NON-CONFRONTATIONAL EXTREMISTS

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Abstract: In many contexts individuals are subject to norms and decisions they disagree with ideologically. What is the effect of regularly being in ideological minority on the propensity to confront majority norms and decisions? We study this in an ideologically-salient field setting – US appeals courts – using exogenous predictors of ideology and random assignment of judges. We find that ideological interaction silences extremists: judges who are ideologically extreme relative to their peers are less confrontational – dissent less often – than other judges despite shaping court decisions the least. Considering many mechanisms, we find that a model of peer pressure can explain the observations.

Keywords: Judicial decision making, group decision making, ideology, peer pressure.

JEL codes: D7, K0, Z1.
1 Introduction

Within politics, work-life and in most social contexts, individuals have to interact with others who disagree with them ideologically. They may thus be subject to norms or decisions they disagree with on an ideological ground. How would an individual’s behavior be affected by being placed in an environment where her peers hold opinions far from her own? Would the individual behave more or less confrontationally than her peers, that is, speak out more or less against decisions and norms? While intuition may suggest that the one in strongest disagreement will be confronting the most, this paper presents empirical results from an ideologically-salient field setting showing just the opposite—individuals with frequent and strong disagreement with their peers confront less than others.

The question of whether holding non-consensual views makes a person more or less prone to challenge norms and decisions has remained empirically open due to unobservability of individual ideology and due to endogeneity of the choice of whom to interact with. These two problems are resolved at the U.S. Courts of Appeals (U.S. Federal Circuit Courts): There exist commonly used and exogenous measures of individual ideology, and assignment of whom an individual interacts with is determined exogenously.

Each Circuit Court consists of a pool of judges from which, for each judicial case, three judges are randomly assigned to sit together on a panel. The panel decides on a verdict (affirming or overturning the lower court verdict) and composes an “opinion” (i.e., a text) motivating the verdict. It is well documented that ideology plays an important role in forming the opinion (e.g., Epstein et al. 2013, Sunstein et al. 2006, Berdejó and Chen 2014) and the ideologically-charged decisions of the Circuit Courts have received increased attention recently, not least following the decision that halted President Trump’s proposed “Muslim-immigration moratorium” (see, e.g., NYTimes 2017).

Roughly speaking, the way by which an individual judge’s action can affect the judicial outcome is twofold: 1) the judge may have a direct effect on the verdict and opinion; 2) if the judge does not manage to directly affect them, she may write a minority opinion
(dissent) thus confronting the majority opinion. Should we expect judges in ideological minority ("extremists") to have an impact on the opinion? If not, should we expect them to behave more or less confrontationally (i.e., dissent more or less) than others? We answer these questions by presenting robust empirical results showing that I) extremists rarely affect the majority opinion and II) ideological disagreement drives dissent against it, yet III) extremists are the least confrontational against the majority opinion. This final observation is surprising given that (I) and (II) imply that these extremists have the most reasons to be confrontational.

Our empirical analysis indicates that this finding—that judges who are in strong ideological disagreement with their peers on the judge pool remain silent—is not about people with extreme ideology per se. Rather, it occurs when a judge is ideologically extreme relative to the current ideological composition of her pool. This suggests that the interaction between peers who are in ideological disagreement is silencing those with opinions far from the mainstream. The implications of this finding may be serious: it may hide undercurrents of dissatisfaction, distort the perception of the distribution of views and create false impressions of consensus where it is absent. In particular, there may appear to be unanimity of opinions while they in fact are under strong disagreement and the strength of court precedents may thus be undeservingly strong.

To understand what may be the reason for this silencing, i.e., to rationalize the empirical observations, we consider two theoretical models: one where confrontation (dissent) has an expressive purpose and one where it has an instrumental purpose (trying to reverse the decision using the supreme court). We derive auxiliary predictions for the two models along dimensions by which they differ. Testing these predictions refutes the instrumental model (which is hence relegated to the appendix, see Sections H.15 and I) and lends support to the expressive-dissent model which has the following features.

Three judges are randomly assigned to a panel where they negotiate the ideological

1Dissenting means stating a disagreement with both the verdict and the opinion. The judge may also concur which is a milder form of disagreement that is used by a judge who agrees with the verdict but disagrees with the motivation, i.e., disagrees with the opinion. See further details in Section 2.1.
2We have also considered a great number of other mechanisms that attempt, but ultimately fail, to explain our observations. These are discussed in Appendix H.
flavor of the majority opinion, with the median judge succeeding to set the opinion to match her own ideology (as is indeed observed in the data). Each judge then decides whether to confront this opinion by formally dissenting. Doing so is costly in terms of time and collegiality. But not dissenting entails a personal cost: the more often a judge signs majority opinions she disagrees with (and the more ideologically distant these opinions are from her own bliss point), the worse she feels. We show that the trade-off for if and when to dissent may drive judges with non-consensual ideology to be less confrontational than judges with more consensual ideologies. However, this result holds only provided that the individual’s ideological cost is sufficiently concave. A concave ideological cost captures that a person is almost indifferent between large and even larger deviations from her bliss point. It is embodied in the notion of the “What the Hell Effect” (Ariely 2012; Baumeister and Heatherton 1996) which has been shown to apply to behaviors ranging from diet-keeping (Baumeister and Heatherton 1996) through truth-telling (Kajackaite and Gneezy 2015; Gino et al. 2010; Hurkens and Kartik 2009) to voting (Kendall et al. 2015). See also Lim (2013) for empirical evidence.

3 A very large literature, including judges’ writings about their own experience, documents a norm of consensus (see e.g. Edwards and Livermore 2008). Epstein et al. (2011) refer to this as “dissent aversion”. This literature attributes the peer pressure largely to collegiality concerns (Fischman 2011; Hettinger et al. 2007; Sunstein et al. 2006).

4 This cost can be interpreted in two main ways: either as a psychological cost of being part of an opinion one disagrees with (cognitive dissonance); or as a loss incurred when not showing ideological purity to one’s ideological supporters (a form of peer pressure arising from one’s ideological faction). Baumeister and Heatherton (1996) have popularized the term “What the Hell Effect” when referring to the breaching of self regulation caused by what is initially perceived to be a one-time deviation from a strictly controlled conduct (diet) but ends up being the trigger for a full-blown violation of that conduct. Kajackaite and Gneezy (2015) show that once the incentives to lie are higher than the cost, subjects switch from telling the truth to lying to the full extent. Earlier research has also been suggestive of a concave cost of lying: The decision whether to lie is often insensitive to the outcome of lying once it is preferred over the outcome of being truthful (Hurkens and Kartik 2009) and so a maximal deviation from the truth will often be chosen by those deciding to lie (Gneezy et al. 2013). Likewise, using a dynamic setting, Gino et al. (2010) showed that once individuals are induced to cheat, they succumb to full-blown cheating. Even in the baseline treatment of Fischbacher and Föllmi-Heusi (2013), who advocated the existence of partial lying, the estimated share of subjects choosing a corner solution (either truth-telling or lying to the maximal extent) was as high as 61%. A concave cost has been argued by Osborne (1995) to be realistic in ideological settings as well. Indeed, when it comes to deviations from one’s ideological bliss point, Kendall et al. (2015) have shown by structural estimation that voters are most likely to have a concave cost of deviating from their ideological bliss point, implying that once their bliss point policy cannot be implemented, they do not care much which of the other policies is implemented.
results from judicial courts.⁶

The intuition for the result is as follows. A judge who is in ideological minority will rarely be the median judge in the panel and will mostly have to decide whether or not to sign opinions far from her ideological bliss point. However, always dissenting on opinions she does not like would imply a very high collegial pressure as she will be facing such opinions virtually all the time, and signing only some opinions while dissenting against others helps little when the perceived cost of signing few unfavorable opinions instead of many is almost the same. Hence, facing a sufficiently high collegial pressure, such an extreme judge will tend to sign virtually all opinions, thus being non-confrontational. In comparison, judges with more consensual ideology will more often be the median of their panels hence will less often need to decide whether to sign unfavorable opinions. So when they do face this problem they dissent, since the cost of deviating from their ideological bliss point is high even if it is only rarely done. Such judges will therefore dissent from time to time. Thus, overall, judges far from the mainstream ideology will be the least confrontational despite having the most reasons to be just that (rarely determining the opinion and often facing ideologically distant opinions). Strictly speaking, our theoretical model predicts and the empirical results show (see visualization in Figure 5) a hill-shaped relationship between a judge’s dissent rate and how extreme she is relative to her peers in her pool. That is, centrist judges rarely dissent, moderately ideological judges often dissent and extremely ideological judges rarely dissent. The above theoretical logic does not hold if the personal cost of bliss-point deviation is linear or convex, and we present necessary and sufficient conditions for the extent of concavity needed for the theory to align with the empirical observations. In the conclusions (Section 5) we discuss the broader empirical and theoretical implications of our finding of a concave ideological cost, should it indeed be true and hold more generally.

The structure of the paper is as follows. The next section outlines the judicial process

⁶Lim (2013) shows that the best cost function to fit her data is one which is first convex (for small ideological deviations) and then concave (for large deviations). For tractability, our model utilizes power functions, whose curvature is fixed. However, in a numerical simulation in appendix G we show that an s-shaped function as considered by Lim (2013) can replicate our empirical findings. Hence, our theoretical finding is very much in line with Lim (2013) and further shows that what matters is that the cost function is concave when deviations become large – individuals do not differentiate between intermediate and large bliss-point deviations, as embodied by the notion of the what-the-hell effect.
more in detail and describes the data. Section 3 presents the main empirical findings. In Section 4 we describe the main model and its results, including some auxiliary predictions. Section 5 concludes. The appendix contains robustness checks, additional empirical results and all proofs. The appendix also contains a long list of mechanisms that might affect decision making in judicial panels and explains why they nevertheless cannot account for the observed data. As part of that list (in Section H.15), we outline a brief yet formal description of the alternative model (where dissent is instrumental) that can produce the main empirical results and a test of auxiliary predictions of that model in which it fails while our main model succeeds.

2 Identification and data

2.1 Institutional background and identification

The U.S. Federal Courts are a system of local level (District Court), intermediate level (Circuit Court), and national level (Supreme Court) councils. Members of these are appointed by the U.S. President and confirmed by the U.S. Senate. They are responsible for the adjudication of disputes involving federal law. Their decisions establish precedent for adjudication in future cases in the same court and in lower courts within its geographic boundaries. Each state has 1–4 District Courts. The 94 U.S. District Courts serve as trial courts with juries. The 12 U.S. Circuit Courts (Courts of Appeals), which are the empirical focus of this paper, take cases appealed from the District Courts. The Circuit Courts have no juries. Each Circuit Court presides over 3–9 states. Figure 1 displays District Court boundaries in dotted lines and Circuit Court boundaries in solid lines.
Figure 1.— Geographical Boundaries of U.S. Federal Courts

Circuit Courts decide cases that provide new interpretations of prior precedents, which expand or contract the space of actions under which an actor can be found liable. State officials regularly update a set of guidelines to identify actions and regulations that may result in costly litigation after Circuit-Court decisions (Frost and Lindquist 2010; Pollak 2001). Circuit Courts rule on the application of federal law, such as the constitutional validity of state laws, among other things. 98% of their decisions are final. Hence, they have a substantial impact on precedence, decision making and policy in the US.

As mentioned, appointment of judges to Circuit Courts is done by the President and confirmed by the U.S. Senate. Each Circuit Court consists of a pool of 8–40 judges (depending on the circuit). The judges have life tenure. Hence, vacant positions appear only when a judge retires (61% of vacancies in our sample), resigns (3%), passes away (12%) or if the number of seats is expanded (24%).\footnote{New seats appeared, for instance, when the 5th Circuit was split into two in 1981.} On average there are 7.9 vacant positions per year in all Circuit Courts in total. This means that the President gets to appoint a judge to any single Circuit Court roughly once every 1.5 years.

In the Circuit Courts, each judicial case gets three randomly assigned judges. We
refer to them as the panel. The judges are drawn from the pool of judges in the Circuit Court. The three judges decide a binary verdict (affirming or overturning the lower court verdict). A majority of two judges is needed to set the verdict.

They also compose an opinion (i.e., a text) motivating the verdict. The opinion serves as precedent for future cases and as such has a large impact on society and policy. Furthermore, being a text, the opinion can reflect the assertiveness of the panel and its ideological composition. A judge has to write a separate (minority) opinion if she either dissents (votes against the binary verdict) or concurs (votes for the verdict but for a different reason, as manifested in her minority opinion). Both dissents and concurrences are costly in terms of time and collegiality and they cannot be cited as binding precedent. Note that, for a judge, dissenting and concurring are two mutually exclusive actions that both imply expressing dissatisfaction with the majority opinion – a form of confrontation.

The random assignment of judges into panels is foundational for our study. Case assignments into panels in Circuit Courts fall into two categories: 1) Once a case arrives, three randomly chosen judges are assigned to the case; 2) Once a year, the calendar is randomly set up in advance determining which judges will sit in which panels on which days in the upcoming year, and when a case comes up it gets assigned to the next panel. It is well established and has been thoroughly tested that both procedures are indeed random. For example, Chen and Sethi (2011) use data from Boyd et al. (2010) and Sunstein et al. (2006), who code 19 case characteristics as determined by the lower court for 415 gender-discrimination Circuit Court cases, and find that case characteristics are uncorrelated with judicial-panel composition. Other papers find that the sequence of judges assigned to cases in each Circuit Court is statistically indistinguishable from a random string. 8

8Given the binary nature of the verdict, at most one judge will dissent, but it can be the case that one judge dissents and another one concurs (where the third judge writes the “majority” opinion).

9Several recent papers employ random assignment of judges (e.g., Aizer and Doyle 2015, Shayo and Zussman 2011, Kling 2006, Belloni, Chernozhukov, and Hansen 2011, Lim, Silveira, and Snyder 2016). Berdejo and Chen (2016) report omnibus tests of whether case and litigant characteristics vary over 4-year cycles, and Chen (2016) does the same for the caseloads and characteristics of judges authoring or sitting on the panel.
2.2 Data and main variables

The data on dissents and concurrences come from Openjurist, which contains all cases from 1950 to 2007. The data was first digitized by one of the authors of the current paper (in Berdejó and Chen 2014) for whether there was a dissenting opinion and whether there was a concurring opinion. The current paper extracts the judge names and merges each judge with his/her ideology score. The ideology score we use is a standard summary measure coming from the Judicial Common Space database (Epstein et al. 2007) that was first coded by Giles et al. (2001). Many papers have used this score which has two main advantages. First, it is exogenous. It exploits the norm of senatorial courtesy by the President and is constructed as follows. If a judge is appointed from a state where the President and at least one home-state Senator are of the same party, the nominee is assigned the score of the home-state Senator (or the average of the home-state Senators if both members of the delegation are from the President’s party). If neither home-state Senator is of the President’s party, the judge receives the score of the appointing President. The score thus assumes that the President does favors to senators from the same party while ignoring the preferences of senators from the other party. Importantly, unlike common measures of Supreme Court judges’ ideology, this score assigns the ideology of the judge before her behavior at the court is observed, which of course is key since we are interested in how a judge’s behavior at the court is affected by her ideology. The second main advantage of this score is its high ability to predict judges’ voting pattern in court, as clearly visualized in Figure 2 (see below for how the voting variable is constructed). The ideology score takes values in between roughly ±0.8—see Appendix Figure 1 for a histogram of the distribution of ideology scores in our

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11 One example is Peresie (2005), which correlates the ideology score of judges with their decisions in sex-discrimination cases. Another is Kim (2009), in a sample of Title VII cases, which uses the ideology score of judges as a measure of their preferences and shows that Circuit-Court-panel decisions are influenced by Circuit preferences, but finds no support for strategic behavior aimed at inducing or avoiding Supreme Court review.

12 The scores of the Senators are located in a two-dimensional space on the basis of the positions that they take in roll-call votes, but only the first of the two dimensions is salient for most purposes. The ideology scores of Presidents are then estimated along this same dimension based on the public positions that they take on bills before Congress.

13 Other papers (e.g. Bailey 2016 and Jacobi and Sag 2009) with a different purpose than ours use non-exogenous ideology scores based on (Supreme Court) judges’ voting at the court (Martin and Quinn 2002).
data. As a robustness check we use the party of appointing President as ideology score and find qualitatively the same results—see Section 3.3.

**Figure 2.** Vote Ideology and the Judicial Common Space database Ideology Score – local polynomial

Notes: x-axis: (Non-demeaned) Judicial Common Space database Ideology score of a judge, where more conservative scores are along the right on the x-axis. y-axis: Vote ideology, demeaned to be centered at zero. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The dashed lines depict the 95% confidence interval. Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).

To examine the ideological color of the majority and minority opinions and of the judge’s vote on each panel, we also employ the U.S. Courts of Appeals Database Project, a random sample of roughly 5% of appeals-courts decisions from 1925 to 2002. This database includes hand-coded information on the ideological content of each coded opinion (liberal = -1, conservative = 1, and mixed or unable to code = 0), to which we will refer as Opinion.

Ideology. This database also reports dissents and concurrences and codes the ideological content of their corresponding minority opinions.

Using the data from the Courts of Appeals database, we create a measure of the ideology of a judge’s voting pattern. This measure is set to equal the ideology of the majority opinion if the judge did not dissent or concur and to equal the ideology of her own minority opinion if the judge dissented or concurred. We call this variable Vote Ideology (this variable is, for instance, used on the y-axis of Figure 2).

Our sample contains 293,868 decisions in Openjurist and 18,686 decisions in the Courts of Appeals database. Overall, 8.5% of opinions in Openjurist have dissents (6.4% have concurrences) while in the Courts of Appeals database 7.9% of opinions have dissents (3.6% have concurrences).

To construct a measure of ideological disagreement between a judge and her peers on the panel of a specific case, we calculate the Score Relative to Panel Median, which is positive when the judge is more conservative than the median and negative when the judge is more liberal than the panel median. We also take the absolute value of this variable and refer to it as Distance to Panel Median.

Next we construct a measure of ideological disagreement between a judge and her peers in the pool, whom she can expect to meet regularly. To do this we begin by calculating the average ideology score of the pool of judges for each Circuit and each year. This average score represents the center of the pool of judges available to be assigned at that Circuit-year, to which we refer as Center of Judge Pool. We then demean the ideology score of a judge by the Center of Judge Pool and get a measure we refer to as Score Relative to Center of

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15 The original coding in the US Courts of Appeals database was the opposite but we flipped it to be in line with our coding of ideology scores, where positive values correspond to conservative views. The Appeals Court Database Project states that for most issue categories, these will correspond to conventional notions of “liberal” and “conservative”. For example, decisions supporting the position of the defendant in a criminal procedure case, the plaintiff who asserts a violation of her First Amendment rights, or the Secretary of Labor who sues a corporation for violation of child labor regulations would all be coded as “liberal”. The directionality codes parallel closely the directionality codes in the Spaeth Supreme Court database.

16 Any analysis requiring the panel median includes only panels where there are no tied or missing scores (panels with tied scores are excluded because the identity of the median judge is not uniquely determined).

17 We use Circuit and year since this represents the ideology of the other judges a specific judge expects to sit with in a panel.
Judge Pool. This measure can take both positive values (if the judge is more conservative than her peers) and negative ones (if the judge is more liberal than her peers) and serves as our main ideological measure. The value of this measure for a given judge may change during her service as the composition of her pool changes. A histogram of Score Relative to Center of Judge Pool is displayed in the center part of Appendix Figure 1. We refer to the absolute value of this measure as **Distance to Center of Judge Pool**.\(^\text{18}\)

Using the data from Openjurist we calculate the **Dissent Rate** and **Concurrence Rate** for each judge in each Circuit-year. These are essentially measures of a judge’s tendency to confront her peers.\(^\text{19}\)

### 3  Empirical Results

In this section we present our four main empirical observations: (1) the median sets the majority opinion; (2) ideological disagreement triggers dissent; (3) dissent is first increasing and then decreasing as a function of a judge’s Distance to Center of Judge Pool; and (4) the most ideological voting is obtained for judges who are moderately leftist or rightist while extreme judges have a less ideological voting pattern.

#### 3.1  Who determines the ideological color of the opinion?

We begin by examining the effect of the ideology score of the judges in the panel on the ideology of the majority opinion. To conduct this analysis we regress the Opinion Ideology (from the U.S. Courts of Appeals Database Project) on a judge’s Score Relative to Center of Judge Pool and its interaction with whether the judge is the median of the panel.

\(^\text{18}\)We also merge in the Martin Quinn Supreme Court scores [Martin and Quinn 2002] and construct **Distance to Supreme Court** when we examine the alternative theory where Circuit Court judges are dissenting to signal the Supreme Court to review and revoke the panel’s decision—see Section 11.15. A histogram of **Score Relative to Supreme Court** is presented in the right panel of Appendix Figure 1.

\(^\text{19}\)As robustness checks, we calculate for each judge the dissent and the concurrence rates averaged over a 2-year bin and over a judge’s lifetime. In these calculations, a judge’s score is the judge’s Score Relative to Center of Judge Pool averaged over all the cases on which she was sitting over the corresponding period. I.e., the judge’s Score Relative to Center of Judge Pool was averaged over all the Circuit-year combinations in which the judge was active over the corresponding period, weighted by the number of cases on which the judge was sitting at each Circuit-year.
in terms of ideology score.\textsuperscript{20}

\begin{equation}
\text{Opinion Ideology}_{pcit} = \alpha + \gamma_1 \text{Score Relative to Center of Judge Pool}_{cit} + \\
+ \gamma_2 1 \left( i \text{ is median}_{pcit} \right) \\
+ \gamma_3 \text{Score Relative to Center of Judge Pool}_{cit} * 1 \left( i \text{ is median}_{pcit} \right) + \nu_{pcit}
\end{equation}

for judge $i$ on panel $p$ in Circuit $c$ and year $t$. If the ideology of a judge influences the opinion, we should expect a positive relationship between the judge’s ideology score (where a high value means a very conservative judge) and the likelihood of a conservative opinion. Table \textsuperscript{I} shows that only the median judge’s ideology score affects the opinion, and it does so positively.\textsuperscript{21} Figure \textsuperscript{3} visualizes the results from the table and shows that the median judge is essentially single-handedly determining the ideological color of the opinion.

\textsuperscript{20}We use scores relative to center of judge pool (and not ideology scores per se) because when the entire circuit moves to the left or to the right, panelists’ scores move accordingly and become slightly correlated with the ideological content of the opinion even without actually affecting it.

\textsuperscript{21}The regression includes only three-judge panels where there are no tied or missing scores and clusters standard errors at the Circuit-year level. Appendix Table \textsuperscript{A.1} presents a robustness check of this result.
Who Determines the Opinion Ideology?

<table>
<thead>
<tr>
<th>Judge is Panel Median</th>
<th>Judge is Not Panel Median</th>
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Score Relative to Center of Judge Pool

No panels with tied or missing scores are included.

Notes: x-axis: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year, where relatively more conservative scores are along the right on the x-axis. y-axis: Opinion ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Each dot represents the average of all opinions in a bin of judges with similar ideology scores. The lines represent the regression coefficients from Table I (in the right schedule the line represents the first coefficient and in the left schedule the sum of the first and third coefficients). The y-axis is demeaned to be centered around zero. Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores.

TABLE I

IDEOLOGY OF OPINION AND IDEOLOGY SCORES OF PANEL MEMBERS

<p>| | | |</p>
<table>
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<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Opinion Ideology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Score Relative to Center of Judge Pool</td>
<td>0.0166</td>
<td>(0.0125)</td>
</tr>
<tr>
<td>Panel Median</td>
<td>0.00118</td>
<td>(0.000775)</td>
</tr>
<tr>
<td>Score Relative to Center of Judge Pool</td>
<td>0.142***</td>
<td>0.0409</td>
</tr>
<tr>
<td>* Panel Median</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>23031</td>
<td></td>
</tr>
<tr>
<td>R-sq</td>
<td>0.001</td>
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</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is opinion ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal.
Our first empirical result is thus:

**Fact 1** *The median of the panel is determining the opinion.*

This is of course not surprising given the pivotal role of the median in a three-judge panel and is consistent with many conventional bargaining models. This result also aligns with a recent experimental study of decision making within groups *(Ambrus et al. 2015)*, finding that the median has the greatest impact on the group’s joint decision. *Ambrus et al.* (2015) further find that the most extreme individuals relative to the median have a particularly small effect on the joint outcome, a result that is largely echoed by our further investigation, see Appendix Figure 3 and description in Appendix B.1. A prior empirical examination of the role of the median judge in Circuit-Court panels (by *Cross* 2007) did not find that the median judge was setting the court’s opinion, when including the score of the median judge and the sum of the scores of all the judges on the panel as two explanatory variables in the same regression. We replicate their specification in Appendix Table A.1 and show that breaking the sum of judges’ scores to the scores of each of the three judges separately (instead of including the median judge twice, as Cross practically does in his specification) corroborates our result that the median sets the decision.

### 3.2 Does ideological disagreement drive dissent?

We proceed now to showing a within-judge property regarding when a judge will choose to dissent. Figure 4 presents a non-parametric visualization of the Dissent Rate by ideology Score Relative to Panel Median using a local polynomial regression. This figure reveals a clear pattern: the more a judge is distant from the panel median, the more likely she is to dissent. This holds both on the left and on the right. We present similar non-parametric visualizations for concurrences in Appendix Figure 5 and when grouping judges with similar scores into 15 separate bins in Appendix Figure 4.

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22Local polynomial regressions are a set of techniques for data description, for estimating a regression curve without making strong assumptions about the shape of the true regression function, and for checking parametric models *(Altman 1992)*. They are data-driven ways of choosing the amount of smoothing *(Fan and Gijbels 1996)*.
Notes: x-axis: Ideology score of a judge demeaned by the median of the panel of judges assigned on the case. y-axis: Rate of dissent. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is rate of dissent. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

To test it in a regression specification, we regress the Dissent and Concurrence rates of each judge-case combination on polynomials of the judge’s Distance to the Panel Median—an absolute value which captures the strength of ideological disagreement with her panel peers. We also add judge fixed effects ($I_i$) to ensure that the result is not driven by the ideology scores of judges per se. We also include Circuit ($C_c$) and year ($T_t$) fixed effects and cluster the standard errors by Circuit-year. The basic regression specification is:

$$\text{Dissent Rate}_{pcit} = \gamma_1 \text{Distance to Panel Median}_{pcit} + \gamma_2 \text{Distance to Panel Median}^2_{pcit} + I_i + C_c + T_t + \nu_{pcit}$$

for judge $i$ on panel $p$ in Circuit $c$ at year $t$. Table III indicates that the frequency of both dissents and concurrences increases in the distance to the panel median, implying that ideological disagreement is a driver of dissent.
### TABLE II

**Dissent and Ideological Distance to Median of Panel**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dissent</td>
<td>Concur</td>
</tr>
<tr>
<td>Distance to Median of Panel</td>
<td>0.00425***</td>
<td>0.00244***</td>
</tr>
<tr>
<td></td>
<td>(0.00119)</td>
<td>(0.000907)</td>
</tr>
<tr>
<td>Distance^2</td>
<td>-0.00142</td>
<td>-0.000868</td>
</tr>
<tr>
<td></td>
<td>(0.00154)</td>
<td>(0.00116)</td>
</tr>
<tr>
<td>Judge Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>541163</td>
<td>541163</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.411</td>
<td>0.414</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Ideology scores are demeaned by the actual center of the panel of judges assigned on a case. The dependent variable is a dummy for whether a judge dissented (column 1) or concurred (column 2) in the panel. Fixed effects include year, circuit, and judge.

Based on these results, our second empirical finding is:

**FACT 2** *The probability that a judge dissents increases in her ideological distance from the panel median.*

This result, that ideological disagreement between judges drives dissent, is in line with previous observations in the literature (e.g., [Wahlbeck et al., 1999](#), [Spriggs et al., 1999](#) and [Hettinger et al., 2004](#)).

### 3.3 Which judges are dissenting the most?

We now turn to examine which judges, in terms of their (relative) ideology score, are behaving most confrontationally. Figure [5](#) presents a non-parametric visualization of Dissent Rate by the ideology Score Relative to Center of Judge Pool using a local polynomial regression. Note that this is mainly a between-judge comparison. As explained earlier, for each Circuit at each year, each judge’s score was recalculated to reflect her location relative to the average ideology in the pool of judges in that circuit and year. That is, Score Relative to Center of Judge Pool measures the ideological disagreement between a judge and all the peers she regularly interacts with.
Intuitively, given Fact 2, one might expect that the more distant a judge is from the pool center, the more she will be inclined to write separate minority opinions (dissent or concur). But Figure 5 reveals a surprising pattern: starting from the left, the most extreme judges rarely dissent, then there is a marked increase in dissent as judges become more moderate, followed by a decrease in dissent rates towards the center of the judicial pool. A similar pattern appears on the right. We will refer to this pattern as a spider pattern, due to the figure’s resemblance of the body and legs of a spider. Appendix Figure 11 shows that the spider pattern is robust to residualizing by circuit and year fixed effects and Appendix Figure 7 presents a similar graph when grouping judges into 15 separate bins according to

23 In Appendix Figure 2 we show that judges who are more distant from the pool center have indeed a higher probability to also be more distant from the panel median.

24 One may note that the x-axis in Figure 5 is narrower than in Figure 4. The reason for this is that Figure 5’s x-axis variable is the ideology score relative to the average of the pool which naturally does not take extreme values as the ideology score relative to the median of a panel, which is on the x-axis in Figure 4.
Score Relative to Center of Judge Pool

As further evidence for the spider pattern, Appendix Figures 8 (bins) and 9 (local polynomial) present the Concurrence Rate of judges according to Score Relative to Center of Judge Pool. Notably, the pattern of the spider is robust: concurrence rates are surprisingly uncommon for the most extreme judges, the rate is higher for judges at moderate distance to the center and lower again for the judges at the very center (though here the pattern is less clear on the right). Finding these patterns for dissents and concurrences separately of course strengthens our confidence in these results.

To rule out the possibility that maybe the extreme judges are simply “picking their battles” (in the sense of compensating for the small number of dissents by putting a lot of effort into writing few but very ideologically extreme minority opinions), we run an additional robustness test. This test shows that the pattern of the spider is robust to using the number of dissent words per case a judge writes in a Circuit-year instead of the yearly rate of concurrence or dissent (Appendix Figure 10). Hence, it is not the case that judges in ideological minority, albeit dissenting less often than others, are spending a larger total effort on dissenting than others.

To test it in a regression specification, we regress the dissent and concurrence rate of each judge on polynomials of her (absolute) distance to the center of the pool of judges in her Circuit-year:

\[
\text{Dissent Rate}_{cit} = \alpha + \gamma_1 \text{Distance to Center of Judge Pool}_{cit} +
\gamma_2 \text{Distance to Center of Judge Pool}^2_{cit} + C_c + T_t + \nu_{cit}
\]

for judge \(i\) in Circuit \(c\) and year \(t\). Circuit and year fixed effects are represented by \(C_c\) and \(T_t\) and standard errors are clustered by Circuit-year. Table III indicates that the spider

\[25\]Note that the share of judges having the same score (or range of scores) has no effect on the pattern of dissent rate. The pattern depicts the average dissent rate for each score, regardless of how many judges have this score, hence there is no distorting effect of weighting by a variable number of judges.

\[26\]We analyze concurrences separately from dissents in order to show the robustness of our results and because they are legally distinct. However, as both require writing a separate minority opinion, we bind them together in our theoretical model and in the further robustness checks reported in the appendix, thus treating them as two alternative manifestations of the same thing: a judge’s decision to confront her panel’s opinion.
pattern is robust: according to the estimated linear and quadratic coefficients in the table, the maximum dissent rate is obtained for Distance to Center of Judge Pool of 0.6 (for dissents) and 0.46 (for concurrences) which are clearly within the bounds of our distribution, which goes from around -0.8 to +0.8.27

### TABLE III

Dissent and Ideological Distance to Center of Judge Pool

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dissent</td>
<td>Concur</td>
</tr>
<tr>
<td>Distance to Center of Judge Pool</td>
<td>0.0404***</td>
<td>0.0285***</td>
</tr>
<tr>
<td></td>
<td>(0.00756)</td>
<td>(0.00570)</td>
</tr>
<tr>
<td>Distance^2</td>
<td>-0.0334***</td>
<td>-0.0313***</td>
</tr>
<tr>
<td></td>
<td>(0.0118)</td>
<td>(0.00862)</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>10043</td>
<td>10043</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.109</td>
<td>0.086</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge's dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

### TABLE IV

Dissent and Alternative Ideology Score

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dissent</td>
<td>Concur</td>
</tr>
<tr>
<td>Distance to Center of Judge Pool</td>
<td>0.0501***</td>
<td>0.0284***</td>
</tr>
<tr>
<td></td>
<td>(0.0157)</td>
<td>(0.0107)</td>
</tr>
<tr>
<td>Score Based on Party of Appointment</td>
<td>-0.0367**</td>
<td>-0.0222*</td>
</tr>
<tr>
<td></td>
<td>(0.0164)</td>
<td>(0.0115)</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>10033</td>
<td>10033</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.106</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores are simply the party of appointment (Republican or Democrat, coded as 1 and 0). The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge's dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

27See Appendix Figure 1 for the distribution of scores.
To verify that the spider pattern is not driven by some distortion in the ideology score, we ran the same regression using an alternative ideology score—the party of appointing President. This score, in its raw form, does not distinguish between judges nominated by Republican Presidents (all have a score of 1). Likewise it does not distinguish between judges nominated by Democrat Presidents (all have a score of 0). To create a relative score based on this raw score we calculate the average ideology in the pool in a circuit-year and calculate a judge’s distance to this average. Hence, the relative score reflects whether a judge is in minority (a large absolute score) or in majority (a small absolute score). Table IV shows the spider pattern is robust to using this alternative score. Furthermore, this score provides a very straightforward reinterpretation of the spider pattern. An “extremist” in this scoring system is simply a judge placed in a pool of judges in which a large majority of the judges were nominated by a President from the other party. Hence, the spider pattern when using this score clearly captures the tendency of judges to stay silent when they are in ideological minority, while behaving the most confrontationally when having a similar number of peers from their own party and from the other party.

Additional robustness checks are reported in Appendix Tables A.2 to A.6. Appendix Table A.2 shows that the results are robust to weighting each judge equally (rather than according to the number of votes); dropping visiting judges; and using higher levels of aggregation (e.g., 2 year or lifetime rates, where in the latter there is one observation per judge). They are also robust to including a cubic term of Distance to Center of Judge Pool, running Logit, and controlling for Distance to Supreme Court or Distance to Median of Panel (i.e., the pattern is not driven by interactions within particular panels). In that table, we also show that when the Distance to Center of Judge Pool is randomly re-assigned to another observation there is no relationship, which further mitigates the concern that the documented relationship is a statistical artifact or is driven by the level of clustering. Appendix Table

For instance, a judge nominated by a Democrat President (that is, score 0) in a circuit-year consisting of a total of 8 Republican-nominated and 2 Democrat-nominated judges will get a relative score of \(0 - \frac{(8 \times (+1) + 2 \times 0)}{10} = -0.8\). A Republican-nominated judge in that same pool will get a relative score of \(1 - \frac{(8 \times (+1) + 2 \times 0)}{10} = 0.2\).
A.3 shows that the results are robust to dropping one Circuit at a time, thus mitigating the concern that the pattern is driven by a particular court or by outliers. Appendix Table A.4 controls for a large set of personal characteristics of judges (which can be seen as a second-best alternative to using judge fixed effects, see more on judge fixed effects below). Appendix Table A.5 re-runs the main quadratic specification in (3) but using two forms of bootstrapping for extreme judges (since there are fewer such observations) and corroborates that the results (and their statistical significance) do not change. Finally, Appendix Table A.6 shows that the pattern is robust to splitting the sample according to whether the case affirmed or reversed the lower court decision. Based on these results we formulate the following empirical finding:

FACT 3  There is a hill-shaped relationship between a judge’s dissent rate and the absolute distance to her pool center.

This result expresses that judges who are extreme relative to their peers are less confrontational than more moderately-distanced judges. It is novel to this paper and, to our knowledge, no paper in any domain has ever examined how being in ideological minority affects one’s tendency to confront decisions of the majority. The result is particularly surprising given the earlier result that a judge who is in ideological minority rarely affects the joint opinion (Fact 1) and given that confrontation is driven by ideological disagreement with this opinion (Fact 2). Yet such a judge remains silent.

It should be noted that this result is driven by a judge’s ideology relative to her peers—the same judges who exhibit the low dissent rate when they are extremists, exhibit a high dissent rate when they are moderates relative to their pool. To show this, Table V reports the coefficients of a regression of dissent rate on polynomials of Distance to Center of Judge Pool with judge fixed effects, using a subsample that contains all the judges who, at a certain point of their career, have Distance to Center of Judge Pool greater than 0.6 (the location of the hump of regression specification 3, see Table III). Table V clearly shows that the spider pattern holds for that subsample of “extreme” judges, indicating that these judges do not remain silent when they are placed in a pool that is ideologically closer to their
bliss point (as reflected in the increasing part of the spider)\textsuperscript{29} Moreover, as can been seen in Appendix Figure \textsuperscript{12} the spider pattern is profoundly attenuated when considering the raw (i.e., not relative) Ideology Score. Hence, we can conclude that the non-confrontational behavior of extreme judges is not driven by their ideology per se or by some related personal characteristics but rather by regularly being in strong ideological disagreement with most of their peers\textsuperscript{30}

\begin{table}
\centering
\caption{Dissent and Ideological Distance to Center of Judge Pool for “Extreme” Judges, Including Judge Fixed Effects} \\
\begin{tabular}{lcc}
\hline
 & (1) & (2) \\
Dissent & 0.0971*** & 0.102*** \\
 & (0.0334) & (0.0291) \\
Concur & 0.102*** & 0.105*** \\
 & (0.0388) & (0.0303) \\
Distance to Center of Judge Pool & -0.102*** & -0.105*** \\
 & (0.0388) & (0.0303) \\
Distance\textsuperscript{2} & Y & Y \\
Circuit Fixed Effects & Y & Y \\
Year Fixed Effects & Y & Y \\
Judge Fixed Effects & N & N \\
N & 1519 & 1519 \\
R-sq & 0.425 & 0.343 \\
\hline
\end{tabular}
\end{table}

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Sample of judges who are extreme relative to their pool at some point of their career (Distance to Center of Judge Pool greater than 0.6 (the location of the hump of regression specification). Ideology scores come from the Judicial Common Space database. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge’s dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year, circuit, and judge. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

3.4 Which judges have the most ideological voting pattern?

Next, we investigate the relationship between a judge’s (relative) ideology score and the ideological color of her votes (Vote Ideology) in the cases she is sitting on.

\textsuperscript{29}For judges who were never “extreme” at some point of their career, using judge fixed effects renders more measurement error (see Greene, 2018) and less meaningful variance to estimate the spider pattern.

\textsuperscript{30}Recall also the reinterpretation of extremism as belonging to a partisan minority when using the alternative party-of-appointing-President score.
We test the relationship between a judge’s ideology Score Relative to Center of Judge Pool and her Vote Ideology using a local polynomial regression. The results are presented in Figure 6. As can be seen, the most ideological voting is obtained for moderately ideological judges and once a judge becomes sufficiently extreme her voting becomes less ideological. To test the statistical significance of this result we run the regression:

\[
\text{Vote Ideology}_{pcit} = \alpha + \gamma_1 \text{Score Relative to Center of Judge Pool}_{cit} + \\
\gamma_2 \text{Score Relative to Center of Judge Pool}^2_{cit} + \\
\gamma_3 \text{Score Relative to Center of Judge Pool}^3_{cit} + \nu_{pcit}
\] (4)

for judge \(i\) on panel \(p\) in Circuit \(c\) and year \(t\). The regression results are presented in Table VI. It confirms (by the negative cubic term) that judges with moderate scores have the most

\footnote{We use polynomial of the third degree in the regression to enable testing for a U-shape on the left and a hill-shape on the right.}
ideological voting pattern while judges with extreme scores have a less ideological voting pattern, implying that being in ideological minority makes a judge behave less ideologically. According to the coefficients in the table, the strongest ideological voting pattern is obtained for judges with scores of -0.36 and +0.47 which are both well within the bounds of our distribution, which goes from around -0.8 to +0.8. In Appendix Table A.8 we also add a quartic term (visualized in Appendix Figure 13) and in Appendix B.4 we employ a number of other robustness checks. As can be seen from these further tests, the result is robust. Appendix Table A.7 reports that the relationship is also robust to using the lifetime average for each judge. The pattern is further robust to splitting the sample according to whether the case affirmed the lower court decision (Appendix Table A.9). It may be noted that the regression in (4) does not contain fixed effects. Adding circuit and year fixed effects makes the results non significant, possibly due to the much smaller sample used here compared to the previous facts.32 The results are also robust to using the alternative ideology score (the one using the appointing President’s party and the share of judges in the pool who are of the other party), see Appendix Table A.11.

TABLE VI

VOTE IDEOLOGY AND IDEOLOGY SCORE OF JUDGE RELATIVE TO CENTER OF JUDGE POOL

<table>
<thead>
<tr>
<th>Score Relative to Center of Judge Pool</th>
<th>Vote Ideology</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.180***</td>
</tr>
<tr>
<td>Score</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0614</td>
</tr>
<tr>
<td>(0.0659)</td>
<td></td>
</tr>
<tr>
<td>Score 3</td>
<td>-0.366***</td>
</tr>
<tr>
<td>(0.125)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>23031</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The independent variables are polynomials of the ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year.

32Recall that in order to construct our measure of Vote Ideology we rely on the Opinion Ideology as coded by the U.S. Courts of Appeals Database Project, which consists of a random sample of only 5% of all cases. Recall also that each of the roughly 10000 observations reported in Table III represents on average about 30 cases.
We summarize these results in the form of the following empirical finding:

**Fact 4** The most ideological voting is obtained for judges who are moderately leftist or rightist.

Also this result is novel to our paper (across all domains, to our knowledge). It shows that ideology is less pronounced in the extreme judges’ voting than it is in the moderately ideological judges’ voting. It is worth noting that also this result disappears when using raw ideology scores (i.e., not relative to the pool)—see Figure 2. Hence, what drives this result is that a judge is ideologically extreme relative to her peers—it is not about extreme ideology per se, but about the interaction between peers who disagree ideologically.

## 4 Theoretical Model

We will now construct a theoretical model to explain Facts 1-4. We keep it as simple as possible in order to highlight the mechanism.

Each judge has an ideology score \( t \) which is public information. The *pool of judges* consists of a continuum of infinitesimal judges with \( t \) uniformly distributed in \([-1, 1]\).\(^{33}\) For each case, three judges (\( t_1, t_2 \) and \( t_3 \)) are randomly and independently drawn from the pool of judges to sit together on a *panel*. The timing of the actions within each case is as follows. First the three judges in the panel bargain to produce an opinion \( v \in \mathbb{R} \). Second, each judge decides whether to sign the opinion or not. We will assume a judge cares about the ideology of the opinion per se and also about whether she herself signs it or not. This could have been captured in a multitude of ways without changing the main results but we will use the simplest possible formulation in order to obtain sharp results.

Bargaining is done through a Condorcet voting process: judges can (indefinitely) propose opinions, and any new proposal competes (by voting of the three judges) against the current leading opinion until a final decision is reached. Let \( V \) denote the (equilibrium) set of opinions and \( k(v) \) its distribution (the pdf). Furthermore, let \( V(t) \subset V \) denote the

\(^{33}\)We use a uniform distribution mainly for tractability but most of our results hold under any single-peaked distribution. The true empirical distribution of judges is presented in Appendix Figure 2.
set of opinions faced by judge \( t \) during her judicial term, and \( k(v|t) \) its distribution.\(^{34}\)

Judges care about the opinion per se and therefore want it to reflect their bliss points \((t)\). Hence, a judge \( t \) gets disutility \( O(V(t), t) \), which is increasing in the distance between each \( v \in V(t) \) and \( t \).\(^{35}\)

Given opinion \( v \), a judge needs to decide whether to sign it or whether to dissent against it.\(^{36}\) When signing an opinion \( v \neq t \), the judge has to bear the inner discomfort associated with actively approving it. This is captured by the cost function

\[
D = D(x)
\]

\[
x = \int |v - t| k(v|t) s(v) \, dv
\]

where \( s(v) \) is an indicator function that equals 1 if the judge chooses to sign opinion \( v \) and equals 0 if she dissents. We assume that \( D \) increases in its argument, which means the ideological cost is increasing in the number of unfavorable opinions a judge signs and the more unfavorable each opinion she signs is. For tractability we model \( D \) as a power function

\[
D(x) = x^\alpha
\]

where \( \alpha > 0 \). \( D \) can be interpreted in two main ways: Either as a judge’s own perceived cost of being part of an opinion she disagrees with; or as a loss when not standing up for the ideology of one’s supporters or ideological faction. Putting the power \( \alpha \) on the aggregate deviation \( x \) instead of putting it on the term \( |v - t| \) captures the idea that judges are optimizing over

\(^{34}\)\( k(v|t) \) is not identical to \( k(v) \) because the verdict is (or at least may be) affected by the judge’s own type \( t \) apart from the types of the two other judges drawn to sit with her on the panel. The one simplifying assumption we make here is that each judge actually meets the continuous distribution of two random judges drawn from \( t \sim U(-1,1) \), implying further that each judge meets the same combinations of two other judges and sits in the same amount of cases (normalized to 1).

\(^{35}\)Strictly speaking, since a judge is sitting in a continuum of cases, the effect of each case on \( O \) is zero. Hence, what we are more precisely assuming is that for any non-zero mass of verdicts \( \tilde{V} \subset V(t) \), if the distance between each \( v \in \tilde{V} \) and \( t \) strictly increases (ceteris paribus), then \( O(V(t), t) \) strictly increases. Note that we do not need to make any additional assumptions on the properties of \( O \), this very general property suffices.

\(^{36}\)In the model we treat dissent and concurrence as the same thing and simply call it \textit{dissent}. 
the long term and not independently per case. Put differently, the judge’s reference point (or ambition) is signing only opinions she agrees with. Then she perceives a cost based on the total deviations from this reference point.\(^{37}\)

In total, a judge \(t\) has the loss function

\[
L = O(V(t), t) + D\left(\int |v - t| k(v|t) s(v) dv\right) + WP(t),
\]

where

\[
P(t) \equiv \int (1 - s(v)) k(v|t) dv
\]

is the judge’s rate of dissent, and the last term in the loss function represents the cost arising from collegial pressure. \(W\) can also be interpreted as capturing the effort of writing a separate opinion. We let this cost be linear in the rate of dissent for tractability, thus \(W\) is the constant marginal cost of dissent.\(^{38}\)

4.1 Bargaining outcome

From the model specification it follows that for any given judicial case and any signing strategy \(s(v)\) chosen by a judge \(t\), an increase in \(|v - t|\) leads to an increase in the judge’s loss \(L\).\(^{39}\) Hence, each judge will strive to minimize \(|v - t|\) during the bargaining process. As a consequence, the opinion in each panel will be solely determined by the bliss point of the median judge in that panel, which we denote by \(t_m\). Though nearly trivial, we present it in the form of a proposition to highlight that it aligns with Fact 1.

**Proposition 1** (Fact 1) In all panels \(v = t_m\).

\(^{37}\)It should be noted that if a judge would optimize case by case (that is, the power would be applied to each deviation) then our main theoretical conclusions would still hold. However, an additional assumption would be needed, namely, that the collegial pressure (see below) is increasing in the extremeness of the dissenting opinion. We prefer to avoid this extra assumption and we also find the assumption of case-by-case optimization less realistic, but we briefly refer to this alternative in Appendix Section H.1.

\(^{38}\)The results would hold qualitatively also for a convex or a slightly concave collegial pressure.

\(^{39}\)The only scenario in which the loss \(L\) could have decreased when replacing \(v_1\) with \(v_2\) such that \(|v_1 - t| < |v_2 - t|\), is one where she signs \(v_2\) but not \(v_1\). This intuitively implausible signing strategy is formally ruled out later (see Section 4.2).
Since the result is simple and intuitive we only provide the following heuristic proof. The result that the median decides follows from the Condorcet voting process and from the fact that \( v = t_m \) is closer (in ideological space) than is any \( v \neq t_m \) to two of the three judges. Hence \( v = t_m \) is always preferred by two judges over any competing alternative. We will now use it as a starting point for the analysis of the individual decision making about dissenting.

4.2 Within-judge variation: To sign or not to sign?

Given the bargaining outcome, it is clear that signing is optimal for the median judge in the panel. As for the other judges in the panel, note first that \( O(V, t) \) is independent of signing or not. Therefore, the first argument in the loss function, \( O(V(t), t) \), does not affect any signing decision made by a judge. Each of these judges therefore minimizes

\[
(6) \quad l(s(v); t) = D \left( \int |v - t| k(v|t) s(v) \, dv \right) + WP(t).
\]

From this specification we get the following result.

**Proposition 2 (Fact 2)** Every judge \( t \) has a unique cutoff \( c(t) \) such that she signs opinion \( v \) if and only if \( |v - t| < c(t) \).

**Proof:** See Appendix \( \Box \) \( Q.E.D. \)

The proposition says that the optimal strategy for a judge is to dissent against opinions that are sufficiently far from her bliss point – opinions within the cutoff distance \( c \) are those the judge tolerates without confrontation. This is of course natural since, otherwise, there would exist two opinions \( v_1 \) and \( v_2 \), such that \( |t - v_1| < |t - v_2| \) yet judge \( t \) is willing to sign \( v_2 \) while refusing to sign \( v_1 \), in which case she could lower her loss by inverting this pair of choices. Put in an empirical context, the proposition says that the probability of dissent should increase in the distance of the judge from the median of the panel (who, as established, sets the opinion). Hence, this result explains Fact 2.

Following this result, the choice of a judge boils down to choosing her cutoff \( c \). Rewrit-
ing the loss function in (6), the minimization problem for judge \( t \) is given by

\[
\min_c l = \min_c \left\{ D \left( \int_{t-c}^{t} |v - t| \, k(v|t) \, dv \right) + W P(c; t) \right\},
\]

where \( P(c; t) \) is the probability of facing opinions beyond judge \( t \)'s cutoff.

We turn now to explicitly express the conditional opinion distribution \( k(v|t) \) in terms of the cumulative distribution of judges’ ideology scores \( F(\cdot) \) and the corresponding density function \( f(\cdot) \). For a given judge \( t \), the probability of having an opinion to her left is

\[
Pr(v < t) = Pr(t_m < t) = [F(t)]^2,
\]

as this happens if and only if both other judges have bliss points below \( t \). Similarly,

\[
Pr(v > t) = Pr(t_m > t) = [1 - F(t)]^2.
\]

In the remaining cases, judge \( t \) is the median, in which case \( |v - t| = 0 \). More generally, the probability that a judge \( t \) faces an opinion \( v \) that is smaller than some \( v' < t \) is

\[
Pr(t_m < v') = [F(v')]^2
\]

and the probability of facing an opinion \( v \) that is larger than some \( v' > t \) is

\[
Pr(t_m > v') = [1 - F(v')]^2.
\]

Differentiating \( Pr(t_m < v') \) and \( Pr(t_m > v') \), the probability density of judge \( t \) encountering opinion \( v' \) is

\[
k(v'|t) = 2F(v') f(v') \]

in the range \( v' < t \) and

\[
k(v'|t) = 2[1 - F(v')] f(v') \]

in the range \( v' > t \).

Consider now a judge \( t \) using a cutoff \( c \). Since the median in each panel determines the opinion, it follows that \( t \) will dissent if and only if both other judges are at a distance \( c \) or more from \( t \) (on the same side of \( t \)). Hence, the probability of dissent for judge \( t \) who uses a cutoff \( c \) is given by

\[
(8) \quad P(c; t) = Pr(t_m < t - c) + Pr(t + c < t_m) = [F(t - c)]^2 + [1 - F(t + c)]^2.
\]

Using the expression for \( D \) in equation (7), we get that

\[
(9) \quad D(x) = D \left( \int_{t-c}^{t} (t - v) \, 2F(v) \, f(v) \, dv + \int_{t}^{t+c} (v - t) \, 2(1 - F(v)) \, f(v) \, dv \right).
\]

\[\text{One may note that both expressions for } k(v|t) \text{ are independent of } t. \text{ This is since the role of } t \text{ here is restricted to determining the switching point between the two expressions.}\]
Using \((8)\) and \((9)\) in \((7)\), the problem of judge \(t\) is thus to choose a cutoff \(c\) to minimize

\[
D \left( \int_{t-c}^{t} (t - v) 2F(v) f(v) \, dv + \int_{t}^{t+c} (v - t) 2(1 - F(v)) f(v) \, dv \right) + W \left[ [F(t - c)]^2 + [1 - F(t + c)]^2 \right]
\]

### 4.3 Between-judge variation: Dissent pattern

We move now to explain the spider pattern of Fact 3—a hill-shaped relationship between the Dissent Rate and the Distance to Center of Judge Pool. The probability that judge \(t\) dissents is determined by two main factors. The first is the chosen cutoff value. For any given judge \(t\), a larger \(c\) implies less dissent. The second factor is the probability density of opinions outside this cutoff. In particular, under a uniform distribution of judges, a judge at the tail of the distribution is bound to encounter more panels where both other judges are on the same side of her and outside a given cutoff \(c\), compared to a judge with the same cutoff but whose bliss point is at the center of the distribution.

The equilibrium function \(c(t)\) has implications for the probability of dissent of each judge as follows.

**Lemma 1**  
If \(c(t)\) is locally weakly decreasing in \(|t|\) then \(P(c(t); t)\) is locally strictly increasing in \(|t|\).

**Proof:** Follows from Lemma 3 in Appendix D. Q.E.D.

The lemma expresses the notion that a judge who is both more extreme and has a smaller cutoff (hence is pickier) will dissent more. The formal proof is in the appendix but the intuition follows directly from the two factors discussed above: A judge who is more extreme encounters more panels in which the median is beyond her cutoff, and more so if the cutoff is smaller. For consistency with Fact 3 it is thus necessary that \(c(t)\) in equilibrium will *not* be decreasing toward the edges of the distribution of judges. This requires a certain property of the function \(D\).

\[\text{Note that this holds under any single-peaked distribution, see the proof of Lemma 1.}\]
Proposition 3  (Fact 3) For any $\alpha < \frac{2}{3}$ there exist values of $W$ such that, in equilibrium, $P(c(t); t)$ has a spider pattern (is first increasing and then decreasing in $|t|$ in the range $[0, 1]$). If $\alpha \geq \frac{2}{3}$, a spider pattern cannot exist in equilibrium.

Proof: See Appendix D. Q.E.D.

The proposition relates to our third stylized fact. It states that the condition for the spider pattern of dissent to arise is that $D$ is sufficiently concave. That is, judges need to be sufficiently picky about small deviations from their ideological bliss points. We will first explain why linear and convex cost functions do not yield a spider pattern and then why a sufficiently concave cost does.

Suppose first that $D$ is linear. Then a judge’s location in the distribution of ideology scores is irrelevant for her choice whether to sign an opinion or not—only the trade-off between the fixed cost $W$ and the distance $|v - t|$ matters. This implies that $c$ would be the same for all judges, and so (by Lemma 1) judges who are more often far from the opinion (i.e., extreme judges) would tend to dissent more than others, contradicting the spider pattern. Now suppose $D$ is convex, so that a judge incurs a sizable personal cost only when on aggregate deviating a lot from her bliss point. Judges at the center of the distribution of ideology scores are rarely allocated to panels where their views are far from the opinion (i.e., far from the median judge), hence will be willing to sign the opinion in these few rare cases since $D$ will be low when doing so. Extreme judges, on the other hand, encounter very unfavorable opinions very often (because they are both rarely the median and are on average further away from the median). When $D$ is convex, signing many of these would imply a marginal cost of signing ($D'$) which is higher than the marginal cost of dissenting ($W$) hence they will dissent against many of the opinions. In particular, the dissent rate will increase as a judge’s ideology becomes more extreme. Hence, a convex $D$ cannot produce a spider pattern.

As the proposition expresses, a sufficiently concave $D$ can give rise to a spider pattern.

42To see this, note that for any given signing strategy, the linearity of $D$ implies that signing another opinion $v$ would impose a cost of $|v - t|$, potentially multiplied by some constant, and dissenting against $v$ would impose a cost $W$. 32
A very concave $D$ implies that a judge feels a high ideological cost even if she signs only few opinions that are only a little bit distant from her bliss point, and that further signing more unfavorable opinions is only slightly more costly. Under such preferences, a judge will, naturally, either sign nothing that does not equal her bliss point, or sign almost anything. To see why the spider pattern can arise under such circumstances, note that the spider pattern requires first an increasing dissent rate when going from centrist judges ($t$ close to 0) to moderately ideological judges ($t$, say, close to ±0.5) and then a decreasing dissent rate as judges become very extreme ($t$ close to ±1). When $|t|$ is small, a judge will often be the panel median and in such cases she will not need to even consider signing unfavorable opinions. In the few cases in which the judge will have to make a decision whether to sign an unfavorable opinion (i.e., when she is not median), the marginal cost of signing will be very high since $D$ is concave. The relatively low frequency in which this happens implies that such judges will choose to virtually always dissent when they are not median. Put differently, $c$ will either be zero or very small. This strict adherence to ideals will hold also for judges who are more ideologically inclined yet not too extreme. However, since these judges are less often median, they will have more opportunities to dissent and we thus get that the probability of dissent is increasing in this range.

As a judge becomes sufficiently extreme, however, always dissenting becomes very costly in terms of peer pressure since that would imply dissenting in almost all panels she is part of (as she is very seldom median). Furthermore, when $D$ is concave, once a judge signs one unfavorable opinion, she might as well sign almost anything. Hence, an extreme judge will surrender ideologically under the collegial pressure ($c$ will be large).

Overall, we get that a concave ideological cost (inner discomfort) drives centrists and moderately ideological judges to stand their ground and confront the opinion (dissent) whenever they disagree with it. But once a judge regularly finds herself very far from the main-

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33

If, in a range of extreme judges, the cutoff $c = |t| + 1$ (which is equivalent to “an infinite” cutoff as there are no opinions further away than that), the dissent rate will be zero at this range. If instead the cutoff is such that also the extreme judges dissent against some opinions, then, recalling the initial discussion in this subsection, the cutoff has to increase sufficiently fast in $|t|$ to keep most opinions from becoming too distant for the most extreme judges (as otherwise they will dissent more than their less extreme colleagues). What creates a quickly increasing (or constant but very large) $c$ is a marginal ideological cost ($D'$) that is very small, which is indeed the case when $D$ is very concave.
stream (i.e., is an extremist), always standing her ground becomes too costly and then the same concave ideological cost makes her cave in and thus become the least confrontational. Thus, a concave cost of bliss point deviations and repeated interaction between individuals who disagree ideologically rationalize our third empirical finding.\footnote{In Appendix G we show by means of a numerical simulation that also a $D$-function which is first convex and then concave can produce a spider pattern. This is in line with the finding of Lim (2013). Hence, what is important for our result is that $D$ is concave when the total deviations become large – a judge does not distinguish between a large and an even larger total deviation.}

The most succinct form of the spider pattern appears when $\alpha$ is very small. For instance, for $\alpha \lesssim 0.3$, judges with small $|t|$ have a cutoff $c = 0$, implying they dissent whenever they are not median, while judges with large $|t|$ have $c = 1 + |t|$, implying they never dissent.\footnote{This leads to a spider pattern with a discontinuous drop to zero dissent. For larger values of $\alpha$ the spider may be smoother.}

This case is particularly tractable (see Appendix D.4.4) and, in the remainder of this theoretical section, we will use it to illustrate how the model replicates Fact 4 and provides additional empirical predictions.

### 4.4 Between-judge variation: Voting pattern

We move now to explain our fourth stylized fact—an S-shaped voting pattern where the moderately ideological judges have the most ideological voting. To explain this fact analytically we construct a measure of the ideology $I(t; v)$ of a vote of judge $t$ as follows:

$$I(t; v) \equiv \begin{cases} 
  v & \text{if } s(v; t) = 1 \\
  t & \text{if } s(v; t) = 0
\end{cases}.$$

In equilibrium, $v = t_m$, hence, by signing the majority opinion, a judge in practice votes for the median’s bliss point. Meanwhile, the minority opinion of a dissenting judge is assumed to equal her own bliss point ($t$) because, in this case, she composes a separate opinion in which she can express whatever she really thinks.\footnote{In the data, the ideology of a vote equals the majority opinion if the judge signs it and otherwise equals the ideology of her own text (the minority opinion), which was coded separately.}

This way, $E_{V(t)}[I(t; v)]$ captures the “ideological bias” of judge $t$’s voting pattern.

**Proposition 4 (Fact 4)** There exists an $\tilde{\alpha} \approx 0.295$ such that, for each $\alpha < \tilde{\alpha}$, there
exists a range of values of $W$ for which $|E[I(t;v)]|$ is maximized for an intermediate value of $|t|$ (i.e., $\text{argmax}_t |E[I(t;v)]| \notin \{-1, 0, 1\}$).

**Proof:** See Appendix E. \hfill Q.E.D.

The proposition expresses a sufficient condition for when moderately ideological judges will have the most ideological voting—$D$ has to be sufficiently concave. The intuition is straightforward. As explained above, centrists and moderately ideological judges will dissent virtually whenever they are not the median, hence will have $I = t$ in almost all cases. This implies that among them we will observe an increase in ideological bias as $|t|$ increases. Conversely, extreme judges almost never dissent, hence their voting pattern mostly reflects the majority opinions they sign, which are determined by the median of their panels, hence tend to be less ideological than the moderates’ voting pattern.\textsuperscript{47} Hence, a concave ideological cost can rationalize our fourth empirical finding.

### 4.5 Additional predictions: The effect of collegial pressure

Now that we have established that a concave $D$—along with collegial pressure—rationalizes the four stylized facts we set out to explain, we will move on to deriving two additional predictions which we will use in Section H.1.5 for testing our model against an alternative model (detailed in Section I). The additional predictions relate to the effect of collegial pressure on the pattern of dissent. In our main model we have the following equilibrium property.

**Proposition 5** Consider the equilibrium described for $\alpha < \tilde{\alpha}$. Then: (i) if $W = 0$, $P(t)$ is monotonically increasing in $|t|$; and (ii) $\text{argmax}_t P(t)$ is decreasing in $W$.

**Proof:** See Appendix F. \hfill Q.E.D.

Part (i) says that, should collegial pressure become very small, the spider pattern will disappear and the dissent rate will be a purely increasing function of a judge’s extremeness. The intuition for this result is that, without collegial pressure, it becomes possible also for extreme judges to be ideologically picky and hence dissent when they dislike the opinion even

\textsuperscript{47}The result of Proposition 4 holds also for other values of $\alpha < 2/3$ but this is harder to show analytically since the pattern of dissent is more complex then (see for instance the summary paragraph of appendix D.4.5).
slightly. Then, given that they are rarely the median in their panels, they will dissent more often than everyone else. Part (ii) of the proposition expresses that, as collegial pressure increases, the range of judges who dissent whenever they are not the median shrinks. This is of course natural since strictly adhering to one’s morals becomes more costly under high peer pressure, hence less judges will do so. In Appendix H.15 these predictions are corroborated empirically.

5 Conclusions and discussion

We study a high-stakes field setting, the U.S. Courts of Appeals, where decisions have an ideological element and judges are repeatedly randomly assigned into panels of three. In this setting, we present a consistent and robust set of evidence that together suggest that judges with non-consensual world views (“extremists”) are less confrontational—despite being less likely to determine the panel’s opinion on the case—and their voting pattern is less reflecting their ideology. Our findings further show that the results are not driven by having extreme ideology per se but rather by being extreme relative to the people one interacts with. Hence, interaction between individuals who disagree ideologically silences those who regularly find themselves far from the mainstream. This may hide undercurrents of dissatisfaction, distort the perception of the distribution of views and create false impressions of consensus where it is absent. To the extent that the results generalize to other settings where ideology is salient, our findings may have important implications, e.g., for the expected behavior of immigrants vis-à-vis native society, for individual differences in confronting social and religious norms and for which factions of society are expected to publicly question the consensus. More narrowly, our results also contribute to the discussion of the balance of powers by raising counter-majoritarian concerns among democracy theorists who advocate the premise that judges should reflect the preferences of the executive branch at the time of their appointment without censoring themselves[48].

To rationalize our empirical observations, we present a simple model of judicial decision making in panels, where dissenting judges are subject to collegial pressure. This model

48 See also Linz (1990), who argues that conflicts arising in presidential systems between the President and Congress can threaten democratic life. Our results raise the question of another conflict.
suggests that the non-confrontational behavior of extreme judges is due to the concavity of ideological preferences which makes them cave in under peer pressure instead of just compromising a little bit, as would have been the case if the ideological cost were convex. Meanwhile, a concave ideological cost induces moderately ideological judges to stand their ground whenever they disagree with an opinion and hence appear more confrontational. A number of alternative models are further considered (see Appendix H). In particular, a competing model about dissents as a way of drawing the attention of the Supreme Court is analyzed and rejected empirically. Our theoretical finding thus suggests that the cost of deviating from one’s principles should be concave.\(^49\)

Naturally, although we have considered a great number of other mechanisms that ultimately fail to explain our empirical findings, one can never be sure that there does not exist another model that can replicate the empirical results. Should our explanation indeed be the true one and should it be shown that concave ideological costs hold also in other field settings, this would have far-reaching implications for the empirical predictions of theoretical models in these domains. For instance, as discussed by Osborne (1995) and shown by Kamada and Kojima (2014), concavity drives polarization in political platforms.\(^50\) Furthermore, Eguía (2013) shows that a concave ideological cost affects socially optimal policy in that, when trying to bridge differences of opinion, disagreeing agents should each get to decide on a subset of issues rather than compromise within each debated issue.\(^51\) The concavity of costs associated with deviations from a moral or political bliss point also affects the sustainability of biased norms (Michaeli and Spiro, 2017); the pattern of revolutions

\(^{49}\)At least for sufficiently large deviations—see Section G.

\(^{50}\)This is since voters do not perceive a difference between a policy slightly away from their bliss point and a policy far away. Hence, unless a candidate adheres very closely to a group of voters’ preferences, these voters will not vote at all. This implies that, in a polarized electorate, political platforms will be polarized when ideological costs are concave but not otherwise.

\(^{51}\)Concave ideological costs imply that an agent would remain very unhappy unless the policy precisely equals her ideological bliss point. Hence, with such preferences, it is pointless to choose a middle ground between disagreeing agents along each dimension. Instead, the full set of issues should be divided between the agents and each agent should get to fully decide on her assigned subset of issues.
Given this wide array of applications, more empirical research into the curvature of moral and ideological costs in various field settings is warranted.

With concave ideological costs, extreme opponents of a political regime or social norm will not stand up for their views. Hence, if a revolution against the regime is started, it will necessarily be initiated by moderate opponents of the regime (Michaeli and Spiro, 2016), and a biased norm, which many strongly disagree with, fosters more obedience and will therefore ultimately be stronger (Michaeli and Spiro, 2017). Similarly, extremists are less likely to be involved in conflict (Gratton and Klose, 2017).
References


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A Additional Empirical Results

APPENDIX FIGURE 1.— Distribution of Ideology Scores

Notes: Ideology scores come from the Judicial Common Space database (Epstein et al. 2007), which provides a summary measure using the voting patterns of the appointing President and home-state Senators. The left panel presents the raw ideology score. In the central panel the ideology score is demeaned by the center of the judge pool in a circuit-year, producing the Score Relative to Center of Judge Pool measure. In the right panel the ideology score is demeaned by the mean ideology score of members of the Supreme Court (The supreme-court ideology comes from Martin and Quinn (2002)).

APPENDIX FIGURE 2.— Distance to Panel Median and Distance to Center of Judge Pool – local polynomial

Notes: x-axis: Absolute value of the distance to the center of the judge pool. y-axis: Absolute value of the distance to the panel median. The figure presents a local polynomial regression with an Epanechnikov kernel of the raw data. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Sample is restricted to panels where scores are available for all three judges.
B Robustness

B.1 Robustness for Fact 1

In Column 1 of Table A.1, we reproduce the finding from Table 6.3 in Cross (2007). Our data sample is slightly smaller than that of Cross (2007) because we drop panels with judges whose scores are tied, but we are able to replicate the finding: the sum of the scores of the judges on the panel is correlated in this specification with opinion ideology, but the score of the median judge is not. In Column 2, we add the Center of Judge Pool and find that the sum of the scores of the judges is no longer significant. Hence, this suggests that the results of Cross (2007), that panel composition determines the opinion, is in fact driven by a variable they have omitted—the ideology of the circuit. In Column 3, we separate the sum of the scores into the score of the left judge and score of the right judge. Now we see that the score of the median judge is the main driver of opinion ideology, though the score of the right-most judge is also correlated with opinion ideology. In Column 4, we show what is arguably the best specification by including all of these measures and, importantly, controlling for the circuit’s ideology, and find that the median judge is the only judge affecting the decision. Columns 3 and 4 are close to the specification in Ambrus et al. (2015), a recent experimental paper examining group-decision making. Controlling for Center of Judge Pool instead of subtracting it from the judge’s ideology score (like we do in Table I) also shows the relevance of the average ideology score of the pool of judges in each Circuit and each year.

We are unaware of other studies (apart from ours and Cross (2007)) examining the median voter theorem in Circuit Courts. Many studies construct the relative position of judges, for example, Peresie (2005) sorts the judges on a panel, but presents a regression where judge votes are the unit of observation and the scores of the judge’s two colleagues (left and right of the pair) are included as regressors. This means that each judge score enters three times in the dataset, once for the score of the judge whose vote is the dependent variable, and twice more as the left score or right score or both.
### APPENDIX TABLE A.1

**Ideology of Opinion and Ideology Scores of Panel Members**

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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<td>Median of Panel Ideology Score</td>
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<td>0.0772</td>
<td>0.149***</td>
<td>0.121***</td>
</tr>
<tr>
<td></td>
<td>(0.0607)</td>
<td>(0.0591)</td>
<td>(0.0308)</td>
<td>(0.0354)</td>
</tr>
<tr>
<td>Sum of Panel’s Ideology Scores</td>
<td>0.0949**</td>
<td>0.0437</td>
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<td></td>
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<tr>
<td></td>
<td>(0.0347)</td>
<td>(0.0331)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Center of Judge Pool Ideology Score</td>
<td>0.249*</td>
<td></td>
<td>0.251*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.134)</td>
<td></td>
<td>(0.133)</td>
<td></td>
</tr>
<tr>
<td>Left of Panel Ideology Score</td>
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<tr>
<td></td>
<td></td>
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<td>(0.0720)</td>
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<tr>
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</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is ideology of opinion, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal.

Next, also as a form of robustness, we investigate the relationship between a judge’s ideology and the ideological color of the (majority) opinion produced by panels she is sitting in. If the median of the panel is the primary driver of the opinion, then extreme judges should rarely influence the outcome of the panel since they are seldom median. Appendix Figure 3 visualizes a local polynomial regression with a judge’s ideology as independent variable and the opinion ideology as a dependent variable. As can be seen, the most ideological opinion is obtained when moderates are involved. Put differently, extreme judges are not affecting the opinions (this result is robust to adding quartic terms and splitting the sample according to whether the lower court decision was affirmed or not) and to using the lifetime average for each judge.
Appendix Figure 3.— Ideology of Opinion and Ideology Score of Judge Relative to Center of Judge Pool – local polynomial

Notes: x-axis: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year, where more conservative scores are along the right on the x-axis. y-axis: Opinion ideology, demeaned to be centered at zero. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is Opinion Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The dashed lines depict the 95% confidence interval. Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample).
B.2 Robustness for Fact 2

Appendix Figure 4 shows the raw data of Dissent Rate by Score Relative to Panel Median when grouping observations with similar relative ideology into separate bins. For each of 15 evenly-spaced bins from the left-most to right-most score, we estimated the average dissent rate for all judge-case combinations in that bin. The dashed lines present the 95% confidence interval around the mean of the dissent rate.

**APPENDIX FIGURE 4.**— Dissent and Ideology Score Relative to Panel Median – bins

Notes: x-axis: Ideology score of a judge demeaned by the median of the panel of judges assigned on the case. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where relatively more conservative scores are along the right on the x-axis. y-axis: Proportion of dissents over all votes in each bin. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

Appendix Figures 5 (local polynomial) and 6 (bins) show that the result is robust to using concurrence instead of dissent.

54The 95% confidence interval comes from a regression of the binary decision whether or not to dissent on a constant for each bin.
Appendix Figure 5.— Concurrence and Ideology Score of Judge Relative to Median of Panel – local polynomial

Notes: x-axis: Ideology score of a judge demeaned by the median of the panel of judges assigned on the case. y-axis: Rate of concur. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is rate of concur. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

Appendix Figure 6.— Concurrence and Ideology Score of Judge Relative to Median of Panel – bins

Notes: x-axis: Ideology scores are demeaned by the median of the panel of judges assigned on a case. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where relatively more conservative scores are along the right on the x-axis. y-axis: Proportion of concurrences over all votes in each bin. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.
B.3 Robustness for Fact 3

Appendix Figure 7 shows the raw data of Dissent Rate by Score Relative to Judge Pool when grouping observations with similar relative ideology into separate bins. These scores were divided from left to right into 15 evenly-spaced bins, where for each bin we estimated the average dissent rate in that bin. We also present the 95% confidence interval around the average dissent rate.\textsuperscript{55} As can be seen the spider pattern appears clearly here too.

**APPENDIX FIGURE 7.**— Dissent and Ideology Score Relative to Center of Judge Pool – bins

![Graph showing dissent rate by ideology score](image)

Notes: x-axis: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where more conservative scores are along the right on the x-axis. y-axis: Mean dissent rate for each bin. The means are weighted averages over all judges and all Circuit-years, accounting for the number of times each judge actually appeared on cases in any given Circuit-year. This is equivalent to presenting the proportion of dissents over all votes in each bin. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

Appendix Figures 8 (bins) and 9 (local polynomial) show the spider pattern appears also when using concurrences instead of dissents, though less markedly on the right.

\textsuperscript{55}The 95% confidence interval comes from a weighted regression of the dissent rate on a constant for each bin, with weights being the number of votes cast by a judge in a Circuit-year.
**Appendix Figure 8.**— Concurrence and Ideology Score of Judge Relative to Center of Judge Pool – bins

Notes: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The x-axis is divided into 15 evenly-spaced bins (the number denotes the midpoint of the bin) from left-to-right in ideology space, where relatively more conservative scores are along the right on the x-axis. y-axis: Mean concurrence rate for each bin. The means are weighted averages over all judges and all Circuit-years, accounting for the number of times each judge actually appeared on cases in any given Circuit-year. This is equivalent to presenting the proportion of concurrences over all votes in each bin. The dashed lines depict the 95% confidence interval. Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

**Appendix Figure 9.**— Concurrence and Ideology Score of Judge Relative to Center of Judge Pool – local polynomial

Notes: x-axis: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year. y-axis: Rate of concur. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is rate of concurrence. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.
Appendix Figure 10 (local polynomial) shows the spider pattern appears also when counting the number of words written in each dissent of a judge (instead of the number of dissents) per case a judge sits in. Hence, it is not the case that extreme judges, albeit choosing a few battles, in total spend a larger amount of effort on dissenting.

**Appendix Figure 10.**— Dissent Word per Case and Ideology Score of Judge Relative to Center of Judge Pool – local polynomial

Notes: x-axis: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year. y-axis: Dissent words per case. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is number of words in dissent by a judge per case in a circuit-year. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

Appendix Figure 11 (local polynomial) shows the spider pattern appears also when residualizing by circuit and year fixed effects.
Notes: x-axis: Ideology score of a judge demeaned by the center of the pool of judges available to be assigned in a Circuit-year, residualized by circuit and year fixed effects. y-axis: Rate of dissent, residualized by circuit and year fixed effects. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is residualized rate of dissent. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.

We further check if the spider is present under different weighting and scores. Appendix Table A.2 Column 1 repeats the main specification using (the absolute) Distance to Center of Judge Pool. Column 2 includes a cubic term (the quadratic and cubic terms jointly produce a spider). Column 3 weights each judge equally but excludes judges who vote less than 10 times. Column 4 does the same but presents a logit model. Column 5 uses a 2-year binned dissent and concur rates. Column 6 uses the lifetime average rate. Column 7 uses the Distance to the Supreme Court. Column 8 uses both Distance to Center of Judge Pool and Distance to the Supreme Court. As can be seen, the spider pattern is robust in these specifications. Column 9 shows that the spider is robust to including polynomials of the distance to panel median as controls, indicating that the spider pattern is not driven by interactions within particular panels. Finally, Column 10 randomly assigns Distance to Center of Judge Pool to a different judge to mitigate the concern of spurious significance or erroneous clustering level. As can be seen the result then disappears implying the result is not driven by spurious significance or by the chosen level of clustering.

We also check if the spider is robust to dropping one Circuit at a time. Appendix Table A.3 Column 1 repeats the main specification using (the absolute) Distance to Center of Judge Pool. Columns 2 to 13 drop one Circuit at a time. As can be seen the spider pattern is robust. This mitigates the concern that the pattern is driven by outliers.
## Appendix Table A.2: Robustness to Alternative Scores

<table>
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<th>(7)</th>
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<th>(10)</th>
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<tbody>
<tr>
<td>Distance to Center of Judge Pool</td>
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<td>-0.0140</td>
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<td>4.074***</td>
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<td>0.0538***</td>
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<td>Distance to Center of Judge Pool 2-year bin</td>
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<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Distance to Center of Judge Pool Lifetime average</td>
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<td>Distance to Supreme Court</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Resampled Distance to Center of Judge Pool</td>
<td></td>
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</tr>
<tr>
<td>Year Fixed Effects</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Circuit Fixed Effects</td>
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<td>Y</td>
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<tr>
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<td>10043</td>
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<td>7744</td>
<td>7744</td>
<td>5836</td>
<td>424</td>
<td>10592</td>
<td>10043</td>
<td>509022</td>
<td>10387</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.124</td>
<td>0.126</td>
<td>0.111</td>
<td>0.147</td>
<td>0.267</td>
<td>0.114</td>
<td>0.131</td>
<td>0.008</td>
<td>0.111</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Data on cases comes from OpenJurist (1950-2007). Judge scores come from the Judicial Common Space database. Absolute value of the distance to the center of the judge pool is the main independent variable. The dependent variable is the judge's sum of dissent rate and concurrence rate in this Circuit-year (with the exception of column 5, which is the rate calculated over two years; column 6, which is the lifetime rate; and column 9, which is the decision to dissent or concur on this panel). Fixed effects include circuit and year (year of appointment for column 6). Observations are weighted by the number of votes cast by the judge in the time-unit of observation (with the exception of columns 3 and 4, which do not weight bin fixed effects). Observations are weighted by the number of votes cast by the judge in the time-unit of observation (with the exception of columns 3 and 4, which do not weight bin fixed effects). Distance to Panel Median is not an alternative score, but is presented as a rejection of judge-specific mechanisms. Resampled Distance to Center of Judge Pool is presented as a rejection of spurious significance, where judicial scores have been randomly reassigned. All columns use robust standard errors clustered at the Circuit-year level, except column 5, which clusters at the Circuit-2-year-bin level. Column 4 runs a logit model and all other columns run linear models. Distance to Center of Judge Pool and Resampled Distance to Center of Judge Pool are presented as options in the table. ** p < 0.05; *** p < 0.01.
APPENDIX TABLE A.3.— Robustness to Dropping One Circuit at a Time

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
<th>(13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>0.0664***</td>
<td>0.0690***</td>
<td>0.0792***</td>
<td>0.0758***</td>
<td>0.0636***</td>
<td>0.0738***</td>
<td>0.0678***</td>
<td>0.0678***</td>
<td>0.0615***</td>
<td>0.0521***</td>
<td>0.0705***</td>
<td>0.0645***</td>
<td>0.0763***</td>
</tr>
<tr>
<td>Distance²</td>
<td>-0.0649***-0.0691***-0.0836***-0.0743***-0.0645***-0.0750***-0.0716***-0.0336**-0.0596***-0.0441***-0.0704***-0.0621***-0.0705***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Drop Circuit</td>
<td>None</td>
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<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
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<td>12</td>
</tr>
<tr>
<td>Year Fixed</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>R-sq</td>
<td>0.124</td>
<td>0.113</td>
<td>0.137</td>
<td>0.124</td>
<td>0.123</td>
<td>0.119</td>
<td>0.129</td>
<td>0.133</td>
<td>0.129</td>
<td>0.131</td>
<td>0.119</td>
<td>0.127</td>
<td>0.114</td>
</tr>
</tbody>
</table>

Notes: Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Absolute value of the distance to the center of the judge pool is the main independent variable. The dependent variable is the judge’s sum of dissent rate and concurrence rate in this Circuit-year. Fixed effects include circuit and year. Observations are weighted by the number of votes cast by the judge in the time-unit of observation. Each one of the 12 columns drops one circuit from the sample. All columns use robust standard errors clustered at the Circuit-year level (* p < 0.10; ** p < 0.05; *** p < 0.01).
Appendix Table A.4 controls for biographical characteristics of the judge. These are controlled using dummy indicators for party of appointment, whether the judge and appointing President were of the same or different political parties, whether government (Congress and President) was unified or divided at the time of appointment, whether the judge was Protestant, Evangelical Protestant, Mainline Protestant, Catholic, Jewish, or non-religious, whether the judge was Black, non-white, or female, whether the judge received a law degree from a public institution, a bachelor’s degree from a public institution, a bachelor’s degree from within the state of appointment, or obtained further graduate studies in law (LLM or SJD), was born in the 1910s, 1920s, 1930s, 1940s, or 1950s, had previous experience as federal district judge, law professor, U.S. attorney, assistant U.S. attorney, Solicitor-General, mayor, state governor, Attorney-General, Deputy or assistant district/city/county attorney, Bankruptcy judge, U.S. Magistrate, Congressional counsel, District/County/City Attorney, Local/municipal court judge, Sub-cabinet secretary, Cabinet secretary, Special prosecutor, State lower court judge, State high court judge, or Local/municipal court judge, or had experience in City council, Department of Justice, Solicitor-General’s office, or served as a member of the State house, State senate, U.S. House of Representatives, or had previous experience in private practice, in government, or in other federal capacity, or received an exceptional rating from the American Bar Association, and, in case the judge was elevated from the district courts, the party of the President who made the district bench appointment. As can be seen, the spider pattern is robust and is not driven by some special characteristics of extreme judges.

APPENDIX TABLE A.4

Dissent and Ideology Score of Judge Relative to Center of Judge Pool

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dissent</td>
<td>0.0459***</td>
<td>0.0288***</td>
</tr>
<tr>
<td>Concur</td>
<td>(0.00784)</td>
<td>(0.00586)</td>
</tr>
<tr>
<td>Distance to Center of Judge Pool</td>
<td>-0.0403***</td>
<td>-0.0324***</td>
</tr>
<tr>
<td></td>
<td>(0.0125)</td>
<td>(0.00886)</td>
</tr>
<tr>
<td>Judge Characteristics</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>8692</td>
<td>8692</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.183</td>
<td>0.173</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Judicial characteristics come from Federal Judiciary Center/Attributes of U.S. Federal Judges Database and controlled for as binary indicators. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge’s dissent rate (column 1) or concurrence rate (column 2) in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year.
Appendix Table A.5 presents the main specification but using two forms of bootstrapping. As can be seen, the spider result is robust and the p-values similar as in the main specification.

**APPENDIX TABLE A.5**

**Dissent and Ideological Distance to Center of Judge Pool – Bootstrap**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Dissents or Concurs Rate</td>
<td></td>
</tr>
<tr>
<td>Distance to Center of Judge Pool</td>
<td>0.0460***</td>
<td>0.0460***</td>
</tr>
<tr>
<td></td>
<td>(0.0107)</td>
<td>[0.0184, 0.0708]</td>
</tr>
<tr>
<td>Distance^2</td>
<td>-0.0433**</td>
<td>-0.0433**</td>
</tr>
<tr>
<td></td>
<td>(0.0170)</td>
<td>[-0.0778, 0.0020]</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>N</td>
<td>7744</td>
<td>7744</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.111</td>
<td>0.111</td>
</tr>
</tbody>
</table>

Notes: Data on cases comes from OpenJurist (1950-2007), but exclude judges with less than 10 votes in a Circuit-year. Ideology scores come from the Judicial Common Space database. The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is the judge’s sum of dissent rate and concurrence rate in a Circuit-year. Fixed effects include year and circuit. Bootstrapped standard errors clustered at the Circuit-year level presented in Column 1, and wild bootstrap estimates clustered at the circuit-year level presented in Column 2 (* p < 0.10; ** p < 0.05; *** p < 0.01).

Appendix Table A.6 presents the main specification but splits the sample according to whether the decision affirmed the lower court verdict. The sample size differs slightly when there are no affirmances or all affirmances for a judges in a Circuit-year. As can be seen, the spider result is robust.
## APPENDIX TABLE A.6

**Robustness to Splitting the Sample by Affirmance**

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Dissent or Concur</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance to Center of Judge Pool</td>
<td>0.0760***</td>
<td>0.0522***</td>
</tr>
<tr>
<td></td>
<td>(0.0109)</td>
<td>(0.0136)</td>
</tr>
<tr>
<td>Distance(^2)</td>
<td>-0.0750***</td>
<td>-0.0495**</td>
</tr>
<tr>
<td></td>
<td>(0.0167)</td>
<td>(0.0215)</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
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<td>Y</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
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<td>Y</td>
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<tr>
<td>Sample</td>
<td>Affirmed</td>
<td>Not Affirmed</td>
</tr>
<tr>
<td>N</td>
<td>9577</td>
<td>9622</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.091</td>
<td>0.072</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. The main independent variable is Ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. Column 1 uses only the sample of decisions that affirmed the lower court opinion and Column 2 uses only the sample of decisions that did not. The dependent variable is the judge’s sum of dissent rate and concurrence rate in this Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the sample of cases in the Circuit-year.

Finally, Appendix Figure 12 shows the raw data (local polynomial) using the *non-demeaned* ideology score as the independent variable. As can be seen, the spider pattern is then strongly attenuated (compared to with the equivalent Figure 5 using relative scores), which indicates that the spider result is driven by the interaction of peers who disagree ideologically rather than by extreme ideology per se.

---

56 Figure 12 has a slightly more narrow range on the x-axis compared to the equivalent figures using relative ideology scores. This is since the distribution of raw scores has a more narrow support than for the relative score (see Appendix Figure 1).
Appendix Figure 12.— Dissent and (non-relative) Ideology Score of Judge – local polynomial

Notes: x-axis: Non-demeaned Ideology score of a judge. y-axis: Rate of dissent. The figure presents a local polynomial regression with an Epanechnikov kernel, where the dependent variable is rate of dissent. The dashed lines depict the 95% confidence interval. Data come from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database.
B.4 Robustness for Fact 4

We check if the S-shape of the voting pattern (Fact 4) is present using lifetime average values for judges. Appendix Table A.7 Column 1 shows that the cubic term is still negative and significant when using the average Score Relative to Center of Judge Pool for a judge during her career.

APPENDIX TABLE A.7
VOTE IDEOLOGY AND LIFETIME IDEOLOGY SCORES

<table>
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<tbody>
<tr>
<td>Vote Ideology</td>
<td></td>
</tr>
<tr>
<td>Score Relative to Center of Judge Pool</td>
<td>0.217***</td>
</tr>
<tr>
<td>Lifetime average</td>
<td>(0.0443)</td>
</tr>
<tr>
<td>Relative Score^2</td>
<td>0.0557</td>
</tr>
<tr>
<td></td>
<td>(0.0860)</td>
</tr>
<tr>
<td>Relative Score^3</td>
<td>-0.489**</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
</tr>
<tr>
<td>N</td>
<td>421</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.056</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. The independent variables are polynomials of the ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year and averaged over a judge’s career in column 1. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is lifetime-average Vote Ideology (each vote is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal). Regression weights are the number of votes cast over a judge’s career.

We next check if the S-shape is present when using quartic terms. Appendix Table A.8 reports the regression coefficients and Appendix Figure 13 shows the predicted pattern when using Score Relative to Center of Judge Pool.
APPENDIX TABLE A.8

Vote Ideology and Ideology Score of Judge Relative to Center of Judge Pool – Quartic

(1) Vote Ideology

<table>
<thead>
<tr>
<th>Score Relative to Center of Judge Pool</th>
<th>0.170***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.0331)</td>
</tr>
<tr>
<td>Relative Score (^2)</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>(0.138)</td>
</tr>
<tr>
<td>Relative Score (^3)</td>
<td>-0.317**</td>
</tr>
<tr>
<td></td>
<td>(0.142)</td>
</tr>
<tr>
<td>Relative Score (^4)</td>
<td>-0.278</td>
</tr>
<tr>
<td></td>
<td>(0.262)</td>
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<tr>
<td>N</td>
<td>23031</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. Main independent variables are Ideology scores demeaned by the center of the pool of judges available to be assigned in a Circuit-year and averaged over a judge’s career. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Vote is the vote of the judge.

APPENDIX Figure 13.— Vote Ideology and Ideology Score of Judge Relative to Center of Judge Pool – predicted pattern from quartic regression

Notes: Data come from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Predicted Vote Ideology (from Table A.8) is plotted for evenly spaced bins of ideology score (demeaned by the center of the judge pool) from left to right. The dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Average of predicted Vote Ideology is displayed for each bin.
We next check if the S-shape in the voting pattern is present when splitting the sample according to whether the decision affirmed the lower court opinion. Appendix Table A.9 reports the regression coefficients showing a negative cubic term implying an S-shape.

**APPENDIX TABLE A.9**

**Vote Ideology and Ideology Score of Judge Relative to Center of Judge Pool, split sample**

<table>
<thead>
<tr>
<th>Score Relative to Center of Judge Pool</th>
<th>(1)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Vote Ideology</td>
<td>0.148***</td>
<td>0.190***</td>
</tr>
<tr>
<td></td>
<td>(0.0384)</td>
<td>(0.0483)</td>
</tr>
<tr>
<td>$\text{Score}^2$</td>
<td>0.128*</td>
<td>0.0298</td>
</tr>
<tr>
<td></td>
<td>(0.0750)</td>
<td>(0.0967)</td>
</tr>
<tr>
<td>$\text{Score}^3$</td>
<td>-0.387**</td>
<td>-0.346*</td>
</tr>
<tr>
<td></td>
<td>(0.163)</td>
<td>(0.199)</td>
</tr>
<tr>
<td>Sample</td>
<td>Affirmed</td>
<td>Not Affirmed</td>
</tr>
<tr>
<td>N</td>
<td>12918</td>
<td>10113</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.001</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. The main independent variable is ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. Column 1 restricts to sample of cases that affirmed the lower court opinion and column 2 restricts to cases that did not.

Next, we show that the result disappears when considering judge scores that are not relative to the pool (Appendix Table A.10 column 2 and a visualization in Figure 2 in the main text). Hence, what is important is that a judge is ideologically extreme relative to her peers—it is not about extreme ideology per se, but about the interaction between peers who disagree ideologically. This result is robust also to quartic terms and lifetime voting.
### APPENDIX TABLE A.10

**Vote Ideology and Ideology Score of Judge Relative to Center of Judge Pool**

<table>
<thead>
<tr>
<th>Score Relative to Center of Judge Pool</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.180***</td>
<td>(0.0308)</td>
</tr>
<tr>
<td><strong>Relative Score</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0614</td>
<td>(0.0659)</td>
</tr>
<tr>
<td>3</td>
<td>-0.366***</td>
<td>(0.125)</td>
</tr>
<tr>
<td>Score</td>
<td>0.141***</td>
<td>(0.0422)</td>
</tr>
<tr>
<td>Score^2</td>
<td>0.0788</td>
<td>(0.0642)</td>
</tr>
<tr>
<td>Score^3</td>
<td>0.145</td>
<td>(0.178)</td>
</tr>
<tr>
<td>N</td>
<td>23031</td>
<td>23031</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.002</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores come from the Judicial Common Space database. Sample includes three-judge panels where there are no tied or missing scores. The dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The main independent variables are ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year (column 1) and non-demeaned ideology score (column 2).

Finally, we show that the results are robust to using the alternative ideology score based on the party of the appointing President and the share of others in the pool that are appointed by a President from the other party (see description in Section 3.3).
## APPENDIX TABLE A.11

### Vote Ideology and Alternative Ideology Score

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vote Ideology</td>
<td></td>
</tr>
<tr>
<td>Score Relative to Center of Judge Pool</td>
<td>0.272***</td>
</tr>
<tr>
<td>Score Based on Party of Appointment</td>
<td>(0.0436)</td>
</tr>
<tr>
<td>Score $^2$</td>
<td>0.0507</td>
</tr>
<tr>
<td></td>
<td>(0.0445)</td>
</tr>
<tr>
<td>Score $^3$</td>
<td>-0.611***</td>
</tr>
<tr>
<td></td>
<td>(0.131)</td>
</tr>
<tr>
<td>N</td>
<td>23031</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from the U.S. Courts of Appeals Database Project (1925-2002 5% Sample). Ideology scores are simply the party of appointment (Republican or Democrat, coded as 1 and 0). The main independent variable is (the absolute value of) ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year. The dependent variable is Vote Ideology, which is coded as 1 for conservative, 0 for mixed or not applicable, and -1 for liberal. The main independent variables are ideology score demeaned by the center of the pool of judges available to be assigned in a Circuit-year (column 1) and non-demeaned ideology score (column 2).
C  Proof of Proposition 2

Suppose by negation that judge \( t \) has a strategy that violates the conditions of the proposition, i.e., there is not a cutoff strategy. Suppose first that at least on one side of \( t \) there are both opinions that \( t \) signs and opinions that she dissents against, and without loss of generality suppose that the violation of the conditions of the proposition happens on her left side (where \( t \) is not necessarily larger than 0). Thus, to the left of \( t \), there must exist two non-overlapping ranges \( \Delta v_s \) and \( \Delta v_d \) such that (1) \( t \) signs all opinions at \( \Delta v_s \) and dissents against all opinions at \( \Delta v_d \); (2) \( \max \{ |v - t| \text{ s.t. } v \in \Delta v_d \} < \min \{ |v - t| \text{ s.t. } v \in \Delta v_s \} \); and (3) \( \int_{\Delta v_s} 2F(v)f(v)dv = \int_{\Delta v_d} 2F(v)f(v)dv \). Condition (1) is just a definition of the two ranges; condition (2) states that there are opinions that \( t \) dissents against and yet are closer to her compared to other opinions she signs, and this is satisfied because any strategy that is not a cutoff strategy as described in the proposition must contain two such ranges; condition (3) equates the probability of the events of having an opinion at \( \Delta v_s \) and at \( \Delta v_d \), and this can always be satisfied by cutting down the region that initially corresponds to a more likely event until both events have equal probabilities. Conditions (2) and (3) together imply that by switching her strategy to signing all opinions at \( \Delta v_d \) and dissenting against all opinions at \( \Delta v_s \), judge \( t \) can strictly decrease her \( D \) cost without changing the dissent cost \( PW \). Thus this supposed strategy cannot be her optimal one. Finally, the only non-degenerate strategy that violates the conditions of the proposition and such that, on any side of her, \( t \) either always signs the opinion or always dissents against it is a strategy in which \( t \) signs \( v \) iff \( t \geq v \) or iff \( t \leq v \). In this case we can still require that ranges \( \Delta v_s \) and \( \Delta v_d \) satisfy conditions (1) and (2), and replace condition (3) with an equivalent one. Without loss of generality, suppose that \( t \) signs \( v \) iff \( t \geq v \). Then replace condition (3) with condition (3’) as follows: \( \int_{\Delta v_s} 2F(v)f(v)dv = \int_{\Delta v_d} 2F(1-v)f(v)dv \). The rest of the proof stays the same – by switching her strategy to signing all opinions at \( \Delta v_d \) and dissenting against all opinions at \( \Delta v_s \), judge \( t \) can strictly decrease her \( D \) cost without changing \( W \) and so this strategy cannot be her optimal one.

D  Proof of Proposition 3

Lemma (1) is proven in Section D.1. The proposition itself is proven:

- For \( \alpha > 1/2 \) in Section D.3
- For \( \alpha < 1/2 \) in Section D.4
- For \( \alpha = 1/2 \) in Section D.5

Unless stated otherwise, the distribution of judges is assumed to be uniform in \([-1, 1]\), and w.l.o.g we consider judges with \( t \geq 0 \). We occasionally refer to \( t \) as the judge’s type.

Lemma 2  Let \( F(\cdot) \) denote any cumulative distribution of judges’ ideology scores. The probability that judge
t is the median is denoted by \( P_m \) and given by

\[
P_m = 2F(t)[1 - F(t)].
\]

**Proof:** Judge \( t \) is the median when one other judge is to her left, which happens with probability \( F(t) \), and the other is to her right, which happens with probability \( 1 - F(t) \). As judge’s types are i.i.d. and the order of the assignment of judges is irrelevant, we get (10). \( Q.E.D. \)

**D.1 Properties of dissent as a function of the cutoff \( c(t) \)**

The probability of dissent for judge \( t \) using cutoff \( c \) is the probability that the opinion is more than \( c \) away from \( t \),

\[
P(c(t); t) = [F(t - c)]^2 + [1 - F(t + c)]^2.
\]

We will occasionally refer to this probability simply as \( P(t) \).

Differentiating by \( t \) yields

\[
P'_c(c(t); t) = 2F(t - c)f(t - c)\left(1 - \frac{dc}{dt}\right) - 2[1 - F(t + c)]f(t + c)\left(1 + \frac{dc}{dt}\right).
\]

The pattern of a spider is defined as follows.

**Definition 1** A spider pattern is when \( P(t) \) is first increasing and then decreasing.

**D.1.1 Proof of Lemma (1)**

Lemma (1) holds more broadly than only for a uniform distribution. Hence its proof here will cover also any distribution of judges that is single-peaked and symmetric around 0.

**Lemma 3** Let the distribution of judges be uniform in \([-1, 1]\) or single-peaked and symmetric around 0. If \( c(t) \) is locally weakly decreasing in \(|t|\) then \( P(c(t); t) \) is locally strictly increasing in \(|t|\).

**Proof:** That \( P(c(t); t) \) is locally weakly increasing in \(|t|\) follows from (12) while noting that, for \( t \geq 0 \), 1) symmetry around 0 implies that \( F(t - c) \geq [1 - F(t + c)] \), 2) a single-peaked or a uniform distribution that is symmetric around 0 satisfies \( f(t - c) \geq f(t + c) \), and 3) \( 1 - \frac{dc}{dt} \geq 1 + \frac{dc}{dt} \) when \( c(t) \) is locally weakly decreasing in \( t \). Then, given that \( F(t - c) \geq [1 - F(t + c)] \) holds with strict inequality for any \( t > 0 \), we get...
that for any non-zero range of values of $t$, $P(c(t); t)$ is locally strictly increasing in $|t|$ \footnote{Technically, if either $F(t - c) = 0$ or $f(t - c) = 0$ (where these two conditions necessarily happen simultaneously for a single-peaked or a uniform distribution), we get that $P'(c(t); t) = 0$ hence $P(c(t); t)$ is not strictly increasing in $t$. However, for this to be the case, the cutoff should be strictly increasing in $t$ (because in this case $c$ is the distance from $t$ to the lower limit of the distribution), which contradicts the initial condition in the lemma that $c(t)$ is locally weakly decreasing.} Q.E.D.

The following two lemmas apply to certain ranges of a uniform distribution of judges and will be useful later on.

**Lemma 4** Let the distribution of judges be uniform in $[-1, 1]$, and suppose the cutoff $c$ is constant at a certain range of types for whom $t \geq 0$, and for each type $t$ at this range $c < 1 - t$. Then the probability of dissent is increasing in $t$ in that range.

**Proof:** Differentiating $P(t)$ and using the properties of a uniform distribution yields

$$P'(t) = 2 \{F(t - c)f(t - c) - [1 - F(t + c)]f(t + c)\} \geq 0,$$

because symmetry implies that $F(t - c) \geq [1 - F(t + c)]$ and the uniform distribution implies that $f(t - c) = f(t + c)$ when $c < 1 - t$. Moreover, given that $F(t - c) \geq [1 - F(t + c)]$ holds with strict inequality for any $t > 0$, we get that the probability of dissent is strictly increasing in the given range of $t$ (except exactly at $t = 0$). \hfill Q.E.D.

**Lemma 5** Let the distribution of judges be uniform in $[-1, 1]$, and suppose the cutoff $c(t)$ satisfies the condition $1 - t < c(t) < 1 + t$ for a certain range of types for whom $t \geq 0$. Then the probability of dissent decreases if and only if $\frac{dc}{dt} > 1$ (when $\frac{dc}{dt}$ is well defined).

**Proof:** Noting that $1 - t \leq c$, hence the dissent occurs only for opinions to the left of $t$, we get from \footnote{Technically, if either $F(t - c) = 0$ or $f(t - c) = 0$ (where these two conditions necessarily happen simultaneously for a single-peaked or a uniform distribution), we get that $P'(c(t); t) = 0$ hence $P(c(t); t)$ is not strictly increasing in $t$. However, for this to be the case, the cutoff should be strictly increasing in $t$ (because in this case $c$ is the distance from $t$ to the lower limit of the distribution), which contradicts the initial condition in the lemma that $c(t)$ is locally weakly decreasing.} that

$$P'(c(t); t) = \frac{1}{2} (1 - c(t) - \frac{dc}{dt}),$$

i.e., $P'(c(t); t) < 0$ iff $\frac{dc}{dt} > 1$. \hfill Q.E.D.

**D.2 Analyzing the loss as a function of the cutoff, $L(c)$**

Denoting

$$z(t) \equiv \int_{t - c}^{t} (t - v)2F(v)f(v)dv + \int_{t}^{t + c} (v - t)2[1 - F(v)]f(v)dv,$$

(13)
we can express the loss of judge $t$ with cutoff strategy $c$ as follows

\begin{equation}
L(c) = \left[ \int_{t-c}^{t} (t-v) 2F(v) f(v) dv + \int_{t}^{t+c} (v-t) 2(1-F(v)) f(v) dv \right]^{a} + WP(t) = z^{a} + WP(t)
\end{equation}

Differentiating by $c$ yields

\begin{equation}
\frac{dL}{dc} = \frac{\alpha z^{a-1} dz}{dc} + W \frac{dP(t)}{dc} \rightarrow
\end{equation}

\begin{equation}
\frac{dL}{dc} = 2cM \alpha z^{a-1} - 2WM = 2M \left[ c \alpha z^{a-1} - W \right],
\end{equation}

where

\begin{equation}
M \equiv F(t-c)f(t-c) + [1-F(t+c)] f(t+c).
\end{equation}

$M$ is positive since $F \in [0,1]$ and $f \geq 0$.

Given that the distribution of judges is $U(-1,1)$, we have $f(t) = 0.5, F(t) = \frac{1+t}{2}$. Then, for $t \geq 0$, we have three possible regions for $c$:

**D.2.1 Case 1: $c \geq 1+t$**

\begin{equation}
z = \frac{1}{2} \left[ \int_{t}^{t} (t-v) (1+v) dv + \int_{t}^{1} (v-t) (1-v) dv \right] = ...
\end{equation}

\begin{equation}
z = \frac{1}{2} \left[ t^{2} + \frac{1}{3} \right]
\end{equation}

Note that $M = 0$ (in \ref{16}) since $F(t-c) = f(t+c) = 0$ under a uniform distribution $c \geq 1+t$. Hence (from \ref{15})

\begin{equation}
\frac{dL}{dc} = 0.
\end{equation}

**D.2.2 Case 2: $1-t \leq c < 1+t$**

\begin{equation}
z = \int_{t-c}^{t} (t-v) 2F(v) f(v) dv + \int_{t}^{1} (v-t) 2(1-F(v)) f(v) dv
\end{equation}

\begin{align*}
&= \int_{t-c}^{t} (t-v) \frac{1+v}{2} dv + \int_{t}^{1} (v-t) \frac{1-v}{2} dv = ...
\end{align*}

\begin{align*}
&= \frac{1}{2} \left[ \frac{(t-c)^{3}}{3} - (t-1)(t-c)^{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right]
\end{align*}

\begin{equation}
\Rightarrow \frac{dL}{dc} = 2M \left[ \alpha c \left( \frac{1}{2} \left[ \frac{(t-c)^{3}}{3} - (t-1)(t-c)^{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right] \right)^{a-1} - W \right].
\end{equation}
Substituting

\[ M = \frac{1}{2} \left[ \frac{1 + t - c}{2} \right] \]

into \( \frac{dL}{dc} \) yields

\[
\begin{aligned}
\frac{dL}{dc} = & \left( \frac{1 + t - c}{2} \right) \left( 2^{1-\alpha} \alpha c \left[ \frac{(t-c)^3}{3} - (t-1)\left(\frac{t-c}{2}\right)^2 + \left(c - \frac{1}{2}\right) t + \frac{1}{6} \right]^{\alpha-1} - W \right).
\end{aligned}
\]

We will interchangeably use different formulations for \( z \):

\[
\begin{aligned}
2z &= \frac{(t-c)^3}{3} - (t-1)\left(\frac{t-c}{2}\right)^2 + \left(c - \frac{1}{2}\right) t + \frac{1}{6} = \ldots \quad (19) \\
2z &= -\frac{1}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} tc^2 + \frac{1}{2} t^2 + \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} = \ldots \quad (20) \\
2z &= \frac{1}{6} (1-t)^3 - \frac{1}{3} c^3 + \frac{1}{2} tc^2 + \frac{1}{2} c^2. \quad (21)
\end{aligned}
\]

**D.2.3 Case 3: \( c \leq 1-t \)**

\[
\begin{aligned}
z &= \int_{t-c}^{t} (t-v) 2F(v) f(v) dv + \int_{t}^{t+c} (v-t) 2(1-F(v)) f(v) dv \\
&= \int_{t-c}^{t} (t-v) \frac{1+v}{2} dv + \int_{t}^{t+c} (v-t) \frac{1-v}{2} dv = \ldots \\
&= c^2 \left[ \frac{1}{2} - \frac{1}{3} c \right] \\
&\Rightarrow \frac{dL}{dc} = 2M \left[ \alpha c \left( c^2 \left[ \frac{1}{2} - \frac{1}{3} c \right] \right)^{\alpha-1} - W \right] \\
&= 2M \left[ \alpha c^{2\alpha-1} \left[ \frac{1}{2} - \frac{1}{3} c \right]^{\alpha-1} - W \right].
\end{aligned}
\]

Using the uniform distribution in equation (16) for \( M \)

\[
\begin{aligned}
M &= \frac{1}{2} \left[ \frac{1 + t - c}{2} + \frac{1 - t - c}{2} \right] \\
&= \frac{1}{2} (1-c)
\end{aligned}
\]

and substituting into (22) yields

\[
\frac{dL}{dc} = (1-c) \left[ \alpha c^{2\alpha-1} \left[ \frac{1}{2} - \frac{1}{3} c \right]^{\alpha-1} - W \right],
\]

\[
\begin{aligned}
\lim_{c \to 0} \frac{dL}{dc} &= \begin{cases} 
+\infty & \text{if } \alpha < 1/2 \\
2^{-1/2} - W & \text{if } \alpha = 1/2 \\
-W & \text{if } \alpha > 1/2
\end{cases}. 
\end{aligned}
\]

67
Lemma 6 \( \frac{dL}{dc} \) is continuous everywhere.

Proof: It is immediate that \( L \) is continuous within each range so we only need to check the transitions.

At \( c = 1 - t \):

\[
\frac{dL}{dc} = \left( \frac{1 + t - c}{2} \right) \left( 2^{1-\alpha} c \left[ \frac{(t-c)^3}{3} - (t-1) \frac{(t-c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right]^{\alpha-1} - W \right)
\]

which equals \( dL/dc \) at case 3 (see equation 23). At \( c = 1 + t \):

\[
\frac{dL}{dc} = \left( \frac{1 + t - c}{2} \right) \left( 2^{1-\alpha} c \left[ \frac{(t-c)^3}{3} - (t-1) \frac{(t-c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} \right]^{\alpha-1} - W \right) = 0
\]

which equals \( dL/dc \) at case 1 (see equation 23).

Q.E.D.

D.2.4 Analyzing \( dL/dc \)

In order to investigate \( \frac{dL}{dc} \), define the function \( g(c,t) \) as follows:

\[
g(c,t) \equiv \begin{cases} 
\frac{c^{2\alpha-1} \left[ 1 - \frac{2}{3}c \right]}{1-\alpha} & \text{if } 0 < c < 1 - t \\
\frac{c \left[ -\frac{4}{5}c^3 - \frac{1}{6}tc^2 + \frac{1}{2}t^2 + \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{5} \right]}{1-\alpha} & \text{if } 1 - t \leq c < 1 + t 
\end{cases}
\]

Lemma 7 The sign of \( \frac{dL}{dc} \) equals the sign of \( 2^{1-\alpha}c g(c,t) - W \).

Proof: Follows immediately from substituting the values of \( g(c,t) \) for the ranges \( 0 < c < 1 - t \) and \( 1 - t \leq c < 1 + t \) in the corresponding expressions of \( \frac{dL}{dc} \).

Q.E.D.

Lemma 8 The sign of \( \frac{\partial g(c,t)}{\partial c} \) is determined by the sign of

\[
E \equiv \begin{cases} 
\left[ 2\alpha(1-c) - 1 + \frac{4}{3}c \right] & \text{if } 0 < c < 1 - t \\
\frac{2}{5}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2}t + \frac{1}{5} + \alpha \left( tc^2 + c^2 - c^3 \right) & \text{if } 1 - t \leq c < 1 + t 
\end{cases}
\]
Note first that \( g(c,t) \) is continuous (since \( g \) is part of \( dL/dc \) which is continuous by Lemma 6) and

\[
\frac{\partial g(c,t)}{\partial c} = \frac{\partial}{\partial c} \left\{ \begin{array}{ll}
c^{2\alpha-1} \left[ 1 - \frac{2}{3} c^{\alpha-1} \right] & \text{if } 0 < c < 1 - t \\
c \left[ -\frac{1}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} t c^2 + \frac{1}{2} t^2 - \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} \right] & \text{if } 1 - t \leq c < 1 + t \\
\end{array} \right. 
\]

\[ \text{(27)} \]

\[
\frac{\partial g(c,t)}{\partial c} = \left\{ \begin{array}{ll}
c^{2\alpha-2} \left[ 2\alpha(1-c) - 1 + \frac{4}{3} c \right] \left[ 1 - \frac{3}{4} c^3 \right] & \text{if } 0 < c < 1 - t \\
\left[ \frac{2}{3} c^3 - \frac{1}{6} t^3 - \frac{1}{2} t c^2 - \frac{1}{2} t^2 - \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha \left( t c^2 + c^2 - c^3 \right) \right] & \text{if } 1 - t \leq c < 1 + t \\
\end{array} \right. 
\]

Note first that \( \left[ 1 - \frac{2}{3} c \right] \geq 0 \) when \( 0 < c < 1 - t \). Then note from equation (21) that \( \left[ \frac{1}{6} (1-t)^3 - \frac{1}{3} c^3 + \frac{1}{2} t c^2 + \frac{1}{2} c^2 \right] = 2z \geq 0 \) (since \( z \) by definition (13) is a sum of two positive integrals). Thus, the sign of \( \frac{\partial g(c,t)}{\partial c} \) is solely determined by

\[ E = \left\{ \begin{array}{ll}
\frac{2\alpha(1-c) - 1 + \frac{4}{3} c} {2\alpha(1-c) - 1 + \frac{4}{3} c} & \text{if } 0 < c < 1 - t \\
\left[ \frac{2}{3} c^3 - \frac{1}{6} t^3 - \frac{1}{2} t c^2 - \frac{1}{2} t^2 - \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha \left( t c^2 + c^2 - c^3 \right) \right] & \text{if } 1 - t \leq c < 1 + t \\
\end{array} \right. 
\]

\[ \text{Q.E.D.} \]

### D.3 Convexity, linearity and weak concavity \( \alpha > \frac{1}{2} \)

This case is divided into three sub-cases which are proven:

- For \( \alpha \geq 1 \) in Section D.3.1
- For \( 2/3 \leq \alpha < 1 \) in Section D.3.2
- For \( 1/2 < \alpha < 2/3 \) in Section D.3.3

But we start with presenting some useful results.

When \( \alpha > \frac{1}{2} \) we have \( \lim_{c \to 0} \frac{dL}{dc} < 0 \), implying that \( c = 0 \) is not a potential solution. Every type has either an inner solution or chooses the corner solution of no dissent \( (c = 1 + t) \).

For \( \alpha = \frac{1}{2} \) equation (26) yields

\[ E = \left\{ \begin{array}{ll}
\frac{1}{2} c & \text{if } 0 < c < 1 - t \\
\frac{1}{6} \left[ c^3 + (1-t)^3 \right] & \text{if } 1 - t \leq c < 1 + t \\
\end{array} \right. 
\]

which is strictly positive. It then follows, given that \( E \) increases in \( \alpha \) at the range \( 1 - t \leq c < 1 + t \)58 that for any \( \alpha \geq \frac{1}{2} \), \( \frac{\partial g(c,t)}{\partial c} \) is positive and hence \( g(c,t) \) increases in \( c \) at the whole range for which it is defined, hence equals \( \frac{W}{2^1-\alpha} \) at most once. Hence \( L(c) \) has at most one inner local min point (where \( dL/dc = 0 \)).

\[ ^{58} \text{Since } (tc^2 + c^2 - c^3) = c^2 (1 + t - c) > 0 \text{ when } c < 1 + t. \]
Corollary 1  Let $\alpha > \frac{1}{2}$. Type $t$ chooses $c = 1 + t$ if $W \geq \alpha 2^{1-\alpha} g(1 + t, t)$, and otherwise she has an inner solution $c \in ]0, 1 + t[$.

Proof: We know from Lemma 6. If $W \geq \alpha 2^{1-\alpha} g(c, t)$ at $c = 1 + t$ then $L'$ never equals zero, hence no inner solution. If $W <\alpha 2^{1-\alpha} g(1 + t, t)$ then she has an inner solution since $L'(1 + t) > 0$ and since $\lim_{c \to 0} L' < 0$. Q.E.D.

Lemma 9  Let $\alpha \geq \frac{1}{2}$. Suppose that some type $t'$ has an inner solution $c(t') \leq 1 - t'$. Then any $t < t'$ has an inner solution $c(t) = c(t') \leq 1 - t'$ too.

Proof: Lemma 7 implies that, in inner solutions, $W = \alpha 2^{1-\alpha} g(c(t'), t')$. For any $t < t'$ we have $g(c, t) = g(c, t')$ at the range $c \in ]0, 1 - t']$, hence $W = \alpha 2^{1-\alpha} g(c(t'), t)$ where $c(t') \leq 1 - t'$ implies that $c(t') < 1 - t$. In other words, $t$ has an inner solution at $c(t')$ as well. Q.E.D.

Lemma 10  Let

$$ h(t) \equiv g(1 + t, t) = (1 + t) \left( \frac{1}{3} + t^2 \right)^{\alpha - 1}. $$

Then $h(t)$ is monotonically increasing if $\alpha \geq \frac{2}{3}$ but has a unique max point at

$$ t_{\text{max}} = \frac{1 - \alpha - \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} $$

if $\alpha \in ]\frac{1}{2}, \frac{2}{3}[$.

Proof:

$$ \frac{dh}{dt} = \left( \frac{1}{3} + t^2 \right)^{\alpha - 1} + 2t(\alpha - 1)(1 + t)\left( \frac{1}{3} + t^2 \right)^{\alpha - 2} = ... $$

$$ = \left[ (2\alpha - 1) t^2 + 2(\alpha - 1)t + \frac{1}{3} \right] \left( \frac{1}{3} + t^2 \right)^{\alpha - 2} $$

which is positive if $\alpha \geq 1$. For $1/2 < \alpha < 1$ note that the bracket determines the sign, it is U-shaped in $t$ and switches sign for $t$ such that

$$ (2\alpha - 1) t^2 + 2(\alpha - 1)t + \frac{1}{3} = 0 $$

$$ t_{1,2} = \frac{-2(\alpha - 1) \pm \sqrt{4(\alpha - 1)^2 - 4\frac{1}{3}(2\alpha - 1)}}{2(2\alpha - 1)} = \frac{1 - \alpha \pm \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1}. $$

70
which has no real solution, and hence the bracket is strictly positive, if \( \alpha \in [\frac{2}{3}, 2] \). Hence \( \frac{dh}{dt} \geq 0 \). Otherwise, if \( \alpha \in ]\frac{1}{2}, \frac{2}{3}[ \), we first note that

\[
\alpha < \frac{2}{3} \Rightarrow 1 - \alpha > 2\alpha - 1 \Rightarrow t_1 > 1,
\]

meaning that the bracket crosses the zero-line at most once for \( t \in [0, 1] \). It crosses it exactly once if there exists a \( t \in [0, 1] \) for which \( \frac{dh}{dt} < 0 \):

\[
\frac{1 - \alpha - \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} < 1 \Leftrightarrow (2\alpha - 1) \left( \alpha - \frac{2}{3} \right) < 0
\]

which indeed holds when \( \alpha \in ]\frac{1}{2}, \frac{2}{3}[ \). Finally, noting that

\[
\frac{1 - \alpha - \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} > 0 \Leftrightarrow \alpha > \frac{1}{2},
\]

we get that, for any \( \alpha \in ]\frac{1}{2}, \frac{2}{3}[ \), \( h(t) \) has a max point at \( t_2 = \frac{1 - \alpha - \sqrt{(\alpha - 2)(\alpha - \frac{2}{3})}}{2\alpha - 1} = t_{\text{max}} \), while \( \frac{dh}{dt} > 0 \) \( \forall t \in [0, 1] \) if \( \alpha \geq \frac{2}{3} \).

Q.E.D.

**Lemma 11**  Let \( \alpha \geq \frac{2}{3} \). A necessary condition for getting the pattern of a spider is to have a range of types \([t, 1]\) with cutoffs \( c \in [1 - t, 1 + t] \) and such that among them the probability of dissent is decreasing in \( t \).

**Proof:** Corollary (24) implies a type has an inner solution iff \( W < \alpha^{2 - \alpha}g(1 + t, t) \). Lemma 10 says that, when \( \alpha \geq \frac{2}{3} \), \( h(t) \equiv g(1 + t, t) \) is monotonically increasing. Hence if some type \( t' \) has an inner solution \( c \), then any type \( t > t' \) has an inner solution too. If \( W \) is sufficiently large so that no type has an inner solution, it follows that no type ever dissents, and this is a degenerate case with no spider. Otherwise, the pattern of a spider requires that among types with sufficiently large \( t \) the probability of dissent will be decreasing. To complete the proof, we will show that a decrease in dissent cannot happen if the inner solutions are such that \( c(t) < 1 - t \). To see that, note that if a given type \( t'' \) has an inner solution \( c(t'') = \hat{c} < 1 - t'' \), then it must be that all types \( t < t'' \) have \( c(t) = \hat{c} \) as their solution too, because \( \forall t \leq t'' \) we have \( \hat{c} < 1 - t \) and because the part of \( g(c, t) \) at the range \( c \in [0, 1 - t''] \) is identical for all these types (see equation (25) in the first range). This implies by Lemma 4 that at the range \( t \in [0, t''[ \) the probability of dissent is increasing in \( t \). Hence, a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent among a range of types whose cutoffs are such that \( 1 - t < c < 1 + t \).

Q.E.D.

**Lemma 12**  Let \( \alpha > \frac{1}{2} \) and suppose there exists a range of types with inner solutions \( c \in [1 - t, 1 + t] \). Then, in this range of inner solutions, \( \frac{dc}{dt} \leq 0 \) if and only if \( \alpha \geq 1 \).
PROOF: For type $t$ with an inner solution at $1 - t < c < 1 + t$ the first order condition \[18\] must hold:

$$2^{1 - \alpha} \frac{c}{3} - (t - 1) \frac{(t - c)^2}{2} + \left( c - \frac{1}{2} \right) t + \frac{1}{6} = W = 0.$$ 

Using the implicit function theorem we get

$$\frac{dc}{dt} = -\frac{\frac{d}{dt} \left[ -\frac{1}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} t c^2 + \frac{1}{2} t^2 + c^2 - \frac{1}{2} t + \frac{1}{6} \right]^{\alpha - 1}}{\frac{d}{dt} \left[ -\frac{1}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} t c^2 + \frac{1}{2} t^2 + c^2 - \frac{1}{2} t + \frac{1}{6} \right]} \to \ldots$$

(30)

$$\frac{dc}{dt} = c \frac{2}{3} c^3 - \frac{1}{6} t^3 - \frac{1}{2} t c^2 + \frac{1}{2} t^2 - c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha (t c^2 + c^2 - c^3).$$

Since we look at the case of $\alpha > \frac{1}{2}$, and noting that

$$tc^2 + c^2 - c^3 = c^2 (t + 1 - c) > 0,$$

it is enough to show that \( \left[ \frac{2}{3} c^3 - \frac{1}{6} t^3 - \frac{1}{2} t c^2 + \frac{1}{2} t^2 - c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha (t c^2 + c^2 - c^3) \right] \) is positive at $\alpha = \frac{1}{2}$ (since the expression increases in $\alpha$\[59\] in order to conclude that the denominator is positive. Indeed, this expression with $\alpha = 1/2$ can be shown to equal $\frac{1}{8} \left[ c^3 + (1 - t)^3 \right] > 0$. It thus follows that for $\alpha \in \left( \frac{1}{2}, 1 \right]$ the sign of $\frac{dc}{dt}$ equals the sign of $-\frac{1}{2} t^2 + \frac{1}{2} c^2 + t - \frac{1}{2}$ while for $\alpha > 1$ the sign of $\frac{dc}{dt}$ is the opposite of the sign of $-\frac{1}{2} t^2 + \frac{1}{2} c^2 + t - \frac{1}{2}$:

(31)

$$-\frac{1}{2} t^2 + \frac{1}{2} c^2 + t - \frac{1}{2} \geq -\frac{1}{2} t^2 + \frac{1}{2} (1 - t)^2 + t - \frac{1}{2} = \ldots$$

$$= -\frac{1}{2} t^2 + \frac{1}{2} - t + \frac{1}{2} t^2 + t - \frac{1}{2} = 0.$$

It thus follows that $\frac{dc}{dt} \leq 0$ if $\alpha \geq 1$ and $\frac{dc}{dt} > 0$ if $\alpha \in [1/2, 1]$.

Q.E.D.

**Lemma 13** Suppose there exists a range of types with inner solutions $c \in [1 - t, 1 + t]$. Then $\frac{dc}{dt} > 1$ iff

(32)

$$G \equiv c^2 \left[ (3 \alpha - 1) c - (2 \alpha - 1) 3 (1 + t) \right] - (1 - t)^2 \left[ (1 - t) + (1 - \alpha) 3 c \right] > 0.$$ 

**Proof:** Using $\frac{dc}{dt}$ from \[30\], $\frac{dc}{dt} > 1$ holds iff

$$\frac{c}{2} \cdot \frac{2}{3} c^3 - \frac{1}{6} t^3 - \frac{1}{2} t c^2 + \frac{1}{2} t^2 - \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6} + \alpha (t c^2 + c^2 - c^3) > 1 \iff \ldots$$

$$c^2 \left[ (3 \alpha - 1) c - (2 \alpha - 1) 3 (1 + t) \right] - (1 - t)^2 \left[ (1 - t) + (1 - \alpha) 3 c \right] > 0.$$ 

Q.E.D.

**D.3.1 Convexity and linearity $\alpha \geq 1$**

**Proposition 6** There cannot be a spider pattern when $\alpha \geq 1$

\[59\] Since $(tc^2 + c^2 - c^3) = c^2 (1 + t - c) > 0$ when $c < 1 + t$.  

72
Proof: Lemma 11 implies that a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent at the range $1 - t < c < 1 + t$. It thus follows from Lemmas 12 and 5 that there cannot be a spider when $\alpha \geq 1$.

Q.E.D.

D.3.2 Very weak concavity $\alpha \in \left[\frac{2}{3}, 1\right]$  

Proposition 7 There cannot be a spider pattern when $\alpha \in \left[\frac{2}{3}, 1\right]$.

Proof: Lemma 11 implies that a necessary condition for getting the pattern of a spider is a decrease in the probability of dissent at the range $1 - t < c < 1 + t$, which together with Lemma 5 implies that $G$ (defined in equation (32)) must be strictly positive somewhere in the range to maintain the possibility of a spider. Investigating $G$, note first that if

\[ ((3\alpha - 1) c - (2\alpha - 1) 3 (1 + t)) \leq 0 \]

then $G \leq 0$. Otherwise, if

\[ ((3\alpha - 1) c - (2\alpha - 1) 3 (1 + t)) > 0 \]

then it gets its max value for $c = 1 + t$, while the part that is deducted from it, $(1 - t)^2 [(1 - t) + (1 - \alpha) 3c]$, is positive and increases in $c$ hence is minimal when $c = 1 - t$. By plugging these values correspondingly we get that

\[ G < (1 + t)^3 [2 - 3\alpha] - (1 - t)^3 [4 - 3\alpha] \leq 0, \]

when $\alpha \in \left[\frac{2}{3}, 1\right]$. Hence there cannot be a spider pattern.

Q.E.D.

D.3.3 Mildly weak concavity $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[\]

We will show that for any $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[\]$ the following pattern of spider exists (though other kinds of spider can be generated too): most types, including those close to 0 and 1, never dissent, while there exists a non-empty range of types in-between that do dissent sometimes.

Proposition 8 For any $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[\]$, there exist values of $W$ for which dissent has the pattern of a spider.

Proof: From Lemma 10 we know that $h(t) = g(1 + t, t)$ has a hill-shape for any $\alpha \in \left]\frac{1}{2}, \frac{2}{3}\right[\]$ with a peak at $t_{\text{max}}$ (defined in equation (28)). We will prove the proposition for $W = \alpha 2^{1-\alpha} [g(1 + t_{\text{max}}, t_{\text{max}}) - \epsilon]$, where $\epsilon$ is very small. In this case, since $W < \alpha 2^{1-\alpha} g(1 + t_{\text{max}}, t_{\text{max}})$. Corollary 1 says inner solutions exist for a small range of types $t \in (t_{\text{max}} - \delta_1, t_{\text{max}} + \delta_2)$, with $c$ very close to $1 + t_{\text{max}}$. For a given $\alpha$, they also depend on $\epsilon$ as a parameter. From corollary 1 we know that all other types, including those close to 0 or close to 1, choose a corner solution $c = 1 + t$ (always dissent). To get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. Given $W$ has been set such that type $t = 0$ has a corner solution, Lemma 9 implies that any type $t$ in the range of types with inner solutions has a solution at the range $c \in [1 - t, 1 + t]$. In this range, we know from Lemma 5 that the probability of dissent decreases if and only if $\frac{dc}{dt} > 1$. 

73
Let us denote the inner min point of $L(c)$ for type $t \in (t_{\text{max}} - \delta_1, t_{\text{max}} + \delta_2)$ by $c_0(t; \varepsilon)$. Then we can expand $c_0(t; \varepsilon)$ into Taylor series in $\varepsilon$ and $t$ around $\varepsilon = 0$ and $t = t_{\text{max}}$ as follows:

\begin{equation}
\Delta c \equiv c_0(t; \varepsilon) - (1 + t_{\text{max}}) = \left( \frac{\partial c_0}{\partial t} \right)_{t=t_{\text{max}}} \varepsilon = 0 \Delta t + \frac{1}{2} \left( \frac{\partial^2 c_0}{\partial t^2} \right)_{t=t_{\text{max}}} (\Delta t)^2 + \left( \frac{\partial c_0}{\partial \varepsilon} \right)_{t=t_{\text{max}}} \varepsilon + \ldots,
\end{equation}

where $\Delta t \equiv t - t_{\text{max}}$. We know, since $t_{\text{max}}$ is the peak of $h(t)$, that at $t_{\text{max}}$ and $c = 1 + t_{\text{max}}$ we have

\[ \frac{dh}{dt} = \frac{\partial g(c, t)}{\partial t} + \frac{\partial g(c, t)}{\partial c} \frac{d}{dt} (1 + t) = \frac{\partial g(c, t)}{\partial t} + \frac{\partial g(c, t)}{\partial c} = 0, \]

hence at $t_{\text{max}}$ we have (from the implicit function theorem)

\begin{equation}
\left( \frac{\partial c_0}{\partial t} \right)_{t=t_{\text{max}}} \varepsilon = 0 = - \left[ \frac{\partial g(c, t)}{\partial t} \right]_{t=t_{\text{max}}, c=1+t_{\text{max}}} / \left[ \frac{\partial g(c, t)}{\partial c} \right]_{t=t_{\text{max}}, c=1+t_{\text{max}}} = 1.
\end{equation}

To find the other derivatives in equation (33) we will expand into series the defining equation

\[ g(c_0(t), t) - g(1 + t_{\text{max}}, t_{\text{max}}) = -\varepsilon. \]

The expansion will be

\begin{align*}
\left( \frac{\partial g}{\partial t} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} \Delta t &+ \left( \frac{\partial g}{\partial c} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} \Delta c + \frac{1}{2} \left( \frac{\partial^2 g}{\partial t^2} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} (\Delta t)^2 + 2 \frac{\partial^2 g}{\partial t \partial c} (\Delta t) (\Delta c) + \frac{\partial^2 g}{\partial c^2} (\Delta c)^2 \
&= \left( \frac{\partial g}{\partial t} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} \Delta t + \left( \frac{\partial g}{\partial c} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} \left[ \Delta t + \frac{1}{2} \left( \frac{\partial^2 c_0}{\partial t^2} \right)_{t=t_{\text{max}}} (\Delta t)^2 + \left( \frac{\partial c_0}{\partial \varepsilon} \right)_{t=t_{\text{max}}} \varepsilon \right] \
&\quad + \frac{1}{2} \left( \frac{\partial^2 g}{\partial t^2} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} (\Delta t)^2 + 2 \frac{\partial^2 g}{\partial t \partial c} (\Delta t) (\Delta c) + \frac{\partial^2 g}{\partial c^2} (\Delta c)^2 + \ldots \
&= -\varepsilon.
\end{align*}

Equation (34) implies that $\Delta c \simeq \Delta t$ hence we can write

\begin{align*}
\left( \frac{\partial g}{\partial t} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} \Delta t + \left( \frac{\partial g}{\partial c} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} \left[ \Delta t + \frac{1}{2} \left( \frac{\partial^2 c_0}{\partial t^2} \right)_{t=t_{\text{max}}} (\Delta t)^2 + \left( \frac{\partial c_0}{\partial \varepsilon} \right)_{t=t_{\text{max}}} \varepsilon \right] \
+ \frac{1}{2} \left( \frac{\partial^2 g}{\partial t^2} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} (\Delta t)^2 + \ldots = -\varepsilon.
\end{align*}

Given that the RHS is independent of $\Delta t$ or $(\Delta t)^2$, and that the coefficient of $\Delta t$ in the LHS,

\[ \left( \frac{\partial g}{\partial t} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} + \left( \frac{\partial g}{\partial c} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}}, \]

\[ \frac{dh}{dt} \left( \frac{\partial g}{\partial t} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}} = \frac{dh}{dt} \left( \frac{\partial g}{\partial c} \right)_{t=t_{\text{max}}, c=1+t_{\text{max}}}, \]

\[ \text{Note that we plug in } \Delta c = \Delta t \text{ only in the expressions for } (\Delta t) (\Delta c)^2 \text{ and } (\Delta c)^2, \text{ where elements of size } \varepsilon \Delta t \text{ or } \varepsilon^2 \text{ in the expansion can be ignored.} \]
is zero, we can conclude from the coefficient of \((\Delta t)^2\) that

\[
\left( \frac{\partial g}{\partial c} \right)_{t=\tau_{\max}, c=1+\tau_{\max}} \left( \frac{\partial^2 c_0}{\partial t^2} \right)_{\varepsilon=0, t=\tau_{\max}} = - \left( \frac{\partial^2 g}{\partial t^2} + 2 \frac{\partial^2 g}{\partial t \partial c} + \frac{\partial^2 g}{\partial c^2} \right)_{t=\tau_{\max}, c=1+\tau_{\max}}
\]

hence

\[
\left( \frac{\partial^2 c_0}{\partial t^2} \right)_{\varepsilon=0, t=\tau_{\max}} = - \left( \frac{\partial^2 g}{\partial t^2} + 2 \frac{\partial^2 g}{\partial t \partial c} + \frac{\partial^2 g}{\partial c^2} \right)_{t=\tau_{\max}, c=1+\tau_{\max}} \left( \frac{\partial g}{\partial c} \right)_{t=\tau_{\max}, c=1+\tau_{\max}}
\]

The denominator is positive because \(g(c, t)\) increases in \(c\) (even for \(c = 1 + t\) – see Lemma 15 below, which applies to any \(\alpha\)). The numerator is the explicit expression of \(\left( \frac{\partial^2 h}{\partial t^2} \right)_{t=\tau_{\max}, c=1+\tau_{\max}}\), which is negative given that \(\tau_{\max}\) is a max point of \(h(t)\). It thus follows that

\[
\left( \frac{\partial^2 c_0}{\partial t^2} \right)_{\varepsilon=0, t=\tau_{\max}} > 0
\]

hence it follows from a Taylor expansion of \(\frac{\partial c_0}{\partial t}\) around \(\tau_{\max}\), using equation (34), that

\[
\left( \frac{\partial c_0}{\partial t} \right)_{\varepsilon=0, t=\tau_{\max}} = 1 + \left( \frac{\partial^2 c_0}{\partial t^2} \right)_{\varepsilon=0, t=\tau_{\max}} \Delta t + \ldots \geq 1 \text{ for } \Delta t \geq 0.
\]

Hence, by Lemma 5, the dissent is first increasing and then decreasing as \(t\) passes \(\tau_{\max}\). We get a spider of the following kind: when \(t\) goes from 0 to 1 the probability of dissent is first 0, then it jumps to some strictly positive probability, then it first increases and then decreases, and finally the probability of dissent decreases abruptly back to 0 and stays there.

\[Q.E.D.\]

### D.4 Strong concavity \(\alpha < \frac{1}{2}\)

This case is divided into two sub-cases depending on whether \(\alpha\) is larger or smaller than an \(\alpha^*\) which is defined in Section D.4.3. The proof is provided

- For \(\alpha \leq \alpha^*\) in Section D.4.4
- For \(\alpha^* < \alpha < 1/2\) in Section D.4.4

We start with some first useful results. When \(\alpha < \frac{1}{2}\) we have by equation (24) \(\lim_{c \to 0} \frac{\partial L}{\partial c} = \infty\), implying that \(\forall t, c = 0\) is a potential solution. Since \(\frac{\partial h}{\partial c}\) is continuous everywhere, there can be an inner solution only if \(\frac{\partial h}{\partial c} = 0\) more than once. In what follows we study some properties of \(g(c, t)\) before splitting the strict concavity case into two sub-cases.
D.4.1 The shape of \( g(c, t) \)

In this subsection we study the properties of \( g(c, t) \) as a function of \( c \).

**Lemma 14** The sign of \( \frac{\partial g(c, t)}{\partial c} \big|_{c \to 1-t} \) equals the sign of \( \frac{\partial g(c, t)}{\partial c} \big|_{c \to 1-t} \).

**Proof:** We start by revisiting Lemma 8 which showed that the sign of \( \frac{\partial g(c, t)}{\partial c} \) is solely determined by \( E \) (defined in equation 26) which we rewrite as follows

\[
E = \begin{cases} 
[2\alpha(1-c) - 1 + \frac{t}{3}c] & \text{if } 0 < c < 1 - t \\
2\alpha c + (\alpha - \frac{1}{2})(1 + t) + \frac{1}{6}(1-t)^3 & \text{if } 1 - t \leq c < 1 + t
\end{cases}
\]

Thus, the sign of \( \frac{\partial g(c, t)}{\partial c} \big|_{c \to 1-t} \) equals the sign of

\[
(1-t)^2 \left[ \left( \frac{2}{3} - \alpha \right) (1-t) + \left( \alpha - \frac{1}{2} \right) (1+t) \right] + \frac{1}{6} (1-t)^3 = ...
\]

\[
= (1-t)^2 \left[ \frac{1}{3} - \frac{4}{3} t + 2\alpha t \right],
\]

where the bracket equals \( E \big|_{c \to 1-t} \). Hence both limits have the same sign. **Q.E.D.**

**Lemma 15** \( \frac{\partial g(c, t)}{\partial c} \big|_{c \to 1-t} > 0 \).

**Proof:** Using Lemma 8 and equation (35) the sign of \( \frac{\partial g(c, t)}{\partial c} \big|_{c \to 1-t} \) equals the sign of

\[
(1+t)^2 \left[ \left( \frac{2}{3} - \alpha \right) (1+t) + \left( \alpha - \frac{1}{2} \right) (1+t) \right] + \frac{1}{6} (1+t)^3
\]

\[
= \frac{1}{6} (1+t)^3 + \frac{1}{6} (1-t)^3 > 0
\]

**Q.E.D.**

**Lemma 16** \( \frac{\partial g(c, t)}{\partial c} \) has at most one local min point (with respect to \( c \)) at the range \([1 - t, 1 + t] \).

**Proof:** By Lemma 8 follows that \( \frac{\partial g(c, t)}{\partial c} \) has a local min point only if \( dE/dc = 0 \). Differentiating equation (35) by \( c \) yields

\[
E' = \begin{cases} 
\frac{4}{3} - 2\alpha & \text{if } 0 < c < 1 - t \\
c \left[ (2 - 3\alpha) c + (2\alpha - 1)(1 + t) \right] & \text{if } 1 - t \leq c < 1 + t
\end{cases}
\]

To learn the sign of \( \frac{\partial g(c, t)}{\partial c} \) throughout the range \( 1 - t \leq c < 1 + t \), we differentiate the expression in (36) at this range, yielding

\[
E'' = 2(2 - 3\alpha) c + (2\alpha - 1)(1 + t) > 0
\]

76
Thus, equation (36) implies that for $c > 0$, since $\alpha < 1/2$, $E$ which by Lemma 8 determines the sign of $\frac{\partial g(c,t)}{\partial c}$ at the range $1 - t \leq c < 1 + t$ has at most one local extremum, at

$$
(38) \quad c = \frac{(1 - 2\alpha)(1 + t)}{2 - 3\alpha},
$$

and equation (37) implies that this is a min point. \hspace{1cm} Q.E.D.

**Lemma 17** \hspace{1cm} $g(c,t)$ does not have a local max point at $0 < c < 1 - t$

**Proof:** We know from equation (36) Lemma 8 that $\frac{\partial g(c,t)}{\partial c}$ cannot turn from positive to negative at the range $0 < c < 1 - t$ (since $\alpha < 1/2$), hence $g(c,t)$ cannot have a local max point there. \hspace{1cm} Q.E.D.

**Lemma 18** If $c_b \equiv \frac{1 - 2\alpha}{4 - 2\alpha} > 1 - t$, then $g(c,t)$ has a U-shape at the range $[0,1+t]$

**Proof:** By Lemma 8 we know $\frac{\partial g(c,t)}{\partial c} = 0$ when $E = 0$. Setting $E = 0$ in equation (35) yields

$$
\begin{cases}
  c = \frac{1 - 2\alpha}{4 - 2\alpha} \text{ if } 0 < c < 1 - t \\
  c^2 \left[\left(\frac{2}{3} - \alpha\right) c + \left(\alpha - \frac{1}{2}\right) (1 + t)\right] = -\frac{1}{6} (1 - t)^3 \text{ if } 1 - t \leq c < 1 + t
\end{cases}
$$

Starting with the first region $(c < 1 - t)$, we get by (26) that $\lim_{c \to 0} E = 2\alpha - 1 < 0$ (when $\alpha < 1/2$). Hence (by Lemma 8) the function $g(c,t)$ is decreasing initially. Then it keeps on decreasing until $c = c_b$ at which $\frac{\partial g(c,t)}{\partial c} = 0$. Thus, if $c_b \equiv \frac{1 - 2\alpha}{4 - 2\alpha} > 1 - t$, we get that $\frac{\partial g(c,t)}{\partial c}$ stays negative throughout the first region, and from Lemma 14 we know that it does not change signs at $c = 1 - t$. Then, Lemmas 16 and 15 imply that $\frac{\partial g(c,t)}{\partial c}$ changes sign exactly once, from negative to positive, and so, overall, $g(c,t)$ has a U-shape at the range $[0,1+t]$. \hspace{1cm} Q.E.D.

**Lemma 19** \hspace{1cm} $g(c,t)$ may have a local max point at the range $1 - t \leq c < 1 + t$ only if $\alpha \in \left[\frac{1}{3}, \frac{1}{2}\right]$ and $t \in \left[\frac{1 - \alpha}{1 - 2\alpha}, \frac{1}{4 - 6\alpha}\right]$

**Proof:** If $c_b \leq 1-t$ (as defined in Lemma 18), then $g(c,t)$ has a local min point at $c_b$ and $\frac{\partial g(c,t)}{\partial c}|_{c\to 1-t} \geq 0$. In that case, Lemmas 16 and 15 imply that, at the range $1 - t \leq c < 1 + t$, it might be that $g(c,t)$ first increases, then decreases, and then increases again. In this case $g(c,t)$ would have a local max point at this range. A necessary condition for $g(c,t)$ to have a local max point is that

$$
c_b \leq 1-t \Rightarrow t \leq 1 - \frac{1 - 2\alpha}{4 - 2\alpha} = \frac{1}{4 - 6\alpha}.
$$

Moreover, the min point $(c = \frac{(1 - 2\alpha)(1 + t)}{2 - 3\alpha})$, as given by equation (38), must exist within the range $[1 - t, 1 + t]$. Otherwise the fact that both $\frac{\partial g(c,t)}{\partial c}|_{c\to 1-t}$ and $\frac{\partial g(c,t)}{\partial c}|_{c\to 1+t}$ are positive would imply that $g(c,t)$ increases
throughout the range \(1-t \leq c < 1+t\) and so cannot have a local max point. The condition \(\frac{(1-2\alpha)(1+t)}{2-3\alpha} < 1+t\) is indeed fulfilled, given that

\[
\frac{1-2\alpha}{2-3\alpha} = 1 - \frac{1-\alpha}{2-3\alpha} < 1
\]

when \(\alpha < 1/2\). The condition \(\frac{(1-2\alpha)(1+t)}{2-3\alpha} > 1-t\) can be rewritten as

\[
(1-2\alpha) (1+t) > (1-t) (2-3\alpha) \quad \Leftrightarrow \quad t > \frac{1-\alpha}{3-5\alpha}.
\]

For \(c \in [1-t,1+t]\) we thus get the necessary condition

\[
\frac{1-\alpha}{3-5\alpha} \leq \frac{1}{4-6\alpha} \quad \Leftrightarrow \quad \alpha \in \left[\frac{1}{3}, \frac{1}{2}\right]
\]

Q.E.D.

**D.4.2 g(1+t,t) as a function of t**

We now turn to study further the properties of \(g(c,t)\) at \(c = 1+t\), as implied by the function \(h(t)\) defined in Lemma 10.

**LEMMA 20** Let \(\alpha \in \left[0, \frac{1}{2}\right]\). Then, as defined in Lemma 10, \(h(t) = g(1+t,t) = (1+t) \left(\frac{1}{3} + t^2\right)^{\alpha-1}\) has a unique inner global max point at \(t_{\text{max}} = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}\).

**PROOF:** The analysis of \(h(t)\) follows the same steps as in Lemma 10 (which was performed for the case of \(\alpha > 1/2\)), up to the analysis of the two roots of the square brackets in (29), which determine the sign of \(h(t)\),

\[(2\alpha - 1) t^2 + 2(\alpha - 1) t + \frac{1}{3}\]

Then, when \(\alpha \in \left[0, \frac{1}{2}\right]\), we get that the first root \(t_1 = \frac{1-\alpha+\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} < 0\), hence \(h(t)\) has a max point at \(t_2 = t_{\text{max}} = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1}\) if this value falls within the range \([0, 1]\). We thus have

\[
\frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-\frac{2}{3})}}{2\alpha-1} < 1 \Leftrightarrow ...
\]

\[(2\alpha - 1) \left(\alpha - \frac{2}{3}\right) > 0\]
which holds for any $\alpha \in ]0, \frac{1}{2}[$. Finally $t_{\text{max}} > 0$ iff

$$1 - \alpha - \frac{\sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3})}{2\alpha - 1} > 0$$

which can be verified to hold for $\alpha < 1/2$. \[Q.E.D.\]

**Lemma 21** $t_{\text{max}} \equiv \frac{1 - \alpha - \sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3})}{2\alpha - 1}$ increases in $\alpha$ at the range $\alpha \in ]0, \frac{1}{2}[$.

**Proof:** The statement holds iff

$$\frac{dt_{\text{max}}}{d\alpha} = \frac{d}{d\alpha} \left\{ 1 - \alpha - \frac{\sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3})}{2\alpha - 1} \right\} > 0$$

$$\Leftrightarrow 2 \left( 1 - \alpha - \sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3}) \right) < (1 - 2\alpha) \left( 1 + \frac{\alpha - 4/3}{\sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3})} \right)$$

$$\Leftrightarrow 2 \left( (1 - \alpha) \sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3}) - (1 - 2\alpha) (\alpha - 2)(\alpha - \frac{2}{3}) \right) < (1 - 2\alpha) \left( \sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3}) + (\alpha - 4/3) \right)$$

$$\Leftrightarrow \frac{\sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3})}{(1 - 2\alpha) (\alpha - 4/3) + 2 (\alpha - 2)(\alpha - \frac{2}{3})} = \frac{4 - 5\alpha}{3}.\]$$

Since both sides are positive, this amounts to proving

$$\alpha^2 - \frac{8}{3}\alpha + \frac{4}{3} < \frac{25\alpha^2 - 40\alpha + 16}{9} \Leftrightarrow 0 < 16\alpha^2 - 16\alpha + 4 = (2\alpha - 1)^2$$

which is evident since $\alpha < 1/2$. \[Q.E.D.\]

**Lemma 22** Let $\alpha \in ]0, \frac{1}{2}[$. Then $t_{\text{max}} \leq \frac{1}{3}$.

**Proof:** Lemma 21 implies that $t_{\text{max}} \equiv \frac{1 - \alpha - \sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3})}{2\alpha - 1}$ reaches its max value for $\alpha \in ]0, \frac{1}{2}[$ when $\alpha = \frac{1}{2}$. Using L’Hopital we get

$$\lim_{\alpha \to \frac{1}{2}} \frac{1 - \alpha - \sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3})}{2\alpha - 1} = \lim_{\alpha \to \frac{1}{2}} \left\{ \frac{1}{2} - \frac{1}{4} \sqrt{2}(\alpha - 2)(\alpha - \frac{2}{3}) \right\} = \frac{1}{3}.\]$$

\[Q.E.D.\]

**D.4.3** Splitting the strong concavity case into two sub-cases

**Lemma 23** Let $\alpha \in ]0, \frac{1}{2}[$ and define $\Omega (t_{\text{max}}(\alpha), \alpha) \equiv \frac{\alpha(1+t_{\text{max}})}{1+3t_{\text{max}}}$. Then $\frac{d\Omega(t_{\text{max}}(\alpha), \alpha)}{d\alpha} > 0$.\[79\]
PROOF: By construction (see Lemma 20) \( t_{\text{max}} \) is the solution to \( \frac{dh}{dt} = 0 \). Setting equation (29) to zero and solving for \( \alpha \) yields

\[
\alpha (t_{\text{max}}) = \frac{(t_{\text{max}} + 1 + 2/\sqrt{3}) (t_{\text{max}} + 1 - 2/\sqrt{3})}{2t_{\text{max}} (1 + t_{\text{max}})}.
\]

Using this in \( \Omega (t_{\text{max}}(\alpha), \alpha) \) yields

\[
\Omega (t_{\text{max}}) = \frac{(t_{\text{max}} + 1 + 2/\sqrt{3}) (t_{\text{max}} + 1 - 2/\sqrt{3})}{2t_{\text{max}} (1 + 3t_{\text{max}}^2)}
\]

where \( t_{\text{max}} \in [2/\sqrt{3} - 1, 1/3] \) (the lower limit follows from substituting \( \alpha = 0 \) in \( t_{\text{max}} = \frac{1-\alpha-\sqrt{(\alpha-2)(\alpha-4)}}{2\alpha-1} \)) while noting that \( t_{\text{max}} \) is increasing in \( \alpha \), as shown in Lemma 21 and from \( \). We will now show that \( \Omega (t_{\text{max}}) \) increases in \( t_{\text{max}} \), which will imply (by Lemma 21) that \( \Omega (t_{\text{max}}(\alpha), \alpha) \) increases in \( \alpha \).

\[
B (t_{\text{max}}) \equiv \frac{d\ln \Omega (t_{\text{max}})}{dt_{\text{max}}} = B_1 (t_{\text{max}}) + B_2 (t_{\text{max}}) + B_3 (t_{\text{max}}) + B_4 (t_{\text{max}})
\]

for

\[
B_1 (t_{\text{max}}) = \frac{1}{(t_{\text{max}} + 1 - 2/\sqrt{3})},
\]

\[
B_2 (t_{\text{max}}) = \frac{1}{(t_{\text{max}} + 1 + 2/\sqrt{3})},
\]

\[
B_3 (t_{\text{max}}) = -\frac{1}{t_{\text{max}}},
\]

\[
B_4 (t_{\text{max}}) = \frac{-6t_{\text{max}}}{(1 + 3t_{\text{max}}^2)},
\]

\[
B_1' (t_{\text{max}}) = -\frac{1}{(t_{\text{max}} + 1 + 2/\sqrt{3})^2},
\]

\[
B_2' (t_{\text{max}}) = -\frac{2}{(t_{\text{max}} + 1 + 2/\sqrt{3})^3} > 0,
\]

\[
B_3' (t_{\text{max}}) = -\frac{2}{(t_{\text{max}}^2)},
\]

\[
B_4' (t_{\text{max}}) = -\frac{108t_{\text{max}}}{(1 + 3t_{\text{max}}^2)^2} > 0.
\]

According to the signs of the second order derivatives, and designating \( t_{\text{low}} = 2/\sqrt{3} - 1 \), \( t_{\text{high}} = 1/3 \), we have for \( t_{\text{max}} \in [t_{\text{low}}, t_{\text{high}}] \)

\[
B_2 (t_{\text{max}}) \geq B_2 (t_{\text{low}}) + B_2' (t_{\text{low}}) (t_{\text{max}} - t_{\text{low}}) = \frac{\sqrt{3}}{4} - \frac{3}{16} (t_{\text{max}} - 2/\sqrt{3} + 1)
\]

\[
B_3 (t_{\text{max}}) \geq B_3 (t_{\text{low}}) + B_3' (t_{\text{high}} - t_{\text{low}}) \frac{t_{\text{max}} - t_{\text{low}}}{t_{\text{high}} - t_{\text{low}}} = -3 - 2\sqrt{3} + \frac{3}{4} (3 + 2\sqrt{3}) (t_{\text{max}} - 2/\sqrt{3} + 1)
\]

\[
B_4 (t_{\text{max}}) \geq B_4 (t_{\text{low}}) + B_4' (t_{\text{low}}) (t_{\text{max}} - t_{\text{low}}) = -\frac{3}{4} (3 + 2\sqrt{3}) (t_{\text{max}} - 2/\sqrt{3} + 1).
\]

Let us designate \( z \equiv t_{\text{max}} - 2/\sqrt{3} + 1 \). Then \( B (t_{\text{max}}) \geq N (z) = 1/z - (3 + 2\sqrt{3}) + (6.5625 + 4.5\sqrt{3}) z \) for \( z \in [0, 4/3 - 2/\sqrt{3}] \). Since \( N' (z) < 0 \) for \( z < 4/3 - 2/\sqrt{3} \) and \( N (4/3 - 2/\sqrt{3}) > 0 \) it follows that \( N (z) > 0 \) for all \( z \) involved, and so \( B (t_{\text{max}}) > 0 \) implying \( \Omega (t_{\text{max}}) \) increases in \( t_{\text{max}} \) and so finally \( \Omega (t_{\text{max}}(\alpha), \alpha) \) increases in \( \alpha \).

\[Q.E.D.\]

**Lemma 24** The functions \( \alpha^{2 - \alpha} h (t_{\text{max}}) \) and \( 2 (1/6)^\alpha \) have one intersection point, denoted by \( \alpha^* \), at the
range $\alpha \in ]0, \frac{1}{2}[$. Furthermore we have

\[
\begin{align*}
\alpha^{2-\alpha} h(t_{\text{max}}) &< 2 (1/6)^\alpha & \text{if } \alpha \in (0, \alpha^*) \\
\alpha^{2-\alpha} h(t_{\text{max}}) &> 2 (1/6)^\alpha & \text{if } \alpha \in (\alpha^*, \frac{1}{2})
\end{align*}
\]

**PROOF:** Using $h(t)$ as defined in Lemma 10 we get

\[
\alpha^{2-\alpha} h(t_{\text{max}}) = 2 (1/6)^\alpha
\]

\[
\Leftrightarrow \alpha (1 + t_{\text{max}}) \left(\frac{1}{6} + \frac{1}{2} t_{\text{max}}^2\right)^{\alpha-1} = \frac{1}{3} (1/6)^{\alpha-1}
\]

\[
\Leftrightarrow 3 \left(1 + 3 t_{\text{max}}^2\right)^{\alpha-1} = \frac{1}{\alpha (1 + t_{\text{max}})}
\]

\[
\Leftrightarrow 3 \left(1 + 3 t_{\text{max}}^2\right)^\alpha = \frac{1 + 3 t_{\text{max}}^2}{\alpha (1 + t_{\text{max}})}
\]

The RHS is the inverse of $\Omega(t_{\text{max}}(\alpha), \alpha)$ as defined in Lemma 23 and we know from that lemma that $\Omega(t_{\text{max}}(\alpha), \alpha)$ increases in $\alpha$ hence the RHS decreases in $\alpha$.

Analyzing the LHS:

Let

\[
\Phi(t_{\text{max}}(\alpha), \alpha) \equiv \left(1 + 3 t_{\text{max}}^2\right)^\alpha
\]

In Lemma 21 we showed that $t_{\text{max}}$ increases in $\alpha$. To show that $\Phi(t_{\text{max}}(\alpha), \alpha)$ is increasing in $\alpha$ it is therefore enough to show that the two partial derivatives of $\Phi(t_{\text{max}}(\alpha), \alpha)$ with respect to its two arguments, $t_{\text{max}}(\alpha)$ and $\alpha$, are both positive.

\[
\frac{\partial \Phi(t_{\text{max}}, \alpha)}{\partial \alpha} = \left(1 + 3 t_{\text{max}}^2\right)^\alpha \ln (1 + 3 t_{\text{max}}^2) > 0
\]

\[
\frac{\partial \Phi(t_{\text{max}}, \alpha)}{\partial t_{\text{max}}} = 6 t_{\text{max}} \alpha \left(1 + 3 t_{\text{max}}^2\right)^{\alpha-1} > 0
\]

Thus, the LHS increases in $\alpha$ while the RHS decreases in $\alpha$, implying that there is a unique intersection point $\alpha^*$. To find which of the functions $\alpha^{2-\alpha} h(t_{\text{max}})$ and $2 (1/6)^\alpha$ is larger below and above $\alpha^*$ we can plug in specific values of $\alpha$. When $\alpha = 0$ the former function goes to 0 (recall that $h$ is bounded) while the latter equals 2 hence is larger. When $\alpha \to \frac{1}{2}$ we know from Lemma 22 that $t_{\text{max}}$ approaches $\frac{1}{3}$ hence the former function approaches

\[
\frac{1}{2} \cdot 2^{1/2} \cdot \frac{4}{3} \left(\frac{4}{9}\right)^{-1/2} = \frac{4}{3} \left(\frac{9}{8}\right)^{1/2} = \sqrt{2}
\]
while the latter equals $2\left(1/6\right)^{1/2} = \sqrt{2/3}$ hence is smaller. This also implies $\alpha^* \in ]0, 1/2[. \quad Q.E.D.$

We will show the pattern of a spider separately for $\alpha \in (0, \alpha^*)$ and $\alpha \in (\alpha^*, 1/2)$. The value of $\alpha^*$ can be numerically calculated to be $\approx 0.3$.

**D.4.4 Very strong concavity $\alpha \in ]0, \alpha^*$**

**Lemma 25** If $\alpha \in ]0, 1/3[$ then $g(c,t)$ has exactly one local min point with respect to $c$.

**Proof:** We have two cases to consider. Case i) is where $c_b > 1 - t$ ($c_b$ is defined in Lemma [18]). Lemma [18] then states that $g(c, \cdot)$ has a U-shape, that is, exactly one local min point. Case ii) is where $c_b \leq 1 - t$. Then (by the proof of Lemma [18]) $g(c, \cdot)$ has one local min point at $c_b \leq 1 - t$, and $\frac{\partial g(c,t)}{\partial c} \bigg|_{c \to 1 - t} \geq 0$. Continuity of $\frac{\partial g(c,t)}{\partial c}$ at $c = 1 - t$ (see Lemma [14]) then implies $\frac{\partial g(c,t)}{\partial c} \bigg|_{c \to 1 - t} \geq 0$. Lemma [15] states that $\frac{\partial g(c,t)}{\partial c} \bigg|_{c \to 1 + t} \geq 0$.

Finally, from Lemma [19] we know that, when $\alpha < 1/3$, $g(c, \cdot)$ does not have a local max point in the range $c \in [1 - t, 1 + t]$, hence it must be that $\frac{\partial g(c,t)}{\partial c} \geq 0$ in this range. Thus, in case ii) there is one local min point. \quad Q.E.D.

**Lemma 26** Type $t$ has an inner solution only if $h(t) > \frac{W}{2^{1/\alpha}}$.

**Proof:** We (from Lemma [25]) know that $g(c,t)$ has a U-shape for all types (because $\alpha^* < 1/3$). Remembering (from Lemma [7]) that

$$\frac{dL}{dc} > 0 \Leftrightarrow g(c,t) > \frac{W}{2^{1-\alpha}},$$

and that (by equation [24])

$$\lim_{c \to 0} \frac{dL}{dc} = +\infty,$$

we get that in order for type $t$ to have an inner solution it must be that $h(t) = g(1 + t,t) > \frac{W}{2^{1/\alpha}}$. \quad Q.E.D.

We will now show the spider for the range of $W$ that satisfy the following conditions:

(i) $t = 0$ prefers $c = 0$ over $c = 1 + t$:

$$L(0,0) = \frac{1}{2}W < \left(\frac{1}{2} \left[t^2 + \frac{1}{3}\right]\right)^{\alpha} = L(t + 1, 0)$$

$$W < 2(1/6)^{\alpha}$$

(ii) $t = 1$ prefers $c = 1 + t$ over $c = 0$:

$$L(c) = z^{\alpha} = \left(\frac{1}{2} \left[t^2 + \frac{1}{3}\right]\right)^{\alpha} = (2/3)^{\alpha} < W$$
∀ \ t \in [0, 1], \ \frac{dL}{dc}|_{c=1+t} < 0. From Lemma 7, this is equivalent to:

∀ \ t \in [0, 1], \ \ h(t) < \frac{W}{2^{1-\alpha}}

\Rightarrow \ W > \alpha 2^{1-\alpha} \max \{ h(t) \}

\Rightarrow \ \{ \text{by Lemma 20} \} \ W > \alpha 2^{1-\alpha} h(t_{\max})

\Rightarrow \ W > \alpha 2^{1-\alpha} (1 + t_{\max}) \left( \frac{1}{3} + t_{\max}^2 \right)^{\alpha - 1}

**Lemma 27** For any \( \alpha \in [0, \alpha^*] \) there exists a range of \( W \) that satisfy conditions (i)-(iii).

**Proof:** The range of \( W \) that satisfy conditions (i)-(iii) is the intersection of the range of \( W \) that satisfy conditions (i) and (ii) and the range of \( W \) that satisfy conditions (i) and (iii) so it is enough to show that, for any \( \alpha \in [0, \alpha^*] \), none of these ranges is empty. Starting with conditions (i) and (ii), we note that

\[
\left( \frac{2/3}{2(1/6)} \right)^{\alpha} = \frac{4^{\alpha}}{2} < 1 \ \forall \alpha < \frac{1}{2},
\]

hence the range of \( W \) that satisfy conditions (i) and (ii) is not empty. Next, the fact that the range of \( W \) that satisfy conditions (i) and (iii) is not empty when \( \alpha < \alpha^* \) follows directly from Lemma 24.

Q.E.D.

**Lemma 28** If \( \frac{dL}{dc}|_{c=1+t} < 0 \) for type \( t = 0 \) then \( \Delta L \equiv L(0) - L(1+t) \) increases in \( t \).

**Proof:** From (17) and (14) we get

\[
\Delta L = L(0) - L(1+t) = W \left[ 1 - P_m \right] - \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha}
\]

\{ by (10) \} \quad = W \left[ 1 - 2F(t) \left[ 1 - F(t) \right] \right] - \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha}

\{ by \ t \sim U (-1, 1) \} \quad = W \left[ 1 - \frac{1}{2} (1 + t) (1 - t) \right] - \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha}

\quad = \frac{1}{2} W \left[ 1 + t^2 \right] - \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha}

\frac{d}{dt} \Delta L = \left[ W - \alpha \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha - 1} \right] t

The term \( \left[ W - \alpha \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha - 1} \right] \) increases in \( t \) (since \( \alpha < 1 \)) and at \( t = 0 \) it equals \( W - 6\alpha \left( \frac{1}{6} \right)^{\alpha} \). If \( \frac{dL}{dc}|_{c=1+t} < 0 \) for type \( t = 0 \) then

\[
\frac{dL}{dc}|_{c=1+t=1} < 0 \Rightarrow \{ \text{Lemma 27} \} \Rightarrow \quad 2^{1-\alpha} g(1, t) < \frac{W}{\alpha} \Rightarrow W > \alpha \left( \frac{1}{6} \right)^{\alpha - 1} = 6\alpha \left( \frac{1}{6} \right)^{\alpha}
\]

Thus, \( \left[ W - \alpha \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right)^{\alpha - 1} \right] > 0 \ \forall t \in [0, 1] \) implying that \( \Delta L \) increases in \( t \).

Q.E.D.

**Lemma 29** Condition (iii) implies that \( \Delta L \) increases in \( t \).
PROOF: Follows directly from applying condition (iii) to $t = 0$ and using Lemma 28. Q.E.D.

PROPOSITION 9 For any $\alpha \in [0, \alpha^*]$ there is a spider for any $W$ at the range of values that satisfy conditions (i)-(iii).

PROOF: Lemma 26 and Condition (iii) imply that every type has a corner solution, either at $c = 0$ or at $c = 1 + t$. Then, conditions (i) and (ii) and Lemma 29 imply that there exists a unique switching point at the range $[0, 1]$ such that types below it choose $c = 0$ while types above it choose $c = 1 + t$. Finally, Lemma 4 implies that dissent rate is increasing in $t$ when $t$ is below the switching point and then it abruptly falls to 0 at the switching point and stays there. Q.E.D.

D.4.5 Mildly strong concavity $\alpha \in ]\alpha^*, \frac{1}{2}[$

Lemma 30 Suppose that $g(c, t)$ has a local max point at some $c \in [1 - t, 1 + t]$. Then $c$ is strictly smaller than 1.

PROOF: A necessary condition for $g(c, t)$ to have a local max point at $c$ is that $\frac{\partial g(c, t)}{\partial c}$ has a min point at some $c' > c$ (because between $1 - t$ and the max point $g(c, t)$ is always increasing while after the max point it starts decreasing). The value of $c$ at the min point of $\frac{\partial g(c, t)}{\partial c}$ at the range $[1 - t, 1 + t]$ is given by equation (38). We will show that this value is smaller than 1. To show that, we need to show

$$\frac{1 - 2\alpha}{2 - 3\alpha} (1 + t) < 1 \Leftrightarrow ...$$

$$t < \frac{1 - \alpha}{1 - 2\alpha}.$$

Lemma 19 implies that $g(c, t)$ has a local max point only if $t \leq \frac{1}{4 - 6\alpha}$. Noting that

$$\frac{1}{4 - 6\alpha} < \frac{1 - \alpha}{1 - 2\alpha} \Leftrightarrow ...$$

$$\Leftrightarrow 6\alpha^2 - 8\alpha + 3 > 0,$$

which can be verified to hold for all $\alpha$, we indeed get that $t \leq \frac{1}{4 - 6\alpha}$ implies that $t < \frac{1 - \alpha}{1 - 2\alpha}$, hence the local max point is strictly smaller than 1. Q.E.D.

Lemma 31 Suppose that $g(c, t)$ has a local max point at some $c_m \in [1 - t, 1 + t]$. Then the value of $g(c, t)$ at the local max point is strictly smaller than $g(1 + t, t)$.

PROOF: Lemma 19 implies that if $g(c, t)$ has a local max point for some $c_m \in [1 - t, 1 + t]$ then $\alpha \in [\frac{1}{3}, \frac{1}{2}]$ and $t \in \left[\frac{1 - \alpha}{3 - 5\alpha}, \frac{1 - \alpha}{4 - 6\alpha}\right]$. So we focus on these values in the remainder of the proof. Lemma 30 further implies that $c_m < 1$. We will prove that the value of $g(c_m, t) < g(1, t)$. This is sufficient because, if this holds, then
together with the fact that \( c_m < 1 \) it implies that at the range \( c > 1 \) the function \( g(c, t) \) must be increasing\(^{61}\) and so \( g(1, t) < g(1 + t, t) \). Focusing on the range \( 1 - t < c < 1 + t \), where (by equation \((25)\))

\[
g(c, t) = c \left( \left( -\frac{1}{3} c + \frac{1}{2} t + \frac{1}{2} \right) c^2 + \frac{1}{6} (1 - t)^3 \right)^{\alpha - 1},
\]

let

\[
X(c; \alpha, t) \equiv [g(c)]^{\alpha - 1} = c^{\alpha - 1} \left[ -\frac{1}{3} c^3 + \frac{1}{2} (t + 1) c^2 + \frac{1}{6} (1 - t)^3 \right]
\]

be defined in the domain \((\alpha, t)\) which corresponds to \( \alpha \in \left[ \frac{1}{3}, \frac{1}{2} \right] \) and \( t \in \left[ \frac{1 - \alpha}{3 - 5 \alpha} \frac{1}{1 - \alpha} \right] \). Noting that

\[
t = \frac{1}{4 - 6 \alpha} \Rightarrow \alpha = \frac{2}{3} - \frac{1}{6 \alpha}
\]

\[
t = \frac{1 - \alpha}{3 - 5 \alpha} \Rightarrow t = \frac{2}{5} \frac{3 - 5 \alpha}{3 - 5 \alpha} + \frac{1}{5} \Rightarrow \alpha = \frac{3}{5} - \frac{2}{25 \alpha - 5},
\]

the domain can alternatively be described as \( t \in \left[ \frac{1}{2}, 1 \right] \) and \( \alpha \in \left[ \alpha_1(t), \alpha_2(t) \right] \), where \( \alpha_1(t) = \frac{2}{3} - \frac{1}{6 \alpha} \) and \( \alpha_2(t) = \frac{3}{5} - \frac{2}{25 \alpha - 5} \). We will show that \( X(c; \alpha, t) \) has an inner min point exactly where \( g(c, t) \) has an inner max point, and that \( X(c; \alpha, t) \) at this min point is larger than \( X(1; \alpha, t) \), which is equivalent to showing that the value of \( g(c, t) \) at the inner max point of \( g(c, t) \) is smaller than \( g(1, t) \)\(^{62}\). Holding \( t \) constant, denoting the value of \( c \) at the inner min point of \( X(c; \alpha, t) \) by \( c_0 \) (if it exists) and exploiting the fact that \( \frac{\partial X}{\partial c} = 0 \) at this min point, we get

\[
\frac{dX(c; \alpha, t)}{d\alpha} \bigg|_{c=c_0} = \frac{\partial X}{\partial c} \frac{\partial c}{\partial \alpha} + \frac{\partial X}{\partial \alpha} = \frac{\partial X}{\partial \alpha} > 0,
\]

implying that \( X(c_0; \alpha, t) \) for given \( t \) reaches its minimum at \( \alpha = \alpha_1(t) \). Hence, given that \( X(1; \alpha, t) \) is independent of \( \alpha \), it is necessary and sufficient to show that \( X(c_0; \alpha, t) > X(1; \alpha, t) \) for \( \alpha = \alpha_1(t) \).

The partial derivative \( \frac{\partial X(c; \alpha, t)}{\partial c} \) is

\[
\frac{\partial X(c; \alpha, t)}{\partial c} = -c^{\alpha - 1} \left\{ c^2 \left( \frac{2}{3} \alpha - \alpha \right) + \left( \alpha - \frac{1}{2} \right) (1 + t) \right\} + \frac{1}{6} (1 - t)^3,
\]

where the expression in the curly brackets is the one determining the sign of \( \frac{\partial g(c, t)}{\partial c} \) (see equation \((35)\)), hence the sign of \( \frac{\partial X(c; \alpha, t)}{\partial c} \) is opposite to that of \( \frac{\partial g(c, t)}{\partial c} \), and so indeed \( X(c; \alpha, t) \) has an inner min point exactly where \( g(c, t) \) has an inner max point. Substituting \( \alpha = \alpha_1(t) \) into the expression for \( \frac{\partial X(c; \alpha, t)}{\partial c} \) and equating

\(^{61}\)To see why it must be increasing, note that \( g \) has at most one inner local max point (this follows from Lemma \(16\) since when there is one local min point of \( \partial g/\partial c \) there can be at most two inner extrema where only one is a max point). Thus, as \( c_m \) is the only inner max point of \( g \) and \( g(c_m, t) < g(1, t) \) then \( g \) must be increasing at \( c = 1 \) and beyond.

\(^{62}\)Since in the expression for \( X \) the power \( \frac{1}{\alpha - 1} < 0 \).
to zero to find a local min point we get

\[
\left(\frac{2}{3} - \frac{2}{3} + \frac{1}{6t}\right) c^3 + \left(\frac{2}{3} - \frac{1}{6t} - \frac{1}{2}\right) (1 + t) c^2 + \frac{1}{6} (1 - t)^3 = 0 \iff ...
\]

\[
[c - (1 - t)] \left[\frac{1}{t} c^2 - (1 - t) c - (1 - t)^2\right] = 0,
\]

whose roots in increasing order are

\[
c_{1,2,3} = \frac{1}{2} t (1 - t) \left[1 - \sqrt{1 + \frac{4}{t}}\right], 1 - t, \frac{1}{2} t (1 - t) \left[1 + \sqrt{1 + \frac{4}{t}}\right].
\]

It can be verified that at \(c_2 = 1 - t\) the sign of \(\frac{\partial g(c,t)}{\partial c}\) turns from positive to negative,\(^{63}\) hence this is a local max point of \(g(c,t)\) and a local min point of \(X(c;\alpha,t)\). Let us now substitute \(c = c_2\) back into \(X(c;\alpha,t)\):

\[
X(c = 1 - t; \alpha_1(t), t) = \frac{1}{3} (1 - t)^{\frac{2t-2t^2}{2t+1}} (2t + 1).
\]

We need to show that this is larger than

\[
X(c = 1; \alpha_1(t), t) = -\frac{1}{3} + \frac{1}{2} (t + 1) + \frac{1}{6} (1 - t)^3 = \frac{2 + 3t^2 - t^3}{6}.
\]

Let

\[
Y(t) \equiv 3 \left[X(1 - t; \alpha_1(t), t) - X(1; \alpha_1(t), t)\right]
\]

\[
= (1 - t)^{\frac{2t-2t^2}{2t+1}} (2t + 1) - \frac{2 + 3t^2 - t^3}{2}.
\]

Then the proof boils down to showing that \(Y(t) > 0 \forall t \in \left[\frac{1}{2}, 1\right]\). Define

\[
y = -\frac{2 - 2t}{2t+1} \ln (1 - t)
\]

Then \(y \geq 0\) for \(t \in \left[\frac{1}{2}, 1\right]\) and the Taylor-Lagrange formula for \(e^{-y}\) implies that \(e^{-y} \geq 1 - y\). Hence

\[
Y(t) \geq Y_1(t) \equiv (2t + 1) \left[1 + \frac{2 - 2t}{2t+1} \ln (1 - t)\right] - \frac{2 + 3t^2 - t^3}{2}
\]

\[
= 2t + 1 + 2 (1 - t) \ln (1 - t) - \frac{2 + 3t^2 - t^3}{2}.
\]

\(^{63}\)Plugging \(c = 1 - t\) and the value of \(\alpha_1(t)\) into equation (36), which is the derivative of the expression determining the sign of \(\frac{\partial g(c,t)}{\partial c}\), yields \(\frac{1}{6t} (1 - t)^2 [1 - 2t]\), which is negative for \(t \in \left[\frac{1}{2}, 1\right]\), implying that the sign of \(\frac{\partial g(c,t)}{\partial c}\) turns from positive to negative at \(c = 1 - t\).
Let us now investigate $Y_1(t)$. We have

$$ Y_1'(t) = -2 [1 + \ln (1 - t)] $$
$$ Y_1''(t) = \frac{2}{1-t} > 0, $$

and it can be verified that $Y_1(1/2) < 0$ while $Y_1(5/9) > 0$, which implies that $Y_1(t) > 0 \ \forall t \geq 5/9$, implying also that $Y(t) > 0 \ \forall t \geq 5/9$. So the only thing left to show is that $Y(t) > 0 \ \forall t \in \left[\frac{1}{2}, \frac{5}{9}\right]$. In the interval $t \in \left[\frac{1}{2}, \frac{5}{9}\right]$ we have

$$ Y(t) \geq Y_2(t) \equiv (2t + 1)(1 - t)^{1/2} - \frac{2 + 3t^2 - t^3}{2} $$

Changing variables as follows: $q = \sqrt{1-t}$, we get

$$ Y_2(q) = q [1 + 2 (1 - q^2)] - \frac{2 + 3 (1 - q^2)^2 - (1 - q^2)^3}{2} $$
$$ = -2 + 3q + \frac{3}{2}q^2 - 2q^3 + q^6 $$

for $q \in \left[\frac{2}{3}, \sqrt{1/2}\right]$. Now, we have $Y_2(2/3) > 0$ and $Y_2'(2/3) > 0$, so it is sufficient to show that $Y_2''(q) > 0$. Indeed,

$$ Y_2''(q) = 3 - 12q + 30q^4 > 0 \ \text{for} \ q \in \left[\frac{2}{3}, \sqrt{1/2}\right], $$

because at the range $q \in \left[\frac{2}{3}, \sqrt{1/2}\right]$ we have

$$ Y_2'''(q) = 12 \left(10q^3 - 1\right) \geq 12 \left(10 \left(\frac{2}{3}\right)^3 - 1\right) > 0 $$

and

$$ Y_2'' \left(\frac{2}{3}\right) = 3 - 12 \left(\frac{2}{3}\right) + 30 \left(\frac{2}{3}\right)^4 > 0. $$

We have thus showed that $Y_2(q) > 0 \ \text{for} \ q \in \left[\frac{2}{3}, \sqrt{1/2}\right]$ and hence $Y(q) > 0 \ \text{for} \ q \in \left[\frac{2}{3}, \sqrt{1/2}\right]$. Q.E.D.

**Lemma 32** For any $\alpha \in \left[\alpha^* - \frac{1}{2}, \alpha^*\right]$ and any $t \in [0,1]$, the loss function $L(c)$ does not have a local min point if $W \geq \alpha 2^{1-\alpha} h(t)$.

**Proof:** Lemma 7 implies that the sign of $\frac{dL}{dc}$ is determined by the sign of $2^{1-\alpha} \alpha g(c,t) - W$. Recalling that $\lim_{c \to 0} \frac{dL}{dc} = \infty$, this implies that $L(c)$ may have a local min point only if there are at least two different values of $c \in [0,1]$ for which $2^{1-\alpha} \alpha g(c,t) = W$. Turning now the focus to $g(c,t)$, the fact that $\lim_{c \to 0} \frac{dL}{dc} = \infty$ implies further that $2^{1-\alpha} \alpha g(0,t) > W$, while it is given that $2^{1-\alpha} \alpha h(t) \leq W$. We thus get
that a necessary (though insufficient) condition for \(L(c)\) to have a local min point is for \(g(c, t)\) to have a local max point in which its value exceeds that of \(h(t)\). This condition does not hold at the range \(c \in [0, 1 - t]\) because \(g(c, t)\) does not have a local max point there (see Lemma \[17\]), and it also does not hold at the range \(c \in [1 - t, 1 + t]\), because Lemma \[31\] states that the value of \(g(c, t)\) at the local max point, if it exists, is strictly smaller than \(h(t)\).

\[Q.E.D.\]

We will show the spider for the range of \(W\) that satisfy the following conditions (where the inequalities in I-III follow from Lemma \[32\]):

(I) There exist types with a potential inner solution (i.e. local min point):

\[W < \alpha 2^{1 - \alpha} h(t_{\text{max}})\]

(II) The type \(t = 0.34\) does not have an inner solution:

\[\alpha 2^{1 - \alpha} h(0.34) \leq W\]

(III) The type \(t = 0\) does not have an inner solution:

\[\alpha 2^{1 - \alpha} h(0) < W\]

(IV) The type \(t = 0\) strictly prefers \(c = 1 + t\) over \(c = 0\) which implies (by plugging the corner options into equation \([14]\)):

\[
\frac{1}{2} W > \left(\frac{1}{2} t^2 + \frac{1}{3}\right)\alpha \\
W > 2 (1/6)^\alpha
\]

(V) There is no type \(t\) for whom \(g(c, t)\) has a local max point in which \(\alpha 2^{1 - \alpha} g(c, t) > W\): Let

\[W_c \equiv \alpha 2^{1 - \alpha} \max \{g(c, t) | c \text{ is local max of } g(c, t)\}.\]

Then we require \(W > W_c\).

**Lemma 33**  
For any \(\alpha \in \left[\alpha^*, \frac{1}{2}\right]\) there exist a range of \(W\) that satisfy conditions (I)-(V)

**Proof:**  
Condition (I) sets an upper bound on \(W\) while the other four conditions set lower bounds, hence we need to show that the intersections of condition (I) and each of the other conditions are not empty. Conditions (I) and (II): Lemma \[22\] shows that \(t_{\text{max}}\) is weakly smaller than \(1/3\), hence \(h(t_{\text{max}}) > h(0.34)\).

Conditions (I) and (III): Lemma \[20\] establishes that \(t_{\text{max}} > 0\), hence \(h(t_{\text{max}}) > h(0)\). Conditions (I) and (IV): Lemma \[24\] implies that \(2 (1/6)^\alpha < \alpha 2^{1 - \alpha} h(t_{\text{max}})\) at the range \(\alpha \in \left[\alpha^*, \frac{1}{2}\right]\). Conditions (I) and (V):
Let \( t_c \) be a type for whom \( W_c = \alpha 2^{1-\alpha} g(c, t) \) at the local max point of \( g(c, t) \). Then Lemma 17 implies that \( g(c, t_c) \) reaches that local max point at some \( c \in [1-t, 1+t] \), and from Lemma 31 we get that the value of \( g(c, t_c) \) at this local max point is strictly smaller than \( h(t_c) \leq h(t_{\max}) \) hence \( W_c < \alpha 2^{1-\alpha} h(t_{\max}) \). \( Q.E.D. \)

**Lemma 34** Let \( \alpha \in ]\alpha^*, \frac{1}{2} [ \) and let \( W \) satisfy conditions (I-V). Then there exists a (non-singleton) neighborhood of \( t_{\max} \) s.t. types at this neighborhood are choosing an inner solution while any other type is choosing \( c = 1 + t \).

**Proof:** Lemma 32 implies that any \( t \) for whom \( W \geq \alpha 2^{1-\alpha} g(1 + t, t) \) has a corner solution to the minimization problem, while condition (V) implies that any \( t \) for whom \( W < \alpha 2^{1-\alpha} h(t) \) has at most one local inner min point.\(^{64}\) Condition (IV) states that type \( t = 0 \) strictly prefers \( c = 1 + t \) over \( c = 0 \), and condition (III) states that \( \alpha 2^{1-\alpha} h(0) < W \), hence \( \frac{dL}{dc}|_{c=1+t} < 0 \) for \( t = 0 \) (Lemma 7), which by Lemma 28 implies that \( \Delta L \) increases in \( t \), hence all types prefer \( c = 1 + t \) over \( c = 0 \). Thus any type with a corner solution chooses \( c = 1 + t \), while all the types for whom \( W < \alpha 2^{1-\alpha} h(t) \) (which by condition (I) contain more than a singleton) choose their unique local min point as a solution because Lemma 7 implies that \( \frac{dL}{dc}|_{c=1+t} > 0 \) for these types hence \( c = 1 + t \) cannot be their global min point. Finally, Lemma 20 implies that \( h(t) \) has a hill shape with a peak at \( t_{\max} \), hence the types for whom \( W < \alpha 2^{1-\alpha} h(t) \) form a neighborhood around \( t_{\max} \).

**Lemma 35** Let \( \alpha \in ]\alpha^*, \frac{1}{2} [ \) and suppose condition (V) holds and some type \( t' \) has an inner solution \( c(t') \leq 1 - t' \). Then any \( t < t' \) has an inner solution \( c(t') \leq 1 - t' \) too.

**Proof:** Lemma 7 implies that, in inner solutions, \( W = \alpha 2^{1-\alpha} g(c(t'), t') \). For any \( t < t' \) we have (by equation 25) \( g(c, t) = g(c, t') \) at the range \( c \in [0, 1-t'] \), hence \( W = \alpha 2^{1-\alpha} g(c(t'), t) \) where \( c(t') \leq 1 - t' \) implies that \( c(t') \leq 1 - t \). In other words, \( t \) has a local min point of \( L \) at \( c(t') \) as well. Condition (V) ensures that this local min point is unique.\(^{65}\) Furthermore, at an inner solution, \( \frac{dL}{dc} \) switches sign from negative to positive, hence \( g(c, t) \) is increasing (by Lemma 7). Condition (V) ensures that this increasing part of \( g(c, t) \) continues until \( c = 1 + t \), implying that \( W < \alpha 2^{1-\alpha} h(t) \) hence \( \frac{dL}{dc}|_{c=1+t} > 0 \) and so the local min point is also the global min point.

**Proposition 10** For any \( \alpha \in ]\alpha^*, \frac{1}{2} [ \) there is a spider for any \( W \) at the range of values that satisfy conditions (I)-(IV)

**Proof:** Lemma 33 established that the conditions imply a non-empty set of \( W \). Lemma 34 established that there exists a non-singleton neighborhood of \( t_{\max} \) where types have inner solutions, while types outside

\(^{64}\)In inner solutions \( \alpha 2^{1-\alpha} g(c, t) = W \) and \( g \) is increasing (so that \( \alpha 2^{1-\alpha} g(c, t) \) crosses the \( W \)-line from below), \( g(c, t) \) can be either U-shaped or double-U-shaped. In the latter case \( \alpha 2^{1-\alpha} g(c, t) \) may cross the \( W \)-line from below twice. Condition (V) ensures that such a crossing from below happens at most once.

\(^{65}\)See proof of Lemma 34 for explanation.
that neighborhood choose \( c = 1 + t \), thus do not dissent. Condition (III) implies that type \( t = 0 \) is among these latter types, and condition (II) implies the same for all types with \( t \geq 0.34 \). Thus we know that the neighborhood of \( t_{\text{max}} \subset [0,1] \). Since \( P = 0 \) for all types who do not dissent, to get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. First note that condition (III) implies that type \( t = 0 \) has no inner solution, and so condition (V) and Lemma 35 imply that any type \( t \) in the range of types with inner solutions has a solution at the range \( c \in [1-t, 1+t] \). In this range, we know from Lemma 5 that the probability of dissent decreases if and only if \( \frac{dc}{dt} > 1 \). Equation (30) gives us the expression of \( \frac{dc}{dt} \),

\[
\frac{dc}{dt} = \frac{(1-\alpha) \left[ -\frac{1}{2}t^2 + \frac{1}{2}c^2 + t - \frac{1}{2} \right]}{\frac{2}{3}c^3 - \frac{1}{6}t^3 - \frac{1}{2}tc^2 + \frac{1}{2}t^2 - \frac{1}{2}c^2 - \frac{1}{2} + \frac{1}{6} + \alpha (tc^2 + c^2 - c^3)},
\]

where equation (31) implies that the numerator is positive. The denominator is the expression that determines the sign of \( \frac{\partial g(c,t)}{\partial c} \) (by Lemma 8) at the range \( c \in [1-t, 1+t] \), hence is positive too given that at an inner solution \( \frac{dL}{dc} \) switches signs from negative to positive, which by Lemma 7 implies that \( g(c,t) \) must be increasing. It thus follows that \( \frac{dc}{dt} > 0 \). Furthermore

\[
\frac{dc}{dt} > 1 \iff \text{as established in Lemma 13}
\]

\[
G = c^2 \left[ (3\alpha - 1) c - (2\alpha - 1) 3 (1 + t) \right] - (1-t)^2 [(1-t) + (1-\alpha) 3c] > 0.
\]

To show the pattern of the spider it is thus sufficient to show that

\[
\frac{dG}{dt} = \frac{\partial G}{\partial t} + \frac{\partial G}{\partial c} \frac{dc}{dt} \geq 0,
\]

which, given that \( \frac{dc}{dt} > 0 \), holds if both partial derivatives, \( \frac{\partial G}{\partial t} \) and \( \frac{\partial G}{\partial c} \), are positive. Indeed,

\[
\frac{\partial G}{\partial t} = 3 \left[ (1-t)^2 + 2 (1-\alpha) c (1-t) - (2\alpha -1) c^2 \right] > 0
\]

for \( \alpha \leq 1/2 \).

\[
\frac{\partial G}{\partial c} = 3 \left[ (3\alpha - 1) c^2 - 2 (2\alpha - 1) (1 + t) c - (1-t)^2 (1-\alpha) \right]
\]

Fixing \( t \) and analyzing the behavior of \( \frac{\partial G}{\partial c} \) as a function of \( c \), we get

\[
\frac{\partial^2 G}{dc^2} = 6 [(3\alpha - 1) c - (2\alpha -1) (1 + t)] = 0
\]

\[
\Rightarrow \quad c = \frac{(2\alpha - 1) (1 + t)}{(3\alpha - 1)}
\]

If \( \alpha \in [1/3, 1/2] \) then \( \frac{\partial G}{\partial c} \) is U-shaped with min point at \( c = \frac{(2\alpha -1)(1+t)}{(3\alpha -1)} < 0 \) (i.e., outside the permissible
range), implying that at the range \( c \in [1 - t, 1 + t] \) it reaches its min at \( c = 1 - t \) where it equals

\[
\frac{\partial G}{\partial c} = 3 \left[ (3\alpha - 1) (1 - t)^2 - 2 (2\alpha - 1) (1 + t) (1 - t) - (1 - t)^2 (1 - \alpha) \right]
\]

\[
= -12 (2\alpha - 1) (1 - t) t > 0
\]

Alternatively, if \( \alpha < 1/3 \), then \( \frac{\partial G}{\partial c} \) is hill-shaped and

\[
c = \frac{1 - 2\alpha}{1 - 3\alpha} (1 + t) > 1 + t
\]

is a max point, implying again that at the range \( c \in [1 - t, 1 + t] \) the function \( \frac{\partial G}{\partial c} \) reaches its min at \( c = 1 - t \) where it was just shown to be positive for any \( \alpha < 1/2 \). Thus, for any \( \alpha \in \left[ \alpha^*, \frac{1}{2} \right] \), \( G \) is increasing, implying that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases, as required for getting the pattern of a spider. Overall, we get a spider of the following kind: when \( t \) goes from 0 to 1 the probability of dissent is first 0, then jumps to some strictly positive probability, then it either increases or decreases, or first increases and then decreases, and finally the probability of dissent decreases abruptly to 0 and stays there.

\[Q.E.D.\]

**D.5 The special case of \( \alpha = \frac{1}{2} \)**

**Proposition 11** Suppose \( \alpha = \frac{1}{2} \). Then there exists a non-empty set of \( W \) such that there is a spider pattern.

**Proof:** We will prove a spider pattern of the following kind exists for some \( W \): when \( t \) goes from 0 to 1 the probability of dissent is first 0, then jumps to some strictly positive probability, then it either increases or decreases, or first increases and then decreases, and finally the probability of dissent decreases abruptly to 0 and stays there. By equation [15] we have

\[
\frac{dL}{dc} = 2M \left[ \frac{1}{2} c^{-1/2} - W \right],
\]

and by [24]

\[
\lim_{c \to 0} \frac{dL}{dc} = \frac{\sqrt{2}}{2} - W.
\]

Next, note that by [25]

\[
g(c, t) = \begin{cases} 
[1 - \frac{2}{3} c]^{-1/2} > 0 & \text{if } 0 < c < 1 - t \\
\frac{c}{\left( -\frac{1}{3} c + \frac{1}{2} t + \frac{1}{2} \right) c^2 + \frac{1}{6} (1 - t)^3} & \text{if } 1 - t \leq c < 1 + t
\end{cases}
\]
\[ \frac{\partial g(c, t)}{\partial c} = \begin{cases} \frac{1}{3} \left[ 1 - \frac{2}{3}c \right]^{-3/2} & \text{if } 0 < c < 1 - t \\ \frac{1}{6} \left(1 - t\right)^3 - \frac{1}{3}c^3 + \frac{1}{2}tc^2 + \frac{1}{2}c^2 \right]^{-3/2} & \text{if } 1 - t \leq c < 1 + t \end{cases} > 0 \]

We know the expression is positive since \[ \left[ \frac{1}{6} \left(1 - t\right)^3 - \frac{1}{3}c^3 + \frac{1}{2}tc^2 + \frac{1}{2}c^2 \right] = 2z \geq 0 \] (since \( z \) by definition (13) is a sum of two positive integrals). So \( g(c, t) \) increases everywhere. If \( W < \frac{\sqrt{2}}{2} \) so that \( \lim_{c \to 0} \frac{d}{dc} > 0 \), then \( L(c) \) is always increasing in \( c \) (by Lemma 7) and since \( \frac{\partial g(c, t)}{\partial c} > 0 \) and so all types choose \( c = 0 \) hence dissent rate is monotonically increasing – no spider. If \( W > \frac{\sqrt{2}}{2} \) so that \( \lim_{c \to 0} \frac{d}{dc} < 0 \), then \( L(c) \) is at least initially decreasing and so all types have either a unique (by Lemma 7 and since \( \frac{\partial g(c, t)}{\partial c} > 0 \)) inner solution or a corner solution at \( c = 1 + t \). Moreover, since \( g(c, t) \) increases everywhere, we know from Lemma 7 that type \( t \) has an inner solution if and only if

\[ h(t) = g(1 + t, t) > \frac{W}{2^{1-\alpha}} = \sqrt{2}W \]

Using \( h(t) = g(1 + t, t) = (1 + t) \left( \frac{1}{3} + t^2 \right)^{-1/2} \) (see Lemma 10), we get

\[ \frac{dh}{dt} = \left( \frac{1}{3} + t^2 \right)^{-1/2} - t(1 + t) \left( \frac{1}{3} + t^2 \right)^{-3/2} = ... \]

\[ = \left( \frac{1}{3} - t \right) \left( \frac{1}{3} + t^2 \right)^{-3/2} . \]

So \( h(t) \) is hill-shaped with a peak at \( t_{\text{max}} = 1/3 \), where it equals

\[ \frac{4}{3} \left( \frac{4}{9} \right)^{-1/2} = 2. \]

Thus, by setting \( W < \sqrt{2} \) we can guarantee that at least types close to \( t_{\text{max}} \) have inner solutions. The hill-shape of \( h(t) \) further implies that we can set \( W \) to be strictly greater than

\[ \max \left\{ \frac{\sqrt{2}}{2} h(0), \frac{\sqrt{2}}{2} h(1) \right\} = \sqrt{3/2} \]

yet strictly smaller than \( \sqrt{2} \), so that, by corollary 1 types close to 0 or to 1 choose \( c = 1 + t \) while there exists a non-empty range of types in-between with inner solutions. To get the pattern of a spider we need to verify that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases. As \( W \) has been set such that type \( t = 0 \) has no inner solution, Lemma 9 implies that any type \( t \) in the range of types with inner solutions has a solution at the range \( c \in [1 - t, 1 + t] \). In this range, we know from Lemma 5 that the probability of dissent decreases if and only if \( \frac{dc}{dt} > 1 \). At the range
c \in [1 - t, 1 + t]$ (setting $g(c, t) = 0$ for inner solutions and applying the implicit function theorem) we have
\[
\frac{dc}{dt} = -\frac{d}{dc} c \left[-\frac{1}{3} c^3 - \frac{1}{6} t^3 + \frac{1}{2} t^2 + \frac{1}{2} c^2 - \frac{1}{2} t + \frac{1}{6}\right]^{-1/2}
\]
\[
= -\frac{\frac{1}{2} \left[-\frac{1}{2} t^2 + \frac{1}{2} c^2 + t - \frac{1}{2}\right]}{6 (1 - t) (1 - t) + c} \geq 0
\]
\[
\Leftrightarrow c^3 \geq (1 - t)^2 \left[2 (1 - t) + 3c\right]
\]
Let
\[
H(t, c) \equiv c^3 - (1 - t)^2 \left[2 (1 - t) + 3c\right].
\]
Then
\[
\frac{\partial H(t, c)}{\partial t} = 6 (1 - t) [(1 - t) + c] \geq 0
\]
and
\[
\frac{\partial H(t, c)}{\partial c} = 3 \left[c^2 - (1 - t)^2\right] \geq 0,
\]
(since we established earlier in the proof that $c > 1 - t$ in inner solutions) and so
\[
\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial c} \frac{dc}{dt} \geq 0,
\]
implying that at the range of inner solutions the probability of dissent is either monotonic or first increases and then decreases, as required for getting the pattern of a spider. \textit{Q.E.D.}
Lemma 36. The functions $\alpha^{2-\alpha} h(t_{\text{max}})$ and $1.8 \left(\frac{2}{9}\right)^{\alpha}$ have one intersection point, denoted by $\tilde{\alpha}$, at the range $\alpha \in ]0, \frac{1}{2}[$. Furthermore we have

$$
\begin{cases}
\alpha^{2-\alpha} h(t_{\text{max}}) < 1.8 \left(\frac{2}{9}\right)^{\alpha} & \text{if } \alpha \in (0, \tilde{\alpha}), \\
\alpha^{2-\alpha} h(t_{\text{max}}) > 1.8 \left(\frac{2}{9}\right)^{\alpha} & \text{if } \alpha \in (\tilde{\alpha}, \frac{1}{2})
\end{cases}
$$

Proof: Using $h(t)$ as defined in Lemma 10 we get

$$
\alpha^{2-\alpha} h(t_{\text{max}}) = 1.8 \left(\frac{2}{9}\right)^{\alpha}
$$

$$
\Leftrightarrow \alpha^{2-\alpha} \left(1 + t_{\text{max}}\right) \left(\frac{1}{3} + \frac{t_{\text{max}}^2}{4}ight)^{\alpha-1} = \frac{9}{5} \left(\frac{2}{9}\right)^{\alpha}
$$

$$
\Leftrightarrow \alpha \left(1 + t_{\text{max}}\right) \left(\frac{1}{6} + \frac{1}{2} \frac{t_{\text{max}}^2}{4} \right)^{\alpha-1} = \frac{2}{5} \left(\frac{2}{9}\right)^{\alpha-1}
$$

$$
\Leftrightarrow \frac{5}{2} \left(\frac{3}{4} + \frac{9}{4} \frac{t_{\text{max}}^2}{4}\right)^{\alpha-1} = \frac{1}{\alpha(1 + t_{\text{max}})}
$$

$$
\Leftrightarrow \frac{5}{2} \left(\frac{3}{4} + \frac{9}{4} \frac{t_{\text{max}}^2}{4}\right) = \frac{3}{4} \frac{1 + 3t_{\text{max}}^2}{4 \alpha(1 + t_{\text{max}})}
$$

$$
\Leftrightarrow \frac{10}{3} \left(\frac{3}{4} + \frac{9}{4} \frac{t_{\text{max}}^2}{4}\right)^{\alpha} = \frac{1 + 3t_{\text{max}}^2}{\alpha(1 + t_{\text{max}})}
$$

The RHS is the inverse of $\Omega \left(t_{\text{max}}(\alpha), \alpha\right)$ which is defined in Lemma 23, where it is also shown to be increasing in $\alpha$, hence the RHS decreases in $\alpha$.

Analyzing the LHS. Let

$$
\xi \left(t_{\text{max}}(\alpha), \alpha\right) \equiv \left(\frac{3}{4} + \frac{9}{4} \frac{t_{\text{max}}^2}{4}\right)^{\alpha}.
$$

In Lemma 21 we showed that $t_{\text{max}}$ increases in $\alpha$. To show that $\xi \left(t_{\text{max}}(\alpha), \alpha\right)$ is increasing in $\alpha$ it is therefore sufficient to show that the two partial derivatives of $\xi \left(t_{\text{max}}(\alpha), \alpha\right)$ with respect to its two arguments, $t_{\text{max}}(\alpha)$ and $\alpha$, are both positive.

$$
\frac{\partial \xi \left(t_{\text{max}}, \alpha\right)}{\partial \alpha} = \left(\frac{3}{4} + \frac{9}{4} \frac{t_{\text{max}}^2}{4}\right)^{\alpha} \ln \left(\frac{3}{4} + \frac{9}{4} \frac{t_{\text{max}}^2}{4}\right) > 0
$$

where the inequality follows since the smallest possible $t_{\text{max}}$ is $\frac{3}{4} \sqrt{2} - 1$ (to see this recall that $t_{\text{max}}$ increases $\alpha$ and hence plug in $\alpha = 0$ in equation 28) which implies $\frac{3}{4} + \frac{9}{4} \frac{t_{\text{max}}^2}{4} > 1$.

$$
\frac{\partial \xi \left(t_{\text{max}}, \alpha\right)}{\partial t_{\text{max}}} = 4.5t_{\text{max}}^2 \alpha \left(\frac{3}{4} + \frac{9}{4} \frac{t_{\text{max}}^2}{4}\right)^{\alpha-1} > 0
$$

Thus, the LHS increases in $\alpha$ while the RHS decreases in $\alpha$, implying that there is a unique intersection point $\tilde{\alpha}$. To find which of the functions $\alpha^{2-\alpha} h(t_{\text{max}})$ and $1.8 \left(\frac{2}{9}\right)^{\alpha}$ is larger below and above $\tilde{\alpha}$, we can plug in specific values of $\alpha$. When $\alpha = 0$ the former function goes to 0 (recall that $h$ is bounded) while the
latter equals 1.8 hence is larger. When $\alpha \to \frac{1}{2}$ we know from Lemma 22 that $t_{\text{max}}$ approaches $\frac{1}{3}$ hence the former function approaches

$$\frac{1}{2} 2^{1/2} \left(\frac{4}{3} \frac{4}{9}\right)^{-1/2} = \frac{4}{3} \left(\frac{9}{8}\right)^{1/2} = \sqrt{2}$$

while the latter equals $1.8 (2/9)^{1/2} = \frac{3}{5} \sqrt{2}$ hence is smaller. This also implies $\tilde{\alpha} \in \left[0, \frac{1}{2}\right]$. The value of $\tilde{\alpha}$ can be numerically calculated to be $\approx 0.295$. Q.E.D.

### E.1 Proof of the proposition

We prove the proposition for $t \geq 0$. Equivalent statements can be made for $t \leq 0$.

The average vote of type $t$, denoted by $J(t) \equiv E_{V(t)}[I(t; v)]$, is a weighted sum of her own type, with probability $P_m + P(t)$, and the median of the judges’ panel otherwise.

$$J(t) = \left[ P_m + P(t) \right] t + \left[ \int_{-c}^{t} 2v F(v) f(v) dv + \int_{t}^{t+c} 2v (1 - F(v)) f(v) dv \right]$$

$$= \left\{ 2F(t) [1 - F(t)] + [F(t - c)]^2 + [1 - F(t + c)]^2 \right\} t + \int_{-c}^{t} 2vF(v) f(v) dv + \int_{t}^{t+c} 2v (1 - F(v)) f(v) dv$$

Let

$$Z \equiv \int_{-c}^{t} 2vF(v) f(v) dv + \int_{t}^{t+c} 2v (1 - F(v)) f(v) dv$$

We analyze separately the three possible regions of $c$ (for $t \geq 0$):

- $c \geq 1 + t$

$$Z = \frac{1}{2} \left[ \int_{-1}^{t} v(1 + v) dv + \int_{t}^{1} v(1 - v) dv \right]$$

$$= \frac{1}{2} \left[ \int_{-1}^{t} (v + v^2) dv + \int_{t}^{1} (v - v^2) dv \right] = ...$$

$$= \frac{t^3}{3}$$

$$\Rightarrow J(t) = \frac{1}{2} (1 - t^2) t + \frac{t^3}{3} = \frac{1}{2} t - \frac{1}{6} t^3$$

- $1 - t \leq c < 1 + t$


\[ Z = \int_{t-c}^{t} \frac{1 + v}{2} dv + \int_{\frac{1}{t}}^{1} \frac{1 - v}{2} dv \]

\[ = \frac{1}{2} \left[ \int_{t-c}^{t} (v + v^2) dv + \int_{1/2}^{\frac{1}{t}} (v - v^2) dv \right] = ... \]

\[ = \frac{t^3}{6} + \frac{1}{12} - \frac{1}{4} t^2 + \frac{1}{2} ct - \frac{1}{4} c^2 + \frac{1}{2} ct^2 - \frac{1}{2} c^2 t + \frac{1}{6} c^3 \]

\[ \Rightarrow J(t) = \left\{ \frac{1}{2} (1 - t^2) + \left[ \frac{1}{2} (1 + t - c) \right]^2 \right\} t + \frac{t^3}{6} + \frac{1}{12} - \frac{1}{4} t^2 + \frac{1}{2} ct - \frac{1}{4} c^2 + \frac{1}{2} ct^2 - \frac{1}{2} c^2 t + \frac{1}{6} c^3 \]

\[ = \ldots = t + \frac{1}{12} (1 - t)^3 + c^2 \left( \frac{1}{6} c - \frac{1}{4} t - \frac{1}{4} \right) \]

\[ c < 1 - t \]

\[ Z = \int_{t-c}^{t} 2vF(v) f(v) dv + \int_{1/2}^{t+c} 2v (1 - F(v)) f(v) dv \]

\[ = \frac{1}{2} \left[ \int_{t-c}^{t} (v + v^2) dv + \int_{1/2}^{t+c} (v - v^2) dv \right] = ... \]

\[ = tc(1 - c) \]

(41) \[ \Rightarrow J(t) = \left\{ \frac{1}{2} (1 - t^2) + \left[ \frac{1}{2} (1 + t - c) \right]^2 + \left[ \frac{1}{2} (1 - t - c) \right]^2 \right\} t + tc(1 - c) \]

\[ = \ldots = \left( 1 - \frac{1}{2} c^2 \right)^t \]

Note: the corner solution of \( c = 0 \) implies that the judge never signs \( v \neq t \) hence \( J(t) = t \). The corner solution of never dissenting \( (c = 1 + t) \) implies that the judge always votes according to the median of the panel, which equals \( \frac{1}{2} t - \frac{1}{6} t^3 \).

An equivalent statement of the proposition is that \( \arg \max J(t) \notin \{0, 1\} \). In Proposition 9 we showed that, for \( \alpha < \alpha^* \) (where \( 0.3 \approx \alpha^* > \tilde{\alpha} \approx 0.295 \)), there exists a unique switching point at the range \([0, 1]\) such that types below it dissent whenever they are not the median of their panel \( (c = 0) \) while types above it never dissent \( (c = 1 + t) \) under the conditions

(42) \[ W < 2 (1/6)^\alpha \]

(43) \[ (2/3)^\alpha < W \]

(44) \[ \alpha 2^{1-\alpha} h(t_{\max}) < W \]

(see (28) for a definition of \( t_{\max} \)) and that these conditions hold for a non-empty set of \( W \). This dissent pattern implies (by (41) and (40)) that there exists some \( \tilde{t} < 1 \) such that

\[ J(t) = \begin{cases} 
  t & \text{for } t \leq \tilde{t} \\
  \frac{1}{2} t - \frac{1}{6} t^3 & \text{for } t > \tilde{t} 
\end{cases} \]
and hence (since \( t \leq 1 \)) that \( J(t) \) is first increasing for \( t \leq \bar{t} \), then drops sharply at \( \bar{t} \), and finally increases again for \( t > \bar{t} \). To show that \( \text{argmax}(J(t)) \notin \{0, 1\} \) it is thus necessary and sufficient that
\[
J(\bar{t}) > J(1) \iff \bar{t} < \frac{1}{3},
\]
hence that the type \( t = 1/3 \) prefers \( c = 0 \) over \( c = 1 + t \):
\[
L(c = 0; t = 1/3) < L(c = 1 + 1/3; t = 1/3) \iff
\]
using (11) with a uniform distribution, (14) and (17)
\[
W \left( \left[ \frac{1+t}{2} \right]^2 + \left[ 1 - \frac{1+t}{2} \right]^2 \right) < \left( \frac{1}{2} \left[ t^2 + \frac{1}{3} \right] \right) ^\alpha \iff \quad \quad \quad
\]
(45)
\[
W < \left( \frac{9}{5} \left( \frac{2}{9} \right) ^\alpha \right).
\]
First we note that \( \frac{9}{5} \left( \frac{2}{9} \right) ^\alpha < 2(1/6) ^\alpha \) for any \( \alpha < \hat{\alpha} \), hence we must show that condition (45) holds along with conditions (43) and (44), i.e., that the set of \( W \) fulfilling the conditions is non-empty. Conditions (45) and (43) yield a non-empty set if
\[
(2/3) ^\alpha < \frac{9}{5} \left( \frac{2}{9} \right) ^\alpha \iff 
\]
\[
\alpha < \frac{\ln \left( \frac{5}{9} \right)}{\ln \left( \frac{2}{9} \right)} \approx 0.54,
\]
which is fulfilled since \( \hat{\alpha} \approx 0.295 \). Conditions (45) and (44) hold together for any \( \alpha < \hat{\alpha} \) by Lemma 36.

**F Proof of Proposition 5**

We prove the proposition for \( t \geq 0 \). Equivalent statements can be made for \( t \leq 0 \). In Proposition 9 we showed that, for \( \alpha < \alpha^* \) (where \( 0.3 \approx \alpha^* > \hat{\alpha} \approx 0.295 \)), there exists a unique switching point at the range \([0, 1]\) such that types below it dissent whenever they are not the median of their panel \( (c = 0) \) while types above it never dissent \( (c = 1 + t) \) under a non-empty set of \( W \).

Proof of part (i): When \( W = 0 \) the objective function for all types becomes \( \text{min} D \) which clearly is achieved by dissenting whenever not being median \( (c = 0) \). Using (11) with a uniform distribution and \( c = 0 \) we get \( P(t) = \left[ \frac{1+t}{2} \right]^2 + \left[ 1 - \frac{1+t}{2} \right]^2 = t^2 + 1 \) which clearly increases in \( t \).

Proof of part (ii): The dissent pattern just described implies that \( \tilde{t} = \text{argmax} P(t) \) equals the largest \( t \) that dissents when not being median. Since under these conditions the alternative is to never dissent, this type must be indifferent between these two options: \( L(c = 0; \tilde{t}) = L(c = 1 + \tilde{t}; \tilde{t}) \). Using (11) with a uniform distribution together with (14) and (17) yields:
\[
W \left( \left[ \frac{1+\tilde{t}}{2} \right]^2 + \left[ 1 - \frac{1+\tilde{t}}{2} \right]^2 \right) = \left( \frac{1}{2} \left[ \tilde{t}^2 + \frac{1}{3} \right] \right) ^\alpha.
\]
Solving for \( W \) it is easy to show that \( \tilde{t} \) decreases in \( W \) if \( \alpha < \hat{\alpha} \approx 0.295 \).
G Simulating an alternative functional form

This appendix aims to show that the spider-pattern (fact 3) can be explained by our main theoretical model (Section 4) when replacing the ideological cost function (inner discomfort) in equation (5) by the following logistic cost function

\[
D(x) = \frac{1}{1 + e^{-kx}}.
\]

This function is convex for small \(x\) and concave for large \(x\). Appendix Figure 14 presents simulation results when using this functional form with \(k = 7\) and \(W = 1.1\) in the main model. As can be seen, the spider pattern appears with such a function as well (upper panel). The lower panel shows the cutoff \(c(t)\). Running the same simulation but varying \(k\) and \(W\) in a large number of ways, suggests the dissent rate is either constant at zero, increasing in \(|t|\), decreasing in \(|t|\), or indeed spider-shaped.

Appendix Figure 14.— Simulation of a logistic \(D\) function

Notes: Upper schedule: Dissent rate per type. Lower schedule: cutoff per type. The simulation uses the functional form in (46) with parameter \(k = 7\) and peer-pressure parameter \(W = 1.1\).

\(^{66}\)The program code is available upon request.
Refuting of alternative theories

This section will discuss in brief a number of alternative theories and why they cannot be the explanation for the patterns we find empirically of extremists being less confrontational than moderately ideological judges. Among these alternative explanations only two can explain our empirical patterns. The first one (Section H.1) only holds if judges have a concave cost of bliss point deviations, hence is basically a variant of our main model. The second is the instrumental model mentioned in the introduction. We treat this model (judges use dissent as an instrument in trying to reverse the majority opinion using the supreme court) in great detail at the end of the list and in the next section by developing a proper theoretical model (Section I), deriving predictions by which it differs from our main model (of peer pressure with a concave cost of bliss-point deviations) and testing these predictions against each other (Section I.15). We find that this instrumental model is refuted by the data. While this, of course, is not a final proof that our main model is correct, the long list of alternative explanations is meant to show that the “usual-suspect” mechanisms can be refuted as explanations for our observations (which of course does not mean that these mechanisms are not valid in general).

H.1 Peer pressure increases in extremeness of the dissenting opinion.

Mechanism: Extreme minority opinions (dissents or concurrences) are sanctioned by other judges in the pool more heavily than moderate minority opinions are. Hence, an extremist judge will find it harder to express her true view by dissenting. Refutation: Extreme judges can always imitate moderate ideologists, thus dissent at least as much as (instead of less than) the moderates do. To make the extremists prefer to dissent strictly less than the moderates, judges must have a concave personal cost of bliss-point deviations also in this mechanism. This way, moderate judges, who are under small peer pressure, would choose to dissent and avoid the personal cost, while extreme judges, who are under severe social pressure, would pay the full personal cost and completely refrain from dissenting. It is thus clear that this mechanism is only a variant of our main model in the body of the paper. We prefer our own model because it does not require the additional assumption of increasing social pressure and because we believe the personal cost is likely to apply to the number of dissents at least as much as it applies to the size of dissent.

H.2 Bargaining.

Mechanism: Through the bargaining process, extreme judges are more successful than moderately ideological judges in pulling the majority opinion in their direction hence have less reasons to dissent. Refutation: This description is inconsistent with Appendix Figure 3 which, on the contrary, shows that at large distances from the center of the pool of judges there is a negative correlation between judges’
ideology and the ideology of the majority opinion. It is also very implausible when considering the alternative ideological score (party of appointing President), because it would suggest that a Republican-nominated judge would have more bargaining power the larger is the share of Democrat-nominated judges in the pool.

**H.3 Unobserved heterogeneity among judges.**

**Mechanism:** Extreme judges have some personal characteristics that are different from the others, and these characteristics make them dissent less. **Refutation:** i) As mentioned in the main text our results are driven by ideology relative to Center of Judge Pool and not by ideology per se (using judge fixed effects in Section 3.3, the attenuation of Fact 3 and disappearance of Fact 4 when using raw ideology score; and the alternative score based on party of appointing President where a judge’s ideology score only depends on her own party and the number of judges of the other party). That is, relative position is not a personal attribute, and when considering extremism per se the empirical patterns are weak to non-existent. ii) Our results are robust to using controls for judge personal characteristics (Table A.4). iii) The judge characteristics driving the result have to be positively correlated with ideology for extreme judges but negatively correlated for less extreme judges (since Facts 3 and 4 show non-monotonicity).

**H.4 Signaling for extremeness.**

**Mechanism:** Judges aim to please voters or party members or those who selected them. Moderately ideological judges need to signal their ideological belonging while everyone knows already that extremists are extreme hence they do not need to use dissent to signal ideology. **Refutation:** i) This does not explain why centrists dissent less than moderately ideological judges. Alternatively, it assumes that centrists, unlike moderates, do not want to signal they are extreme. ii) As explained under H.3 our results are driven by ideology relative to Center of Judge Pool and not by ideology per se and they hold also when using the alternative score based on party of appointing President (where a judge cannot be more “extreme”, only more or less in minority).

**H.5 Signaling for conformity.**

**Mechanism:** A model where dissent is a signal of being an extremist (which is supposedly a bad thing) and where peer pressure is applied to judges based on the type they are perceived to be (in expectation in equilibrium) would supposedly imply extreme judges dissent less. **Refutation:** [Bernheim (1994)] shows this kind of model will produce dissent (non-conformity in his model) that is increasing in the extremeness of the type, hence cannot explain why extreme judges dissent less.

**H.6 Tie the hands of the supreme court.**

**Mechanism:** Extreme judges join the majority opinion in order to be able to add facts to it and thereby tie the hands of the supreme court. **Refutation:** i) If extremists do this, then moderates may want
to do this too. ii) The facts added should affect the color of the opinion as the sample becomes large but this is inconsistent with Appendix Figure 3.

H.7 Log-rolling.

**Mechanism:** An extreme judge joins an unfavorable opinion in return for getting the other judges to agree on the extremist’s view in the next case. **Refutation:** This description is inconsistent with Appendix Figure 3 which, on the contrary, shows that at large distances from the center of the pool of judges there is a negative correlation between judges’ ideology and the ideology of the majority opinion. If extreme judges refrain from dissenting in return for getting to determine future opinions, then this influence should take away that negative correlation.

H.8 Score bias.

**Mechanism:** Our ideology score is constructed by the voting behavior of the appointing President and home state senators (see Section 2). This score may be flawed if extreme Presidents appoint non-extreme judges to show they are non–biased. **Refutation:** i) Figure 2 shows an almost linear relationship between ideology of voting and judge’s ideology score that is not demeaned by the Center of Judge Pool. This is an indication that the scoring system we use is indeed a good proxy of judges’ ideology. ii) The results are robust to the alternative ideology score based on party of appointing President, i.e., a score that is not dependent on how extreme the President is. iii) This mechanism cannot explain non-monotonicity unless one assumes that moderately ideological Presidents for some reason do not at all need to signal that they are not biased.

H.9 Moderates as official party spokesmen.

**Mechanism:** Moderate judges are the modal judges in their corresponding (liberal or conservative) group and are very close to their party leadership, while centrist and extremists are located at the tails of their party of affiliation. The moderates thus tend to signal the official party line and act accordingly by being vocal (i.e. dissent more and signal the party position on the issue). **Refutation:** i) Figure 2 shows a clear monotonic relationship between ideology of voting and a judge’s ideology score that is not demeaned by the Center of Judge Pool. This is an indication that, abstracting from the censoring effect of the environment, extreme judges adopt a more ideological line than moderate judges. ii) The judge fixed effects exercise in Section 3.3 shows that extreme judges are dissenting more when the pool changes so they become “moderates”. So it cannot be that one type of judge is designated to be the main defender of the party line since (by the judge fixed effects exercise) this person changes behavior when the environment changes. iii) Our results are robust to using party of appointing President as ideology score. Here, all judges in the pool who are from the same party get the same score, and “moderate” judges are simply (all) the judges who happen to sit in a well-balanced pool.
H.10 Results driven by outliers.

**Mechanism:** The result of low dissent rate for extreme judges could be driven by outliers **Refutation:** i) The pattern is robust to using concurrences rather than dissents (see Appendix Figure 9). ii) There is a lower bound of dissent at zero, hence single outliers cannot pull down the average very much. iii) We test robustness for outlier Circuits: Appendix Table A.3 shows that the results are robust to dropping one Circuit at a time.

H.11 Random opinion writing.

**Mechanism:** The opinion is written by a random judge who gets to decide the content, hence extreme judges are not under-represented among the judges who set the court’s opinion. **Refutation:** i) This cannot explain the spider. ii) It is false since we show that the median determines the ideological color of the opinion (Fact 1).

H.12 Outlier circuits.

**Mechanism:** The results are driven by a few circuits with many judges since only large circuits would have large enough variation in judges to include extremists **Refutation:** i) This is not an explanation for non-monotonicity. ii) Appendix Table A.3 shows that the results are robust to dropping one Circuit at a time.

H.13 Risk of impeachment.

**Mechanism:** The risk of impeachment, given dissent, could be increasing in extremeness of the dissenting opinion. **Refutation:** Impeachment is essentially a form of pressure, hence the refutation in mechanism H.1 applies.

H.14 Extreme judges dissenting on moderate cases are viewed as unreasonable.

**Mechanism:** A judge who dissents against an opinion that is close to her type is viewed as unreasonable. Since there are more moderate cases than there are extreme cases, the extreme judge dissents less than moderate judges do. **Refutation:** Under this mechanism, moderate judges would also be viewed as unreasonable when dissenting on slightly unfavorable cases. Considering this, the logic of the mechanism is incorrect since it disregards the opinions on the whole political spectrum. Considering all opinions (on the same side and the opposite side of the political spectrum), an extreme judge will statistically need to more often consider signing or dissenting against very unfavorable opinions than a moderate does and would therefore dissent more under this mechanism.
H.15 Empirical testing of our model against an alternative model that can explain the spider pattern

This section describes an alternative model that may explain the main stylized fact (Fact 3), derives two predictions from this model that differ from the two predictions outlined in Section 4.5 and tests the different predictions against each other empirically.

The alternative theory is one where a judge dissents, at a cost of collegial pressure, in the hope that the U.S. Supreme Court (SCOTUS) will use this as a signal to review the case and overturn the (binary) verdict. Majority voting within panels implies also here that the median judge decides the verdict for the panel. In Appendix I we develop a simple model capturing this mechanism (the model is inspired by the model in Beim et al. 2014). The intuition for why this alternative SCOTUS model can produce a spider-shaped dissent pattern is as follows. A judge compares, case by case, the cost of dissent \( W \) with how wrongful she thinks a certain verdict is. This means that two prerequisites need to be in place for a judge to dissent: i) she needs to think that the verdict is sufficiently bad to warrant the cost of dissent and ii) she needs to have the Supreme Court on her side as otherwise the verdict will not be overturned anyway. Here, centrists on the one hand usually have the Supreme Court on their side but on the other hand, often being the median, seldom encounter verdicts that are too far from what they think is right. Hence they rarely dissent. Conversely, extremists often dislike the verdict sufficiently to dissent but rarely have the Supreme Court on their side, hence dissent seldom too. Finally, moderately ideological judges may have a larger set of cases where they both sufficiently oppose the verdict and have the Supreme Court on their side. In the appendix we show that this may create a spider pattern of dissent. Two additional predictions (the equivalent of Proposition 5) can be derived from the SCOTUS model.

**Proposition 12** Consider the SCOTUS model. Then: (i) if \( W = 0 \), \( P(t) \) is monotonically decreasing in \( |t| \); and (ii) \( \arg \max_{|t|} P(|t|) \) is increasing in \( W \).

**Proof:** See Appendix I. \( Q.E.D. \)

Prediction (i) of the SCOTUS model says that, as the collegial cost of dissent (\( W \)) approaches zero, the dissent rate becomes a decreasing function of judge’s extremeness. This is intuitive since, when the collegial pressure is low, the only factor that determines whether a judge dissents is whether she has the Supreme Court on her side (because there is no collegial pressure against dissenting). This means that centrist judges will dissent very often. Furthermore, the more extreme a judge is, the less likely it is that her preferences will be aligned with the Supreme Court, which means that the dissent rate falls. This way, the prediction of the SCOTUS model is opposite to the prediction of our main model (with a concave ideological

\(^{67}\)2% of the Circuit Court decisions are appealed to the Supreme Court, of which 30% are affirmed.
cost) where, as the collegial pressure goes to 0, the dissent rate increases with how extreme the judge is (Proposition 5 part i).

Prediction (ii) of Proposition 12 refers to the consequences of an increase in the cost of dissent. In the SCOTUS model, the judge at the peak of the spider pattern is the one for whom the threshold cutoff for dissent, as determined by the cost of dissent, exactly equals her ideological distance to the Supreme Court, implying that she dissents against any verdict that both she and the Supreme Court view as biased to the “wrong” side. Judges who are more extreme dissent in these cases as well, but in total they are predicted to dissent less because, compared to the judge at the peak, they have less objection to verdicts that manifest extreme ideology on their side of the ideological spectrum, yet do not have the Supreme Court on their side for overturning verdicts they consider to be too moderate. If the cost of dissent increases, judges have to censor themselves more, hence the cutoff for dissent is larger, implying that a judge has to be more ideologically extreme to be at the peak of the spider pattern. Therefore, the prediction of the SCOTUS model is that, as the cost of dissent increases, the spider peak would move outwards. In the main model we had the opposite prediction: an increase in the cost of dissent would have pushed the peak of the spider inwards (Proposition 5 part ii).

H.15.1 Testing predictions for $W \rightarrow 0$

When testing these opposite predictions (part i of propositions 5 and 12), it is important to note that in the main model it is the Distance to Center of Judge Pool that measures the extremeness, while in the SCOTUS model it is the Distance to SCOTUS that measures a judge’s extremeness. To get a measure of the Distance to SCOTUS, we use the Martin Quinn Supreme Court scores to put the judicial scores of the Circuit Court and the Supreme Court on the same metric (Martin and Quinn 2002).

To test the predictions, we use retired status of a judge as a proxy for a very low cost of dissent. The motivation for this is as follows. Firstly, judges who have retired take a reduced caseload, hence have more time to write dissents. Secondly, they arguably have lower collegial pressure from colleagues or are less sensitive to such pressure. We first verify that judges who have retired have a discontinuous drop in caseload from about 100 per year to 30 per year (Figure 15) and that caseload continues to decline gradually thereafter. Next, we show that retired judges dissent more, discontinuously at the year of retirement (Figure 16), and verify that the increase in dissents is not due to age. In fact, older judges are less likely to dissent, which also explains the decline in dissent before retirement. Figure 16 visualizes the following regression:

\[
\text{Dissent Rate}_{it} = a + b \times \mathbb{1}(\text{Years after Retirement} \geq 0)_{it} + c \times \text{Years after Retirement}_{it} + d \times \text{Years after Appointment}_{it} + \nu_{it}
\]

for judge $i$ and year $t$.

\[68\] In appendix 1.2 we show that age and experience vary smoothly around the retirement decision and we also visualize the results of the regression presented in Table A.12.
**APPENDIX FIGURE 15.**— Caseload and Years from Retirement

![Graph showing caseload per judge against years after retirement.](image)

Notes: Each dot represents the average caseload of judges with the same number of years relative to retirement. Data on cases comes from OpenJurist (1950-2007).

**APPENDIX FIGURE 16.**— Dissent or Concurrence and Years from Retirement vs. Age

![Graphs showing sum of dissent and concurrence rates against years after retirement and age.](image)

Notes: Each dot represents the average sum of dissent rate and concurrence rate for judges with the same number of years relative to retirement (left panel) or the same age (right panel) in a Circuit-year. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. Data on cases comes from OpenJurist (1950-2007).
To test the opposing prediction (i) of Propositions 5 and 12, we run the regression in equation (3), limiting the sample to retired judges only.

As can be seen in Table A.12, for retired judges, the rate at which judges dissent or concur is positively correlated with Distance to Center of Judge Pool (columns 1 and 5), which supports our main model. The rate at which judges dissent or concur is also positively correlated with Distance to SCOTUS (columns 3 and 7), which goes against the prediction of the SCOTUS model. Note also that the spider pattern disappears (columns 2, 4, 6 and 8), as predicted in Proposition 5. In total, these results provide support for the main model and go against the prediction of the SCOTUS model.

### APPENDIX TABLE A.12

**Dissent or Concurrence and Ideology Score among Retired Judges**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
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<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
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<td>Distance to Center of Judge Pool</td>
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<td>0.000569</td>
<td>0.0377***</td>
<td>0.0115</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.00810)</td>
<td>(0.0244)</td>
<td>(0.00814)</td>
<td>(0.0250)</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Distance to Center of Judge Pool$^2$</td>
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<tr>
<td></td>
<td>(0.0418)</td>
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<td></td>
<td></td>
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<tr>
<td>Distance to Supreme Court</td>
<td>0.0226***</td>
<td>0.0488*</td>
<td>0.0253***</td>
<td>0.0427</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.00849)</td>
<td>(0.0266)</td>
<td>(0.00848)</td>
<td>(0.0264)</td>
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<td></td>
</tr>
<tr>
<td>Distance to Supreme Court$^2$</td>
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<td></td>
<td></td>
<td>-0.0296</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.0466)</td>
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<td>Control for Age and Experience</td>
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<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Circuit Fixed Effects</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
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<tr>
<td>Year Fixed Effects</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
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<td>3353</td>
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<td>3673</td>
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<td>3353</td>
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<td>3673</td>
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<tr>
<td>R-sq</td>
<td>0.090</td>
<td>0.091</td>
<td>0.081</td>
<td>0.082</td>
<td>0.094</td>
<td>0.094</td>
<td>0.087</td>
<td>0.088</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Ideology scores come from the Judicial Common Space database. Absolute values of the distance to the center of the judge pool or the Supreme Court are the main independent variables. The dependent variable is the judge’s sum of dissent rate and concurrence rate in a Circuit-year. Fixed effects include year and circuit. Observations are weighted by the number of votes cast by the judge in the Circuit-year. Columns 1-4 use the same set of controls as in Table III.

### H.15.2 Testing predictions about the most dissenting judge

Our second test examines the effect of an increase in the cost of dissent on who the most dissenting judge is (part ii of propositions 5 and 12). To test this we follow Berdejo and Chen (2014), who find that dissents decrease during wartime (Figure 17). For the purpose of our test, our assumption is that this decrease is due to wars tending to increase social cohesion (increase $W$). We test the main (concave ideological cost) model’s prediction by examining how the dissent rate is affected by the interaction between a judge’s Distance
to Center of Judge Pool and wartime.

\[
\text{Dissent Rate}_{cit} = a + b \cdot \text{Distance}_{cit} + c \cdot \text{Distance}^2_{cit} + d \cdot \text{Distance}_{cit} \cdot \text{wartime}_t + e \cdot \text{Distance}^2_{cit} \cdot \text{wartime}_t + f \cdot \text{wartime}_t + \nu_{cit}
\] (47)

for judge \(i\) in Circuit \(c\) and year \(t\).

**Appendix Figure 17.**— The Effect of Wartime on Dissents

Notes: Each dot represents the proportion of dissents over many votes on cases with the same publication year. Figure reproduced from Berdejó and Chen (2014).

We test the SCOTUS model’s prediction by running the same regression but using Distance to SCOTUS as a measure of extremeness. The peak of the spider is determined by the first-order condition of the regression equation. Therefore, to test for a shift in the peak of the dissent rate, we test for a significant difference between \(-\frac{b}{2c}\) and \(-\frac{(b+d)}{2(c+e)}\). If wartime shifts the peak inwards (\(-\frac{b}{2c} > -\frac{(b+d)}{2(c+e)}\)), this would corroborate the main model and weaken the SCOTUS model and vice versa. The regressions, the ratios, and the test statistics for the equality of the coefficient ratios are reported in Table A.13. Using **Distance to Center of Judge Pool**, Column 2 reports that during war there is a significant inward shift of the peak of the spider, which is consistent with the main model. Using **Distance to Supreme Court**, Column 4 rejects the significant outward shift that is predicted by the SCOTUS model (in fact the coefficient even has the wrong sign). Judged together, the two tests seem to refute the SCOTUS model while supporting the main model presented in this paper.

Appendix Table A.14 shows the results are the same if we add controls for judge age and experience.
APPENDIX TABLE A.13
Dissent or Concur and Ideology Score among Judges: Tests for changes in who dissents the most due to increase in dissent costs during wartime

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>0.0806***</td>
<td>0.0962***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0120)</td>
<td>(0.0103)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance^2</td>
<td>-0.0819***</td>
<td>-0.137***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0182)</td>
<td>(0.0160)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance * Wartime</td>
<td>-0.0861***</td>
<td>-0.127***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0255)</td>
<td>(0.0255)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance^2 * Wartime</td>
<td>0.127***</td>
<td>0.172***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0450)</td>
<td>(0.0379)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test for Difference in Ratio of Coefficients</td>
<td>-0.862**</td>
<td>0.182</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.414)</td>
<td>(0.284)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Results of regression 47. Robust standard errors clustered at the circuit-year level in parentheses (* p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Absolute value of the distance to the center of the judge pool (columns 1 and 2) and absolute value of the distance to supreme court (columns 3 and 4) are the main independent variables of interest. The dependent variable is the sum of a judge’s rates of dissent and concurrence in a Circuit-year. Observations are weighted by the number of votes cast by the judge in the Circuit-year.

I The SCOTUS model

In this section we present a hierarchical model that is able to produce the spider-shaped pattern of dissent rate as a function of ideological distance to SCOTUS and we derive the predictions that are presented in Proposition 12 and are tested empirically in Section H.15. The hierarchical model presented here is developed along the lines of the model of Beim et al. (2014).

Every period, three judges are randomly and independently drawn from a uniform distribution of types $t \sim U (-1,1)$ to sit together on a panel. The panel produces a binary verdict ("conservative" or "liberal") for a case with characteristics $x$, where $x \in [-1,1]$. A judge of type $t$ prefers a conservative verdict over a liberal one iff $x < t$. The panel determines the verdict by a majority voting, implying that the verdict is conservative if and only if the median judge, denoted $t_m$, is such that $x < t_m$.

A panel member may also dissent. Upon noticing a dissent, the Supreme Court may decide to review the case. The bliss point of the Supreme Court is normalized to 0. Thus, the Supreme Court rules conservatively on a reviewed case iff $x < 0$. The cost of dissenting is denoted $W$ (and represents writing costs of the minority opinion or collegial pressure). When a judge $t$ is able to reverse the binary verdict in
case where her utility gain is $|t - x|$. Hence, a judge will never dissent if $|t - x| \leq W$. We can calculate the type-dependent probability of dissent $P(t)$ while considering only a judge with $t > 0$ (by symmetry the same applies to judges $t < 0$).

Under this framework, judge $t > 0$ may dissent in two scenarios:

1. $t_m > 0$ and $x \in [t, t_m]$, so that both $t$ and the Supreme Court prefer a liberal verdict while the panel produces a conservative verdict.

2. $t_m < 0$ and $x \in [t_m, 0]$, so that both $t$ and the Supreme Court prefer a conservative verdict while the panel produces a liberal verdict.

Under scenario 1, the judge indeed dissents if $t - x > W$, i.e., if $x \in [t + W, t_m]$. Under scenario 2, the judge indeed dissents if $t - x > W$, i.e., if $x \in [t_m, \min\{0, t - W\}]$. We will now show that the model produces a spider-shaped pattern of dissent rate for $W < 1/2$. In this case, we have $W < 1 - W$.

Figure 18 is helpful in distinguishing between three regions of $t$.

**APPENDIX FIGURE 18.— Regions in SCOTUS model**

A judge $t$ in region I dissents if either $x \in [t + W, t_m]$ or $x \in [t_m, t - W]$.

A judge $t$ in region II dissents if either $x \in [t + W, t_m]$ or $x \in [t_m, 0]$.

A judge $t$ in region III dissents if $x \in [t_m, 0]$.

Calculating the type-dependent probability of dissent $P(t)$, we get (each line represents one region in the graph)

$$P(t) = \begin{cases} \frac{1}{3} \left[ \frac{1}{2} (1 - (t + W)) \right]^3 + \frac{1}{3} \left[ \frac{1}{2} (t - W) - (-1) \right]^3 & \text{if } t \in [0, W] \\ \frac{1}{3} \left[ \frac{1}{2} (1 - (t + W)) \right]^3 + \frac{1}{3} \left( \frac{1}{2} \right)^3 & \text{if } t \in [W, 1 - W] \\ \frac{1}{3} \left( \frac{1}{2} \right)^3 & \text{if } t \in [1 - W, 1] \end{cases}$$

To understand the calculations of the expression of $P(t)$, note first that the event $x \in [t_m, 0]$ is independent of $t$ and it occurs iff $\min t < t_m < x < 0$. As $\min t, t_m$ and $x$ are all drawn from a uniform distribution over $[-1, 1]$, the probability that all three of them are negative is $\left( \frac{1}{2} \right)^3$, and the probability that $x$ is the largest among the three is $1/3$, yielding the expression $\frac{1}{3} \left( \frac{1}{2} \right)^3$. Next, the event $x \in [t_m, t - W]$ is an adjustment of this calculation for the event $x \in [t_m, t - W]$. In particular, we now need $\min t < t_m < x < t - W$, so
needs to be the largest of the three uniformly-distributed variables, which all need to be in the region 
\([-1, t - W]\), and this event corresponds to probability \(\frac{1}{3} \left[ \frac{1}{2} [(t - W) - (-1)] \right]^3\) (i.e., the probability of being 
negative, \(1/2\), is replaced with the probability of being smaller than \(t - W\), which is \(\frac{1}{2} [(t - W) - (-1)]\)). 

Finally, the event \(x \in [t + W, t_m]\) occurs iff \(t + W < x < t_m < \max t\). So \(x\) needs to be the smallest of three 
uniformly-distributed variables, which all need to be in the region \([t + W, 1]\), and this event has probability 
\(\frac{1}{3} \left[ \frac{1}{2} [1 - (t + W)] \right]^3\).

Differentiating with respect to \(t\) yields

\[
\frac{dP(t)}{dt} = \begin{cases} 
-\frac{1}{8} [1 - (t + W)]^2 + \frac{1}{8} [(t - W) + 1]^2 & \text{if } t \in [0, W] \\
-\frac{1}{8} [1 - (t + W)]^2 & \text{if } t \in [W, 1 - W] \\
0 & \text{if } t \in [1 - W, 1]
\end{cases}
\]

It is immediate to see that \(dP(t)/dt\) is negative in region II. To get the sign of \(dP(t)/dt\) in region I, note 
that \(t > 0\) implies that \(t + W\) is closer to the right edge of the type distribution (1) than \(t - W\) is to the left 
edge of the type distribution (-1), implying that 

\[
[1 - (t + W)]^2 < [(t - W) + 1]^2,
\]

implying that \(dP(t)/dt > 0\) in region I. Overall, we get that \(P(t)\) increases in region I and then decreases in 
region II and stays flat in region III, implying a spider-shaped pattern of dissent rate.

### I.1 Proof of Proposition 12

(i) When \(W = 0\), regions I and III disappear and we are left only with region II where \(dP(t)/dt\) is 
negative. Symmetry implies that for any type \(t\), \(P(t)\) is decreasing in \(|t|\).

(ii) The value of \(t\) for which \(P(t)\) is maximal is the border between regions I and II, i.e. \(t = W\). It 
is thus immediate that \(\arg \max_t P(t)\) increases in \(W\).
I.2 Robustness regarding retired judges

Appendix Figure 19 shows that age and experience vary smoothly at the time of retirement.

APPENDIX FIGURE 19.— Age and Experience at Retirement

Notes: Each dot represents the average age (left panel) or experience (right panel) of judges with the same number of years relative to retirement. Data come from biographical data on judges (1950-2007).
Appendix Figure 20 shows that the relationship between rate of dissent or concurrence and ideology score among retired judges does not appear to be driven by outliers.

APPENDIX FIGURE 20.—Retired Judges: Dissent or Concurrence and Ideological Distance

Notes: Rates of dissent or concurrence are calculated at the Circuit-year level. Each dot represents the average yearly sum of dissent rate and concurrence rate for retired judges sharing a similar score. The average is a weighted average to account for the number of times the judge actually appeared on cases in that Circuit-year. On the left panel, the score is ideological distance to the center of the judge pool and on the right panel, the score is ideological distance to the Supreme Court. Data on cases comes from OpenJurist (1950-2007).
### I.3 Robustness regarding wartime

Appendix Table A.14 shows that age and experience do not affect the wartime results.

**APPENDIX TABLE A.14**

<table>
<thead>
<tr>
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<td>Distance (^2)</td>
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<tr>
<td>Distance (^*) Wartime</td>
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<td></td>
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<tr>
<td>Distance (^2) (*) Wartime</td>
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<td></td>
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\(^1\)Distance to: Controls for Age and Experience Y Y Y Y, Circuit Fixed Effects Y Y Y Y, Year Fixed Effects Y Y Y Y, N 8755 8755 8755 8755, R-sq 0.130 0.127

Notes: Results of regression 47. Robust standard errors clustered at the circuit-year level in parentheses (\(*) p < 0.10; ** p < 0.05; *** p < 0.01). Data on cases comes from OpenJurist (1950-2007). Absolute value of the distance to the center of the judge pool (columns 1 and 2) and absolute value of the distance to supreme court (columns 3 and 4) are the main independent variables of interest. The dependent variable is the sum of a judge’s rates of dissent and concurrence in a Circuit-year. Observations are weighted by the number of votes cast by the judge in the Circuit-year.