \frac{\beta_1 1 D_2}{p_i(\alpha_1 + \beta_1 1 D_2)} (TT - h_1)$$  \hspace{1cm} (A.21)
\[ f_{T-1}^c = \frac{\beta_3 T + 1}{p_f(\alpha_2 + \beta_2 T + 1 + \beta_3 T + 1)} (w_{T-1} + I_{T-1}) \]  
\[ f_{T-2}^c = \frac{\beta_3 T - 1}{p_f(\alpha_2 + \beta_2 T - 1 + \beta_3 T - 1)} (w_{T-2} + I_{T-2}) \]  
\[ \vdots \]
\[ f_t^c = \frac{\beta_3 T + 1}{p_f(\alpha_2 + \beta_2 T + 1 + \beta_3 T + 1)} (w_t + I_t) \]  
\[ f_2^c = \frac{\beta_3 T + 3}{p_f(\alpha_2 + \beta_2 T + 3 + \beta_3 T + 3)} (w_2 + I_2) \]  
\[ f_1^c = \frac{\beta_3 T + 1}{p_f(\alpha_2 + \beta_2 T + 1 + \beta_3 T + 1)} (w_1 + I_1) \]  

Equation (A.25) is equal to (8) in the main text, while Equation (A.31) corresponds to (9) in the text. The sequence of values for \( D_{t+1} \) is reported in (10).

Having found the solutions for the time allocation and non-parental child care decisions, the solution for the mother’s labor supply can be computed using the same backward procedure. Equation (11) represents the optimal labor supply in each period as a function of \( \tau_t, i_t \), and \( f_t \); substituting (7), (8) and (9), it yields the optimal labor supply choice for each period \( t \), as defined by (12).

**Appendix B. The PSID data and the CDS-TD supplements**

The dataset is composed of different supplements of the Panel Study of Income Dynamics (PSID) gathered in the period 1985-2007. Table B.1 summarizes the main information on availability and sources of data.

To merge PSID and CDS data I exploit the information on the relationship of each CDS child with respect to the head of the household and the primary caregiver. The final sample is made up of all children aged 0-12 in 1997 without siblings and with both parents living in the household, without missing information on child’s and parents’ characteristics and with at least one test score observation. As summarized in Table B.2, the birth cohorts of children in this sample range from 1984 to 1996, while the terminal period of the model \( T = 13 \) corresponds to 1997 for those born in 1984 and to 2009 for those born in 1996.

Table B.3 summarizes the available data for a child born in 1996. This table stresses the existence of a long time-gap of missing data, because of the structure of the surveys and the timing of the interviews. In particular, data on maternal time and child’s cognitive outcomes are available only in the years of the TD and CDS supplements, i.e., 1997, 2002 and 2007. Data on non-parental child care suffers from the same issue after kindergarten age, since that information is available only for the year of the CDS survey.

Table B.4 shows the average characteristics of the sample used for the estimation \( N = 417 \)
### Table B.1
Availability and sources of data.

<table>
<thead>
<tr>
<th>Set of Variables</th>
<th>Source</th>
<th>Survey Years</th>
<th>Additional Info</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formal and informal child care</td>
<td>CDS</td>
<td>1997-2002-2007</td>
<td>Retrospective questions on the most used arrangements from birth until kindergarten and questions on the most used arrangements at the time of the survey.</td>
</tr>
<tr>
<td>Child cognitive outcomes</td>
<td>CDS</td>
<td>1997-2002-2007</td>
<td>Only for children older than 3</td>
</tr>
<tr>
<td>Child demographic characteristics</td>
<td>CDS</td>
<td>1997-2002</td>
<td>Time-invariant (except age)</td>
</tr>
<tr>
<td>Maternal time with the child</td>
<td>CDS-TD</td>
<td>1997-2002</td>
<td>Available only for the year of the survey</td>
</tr>
<tr>
<td>Parents’ demographic characteristics</td>
<td>PSID</td>
<td>1997</td>
<td>Time-invariant (except age)</td>
</tr>
</tbody>
</table>

and of the total sample of children in CDS, for whom it has been possible to derive information on their parents (3243 observations). This comparison sample includes both families with only one child and families with more children.

### Appendix C. Estimation

The estimation is done in two-stages: the parameters of the income process are estimated in the first stage, while all remaining parameters are estimated in the second stage. After computing the statistics defined in Table 4 for the actual data, I proceed with the first-stage estimation of the income parameters. This involves the simulation of the income process, after drawing from a standard normal distribution $N \times R$ times, for every period, with $N = 417$ and $R = 5$. The statistics used to estimate these parameters are the average and standard deviation of income for all the periods, as well as the average household income by a father’s level of education, race and age. I compute these points for both the actual and the simulated income processes. The Method of Simulated Moments estimator for this first stage minimizes an objective function where each moment condition is the distance between the income data...
moments and their simulated counterparts. Each moment condition is weighted using the inverse of the corresponding statistics in the data.

The second-stage involves the estimation of all remaining parameters using the same estimator. First of all, I simulate the data according to the DGP implied by the model, taking $N \times R \times T$ draws for wage and income and $N \times R$ draws for the child’s initial ability shock, the mother’s skills, as well as the mother’s preferences, with $N = 417$, $R = 5$ and $T = 13$. Following Keane and Moffitt (1998), I re-draw the errors to simulate the income distribution using the parameters estimated in the first stage. In each period, the values for the mother’s labor supply, formal and informal child care and maternal time are derived using the optimal solutions implied by the model. Then, after having simulated the data for all the periods, I compute the statistics defined in Table 4 from the simulated data.

The estimator used in this second-stage minimizes an objective function where each moment condition is the distance between the data statistics and the simulated counterparts:

$$\hat{\theta} = \arg \min \hat{g}(\theta)' W \hat{g}(\theta)$$

(C.1)

where

$$\hat{g}(\theta) = \bar{m} - \bar{M}(\theta)$$
Table B.4
Mean characteristics of the sample with respect to PSID-CDS data.

<table>
<thead>
<tr>
<th></th>
<th>PSID-CDS</th>
<th>Sample</th>
<th>T-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mother’s hours of work</td>
<td>23.61</td>
<td>27.30</td>
<td>−10.71***</td>
</tr>
<tr>
<td>Mother’s time with the child</td>
<td>25.83</td>
<td>21.16</td>
<td>5.42***</td>
</tr>
<tr>
<td>Formal child care</td>
<td>8.14</td>
<td>10.26</td>
<td>−6.99***</td>
</tr>
<tr>
<td>Informal child care</td>
<td>4.94</td>
<td>5.84</td>
<td>−3.48***</td>
</tr>
<tr>
<td>Mother’s wage before child’s birth</td>
<td>11.01</td>
<td>11.31</td>
<td>−1.25</td>
</tr>
<tr>
<td>Household income</td>
<td>674.16</td>
<td>791.36</td>
<td>−7.56***</td>
</tr>
<tr>
<td>Mother’s education</td>
<td>12.99</td>
<td>13.27</td>
<td>−7.03***</td>
</tr>
<tr>
<td>Mother’s age at child’s birth</td>
<td>26.99</td>
<td>28.20</td>
<td>−14.43***</td>
</tr>
<tr>
<td>Mother’s race: white</td>
<td>0.61</td>
<td>0.61</td>
<td>0.33</td>
</tr>
<tr>
<td>Child’s gender: male</td>
<td>0.51</td>
<td>0.51</td>
<td>0.29</td>
</tr>
<tr>
<td>Child’s birth weight</td>
<td>3315.53</td>
<td>3387.16</td>
<td>−7.77***</td>
</tr>
</tbody>
</table>

* Monetary variables deflated into 1997 US$.

** Mother’s wage before childbirth refers to the year before the child was born.

*** Difference statistically significant at the p < 0.01 level.

$n$ is the vector of statistics defined from the actual data, while $\hat{M}(\theta)$ is the vector of simulated statistics according to the model that are functions of the structural parameters to be estimated. $W$ is a positive definite diagonal weighting matrix. The most efficient minimum distance estimator uses a weighting matrix whose elements are estimates of the inverse of the covariance matrix of the vector $n$; this is the so-called optimal minimum distance (OMD) estimator (Cameron and Trivedi, 2005, pag. 203). Since Altonji and Segal (1996) provide evidence of small sample biases in the OMD estimator, I use the diagonally weighted minimum distance estimator proposed by Blundell, Pistaferri, and Preston (2008). Given $S$ number of moments, the weighting matrix is then defined as:

$$W = \begin{pmatrix}
\hat{V}[\hat{n}_1]^{-1} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & \hat{V}[\hat{n}_S]^{-1}
\end{pmatrix}$$

where $\hat{V}[\hat{n}]$ is estimated with non-parametric bootstrap and according to the formula (Davidson and MacKinnon, 2003, p. 208):

$$\hat{V}[\hat{n}] = \left[ \frac{1}{B} \sum_{b=1}^{B} (\hat{m}_b^* - \bar{m}^*) (\hat{m}_b^* - \bar{m}^*)' \right]$$

(C.2)

Non-parametric bootstrap (with replacement) is implemented following Wooldridge (2002, p. 379): I use a random number generator to obtain $N$ integers, where $N = 417$ represents the sample size of the actual data, and these integers index the observations drawn from the actual distribution of data. Repeating this process $B$ times, it yields $B$ bootstrap samples on which the statistics defined in Table 4 can be computed: $\hat{m}_b^*$ represents a statistic computed for the
sample $b$, while $\bar{m}^*$ is the average of the statistics across the $B$ samples.\textsuperscript{38}

Appendix C.1. Standard errors

Non-parametric bootstrap with replacement is also used to compute the standard errors. After having drawn $B_{se}$ samples from the actual data, I repeat the estimation of the parameters for each sample, by using different starting values for each bootstrap iteration.\textsuperscript{39} This yields an empirical distribution of the parameters estimates, from which I can recover a bootstrap estimate of the variance, using the formula (Train, 2009, pag. 201):

$$
\hat{V}\left[\hat{\theta}\right] = \left[\frac{1}{B}\right] \sum_{b=1}^{B} \left(\hat{\theta}_b^* - \bar{\theta}^*\right) \left(\hat{\theta}_b^* - \bar{\theta}^*\right)'
$$

(C.3)

Taking the square root of (C.3) yields the bootstrap estimate of the standard errors $se_{\hat{\theta}}$.

Appendix C.2. Identification

This subsection presents a list of figures that show the variation in the objective function and in the main moment conditions induced by the perturbation of the estimated parameters, in order to test the goodness of the selected moments for the identification of the parameters.

Figure C.1 shows the variation in the objective function (Equation (C.1)) induced by the perturbation of each estimated parameter in the vector $\hat{\theta}$. Figure C.2 reports the variation in the moments used to identify the formal and informal child care cost equations: these moments represent the correlation between the cost of each child care type and the corresponding cost determinant, i.e., state funding for center-based child care for formal child care and presence of family members in the neighborhood for the informal one, respectively. Figures C.3 and C.4 refer instead to the moments used for the identification of the productivity parameters: Figure C.3 shows the variation in the moments used to identify the slope parameter in the productivity of a mother’s child-care time, and the contribution of a mother’s college education to the same productivity parameter; Figure C.4 shows instead the variation in the moments used to identify the slope parameters in the productivity of informal and formal child care. Figure C.5 reports the variation in the moment conditions used to identify the mother’s unobserved productivity types in the labor market, by perturbing the estimated values. Finally, Figure C.6 reports the variation in the moments used for the identification of the initial ability, as specified in Section 5, that is induced by the perturbation of the intercept and shock parameters in the initial level of ability. The presence of a variation in all the above-mentioned moments confirms that such statistics are informative of the estimated parameters and can be used for identification purposes.

Appendix D. Estimated parameters and fit of moments

Table D.1 reports the untransformed parameters in the mother’s utility function (Panel A), and in the child’s cognitive ability production function (Panel B). Table D.2 reports the estimated

\textsuperscript{38}B = 200.

\textsuperscript{39}B_{se} = 50
Figure C.1
Variation in the objective function around the estimated parameters.

NOTE. This graph reports the values of the objective function perturbing each parameter by 2 standard deviations up and down with respect to the estimated value.

Figure C.2
Variation in the moment conditions used to identify the parameters in the informal and formal child care cost equations, by perturbing the estimated parameters.

NOTE. This graph reports the values of the moment conditions obtained from (i) the correlation between the informal child care cost and the presence of family members in the neighborhood (Left), and (ii) the correlation between the formal child care cost and the state funding for center-based child care (Right), by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated values.

Figure C.3
Variation in the moment conditions used to identify the productivity of a mother’s time with the child and the contribution of a mother’s education, by perturbing the estimated parameters.

NOTE. This graph reports the values of the moment conditions obtained from (i) the correlation between a mother’s time with the child in t and the child’s scores in t+5, conditional on whether the score is LW or AP (Left), and (ii) the correlation between a mother’s education and a child’s score, conditional on whether the score is LW or AP and on a mother’s wage (Right), by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated values. The parameters represent the elasticity of a child’s ability with respect to a mother’s time with the child (Left) and the contribution of a mother’s education to such elasticity (Right).
Figure C.4
Variation in the moment conditions used to identify the productivity of informal and formal child care, by perturbing the estimated parameters.

![Graph showing variation in productivity](image)

NOTE. This graph reports the values of the moment conditions obtained from the correlation between informal (Left) and formal (Right) child care hours in $t$ and the child's scores in $t + 1$, conditional on whether the score is LW or AP, by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated values. The parameters represent the elasticity of a child’s ability with respect to informal (Left) and formal child care (Right).

Figure C.5
Variation in the moment conditions used to identify a mother’s unobserved productivity in the labor market, by perturbing the estimated parameters.

![Graph showing variation in productivity](image)

NOTE. This graph reports the values of the moment conditions obtained from the variance (Left column) and serial correlation (Right column) of the residuals from a OLS regression of a mother’s wage on a mother’s education, race, age, year of birth, area of residence and the interaction between the latter two, by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated value. The parameters represent the proportion of mothers in each group identified by a level of unobserved skills in the labor market ($\text{MomTypeLow}$ and $\text{MomTypeHigh}$) and a level of preference for a child’s ability ($\text{Gamma3Low}$ and $\text{Gamma3High}$).

Figure C.6
Variation in the moment conditions used to identify the intercept and shock in the child’s initial ability, by perturbing the estimated parameters.

![Graph showing variation in productivity](image)

NOTE. This graph reports the values of the moment condition obtained from the variance of the residuals from a OLS regression of a child’s first test score observation on a dummy indicating whether the test is LW or AP and a child’s age fixed effects, by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated values. The parameters represent the intercept (Left) and the standard deviation of the shock (Right) in the initial level of ability of a child.
parameters in the household income function (Panel A), and the estimated parameters in the initial level of ability of the child and in the test score specification (Panel B). Finally, Table D.3 reports the fit for the targeted unconditional moments used for the estimation of the model.

Table D.1
Estimated untransformed parameters in the mother’s utility function and the child’s cognitive ability production function.

<table>
<thead>
<tr>
<th>Panel A. Utility function</th>
<th>Estimate</th>
<th>Std. Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_2$ Taste for consumption Type I</td>
<td>-0.0218</td>
<td>0.2020</td>
</tr>
<tr>
<td>$\gamma_2$ Taste for consumption Type II</td>
<td>-0.0179</td>
<td>0.2872</td>
</tr>
<tr>
<td>$\gamma_3$ Taste for child ability Type I</td>
<td>-0.6952</td>
<td>0.2052</td>
</tr>
<tr>
<td>$\gamma_3$ Taste for child ability Type II</td>
<td>-0.1238</td>
<td>0.1391</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Cognitive ability production function</th>
<th>Estimate</th>
<th>Std. Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_{tfp}$ Total factor productivity. Intercept</td>
<td>-0.4371</td>
<td>0.1384</td>
</tr>
<tr>
<td>$\xi_{tfp}$ Total factor productivity. Slope</td>
<td>0.0919</td>
<td>0.0160</td>
</tr>
<tr>
<td>$\xi_{t}$ Productivity of a mother’s time with the child. Intercept</td>
<td>0.2623</td>
<td>0.2981</td>
</tr>
<tr>
<td>$\xi_{Edu}$ Productivity of a mother’s time with the child. Effect of a mother’s education</td>
<td>0.6135</td>
<td>0.6779</td>
</tr>
<tr>
<td>$\xi_{t}$ Productivity of a mother’s time with the child. Slope</td>
<td>-0.3036</td>
<td>0.0393</td>
</tr>
<tr>
<td>$\xi_{n}$ Productivity of informal child care. Intercept</td>
<td>-0.0060</td>
<td>0.2740</td>
</tr>
<tr>
<td>$\xi_{n}$ Productivity of informal child care. Slope</td>
<td>-0.6362</td>
<td>0.0648</td>
</tr>
<tr>
<td>$\xi_{f}$ Productivity of formal child care. Intercept</td>
<td>0.3470</td>
<td>0.3305</td>
</tr>
<tr>
<td>$\xi_{f}$ Productivity of formal child care. Slope</td>
<td>-0.6709</td>
<td>0.0501</td>
</tr>
<tr>
<td>$\xi_{A}$ Productivity of a child’s ability in the previous period. Intercept</td>
<td>-0.3047</td>
<td>0.0667</td>
</tr>
<tr>
<td>$\xi_{A}$ Productivity of a child’s ability in the previous period. Slope</td>
<td>-0.1653</td>
<td>0.0312</td>
</tr>
</tbody>
</table>

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap iteration. See Appendix C for further details on the estimation.

Table D.2
Estimated parameters for the household income function, the child’s initial ability and the test score specification.

<table>
<thead>
<tr>
<th>Panel A. Household income function</th>
<th>Estimate</th>
<th>Std. Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{inc}$ Intercept</td>
<td>-0.3759</td>
<td>0.3067</td>
</tr>
<tr>
<td>$\mu_{inc}$ Coefficient for father’s years of education</td>
<td>0.1263</td>
<td>0.0145</td>
</tr>
<tr>
<td>$\mu_{inc}$ Coefficient for father’s race</td>
<td>0.2162</td>
<td>0.0529</td>
</tr>
<tr>
<td>$\mu_{inc}$ Coefficient for father’s age</td>
<td>0.0102</td>
<td>0.0054</td>
</tr>
<tr>
<td>$\sigma_{inc}$ Std deviation income shock</td>
<td>0.6185</td>
<td>0.0366</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Initial ability and test score specification</th>
<th>Estimate</th>
<th>Std. Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_0$ Intercept</td>
<td>-17.1175</td>
<td>9.2067</td>
</tr>
<tr>
<td>$\eta_1$ Coefficient for birth weight</td>
<td>-13.2826</td>
<td>22.0854</td>
</tr>
<tr>
<td>$\eta_2$ Coefficient for gender</td>
<td>-20.8972</td>
<td>18.8766</td>
</tr>
<tr>
<td>$\eta_3$ Coefficient for a mother’s age at birth</td>
<td>-18.2699</td>
<td>6.6867</td>
</tr>
<tr>
<td>$\sigma_v$ Std deviation initial ability shock</td>
<td>16.0095</td>
<td>0.8058</td>
</tr>
<tr>
<td>$\kappa$ Coefficient for LW test scores (vs AP)</td>
<td>0.1748</td>
<td>0.0317</td>
</tr>
</tbody>
</table>

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap iteration. See Appendix C for further details on the estimation.

Appendix E. Sensitivity analysis

This section presents two exercises that I perform in order to understand the implications of omitting the schooling time and the father’s time with the child from the specification of the
after age five, I assign that to all ages afterwards; to those, instead, who just have a schooling
reason, assumptions should be made on how to assign these values to the missing periods. For
and for many children available only at one point in time (i.e., either 1997 or 2002). For this
the use of this variable is that it is cross-sectional (i.e., available only for the year of the survey),
in the Time Diary component of the PSID, gathered in 1997 and 2002. The main problem with
increase the productivity of this specific input.

inputs. On the other hand, the fact that time at school is summed up to formal child care may
affect the estimated productivity parameters for a mother’s child-care time and non-parental
child care in two opposite directions. On the one hand, the fact that another educational input
is included in the CAPF may lower the importance of the mother’s investment decisions, and
thus the role played by a mother’s education or formal child care in providing high-quality
inputs. On the other hand, the fact that time at school is summed up to formal child care may
increase the productivity of this specific input.

For the implementation of this exercise, I use the information on time spent at school provided
in the Time Diary component of the PSID, gathered in 1997 and 2002. The main problem with
the use of this variable is that it is cross-sectional (i.e., available only for the year of the survey),
and for many children available only at one point in time (i.e., either 1997 or 2002). For this
reason, assumptions should be made on how to assign these values to the missing periods. For
this exercise, I recode the variable as follows: for the children who have a schooling information
after age five, I assign that to all ages afterwards; to those, instead, who just have a schooling

<table>
<thead>
<tr>
<th>Table D.3</th>
<th>Fit for targeted unconditional moments.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td><strong>Simulation</strong></td>
</tr>
<tr>
<td>Corr mother’s wage and mother’s hours of work</td>
<td>0.0054 0.0858</td>
</tr>
<tr>
<td>Corr household income and mother’s hours of work</td>
<td>−0.3147 −0.7119</td>
</tr>
<tr>
<td>Corr mother’s wage and mother’s time with the child</td>
<td>0.2565 0.2168</td>
</tr>
<tr>
<td>Corr household income and mother’s time with the child</td>
<td>−0.0986 −0.0421</td>
</tr>
<tr>
<td>Corr mother’s wage and formal child-care time</td>
<td>0.7460 0.3814</td>
</tr>
<tr>
<td>Corr mother’s wage and informal child-care time</td>
<td>0.3898 0.2263</td>
</tr>
<tr>
<td>Corr household income and formal child-care time</td>
<td>0.9607 0.2364</td>
</tr>
<tr>
<td>Corr household income and informal child-care time</td>
<td>0.5115 0.1919</td>
</tr>
<tr>
<td>Corr mother’s hours of work and mother’s time with the child</td>
<td>−0.0447 −0.5757</td>
</tr>
<tr>
<td>Corr mother’s hours of work and formal child-care time</td>
<td>0.4393 0.0838</td>
</tr>
<tr>
<td>Corr mother’s hours of work and informal child-care time</td>
<td>0.2429 0.0704</td>
</tr>
<tr>
<td>Coefficient of mother’s time with the child in t − 5 in a OLS reg on test score in t, cond. on a dummy for LW</td>
<td>0.5880 0.4110</td>
</tr>
<tr>
<td>Coef of a dummy for high-educated mother on child’s test score, cond. on child’s age FE, a dummy for LW and mother’s wage</td>
<td>1.5746 2.5506</td>
</tr>
<tr>
<td>Coef of a dummy for high-educated mother on mother’s time with the child, cond. on child’s age FE and mother’s wage</td>
<td>1.5311 8.6370</td>
</tr>
<tr>
<td>Coef of a dummy for high-educated mother on mother’s hours of work, cond. on child’s age FE and mother’s wage</td>
<td>−1.4386 −7.4585</td>
</tr>
<tr>
<td>Coef of formal child care in t − 1 in a OLS regression on test score in t, cond. on a dummy for LW</td>
<td>0.3443 0.0091</td>
</tr>
<tr>
<td>Coef of informal child care in t − 1 in a OLS regression on test score in t, cond. on a dummy for LW</td>
<td>0.0979 0.0088</td>
</tr>
<tr>
<td>Var of residuals from child’s test score OLS reg on a dummy for LW and child’s age FE</td>
<td>39.9551 35.5324</td>
</tr>
<tr>
<td>Mean mother’s wage</td>
<td>14.8059 4.0804</td>
</tr>
<tr>
<td>Std deviation mother’s wage</td>
<td>10.2725 18.0704</td>
</tr>
<tr>
<td>Var of the residuals from a mother’s wage OLS reg on mother’s charact.</td>
<td>0.2199 0.2314</td>
</tr>
<tr>
<td>Coef of residuals from a mother’s wage OLS reg on mother’s charact. in t on the residuals in t − 1 (autocorr)</td>
<td>0.6729 0.5174</td>
</tr>
<tr>
<td>Mean price formal child care</td>
<td>2.0769 2.9485</td>
</tr>
<tr>
<td>Std deviation price formal child care</td>
<td>3.5989 4.5844</td>
</tr>
<tr>
<td>Mean price informal child care</td>
<td>1.2788 2.4187</td>
</tr>
<tr>
<td>Std deviation price informal child care</td>
<td>1.2928 3.5448</td>
</tr>
<tr>
<td>Corr price formal child care and state funding for center-based child care</td>
<td>0.4572 0.6029</td>
</tr>
<tr>
<td>Corr price informal child care and family in neighborhood</td>
<td>−0.0409 0.0886</td>
</tr>
<tr>
<td>IV reg of formal child-care hours on the price of formal child care, instrumented by state funding for center-based child care</td>
<td>−1.0439 −1.2683</td>
</tr>
<tr>
<td>Mean household income</td>
<td>7.9136 7.9036</td>
</tr>
<tr>
<td>Std deviation household income</td>
<td>6.4406 6.4411</td>
</tr>
</tbody>
</table>

NOTE. Actual data represent PSID-CDS data on children aged 0-12 in 1997, without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

CAPF in the baseline model. For the sake of brevity, I only report and discuss the productivity
parameters for a mother’s time with the child, and for formal and informal child care.

Appendix E.1. Time at school

The baseline analysis relies on a specification of the CAPF (Equation (4)) without any inputs
related to the time the child spends at school. In order to check how this affects the estimated
productivity parameters, I re-estimate the model by adding an amount corresponding to the time
spent at school to the measure of formal child care. The inclusion of this additional value may
affect the estimated productivity parameters for a mother’s child-care time and non-parental
child care in two opposite directions. On the one hand, the fact that another educational input
is included in the CAPF may lower the importance of the mother’s investment decisions, and
thus the role played by a mother’s education or formal child care in providing high-quality
inputs. On the other hand, the fact that time at school is summed up to formal child care may
increase the productivity of this specific input.

For the implementation of this exercise, I use the information on time spent at school provided
in the Time Diary component of the PSID, gathered in 1997 and 2002. The main problem with
the use of this variable is that it is cross-sectional (i.e., available only for the year of the survey),
and for many children available only at one point in time (i.e., either 1997 or 2002). For this
reason, assumptions should be made on how to assign these values to the missing periods. For
this exercise, I recode the variable as follows: for the children who have a schooling information
after age five, I assign that to all ages afterwards; to those, instead, who just have a schooling

55
information before age five, which is equal to zero because they do not attend school yet, I assign them the median amount of time in school after age five in the sample. I then re-estimate the model by using a measure of formal child care time that incorporates the amount of time the child spends at school after age five. The estimated productivity parameters are reported in Figure E.1: the graph to the Left shows a productivity of a mother’s time with the child for high-educated mothers which is lower than the one estimated in the baseline analysis, while the graph to the Right shows only a lower productivity of formal child care in the early years, with the estimates going back to the values estimated in the baseline analysis at later ages. These results seem to suggest that once schooling time is taken into account, the educational investments made by the mother, either through her child-care time or through high-quality non-parental child care, become less relevant. Thus, the absence of this input seems to induce an upward bias in the estimates of the productivity parameters for a mother’s child-care time and non-parental child care.

Finally, it should be noticed that this exercise does not take into account that schools may be heterogeneous in the quality of education they provide. The CDS data only provides information on the type of school (i.e., public or private) attended by the child in 1997, 2002 and 2007, and this information suffers of the same issue that affects the schooling time variable used above. In addition, almost 90 percent of the sample attend a public school for the entire period (from age 6 until 13), and I do not detect differences in schooling decisions by a mother’s employment status or a mother’s education. For this reason, for the purpose of this exercise, I have not included this additional heterogeneity in the specification of schooling time.

Figure E.1
Estimated productivity parameters, in case non-parental child care also includes time at school.

NOTE. This graph represents the productivity parameters for maternal time ($\tau_t$), informal child care ($i_t$), and formal child care ($f_t$), as a function of child’s age $t = 1, 2, \ldots, 13$, in a model where $f_t$ includes the time spent in formal child care and at school.

Appendix E.2. Father’s time investments

According to the baseline specification of the model, only the mother’s time is productive for the child cognitive development, while the father’s contribution only comes through his labor income that affects the mother’s investment decisions. This specification rules out the possibility that the time the father spends alone with the child may represent an additional input in the child cognitive development process. However, it could be the case that fathers become more involved in the child-care activities, especially as the child grows up, and that this time also
contributes to the cognitive development of the child later on. If that is the case, the time-decreasing productivity of a mother’s child-care time that I observe in the baseline results may be a consequence of an augmented father’s involvement. In addition, fathers married with more educated women may be more likely to be involved with the child, as a consequence of assortative mating: this may result in a biased estimate for the time productivity of high-educated mothers, as well as of the alternative forms of care.

In order to understand how the omission of a father’s child-care time in the CAPF affects the estimated parameters, I re-estimate the model, by keeping the baseline definition of a mother’s time with the child, but I focus on the subsample of children who spend more time with their father, i.e. the child-care time of the father (alone) is above the median, which is 2.08 hours per week.\textsuperscript{40} This analysis aims at checking whether the results found in the baseline sample hold for a subsample in which the father is highly involved with the child. The results are reported in Figure E.2. By comparing Figure E.2-Left with Figure 3-Left, it can be observed that, in the new sample, the elasticity of a child’s cognitive ability with respect to maternal child-care time of high-educated mothers is lower than in the baseline results; moreover, the difference in productivity between high- and low-educated mothers is less apparent than in the baseline. Similarly, Figure E.2-Right shows a less relevant difference between the productivities of formal and informal child care. These results seem to suggest that assortative mating plays a role, and that if fathers are more involved with their child, high-quality child-care time of the mother, as well as high-quality non-parental child care play a less important role for the child’s cognitive development.

It should be kept in mind that the sample on which this exercise has been performed is very selected, thus the true productivities of maternal and non-parental child care are very likely to lie between the values reported in Figure E.2 and the values estimated in the baseline analysis. In conclusion, the absence of a father’s time as an input in the CAPF is very likely to generate an upward bias in the estimated productivities of a mother’s child-care time and non-parental child care.

\textsuperscript{40}This results in an estimation sample of 250 observations. The information about the amount of time spent with the father still comes from the Time Diary components of the PSID-CDS, held in 1997 and 2002. For the children for whom such information is available in both surveys, hence at two points in time, I take the highest value: this implies that a child is included in the sample if the father’s child care time (alone) is larger than the median at least once.
Figure E.2
Estimated productivity parameters for the subsample of children with above-median time with the father.

NOTE. This graph represents the productivity parameters for maternal time ($\tau_t$), informal child care ($\nu_t$), and formal child care ($f_t$), as a function of child’s age $t = 1, 2, \ldots 13$, for the subsample of children whose father’s time is above the median in the sample, i.e. larger than 2.08 hours per week.