Two-sided reputation in certification markets*

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Abstract

In a market where sellers solicit certification to overcome asymmetric information, we show that the profit of a monopolistic certifier can be hump-shaped in its reputation for accuracy: a higher accuracy attracts high-quality sellers but sometimes repels low-quality sellers. As a consequence, reputational concerns may induce the certifier to reduce information quality, thus depressing welfare. The entry of a second certifier impacts reputational incentives: when sellers only solicit one certifier, competition plays a disciplining role and the region where reputation is bad shrinks. Conversely, this region may expand when sellers hold multiple certifications.

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1 Introduction

Certifiers play a critical role in markets plagued by information asymmetries: by providing an expert and independent opinion, they bridge the informational gap between buyers and sellers and boost gains from trade. Many markets would actually be extremely thin in the absence of certification mechanisms. For instance, the 2011 final report of the US Financial Crisis Inquiry Commission (2011) emphasizes that "without the active participation of the rating agencies, the market for mortgage-related securities could not have been what it became." In financial markets, the central role of certifiers is reinforced by regulations that rely on their seal of approval.[1] However, because certifiers are themselves subject to incentive problems, they are not a perfect solution to information frictions. The unfolding of the financial crisis in 2008 suggests that credit rating agencies (henceforth CRAs) have been instrumental in the misallocation of capital.[2] In February 2013, the U.S. Department of Justice lodged a complaint against Standard & Poors, claiming that "S&P's desire for increased revenue and market share in the RMBS and CDO markets led S&P to downplay and disregard the true extent of the credit risks posed by RMBS and CDO."[3] A report of the Securities and Exchange Commission in September 2011 still casts doubt on CRAs' incentives to provide unbiased information.[4]

In this paper, we investigate a fundamental incentive mechanism for certifiers: reputation. Reputation has been a central defense of CRAs against accusations of conflict of interest and misaligned incentives. In the words of Thomas McGuire, former executive vice-president of Moody's, "what's driving us is primarily the issue of preserving our track record. That's our bread and butter."[5] However, Mark Froeba, former senior vice-president of Moody's, suggests that rating agencies have striven in the same breath to develop a reputation for being accommodating with security issuers: "This was a systematic and aggressive strategy to replace a (...) getting-the-rating-right kind of culture with a culture that was supposed to be "business-friendly," but was consistently less likely to assign a rating that was tougher than our competitors."[6] This suggests that reputation plays

[1]The U.S. Federal Reserve has been using ratings to regulate commercial banks since the 1930s; the SEC has been using ratings for the regulation of broker-dealers or money-market funds.

[2]The same report from the FCIC states: "We conclude that the failures of credit rating agencies were essential cogs in the wheel of financial destruction. The three credit rating agencies were key enablers of the financial meltdown."

[3]RMBS stands for Residential Mortgage-Based Security, CDO stands for Collateralized Debt Obligation. Both are structured financial products that essentially consist of bundles of repackaged debt claims.

[4]See, for instance, "SEC critical of rating agency's controls," Financial Times, September 30, 2011. Cornaggia et al. (2011) provide empirical evidence of CRAs' systematic bias towards high-turnover issuers.

[5]Quoted in Becker and Milbourn (2011).

[6]"How Moody's sold its ratings - and sold out investors", McClatchy Newspapers, October 19, 2009. See also Kisgen et al. (2016) for evidence of conflicting incentives within CRAs.

a dual role: on the one hand, CRAs need to maintain a reputation for accuracy for their ratings to have any value; on the other hand, a reputation for leniency helps attract a larger number of issuers. The objective of this paper is to examine how certifiers strike a compromise between these two conflicting aspirations. Rating agencies constitute a natural illustration of our framework to which we repeatedly come back in the paper.[7] However, our model is also suited to analyze any market in which agents resort to certification bodies or standard-setting organizations to overcome a market failure (Lerner and Tirole (2006); Farhi et al. (2005, 2013)). Examples of such markets include financial audit, technical standards (e.g., ISO, CEN), school accreditations (e.g., EQUIS, AACSB) or individual proficiency tests (e.g., GMAT, GRE, TOEIC).

We first develop a static model in which asymmetric information about product quality results in market breakdown, which generates a demand for certification from sellers. Sellers (endogenously) hold products of different qualities, which creates heterogeneity in their preferences for the accuracy of certification. A higher accuracy unambiguously attracts high-quality sellers who have nothing to hide, and are able to charge higher prices when certificates are more reliable signals of quality. Sellers with low-quality products also care about accuracy because it gives credibility to certification by attracting high-quality sellers, but trade this benefit of being pooled with high-quality sellers when undetected against the increased risk of their true quality being revealed. It follows that too precise a certifier at some point deters low-quality sellers from soliciting certification.

When sellers are subject to a financial friction that constrains their ability to pay for certification, we show that a monopolistic certifier maximizes profit by attracting both sides of the market, i.e., high- as well as low-quality sellers.[8] This, in turn, translates into a preference for a balanced reputation: the certifier's profit is maximum when perceived as neither too accurate nor too inaccurate. We then introduce reputation building, and show that the desire to achieve an intermediate reputation gives rise to a rebalancing effect: when perceived as inaccurate, the certifier provides effort to increase the precision of its signal and gain credibility; conversely, a certifier with a high reputation for accuracy lowers the precision of the information it provides in order to appeal to a wider range of (future) sellers. Therefore, reputational concerns sometimes discipline the certifier relative to the static case, but can also worsen information provision, and be

[7]The financial crisis has engendered a stream of papers on credit rating agencies, some of them involving reputational concerns. For an overview of the recent literature on CRAs, see Jeon and Lovo (2013).

[8]We also discuss in Section 5 how a regulator concerned with excessive market power in the certification market would want to impose a cap on certification fees, which has the same effect as that of the financial constraint we consider here.

accordingly welfare-decreasing ("bad reputation").[9]

We then proceed to the case where two certifiers coexist in the market. Specifically, we consider a monopolist facing the entry of a second certifier, and derive the existence of two types of equilibrium corresponding to different market structures. In the first one, sellers who buy certification solicit one certifier only. In this equilibrium, which we call "singlehoming," only the more precise certifier is active, although its pricing is constrained by the presence of the entrant. Intuitively, even though the monopoly profit of the certifier is maximized when it attracts a mix of high and low-quality sellers, high-quality sellers drive the whole demand because they exert a positive externality on low-quality ones: a certifier receives no demand from low-quality sellers unless it attracts some high-quality sellers. Since high-quality sellers have a clear preference for accuracy, the more precise certifier is always able to price in such a way to attract those sellers, thereby driving the competitor out of the market. It follows that the incumbent certifier is less likely to be ousted by the entrant when its reputation is higher, which shifts its preferred reputation towards a higher precision. As a result, competition attenuates bad reputation effects: a monopolistic certifier which decreases precision for reputational motives provides more precise information when it is facing entry.

In the second type of equilibrium, sellers who buy certification simultaneously solicit both certifiers, which we refer to as "multihoming." While some of the intuitions of the singlehoming equilibrium carry over, we show that multihoming generates a distinct and countervailing effect on reputational incentives. Since any seller who applies for certification requests two certificates, both certifiers now share the same clientele and the demand for certification depends on their *collective* precision. The joint profit of the two certifiers is maximized when their combined precision is equal to the optimal precision of a monopolist. Accordingly, the precision that is *individually* optimal is lower than the monopolistic one. We show that the entry of a second certifier, by increasing the precision of the overall certification process, may depress the incumbent's incentives to develop a reputation for precision, which exacerbates bad reputation effects.

Our paper belongs to the literature on the reputation and credibility of experts. After Sobel (1985), Benabou and Laroque (1992) and Mathis et al. (2009) have shown that reputational concerns improve information transmission, but might not be sufficient to ensure perfect communication. These papers are based on a trade-off between short-term incentives to manipulate information in order to inflate current profits and long-term incentives to build a reputation. By contrast, we show

[9] In the remainder of the paper, we use "bad reputation" to describe situations where reputational concerns are welfare-decreasing, as in Ely and Välimäki (2003), although the mechanics of their model is formally different from ours.

that, even in the absence of an immediate reward from information manipulation, reputation itself can lead a certifier to decrease the quality of information. Therefore, reputation can be welfare-reducing, while it is welfare-enhancing in those two papers. This feature relates our work to another stream of papers where reputational concerns can be "bad," i.e., welfare-reducing (Morris, 2001; Ely and Välimäki, 2003; Ely et al., 2008). There, a type with a preference for truth-telling distorts information to separate himself from systematically biased types, and be perceived as reliable in the future. On the contrary, in our model, the certifier lowers information quality because it wants to signal leniency to future sellers.

Our paper also contributes to the literature on reputation or signaling with heterogenous audiences (e.g., Gertner et al. (1988); Austen-Smith and Fryer (2005); Shapiro and Skeie (2015); Levit and Malenko (2016)). Within this literature, three papers are more closely related to ours. Frenkel (2015) models the reputational incentives of a CRA facing two audiences (security issuers and investors), but focuses on a case where one of the audiences (issuers) observes more information about the behavior of the CRA. This provides incentives for the CRA to build a "double reputation," i.e., to try and signal different traits to different audiences, possibly resulting in (inefficient) ratings inflation. The decision to pursue one single or two reputations in a two-audience context is central in Bar-Isaac and Deb (2014a), who study how separate or common observation of actions affect reputational incentives, and show that reputation-building under common observability always increases welfare. By contrast, all players observe the same information about the certifier in our model, but the desire of the certifier to maintain a single reputation vis-à-vis audiences with heterogenous preferences may still decrease welfare. In a career-concern setup similar to ours, Bar-Isaac and Deb (2014b) show that the profit of a monopolist who sells to two horizontally differentiated customer segments may be non-monotonic in the market perception of the product's positioning. A distinctive feature of our framework with respect to theirs, and more generally within this literature, is that the different types that the certifier tries to attract are connected not only through its actions (hence its reputation), but also directly through participation externalities: a higher demand for certification from good sellers increases the credibility of certificates, which encourages bad sellers to participate; conversely, the presence of bad sellers in the pool of applicants lowers the benefits which good sellers derive from certification.

The importance of externalities across different types of users is central to the literature on two-sided markets. While this literature has mostly focused on the pricing structure which platforms design to attract different groups who exert an externality on each other (see, e.g., Rochet and Tirole

(2006)), our paper suggests that reputation may also be instrumental in pursuing this objective, and examines how reputation, in turn, affects pricing strategies. We also highlight how reputation interacts with pricing when certifiers compete both in prices and in reputations. Our paper therefore contributes to linking a literature on the competitive structure of certification markets (Lerner and Tirole, 2006; Bolton et al., 2012; Farhi et al., 2013) to the literature on the reputation of experts. In particular, we are, to the best of our knowledge, the first to examine how the possibility that sellers may hold multiple certifications (adversely) affects reputational incentives.

The remainder of the paper is organized as follows. Section 2 provides a description of the model. In Section 3, we study how reputational incentives affect the precision of a monopolistic certifier. In Section 4, we allow for the entry of a second certifier, and analyze the impact of competition on reputational concerns. Section 5 provides a discussion on modeling choices and the robustness of some assumptions. Section 6 concludes. All the proofs are in the Appendix.

2 The model

2.1 The product market

We consider a market populated by a risk-neutral seller and (many) risk-neutral buyers. The value of the product to the seller is normalized to 0, while its value to buyers depends on the quality of the product $\theta \in \{\theta_b, \theta_g\}$, which is endogenously determined. The seller can offer at no cost a product of low quality θ_b worth 0 to buyers. Alternatively, he can supply a product of high quality θ_g worth 1 to buyers. This involves a (non-monetary) cost λ, where λ is private information of the seller and is assumed to be uniformly distributed on $[0, 1]$.[10] We hereafter refer to λ as being the "type" of the seller.

Note that supplying high quality is efficient for any seller, although sellers differ as to how efficient they are.[11] As a result, in a frictionless market where quality is public information, the seller chooses high quality irrespective of his cost. By contrast, if product quality is private information to the seller, which we assume from now on, the seller chooses low quality regardless of his cost. Therefore, the informational friction we introduce precludes any gain from trade, which calls for a certification mechanism.

This setup is stylized enough to fit many markets in which certification plays a key role. One

[10]Our results hold for any log-concave distribution of λ.

[11]The addition of "inefficient" types ($\lambda > 1$) is irrelevant to the analysis of certification.

example is the market for structured financial products such as Mortgage-Based Securities (MBS), in which financial firms originate loans or mortgages with the purpose of distributing them to investors who value the risk exposure they provide. There, the ability of originators to create surplus depends both on intrinsic skills (access to market, ability to screen loan applicants, or to adequately package and structure loans into tradable securities), which we capture through λ, and on the willingness of the originator to use costly and limited resources to deploy these skills, which we capture through quality choice. Both theoretical and empirical work point at moral hazard and private information as central issues in the market for structured financial products (Keys et al. (2010); Hartman-Glaser et al. (2012)).

2.2 The certification process

A risk-neutral certifier is endowed with a technology which produces a signal $\sigma \in \{H, M, L\}$ on product quality, with the following conditional distributions

$$\Pr(H|\theta_g) = \Pr(L|\theta_b) = \alpha + e \qquad \text{and} \qquad \Pr(M|\theta_g) = \Pr(M|\theta_b) = 1 - (\alpha + e),$$

where α is an enduring technological parameter of the certifier (its type), and e is some unobservable effort which the certifier exerts. We assume that $\alpha \in \{\alpha_L, \alpha_H\}$ with $\alpha_H = 1 - \alpha_L > \frac{1}{2}$, and $e \in [-\varepsilon, \varepsilon]$, where $0 < \varepsilon < \alpha_L$. Exerting effort involves a cost $\frac{1}{2}ce^2$, even for negative values of e : while increasing the precision of the signal takes extra effort and resources, deliberately lowering precision can increase the probability for the certifier to be exposed to lawsuits or regulatory sanctions ex post.[12] The assumption that the cost of effort is symmetric in the positive and negative ranges is made for analytical convenience, but is inessential: we discuss in Section 5 how our results extend to the case where the cost of decreasing the precision of the signal is arbitrarily smaller than the cost of increasing it.

The certifier charges a fee ϕ in exchange for its services. We make three related assumptions on the pricing of certification. First, and in line with common practice in most certification markets, including credit ratings, we assume that certifiers only charge sellers.[13] Second, we impose that the fee charged by the certifier to the seller be paid upfront. That is, ϕ cannot be contingent on the

[12] In February 2015, the U.S. Department of Justice announced a \$1.5 billion settlement of its lawsuit against S&P in which the CRA admitted having relaxed its rating standards for structured financial products between 2004 and 2007 (see, e.g., "How the Justice Department, S&P Came to Terms," in the Wall Street Journal, February 2nd, 2015).

[13] The most common justification for charging sellers lies in the public good nature of information which makes it difficult to prevent information leakages across buyers. See also Stahl and Strausz (2011) on this issue.

outcome of the certification process, which could in itself create a direct incentive for the certifier to issue signals more favorable to the seller. Averting such wrong incentives was a rationale behind the agreement between the New York State Attorney General Andrew Cuomo and the three main credit rating agencies in the Spring 2008, which imposed that credit rating agencies be compensated "regardless of whether the investment bank ultimately selects them to rate a RMBS."[14] In this context, the precision of certification is purely driven by long-term reputational concerns, the focus of this paper. Third, we assume that sellers have limited cash on hand ω, and cannot pledge the proceeds from the possible sale of their products to the certifier to raise extra funds. In the CRA example, this financial constraint is consistent with debt issuers resorting to certification because they lack internal funding and need credibility to raise funds externally as in Opp et al. (2013) or Kashyap and Kovrijnykh (2016).[15] We assume ω to be distributed according to a continuous distribution with full-support density $f(.)$ on $[0, \overline{\omega}]$, and to be independent of λ. We show in Section 5 that the cap on certification fees which the financial constraint imposes could also result from the intervention of a regulator in a non-competitive market for certification. In either case, we will make clear how this friction in the certifier's ability to adjust prices impacts its reputational concerns.[16]

2.3 Timing and information

We conclude this description of the model with the timing of the game. There are two periods. Within each period t, the game unfolds as follows:

a. ω_t is publicly realized,

b. The certifier posts a fee ϕ_t,

c. The seller privately learns his cost λ and privately decides on product quality,

d. The seller decides whether to solicit certification or not,

[14] See the press release of the NY State Office of the Attorney General on June 5th, 2008. Imposing that CRAs charge fees upfront was also part of the Franken amendment to the Dodd-Frank Act. This amendment was eventually removed to allow for an extended review by the SEC (see, e.g., Ozerturk (2014)).

[15] In these two papers, the CRA's fee is funded out of the proceeds from the debt issuance. This makes the payment to the agency de facto contingent on the rating, since firms are more likely to issue debt following a good rating. Our non-pledgeability assumption rules this out and is therefore consistent with the fee ϕ being paid ex ante.

[16] For an analysis of an optimal payment structure for CRAs (in the absence of reputational concerns), see Faure-Grimaud et al. (2009) and Kashyap and Kovrijnykh (2016).

e. If the seller solicits certification, the certifier exerts effort e_t, and produces a signal $\sigma_t \in \{H, M, L\}$ observable to all parties,

f. Buyers independently submit bids for the product in a second-price auction.

The certifier is long-lived and does not discount future revenues and costs (for simplicity). Buyers and sellers only live one period. In addition, the financial constraint of the seller ω_t is i.i.d. across periods. While effort is private information of the certifier, the signal σ_t is verifiable. In other words, the certifier can control the ex-ante precision of the signal through costly effort, but cannot manipulate the signal ex post. This captures the idea that manipulations of the technology are more difficult to detect than manipulations of the signal produced. In particular, it is consistent with reports on how credit rating agencies have been adjusting the information they provide to markets: rather than directly manipulating the outcome of their credit risk models (the rating itself) they adjust their models or the type of information inputted into these models (see, e.g., the 2008 SEC Report of Issues Identified in the Examinations of Select Credit Rating Agencies). We also rule out that the seller manipulate the signal produced (unlike, for instance, in Cohn et al. (2013), who focus on seller reputation). Finally, we assume for simplicity that the signal σ_t is public if the seller requires certification.[17]

Importantly, we assume that the certifier does not have private information on α, as in Holmström (1999). In the beginning of period 1, all players share the common belief that $\Pr(\alpha = \alpha_H) = \rho_1$. If certification takes place in period 1, all the period-2 players observe the certification outcome σ_1. We denote $\rho_2 = \Pr(\alpha = \alpha_H | \sigma_1)$ and will henceforth refer to ρ_t as the certifier's reputation in period t. Note that given the signal structure, observing the signal produced by the certifier is sufficient to update one's belief about its type, that is, observing product quality ex post would not provide any additional information.

Finally, because the seller has private information, applying for costly certification is in itself informative. As usual in signaling games, the indeterminacy of out-of-equilibrium beliefs generates equilibrium multiplicity in the subgame where the certification fees are fixed and the seller decides on quality and certification. In particular, equilibria where a certifier is inactive can be supported by the out-of-equilibrium belief that only high-cost sellers holding low-quality products apply to that certifier. To impose some discipline on out-of-equilibrium beliefs, we restrict attention throughout

[17] A simple unravelling argument applies here: even if the seller could conceal both his decision to solicit certification and the certification outcome itself, he would be strictly better off disclosing the signal if $\sigma_t \in \{M, H\}$. Indeed, buyers believe by default (i.e., absent certification) the product to be of low quality.

the whole analysis to equilibria which satisfy D1, i.e., we impose that a deviation to a certifier who is inactive in equilibrium be interpreted as coming from the seller type(s) who benefit the most from this deviation.

3 Monopolistic certifier

We first derive the equilibrium of the period-2 game where the certifier has no reputational concerns, and then turn to period 1 to study how reputation-building affects the precision of certification, as well as welfare.

3.1 Period 2: Market equilibrium with certification

The certifier's expected precision in period t, i.e., the probability that it perfectly identifies quality, is a function of its reputation and effort:

$$\Pr(\sigma_t = H|\theta = \theta_g) = \Pr(\sigma_t = L|\theta = \theta_b) = \rho_t \alpha_H + (1 - \rho_t)\alpha_L + e_t \equiv q(\rho_t, e_t). \tag{1}$$

In period 2, the certifier has no reputational concerns, hence provides no effort, $e_2 = 0$, so that the technological parameter α only drives the precision of its signal.

Dropping the time subscript, we characterize the market equilibrium as a function of the precision of the signal q in two steps. We first derive the perfect Bayesian equilibrium of the subgame in which ϕ is given and the seller decides on product quality and a certification strategy as a function of his type λ. This equilibrium generates a demand for certification which we then use in a second step to derive the optimal pricing policy.

3.1.1 The demand for certification

In equilibrium, the seller's decisions to invest in quality and to solicit certification are jointly determined. In particular, since high quality is both costly and unverifiable, if a seller does not solicit certification, he always chooses low quality. Therefore, the only possible price for an uncertified product is zero. A seller who bears the cost λ of high quality then always purchases certification and gets a payoff

$$q + (1 - q)P_M - \phi - \lambda. \tag{2}$$

Indeed, with probability q, the certifier perfectly identifies the product as high quality, $\sigma = H$, and the seller can sell it at price $P_H = 1$. With probability $1-q$ the certifier's signal is $\sigma = M$ and the seller can sell at a price P_M which is determined in equilibrium: P_M reflects the average quality in the pool of sellers who solicit certification and obtain a certificate $\sigma = M$, which in turn depends on the seller's strategy.

Alternatively, the seller can choose low quality and still apply for certification: he then obtains 0 if he is perfectly identified (i.e., $\sigma = L$), but can charge P_M if $\sigma = M$, so that his expected payoff is

$$(1-q)P_M. \tag{3}$$

Finally, the seller can simply stay out of the market and get 0.

Combining these observations, a seller chooses high quality if and only if

$$q + (1-q)P_M - \phi - \lambda \geq \max\{(1-q)P_M - \phi, 0\}. \tag{4}$$

Therefore, there exists a cutoff $\overline{\lambda}$ below which every type chooses high quality and applies for certification. Types above $\overline{\lambda}$ choose low quality, but may still solicit certification with a strictly positive probability when it is profitable, i.e., when $(1-q)P_M - \phi \geq 0$. It follows that, for fixed q and ϕ, the equilibrium strategy is characterized by $(\overline{\lambda}, \gamma)$, where $\gamma \in [0,1]$ denotes the fraction of low-quality sellers who solicit certification.[18] The total demand for certification then reads

$$D(\phi) \equiv \overline{\lambda}(\phi) + [1 - \overline{\lambda}(\phi)]\gamma(\phi), \tag{5}$$

where we ignore, in a first step, the financial constraint of the seller ω. Solving for the Perfect Bayesian Equilibrium, we show that the demand for certification is uniquely characterized.

Lemma 1. *For a given precision q, there is a unique Perfect Bayesian Equilibrium.*

1. *If $\phi \leq q(1-q)$, then $\overline{\lambda}(\phi) = q$, $\gamma(\phi) = 1$ and total demand is $D(\phi) = 1$.*

2. *If $q(1-q) < \phi < 1-q$, then $\overline{\lambda}(\phi) = q$, $\gamma(\phi) = \frac{q(1-q-\phi)}{(1-q)\phi} \in (0,1)$ and total demand is $D(\phi) = \frac{q(1-q)}{\phi}$.*

3. *If $1-q \leq \phi < 1$, then $\overline{\lambda}(\phi) = 1-\phi$, $\gamma(\phi) = 0$ and total demand is $D(\phi) = 1-\phi$.*

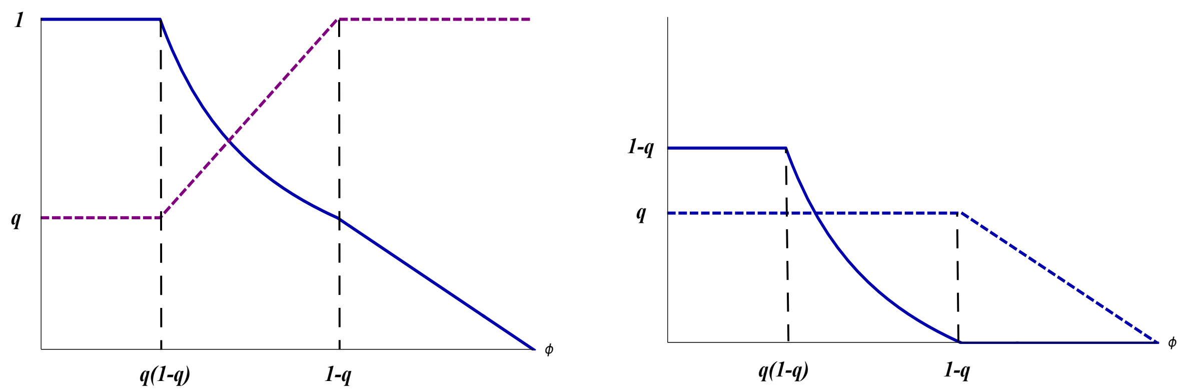
[18] We could allow the strategies of types $\lambda > \overline{\lambda}$ to depend on λ, but since λ does not affect the payoff of sellers who produce low quality, one may without loss of generality focus on the total fraction of low quality sellers who solicit certification.

4. If $\phi \geq 1$, then $\overline{\lambda}(\phi) = \gamma(\phi) = D(\phi) = 0$.

Figure 1 illustrates the results of this Lemma. The left panel plots the total demand for certification as a function of ϕ, as well as the price P_M following a certificate $\sigma = M$. P_M obtains from Bayes' rule,

$$P_M = \frac{\overline{\lambda}(1-q)}{\overline{\lambda}(1-q) + (1-\overline{\lambda})\gamma(1-q)} = \frac{\overline{\lambda}}{\overline{\lambda} + (1-\overline{\lambda})\gamma} \text{ for } (\overline{\lambda}, \gamma) \neq (0,0), \tag{6}$$

and hence, endogenously reacts to changes in composition of the pool of applicants, which we depict in the right panel, where the total demand for certification is decomposed between high and low-quality sellers.



(a) Total demand for certification $D(\phi)$ (solid) and price $P_M(\phi)$ following certificate $\sigma = M$ (dashed).

(b) Demand decomposition among high-quality (dashed) and low-quality sellers (solid).

Figure 1: The demand for certification
Parameter specification: $q = 0.4$.

When certification is cheap, $\phi \leq q(1-q)$, all sellers apply for certification ($\gamma = 1$). As a result, P_M reflects the average quality in the entire pool of sellers, that is, q. When the certification fee increases, high-cost types ($\lambda > q$) are the first ones to drop out: because they hold low-quality products, they are less likely to trade, and therefore value certification relatively less. Specifically, as ϕ increases from $q(1-q)$ to $1-q$, the mass of high-cost sellers who apply for certification declines from $1-q$ to 0 (as γ decreases from 1 to 0). At the same time, because the average quality in the pool of applicants improves, the price P_M rises from q to 1. As ϕ increases beyond $1-q$, low-quality sellers are completely excluded from certification ($\gamma = 0$), and the mass of low-cost sellers ($\lambda \leq q$) who supply high quality starts to decrease as well. Indeed, even if high-quality sellers can then sell at a price of 1 for sure, the increased certification fee lowers the return to investing in quality.

Note finally that the uniqueness of the Perfect Bayesian Equilibrium derived in Lemma 1 makes use of D1. Absent this refinement, this equilibrium coexists with a no-certification equilibrium when ϕ is high enough, supported by the out-of-equilibrium belief that a seller who applies for certification holds a low-quality product.

3.1.2 Monopoly pricing of certification and the value of precision

To build intuition, we start by deriving the optimal monopoly pricing, ignoring the financial constraint ω. From Lemma 1, the demand for certification has the following properties.

1. If $\phi \leq q(1-q)$, the elasticity of demand is nil, so that a price increase raises the certifier's profit.

2. If $q(1-q) < \phi < 1-q$, the demand is unit elastic, so that a price increase leaves the certifier's profit unchanged: the loss of the marginal (high-cost) seller is exactly offset by the price increase on infra-marginal types.

3. If $\phi \geq 1-q$, the price elasticity of demand is lower than 1 if and only if $\phi \leq \frac{1}{2}$. That is, the certifier's profit, equal to $\phi(1-\phi)$, is increasing in ϕ for $\phi \leq \frac{1}{2}$ and decreasing afterwards.

The certifier's optimal strategy follows from standard principles of monopoly pricing. First, it is suboptimal to price in the region where the demand is inelastic, that is, ϕ should be higher than $q(1-q)$. In addition, since profit is constant for $\phi \in (q(1-q), 1-q)$, it is optimal raise ϕ above $1-q$ if and only if the demand is inelastic at $\phi = 1-q$, i.e., if $1-q < \frac{1}{2}$. It follows that the unconstrained monopoly fee is

$$\begin{cases} \phi^m \in (q(1-q), 1-q) & \text{if } q < \frac{1}{2}, \\ \phi^m = \frac{1}{2} & \text{if } q \geq \frac{1}{2}. \end{cases} \tag{7}$$

Intuitively, when the certifier's precision is low ($q < \frac{1}{2}$), pricing out high-cost types requires a high fee $\phi > 1-q > \frac{1}{2}$. This is suboptimal because, in that region, the demand of low-cost types is highly elastic. On the contrary, when the certifier's precision is high ($q > \frac{1}{2}$), the demand from low-cost sellers is sufficiently inelastic at $\phi = 1-q$ that the certifier finds it profitable to price above $1-q$ to extract more surplus from them, which drives high-cost sellers out of the market. In the latter case, the exercise of monopoly power results in distortionary pricing, hence lower gains from trade, which we discuss further in Section 5.

The optimal fee in (7) generates the following (unconstrained) monopoly profit,

$$\pi^m \equiv \phi^m D(\phi^m) = \begin{cases} q(1-q) & \text{if } q < \frac{1}{2}, \\ \frac{1}{4} & \text{if } q \geq \frac{1}{2}. \end{cases} \tag{8}$$

When precision is low, $q < \frac{1}{2}$, the certifier's profit is increasing in q. The intuition is as follows. Keeping ϕ constant, a marginal increase in q increases the mass of types who choose high quality and apply for certification, i.e., $\overline{\lambda} = q$ goes up. The effect on low-quality sellers ($\lambda > q$) is a priori ambiguous: their probability of selling, $1-q$, decreases, but as $\overline{\lambda}$ goes up, the average quality in the pool of applicants improves, so that the price P_M they obtain conditional on selling increases. When q is small, the magnitude of this second effect is important because any marginal increase in P_M is magnified by a large probability $1-q$ for low-quality sellers of obtaining $\sigma = M$. Hence, when precision improves starting from a low level, the credibility benefit (the impact of the increase in P_M) is relatively strong for types with a low-quality product relative to the cost of being screened out more often. This limits the drop in γ, and the overall demand for certification increases.

Conversely, when $q > \frac{1}{2}$, this credibility gain is small relative to the cost of being detected with a higher probability, so the certifier is less attractive to low-quality sellers. This induces the certifier to price in the region where the marginal seller holds a high-quality product ($\gamma = 0$, $\overline{\lambda} = 1-\phi < q$). Therefore, the mere fact that a seller buys certification signals high quality and a marginal change in the certifier's precision does not affect the credibility of certificates. As a result, a monopolistic certifier solves a simple rent-extraction problem where the demand for certification is driven by the trading profit of the marginal high-quality seller, and is independent from q, so the certifier's profit is constant in q.

Consider now the effect of the seller's financial constraint ω. Since the certifier's unconstrained profit is quasi-concave in ϕ, the optimal pricing in the presence of the constraint ω is simply

$$\phi^* \equiv \min(\phi^m, \omega).$$

Imposing this constraint when $q < \frac{1}{2}$ may lower the certifier's profit, but does not affect the monotonicity of that profit with respect to precision q. If $q < \frac{1}{2}$, the certifier's unconstrained price ϕ^m is always below $1-q$, in the region where sellers with low-quality products participate with positive probability ($\gamma > 0$). This is a fortiori true at the constrained price $\phi^* \leq \phi^m$. As discussed above, in that region where q is small and $\phi < 1-q$, the overall demand for certification increases in q, which drives profits up.

The key effect of the seller's financial constraint ω arises when the seller's precision is high, $q > \frac{1}{2}$. There, the constraint forces the certifier to lower its price down to the region in which (some) low-quality sellers participate, while it would optimally exclude them ($\phi^m > 1 - q$). But because q is large, the marginal benefit of a higher q (i.e., a higher P_M) is small for low-quality sellers who rarely obtain a certificate $\sigma = M$, relative to the marginal cost of getting that certificate with a lower probability. As a result, the demand from high-cost types can drop enough that the aggregate demand for certification decreases.

Formally, if $\omega < \frac{1}{2}$, then the certifier cannot set the unconstrained price ϕ^m for $q > \frac{1}{2}$ (see Equation (7)). As a result, its profit, $\phi^* D(\phi^*)$ is strictly *decreasing* in q on $(\frac{1}{2}, 1-\omega)$, and constant afterwards. That is, the certifier's profit is *hump-shaped* in its precision q. We focus from now on on the case where the seller's financial constraint always binds when $q > \frac{1}{2}$, by assuming that ω is always smaller than $\frac{1}{2}$.

Assumption 1. $\overline{\omega} = \frac{1}{2}$.

Having ω randomly drawn on $[0, \overline{\omega}]$ allows to smooth the certifier's profit function,

$$\pi^*(q) \equiv \int_0^{\overline{\omega}} \phi^* D(\phi^*) f(\omega) d\omega,$$

which inherits the non-monotonicity discussed above.

Proposition 1. *The certifier's expected profit $\pi^*(q)$ is continuously differentiable, strictly increasing on $\left(0, \frac{1}{2}\right)$ and strictly decreasing on $\left(\frac{1}{2}, 1\right)$.*

As is clear from the previous discussion, the financial constraint ω plays a crucial role in generating this non-monotonicity: by limiting the certifier's ability to extract surplus from low-cost types, it makes the certifier's profit more sensitive to the demand from high-cost applicants with low-quality products, whose willingness to pay for certification decreases in the precision of the signal when $q > \frac{1}{2}$. While the cap on the certifier's fee stems here from a financial constraint on the seller side, other limits to the certifier's ability to raise prices could generate the non-monotonicity described in Proposition 1. In particular, a natural reason why a monopolistic certifier's ability to raise prices could be restrained is the (actual or potential) intervention of a regulator. We argue in section 5 below that a cap on certification fees can indeed improve welfare, so that the non-monotonicity of $\pi^*(q)$ can also result from a regulatory intervention. Notice finally that the case where the pricing friction vanishes and the certifier's profit is monotonic in its accuracy (see Equation (8)) would generate similar reputational incentives as in earlier work by Benabou and Laroque (1992) or Mathis et al. (2009). We focus here on the novel case where a higher reputation for accuracy is not always more valuable, and on the implications of such ambiguous reputational concerns for incentives to improve accuracy.

3.2 Period 1: Reputation building and Welfare

Unlike in period 2, the certifier has an incentive to provide effort in period 1 because the signal it produces affects its future reputation, ρ_2. In turn, ρ_2 affects its perceived precision $q(\rho_2, 0) = \rho_2\alpha_H + (1-\rho_2)\alpha_L$, hence its profit $\pi^*[q(\rho_2, 0)]$ in period 2.

More precisely, if the certifier is expected to provide effort e_1, the probability that $\alpha = \alpha_H$ conditional on revealing the true quality of the product goes up from ρ_1 to

$$\begin{aligned}\rho^+(\rho_1, e_1) &\equiv \Pr(\alpha = \alpha_H|\sigma_1 = H) = \Pr(\alpha = \alpha_H|\sigma_1 = L) \\ &= \frac{\rho_1(\alpha_H + e_1)}{\rho_1\alpha_H + (1-\rho_1)\alpha_L + e_1} \geq \rho_1. \end{aligned} \tag{9}$$

Conversely, when the certifier produces a signal $\sigma_1 = M$, its reputation goes down from ρ_1 to

$$\begin{aligned}\rho^-(\rho_1, e_1) &\equiv \Pr(\alpha = \alpha_H|\sigma_1 = M) \\ &= \frac{\rho_1(1-\alpha_H - e_1)}{1-\rho_1\alpha_H - (1-\rho_1)\alpha_L - e_1} \leq \rho_1. \end{aligned} \tag{10}$$

The certifier chooses e_1 to maximize its expected profit in period 2 net of effort costs, taking as given other players' expectation on effort. In equilibrium, this expectation has to be correct. It follows that the equilibrium level of effort e_1^* solves

$$\max_{e_1 \in [-\varepsilon, \varepsilon]} q(\rho_1, e_1)\pi^*[q(\rho^+(\rho_1, e_1^*), 0)] + [1 - q(\rho_1, e_1)]\pi^*[q(\rho^-(\rho_1, e_1^*), 0)] - c\frac{e_1^2}{2}. \tag{11}$$

Under the assumption that c is sufficiently large, the equilibrium level of effort is as follows.[19]

Proposition 2. *In period 1, there is a unique equilibrium level of effort, $e_1^*(\rho_1)$.*
$e_1^*(\rho_1)$ is continuous on $[0, 1]$, and there exists a threshold $\overline{\rho} \in (0, 1)$ such that:

- $e_1^*(\overline{\rho}) = e_1^*(0) = e_1^*(1) = 0,$

[19]See the proof of Proposition 2 in Appendix A for a formal condition, and Section 5.2 for a discussion.

- $e_1^*(\rho_1) > 0$ *for* $\rho_1 \in (0, \overline{\rho})$,

- $e_1^*(\rho_1) < 0$ *for* $\rho_1 \in (\overline{\rho}, 1)$.

Proof. In the Appendix. □

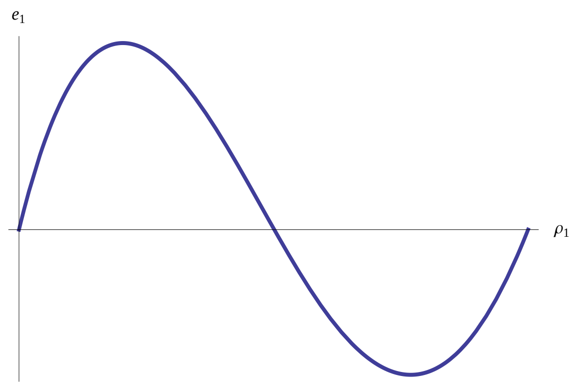


Figure 2: Equilibrium effort in period 1
Parameter specification: $\alpha_H = 1 - \alpha_L = 0.6, c = 0.6, \epsilon = 0.1, \omega$ *uniform on* $[0, \frac{1}{2}]$.

From (1), the certifier's perceived precision $q(\rho_2, 0)$ in period 2 is an increasing function of its reputation ρ_2, so the expected profit of the certifier in period 2 is hump-shaped in ρ_2, just as $\pi^*(q)$ is hump-shaped in q (Proposition 1). As a result, the direction in which reputational incentives affect period-1 effort depends on the initial reputation of the certifier ρ_1. When that reputation is low, reputational concerns have a disciplining effect, $e_1^*(\rho_1) > 0$, as the certifier tries to build up credibility. Meanwhile, when ρ_1 is high, the certifier's precision is lower than in the static benchmark, $e_1^*(\rho_1) < 0$, as the seller tries to appeal to future sellers of low-quality products.

The effects of reputational concerns in our model differ from existing models of reputation for experts in several ways. First, the certifier's incentives to manipulate the precision of its signal are purely driven by reputation, that is, there are no short-term incentives to distort information. This contrasts with Benabou and Laroque (1992) or Mathis et al. (2009), in which the expert trades the long-term benefits from maintaining a reputation against short-term incentives to distort information in order to reap immediate profits. In these models, the expert always prefers to be perceived as more accurate, but is, at some point, willing to milk his reputation to enjoy higher current benefits. Accordingly, while reputation might not be enough to perfectly discipline him, there is more information transmission when he cares about his reputation than when he does not. Instead, in our model, the precision of the certifier's signal is lower if it cares about its reputation when $\rho_1 > \overline{\rho}$. Second, in our model, reputational concerns have a weak impact on effort not only for extreme values of ρ_1 (close to 1 or 0) but also in the intermediate region around $\overline{\rho}$. By contrast, in Benabou and Laroque (1992) where the value of reputation is monotone, the distortion from the static preferred action is highest in the intermediate reputation range because the prior uncertainty about the certifier's type is maximum and beliefs are more malleable. In our model where the value of reputation is single-peaked, effort is low in this region because the certifier is equally concerned with catering to good and bad sellers. Finally, while Frenkel (2015) shows that a CRA's desire to build a reputation vis-à-vis two audiences may result in ratings inflation, his result hinges on the assumption that the two audiences observe different signals, hence hold different beliefs about the CRA. On the contrary, our model predicts that bad sellers may be more likely to be certified even when all parties share the same beliefs about the certifier.

We close this section by pointing out the welfare implications of reputation-building. Given a precision $q(\rho_1, e_1)$, the monopoly pricing derived above implies that gains from trade in the product market in period 1 are given by

$$\int_0^{\overline{\omega}} \int_0^{\min\{q(\rho_1,e_1),1-\omega\}} (1-\lambda) f(\omega)\, d\lambda\, d\omega. \tag{12}$$

The marginal type $\overline{\lambda} = \min\{q(\rho_1, e_1), 1-\omega\}$ is nondecreasing in $q(\rho_1, e_1)$. Since $q(\rho_1, .)$ is in turn increasing in e_1, higher effort directly translates into higher allocative efficiency in the product market. On the contrary, in the region where effort is negative ($\rho_1 > \overline{\rho}$), the lower precision resulting from reputation-building weakens sellers' incentives to improve quality, thereby creating a substitution effect: some sellers who would have incurred the cost of producing high quality products had certification been more precise now switch to low quality, at the expense of efficiency. This goes through two complementary channels. First, the probability for a high-quality seller of being revealed as such is lower when precision decreases. Second, a lower precision becomes more attractive to low-quality sellers and increases the probability γ that they apply for certification (this is the very reason why the certifier prefers a lower reputation in this region). That is, the quality in the pool of applicants deteriorates, which decreases the price P_M which a good seller obtains when not certified as such.

Notice that total welfare in period 1 equals

$$\int_0^{\overline{\omega}} \int_0^{\min\{q(\rho_1,e_1),1-\omega\}} (1-\lambda) f(\omega)\, d\lambda\, d\omega - c\frac{e_1^2}{2}, \tag{13}$$

which implies that the efficient level of effort in period 1 is positive. Accordingly, the fact that the certifier is willing to sacrifice resources in order to lower his reputation is unambiguously welfare-decreasing.

Notice also that this inefficiency driven by dynamic (reputational) incentives coexists with the standard static inefficiency due to distortionary pricing. The latter would exist absent reputational concerns, that is, for a given perceived precision of certification q. We discuss in greater detail in Section 5 how such a distortion is a rationale for regulatory intervention, notably price caps on certification.

4 Multiple certifiers

Let us now assume that certifier A acts as a monopoly in the first period but faces in period 2 the entry of a second certifier, B, with a precision q_B drawn from a continuous distribution with continuous c.d.f. G and full-support density on $[\underline{q}_B, \overline{q}_B]$, where we set $\underline{q}_B = 0$ and $\overline{q}_B = 1$ by default.[20]

The sequence of actions in period 1 is unchanged. In period 2, the certification game unfolds as follows (we drop time subscripts for simplicity):

1. ω and certifier B's precision q_B are realized and publicly observed,
2. A and B simultaneously post fees ϕ^A and ϕ^B,
3. The seller chooses product quality,
4. The seller decides whether to apply for certification from A and/or from B,
5. If solicited, certifier i produces a signal $\sigma^i \in \{H, M, L\}$ observable to all parties,
6. Trade takes place.

For simplicity, A and B produce independent signals (conditional on product quality). As discussed above, the subgames where the fees (ϕ^A, ϕ^B) are fixed generically admit multiple equilibria. D1 allows to rule out some of these equilibria, but unlike in the monopoly case, is not always sufficient to guarantee uniqueness. When several equilibria survive the refinement in the subgame, we select the one which maximizes the profit of the more precise certifier.

[20]We will restrict the support of q_B in subsection 4.2 below to isolate and illustrate the reputational effects specific to multihoming. Assuming a random quality of the entrant only plays the role of smoothing profit functions, which in turn ensures equilibrium existence in the reputation game, but does not alter the results qualitatively.

Once we have pinned down and selected an equilibrium in all the subgames, we analyze the full game where certifiers set fees optimally. There also, multiple equilibria generically coexist, among which we contrast two distinct equilibrium configurations. In the first one, to which we refer as "singlehoming equilibrium," sellers apply to one certifier only (the more precise one), while sellers who buy certification do so from both certifiers in the second one, to which we refer as "multihoming equilibrium." Note that we do not make any a priori restriction on the seller's decision to singlehome or multihome but rather derive the coexistence of these two certification strategies as equilibrium behaviors. In these two market configurations, reputational incentives may have opposite implications for effort provision. Under singlehoming, the threat of being displaced by a more accurate competitor makes lower reputations less appealing, thereby limiting the negative impact of the incumbent's reputational concerns on precision. Under multihoming, the additional precision that the entrant brings to the certification process can weaken the benefit of higher reputations for the incumbent, thereby worsening its reputational incentives.

4.1 Singlehoming

We start by showing the existence of an equilibrium in which a seller who buys certification solicits one certifier only. In this configuration, competition has the familiar effect of eroding the profit of the incumbent relative to its monopoly payoff.

Lemma 2. *There exists a singlehoming equilibrium in period 2. In any singlehoming equilibrium, the expected profit of certifier A reads*

$$\pi^{sh}(q_A, q_B) \equiv \begin{cases} \pi^*(q_A) & \text{if } q_B < \min\{\frac{1}{2}, q_A\}, \\ \pi^*(q_A) - \delta(q_B) & \text{if } \frac{1}{2} < q_B < q_A, \\ 0 \ (A \text{ is inactive}) & \text{if } q_B > q_A, \end{cases} \tag{14}$$

where $\delta(.)$ is increasing in q_B and $\delta\left(\frac{1}{2}\right) = 0$. Symmetrically, B's expected profit is $\pi^{sh}(q_B, q_A)$.

In a singlehoming equilibrium, sellers solicit one certifier at most. In particular, high-quality sellers, who drive the whole demand for certification, flock to the more attractive certifier. There are two ways for a certifier to be attractive to high-quality sellers: a better reputation, and a lower fee. When B's precision is weak ($q_B < \min\{\frac{1}{2}, q_A\}$), B is never attractive enough in terms of reputation to appeal to high-quality sellers, so that B's clientele can only consist of low-quality types. Accordingly, A is not threatened by the presence of B and can charge its monopoly price and enjoy the monopoly profit.

As B's precision increases ($\frac{1}{2} < q_B < q_A$), monopoly pricing becomes unsustainable for A. Intuitively, buyers now recognize that the decision to solicit B might not be motivated solely by the prospect of being detected with a lower probability, but also reflects the relative attractiveness of B's fee as compared to A's. Specifically, if A sticks to the monopoly price, buyers interpret a deviation to B as coming from a seller with a high-quality product if ϕ^B is sufficiently low. As a result, A has to lower its price to make sure that B never attracts sellers. In this limit pricing equilibrium, A's profit is positive, but strictly lower than under monopoly, and the profit erosion $\delta(q_B)$ increases as B gets more precise. Finally (and symmetrically), if B's precision improves beyond A's, A is excluded from the market, as B could then always exploit its comparative advantage in terms of reputation to price in such a way to exclude A.

Turn now to the effect of singlehoming competition in period 2 on reputational incentives in period 1. From Lemma 2, the expected singlehoming profit of an incumbent A with precision q_A is

$$\Pi^{sh}(q_A) \equiv \int_0^1 \pi^{sh}(q_A, q_B)\, dG(q_B) = G(q_A)\pi^*(q_A) - \int_{\frac{1}{2}}^{\max\{\frac{1}{2}, q_A\}} \delta(q_B)\, dG(q_B). \tag{15}$$

From Proposition 1, the certifier's monopoly profit $\pi^*(q_A)$ is hump-shaped and reaches a maximum at $q_A = \frac{1}{2}$. Using (15), $\Pi^{sh}(q_A)$ is then strictly increasing on $(0, \frac{1}{2})$. Furthermore, since $\delta\left(\frac{1}{2}\right) = 0$, one has $\Pi^{sh\prime}(\frac{1}{2}) = G'(\frac{1}{2})\pi^*(\frac{1}{2}) > 0$, so the precision which maximizes $\Pi^{sh}(q_A)$ is strictly higher than $\frac{1}{2}$: the bliss reputation of the incumbent is higher under singlehoming than under monopoly.

Although reputational incentives in period 1 depend on the entire shape of the profit function, and not only on its maximum, the intuition that singlehoming competition makes lower reputations less desirable remains valid, as the next Proposition shows.

Specifically, let $e_1^{sh}(\rho_A)$ denote the incumbent's equilibrium effort in period 1 when its reputation is ρ_A and it expects singlehoming in period 2.

Proposition 3. *Singlehoming competition mitigates bad reputation effects:*

$$e_1^*(\rho_A) < 0 \Rightarrow e_1^*(\rho_A) < e_1^{sh}(\rho_A).$$

In the region where reputation generates wrong incentives, competition shifts reputational incentives in a way that improves precision. That is, it lowers incentives for the certifier to pander to low-quality sellers in period 1. A monopolistic certifier may prefer a lower reputation for precision because when the financial constraint binds (i.e., for low realizations of ω), it can make up for the revenue it cannot extract from high-quality sellers by attracting low-quality sellers. This leads to negative effort in period 1 to increase the likelihood of a drop in reputation. Instead, competition makes the profit sensitive not only to the absolute accuracy of certificates, but also to the relative accuracy compared to the competitor. In particular, the threat of being perceived as less precise than the competitor, hence of becoming inactive, lowers the willingness to pay for a lower reputation.[21] The resulting reduced investment in reputation materializes in a lower effort in absolute value, i.e., a higher effort in the region where effort is negative.

Notice that competition has the additional effect of rendering higher reputations less valuable as well. Indeed, under monopoly, high reputations are more valuable when sellers are less cash-constrained (i.e., for high realizations of ω), as the certifier can then extract more surplus from high-quality sellers. Since such market power is limited in the presence of the entrant, competition could in principle, by making higher reputations less attractive, worsen reputational incentives. However, competition constrains the pricing of the incumbent the most when the reputation of the entrant is high. This is precisely in those instances that the threat for the incumbent of being ousted by the entrant if its reputation deteriorates is the highest. Proposition 3 shows that this second effect dominates for any distribution of the entrant's precision, so that competition ultimately disciplines the certifier in the region where reputation creates wrong incentives. This second effect makes the impact of singlehoming competition on effort potentially ambiguous in the region where $e_1^*(\rho_A) > 0$. There, whether the erosion of future profits has a stronger impact on incentives than the threat of being displaced, depends on the distribution of the entrant's precision.

4.2 Multihoming

We now consider equilibria where a seller who applies for certification solicits both certifiers. Note that such practice is in line with the empirical observation that multi-rated issuances are pervasive in the market for corporate credit ratings: Chen et al. (2014) reports that the overwhelming majority of large corporate bond issues involve at least two ratings.[22] We show that, while the disciplining effect of singlehoming is still at play when sellers solicit multiple certificates, multihoming generates a distinct and sometimes countervailing effect on reputational incentives. To illustrate this point,

[21] This implies that competitive pressure alone would not make A's profit non-monotonic in its precision q_A.

[22] In their sample of 8,767 bonds taken from the Barclays Capital Bond Index, 99.5% of bond issues are rated by S&P and Moody's and 70% are rated by Fitch. On this, see also Bongaerts et al. (2012). Doherty et al. (2012) introduce the possibility for a previously rated issuer to apply for a second rating when another rating agency enters the market. However, they do not study the decision to apply for multiple ratings simultaneously.

we focus on the multihoming equilibrium where the profit of the most precise certifier is the highest.

This multihoming equilibrium exhibits an intuitive relation to the singlehoming equilibrium in Lemma 2:

Lemma 3. *There exists a multihoming equilibrium in period 2 in which a seller who applies for certification always solicits A and B and where certifier A's expected profit is*

$$\pi^{mh}(q_A, q_B) = \begin{cases} \pi^{sh}(q_A + q_B - q_A q_B, q_B), & \text{if } q_A > q_B, \\ 0 & \text{if } q_A < q_B. \end{cases}$$

where $\pi^{sh}(.,.)$ is as defined in Lemma 2. Symmetrically, certifier B's expected profit is $\pi^{mh}(q_B, q_A)$.

The multihoming profit equals the singlehoming profit which the more accurate certifier would obtain if its precision combined the precision of both certifiers, i.e., was equal to $q_A + q_B - q_A q_B$. Intuitively, when both certifiers are simultaneously active, the demand for certification depends on the joint precision of the certification process, that is, the probability $q_A + q_B - q_A q_B$ that *at least* one certifier perfectly identifies product quality. In other words, certifiers exert an externality on each other's demand.[23] While the more accurate certifier still captures all the revenue from the certification market, as in the singlehoming case, the less accurate one, despite making no profit, now modifies the (joint) demand for certification.

To understand the direction in which multihoming alters the reputational incentives of the incumbent, let us start from the case where the entrant's precision is nil, $q_B = 0$. Then, the incumbent earns its monopoly profit $\pi^*(q_A)$, which is maximized for $q_A = \frac{1}{2}$. Now, suppose q_B increases by a small amount. From Lemmas 2 and 3, A's profit is

$$\pi^{sh}(q_A + q_B - q_A q_B) = \pi^*(q_A + q_B - q_A q_B),$$

and therefore the *joint* precision of the certification process that maximizes A's profit is still $\frac{1}{2}$. However, since B now adds valuable information to this process, the optimal *individual* precision for A is such that

$$q_A + q_B - q_A q_B = \frac{1}{2} \Leftrightarrow q_A = \frac{1 - 2q_B}{2(1 - q_B)} < \frac{1}{2},$$

and this bliss point decreases in the accuracy of B as long as A remains the more accurate certifier. In fact, as q_B increases, A's entire profit shifts towards the left, as illustrated in Figure 3.

[23] Remark that, since the profit function of a monopolist is single-peaked, this externality can be positive or negative.

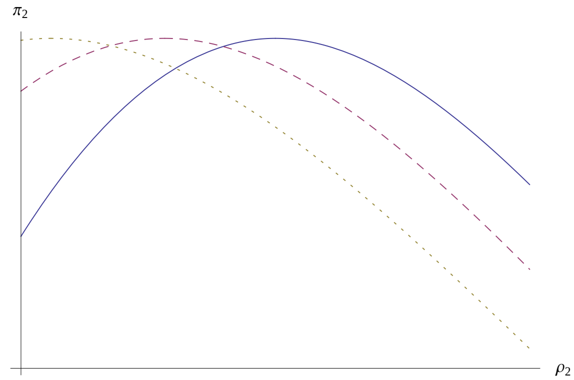


Figure 3: Equilibrium profits in the multihoming equilibrium.
Parameter specification: $\alpha_H = 0.6, q_B = 0$ *(solid),* $q_B = 0.08$ *(dashed),* $q_B = 0.15$ *(dotted).*

This multihoming externality, which tends to make lower reputations more desirable than under monopoly, coexists with the disciplining effect already discussed in the singlehoming case, which tends to make a higher precision more desirable (i.e., it is still true in the multihoming equilibrium that the more precise certifier makes higher profits). Which effect dominates ultimately depends on the distribution of the entrant's precision q_B. Intuitively, the former effect, which is specific to multihoming, is more likely to dominate when the entrant is relatively imprecise in comparison to the incumbent, so that the risk of being ousted is low. To formally make this point, and to highlight how multihoming adversely impacts reputational incentives, we make the following simplifying assumptions:

Assumption 2. ω *is uniformly distributed on* $[0, \frac{1}{2}]$.

Assumption 3. $\overline{q}_B < \frac{\alpha_L - \frac{3}{8}}{\alpha_L}$.

Assumption 2 only simplifies the analysis by generating more tractable analytical forms than a generic distribution for the financial constraint ω. Assumption 3, by imposing that the upper bound of the entrant precision $\overline{q}_B$ be low enough relative to the lower bound of the incumbent's precision α_L, essentially ensures that the entrant is never able to "steal" the certification market revenue from the incumbent, which shuts down the disciplining effect. Therefore, in what follows, reputational effects are purely driven by the new externality which multihoming introduces relative to singlehoming.

Let $e_1^{mh}(\rho_A)$ denote the incumbent's equilibrium effort in period 1 when its reputation is ρ_A and it expects a multihoming equilibrium in period 2.

Proposition 4. *Under Assumptions 2 and 3,* $e_1^{mh}(\rho_A) < e_1^*(\rho_A)$ *for all* $\rho_A \in (0, 1)$.

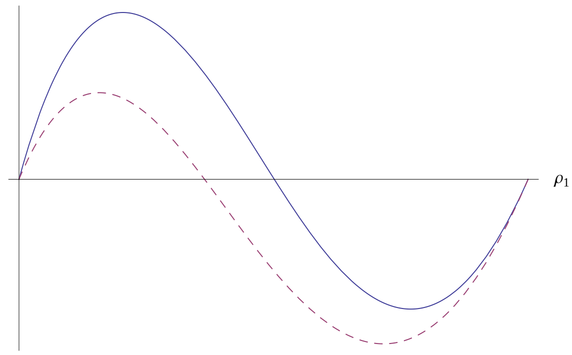


Figure 4: Comparison of equilibrium efforts under monopoly (solid) and multihoming (dashed). *Parameter specification:* $\alpha_H = 1 - \alpha_L = 0.6, c = 0.6, \epsilon = 0.1, \omega$ *uniform on* $[0, \frac{1}{2}]$.

Proposition 4 and Figure 4 illustrate how multihoming leads to lower information quality. The intuition follows from the above discussion. When the seller multihomes, certifiers, instead of competing to attract sellers, share the same customer base. As a result, the additional information brought by the entrant makes the value of precision to the incumbent sometimes lower than in the monopoly case, and definitely less valuable than under singlehoming. This, in turn, feeds back into the incumbent's incentives to build a reputation for accuracy and depresses its effort in period 1.

As discussed above, Assumption 3 is designed to insulate the effect of the multihoming externality on reputational incentives. In the more general comparison between monopoly and multihoming, the conclusion of Proposition 4 would have to be qualified. First, when the precision of the entrant q_B gets large, competition plays a disciplining role, as under singlehoming.[24] Second, multihoming comes with the added benefit of a second signal on product quality, which in itself improves the informativeness of certification and therefore boosts gains from trade. For tractability, we have modelled competition as entry in period 2, but in a steady state where both certifiers are present in every period, the potentially detrimental effect of competition on reputational incentives would also have to be weighted against the benefit of obtaining two independent signals.[25]

We close this section with a remark on our assumption that sellers *simultaneously* solicit A and B, should they choose to multihome: we do not allow the seller to make the purchase of a second certificate conditional on the outcome of the first certification.[26] This assumption is made

[24] Intuitively, the more precise certifier still holds an advantage over the less precise one even in a multihoming equilibrium because if, out of equilibrium, the more imprecise certifier raises its fee, which in turn forces the seller to singlehome, then the seller is more likely to flock to the more precise certifier.

[25] Notice that considering longer horizons also raises the interesting question of the incentives for the entrant to build a reputation. See Jeon and Lovo (2012) on this issue.

[26] "Rating shopping" has been analyzed notably in Skreta and Veldkamp (2009) and Bolton et al. (2012).

for tractability and relaxing it would simply make multihoming less likely to arise in the current setting. For instance, a high-quality seller would always prefer sequential certification because a H-signal from the first certifier identifies perfectly his quality, making a second certificate worthless. However, if a H-signal could come from a low-quality seller, then an equilibrium in which high-quality sellers always multihome could be sustained by buyers' beliefs (consistent with D1) that a seller who holds only one H-certificate has a low-quality product. This, in turn, would imply that high-quality sellers could sell only if they multihome.

5 Discussion

We close with a discussion of our modeling choices, and of alternative ways to motivate or interpret some of our assumptions.

5.1 Price regulation

We argue here that the cap on the certifier's fee could also be imposed by a regulator. From Section 3, a certifier with precision q has the ability to generate gains from trade for any seller with cost $\lambda \leq q$. Whether the certifier allows these gains from trade to be fully realized depends on its pricing policy. The unconstrained monopoly pricing in (7) is such that when $q > \frac{1}{2}$, the certifier sets $\phi > 1 - q$, so that only types in $(0, 1 - \phi) \subset (0, q)$ trade. In that case, monopoly pricing generates the familiar deadweight loss: types in $(1 - \phi, q)$ are priced out of the certification process, and as a result, some gains from trade fail to be realized.

In the model, the financial constraint of the seller, ω, which caps the certifier's fee is key in generating the non-monotonicity in the certifier's profit $\pi^*(q)$. The above discussion suggests an alternative reason why the price for certification could be capped: a regulator who cares about total welfare would want to limit the certifier's ability to exert market power. In March 2009, Connecticut Attorney General Richard Blumenthal launched an investigation into the three largest credit rating agencies arguing that they were exploiting the lack of competition (due in part to their status of SEC-approved CRAs) to extract rents. In a response to Blumenthal, Standard & Poors indicated that the firm was imposing a cap on its fees for asset-backed security deals.[27] More generally, the regulators' concern with excessive concentration in the credit rating industry, explicit in the Credit Rating Agency Reform Act of 2006, may bound the temptation of CRAs to raise their

[27] See "Connecticut attorney probes ratings bailout windfall," Reuters Business UK, April 7, 2009.

fees, and then risk triggering regulatory intervention (Glazer and McMillan (1992)).

In our model of a monopolistic certifier (with no financial constraint), imposing a cap $\overline{\phi}$ on fees is welfare-enhancing for any given precision. In particular, setting $\overline{\phi} = 1 - q$ maximizes gains from trade in the current period, while generating a hump-shaped profit $q(1-q)$ for the certifier over the interval $[0, 1]$. Even in the more realistic case where the cap cannot be perfectly adjusted to the certifier's precision, any $\overline{\phi} < \frac{1}{2}$ weakly increases efficiency (strictly so for some realizations of q), and keeps the certifier's profit hump-shaped in its reputation. In this case, as $\overline{\phi}$ decreases, welfare improves, an insight which also applies to the competitive case with two certifiers: in the equilibria derived in Propositions 2 and 3, because certifiers offer differentiated products, the more accurate certifier may still exert market power, at the expense of social welfare.

While market power provides a rationale for caps on certification fees, our analysis hints that price caps also impact welfare through reputational effects. Capping fees increases static efficiency for a given precision but may also weaken certifiers' reputational incentives by making their profit more sensitive to lower-quality sellers who value precision relatively less. This suggests that a regulator can face a trade-off between static and dynamic efficiency.

To illustrate this, consider the monopoly case and suppose that $q[\rho^-(\rho_1, \varepsilon)] > \frac{1}{2}$, that is, for any level of effort, the perceived precision of the certifier in period 2 q_2 is above $\frac{1}{2}$. Let us compare two polar opposite policies: in the first one, the regulator does not control prices, while in the second one he sets a price cap $\overline{\phi} = 1 - q_2$, i.e., the price cap which maximizes welfare in period 2. While the price control increases period 2 surplus, it lowers period 1 surplus. Actually, with no price control, $\pi^*(q_2)$ is constant on the interval $\left[\frac{1}{2}, 1\right]$ which by assumption contains all the perceived precisions the certifier may reach. It follows that its period-1 effort is $e_1^* = 0$. By contrast, with the price control, $\pi^*(q_2) = q_2(1 - q_2)$ which is strictly decreasing on $\left[\frac{1}{2}, 1\right]$. Accordingly, the certifier would like its reputation to deteriorate, hence a negative period-1 effort. This lowers the precision of period-1 certification, and in turn adversely impacts welfare (Section 3.2).

5.2 Certifier's effort

In our model, the certifier can affect the precision of its signal through costly effort. Throughout the paper, we assume that the costs of increasing and decreasing precision are symmetric, and that the impact of effort is transitory, i.e., limited to the current period. We discuss here the consequences of relaxing these assumptions.

First, suppose that the marginal cost of effort is $c_- e$ when e is negative and $c_+ e$ when e is positive with $c_- < c_+$, which captures the idea that dumbing down the signal is cheaper than improving its precision. To the extent that both c_+ and c_- are large enough that the reputation game still has a unique equilibrium (as in Proposition 2), introducing this cost asymmetry only makes bad reputation effects more pronounced: precision decreases more in the region where effort is negative than it increases in the region where effort is positive. If the cost of negative effort c_- becomes nil (or sufficiently small), multiple equilibria coexist in the region where effort is negative. Intuitively, when the market expects highly negative effort, it strongly updates beliefs upward following a signal $\sigma \in \{H, L\}$, which hurts the certifier in this region, and in turn justifies highly negative effort. Conversely, one can sustain an equilibrium with lower (in absolute value) negative effort. Eventually, the equilibrium effort retains some indeterminacy, but remains unambiguously negative when the reputation of the certifier is above some threshold.

Second, suppose that the impact of period-1 effort carries over to period 2. To fix ideas, assume that the precision in period 2 is given by $q_2 = \alpha + \eta e_1 + e_2$, where $\eta > 0$ parametrizes the persistence of effort. The predictions of the model are qualitatively unchanged. In particular, the sign of $e_1^*(\rho_1)$, hence the region in which reputation is bad, does not depend on η. Intuitively, while e_1 affects period-2 profits through its direct effect on reputation, the updating process on α is only based on the period-1 signal, and hence independent of η for a given effort e_1. In addition, at the reputation such that the equilibrium level of effort is $e_1^* = 0$ ($\overline{\rho}$ in Proposition 2), the persistence of effort is irrelevant. It follows that the equilibrium condition at $\overline{\rho}$ is independent of η, that is, the threshold above which effort is negative does not depend on the persistence of effort.[28]

6 Conclusion

Certification plays an essential role in markets where products are complex, information is costly to produce and unevenly distributed across market participants. In addition, regulatory oversight often relies on certifiers, as in the financial industry. A few certification intermediaries may accordingly exert a disproportionate influence on the allocation of resources, and the question of their ability and their incentives to produce accurate information is critical. In this paper, we argue that reputation in certification markets is two-sided, in that certifiers use their reputation to attract different types of sellers with conflicting preferences for precision. As a result, reputational concerns

[28]Notice that the magnitude of effort does depend on η because e_1 shifts the expected precision in period-2 not only through the reputation, but also directly. The marginal benefit of effort differs from the non-persistent case only through this extra effect. Therefore, whether a higher persistence of effort increases or decreases period 1 effort simply depends on the curvature of the certifier's profit function in period 2, $\pi^*(.)$.

may lead certifiers to curb precision in order to expand their (future) market, and competition between certifiers may actually worsen reputational incentives even further.

The idea that reputational concerns are multi-sided, in that they reflect the desire to attract different clienteles that exert an externality on each other, has other possible applications. In particular, two-sided markets where a platform connects two types of agents (e.g., media, operating systems, social networks) would constitute a natural application. While the literature on two-sided markets has mostly focused on the determinants of pricing on each side of the market (see, e.g., Rochet and Tirole (2006)), the current paper suggests that reputation might also play an important role in shaping the demand from groups of users who value each other's participation to a common platform, but might have different preferences or tastes. We believe this is a promising venue for future research.

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Appendix A: Monopoly

Proof of Lemma 1

A Perfect Bayesian Equilibrium is characterized by a triplet $\left(\overline{\lambda}, \gamma, P_M\right)$ such that:

$$\overline{\lambda} = \max\left\{0, q + \min\left\{(1-q)P_M - \phi, 0\right\}\right\} \tag{16}$$

$$\begin{cases} \gamma = 1 & \text{if } (1-q)P_M - \phi > 0, \\ \gamma \in [0,1] & \text{if } (1-q)P_M - \phi = 0, \\ \gamma = 0 & \text{otherwise.} \end{cases} \tag{17}$$

$$P_M = \frac{\overline{\lambda}}{\overline{\lambda} + (1-\overline{\lambda})\gamma} \text{ for } (\overline{\lambda}, \gamma) \neq (0,0) \tag{18}$$

(16) pins down the highest type who chooses high quality, and is derived from (4), while (17) states that types who choose low quality ($\lambda > \overline{\lambda}$) choose the certification strategy that maximizes their payoff given an equilibrium price P_M. Finally, (18) states that P_M derives from Bayes' rule whenever applicable.

The total demand D is given by

$$D = \overline{\lambda} + (1-\overline{\lambda})\gamma \tag{19}$$

Consider the following candidate equilibria:

1. $\overline{\lambda} = 0$.

   From (16), this implies $(1-q)P_M - \phi < 0$ and hence there is no demand for certification from sellers with low-quality products: $\gamma = 0$. If $\overline{\lambda} = \gamma = 0$ is an equilibrium strategy, then each seller type gets 0 in equilibrium, and the benefit from deviating to certification is $q + (1-q)P_M - \lambda - \phi$ if $\lambda \leq q$ and $(1-q)P_M - \phi$ if $\lambda > q$.[29] Hence the type who benefits the most from deviating is $\lambda = 0$, and if he deviates, he chooses high quality so that D1 imposes $P_M = 1$ out of equilibrium. If $\phi < 1$, then the deviation is profitable for types $\lambda < 1 - \phi$ while if $\phi \geq 1$, there is no profitable deviation. Therefore, $\overline{\lambda} = \gamma = 0$ can be part of an equilibrium if and only if $\phi \geq 1$.

2. $\overline{\lambda} > 0$ and $\gamma = 0$.

[29] From the discussion in the text, conditional on applying for certification (in or out of equilibrium), it is optimal to choose high quality if and only if $\lambda \leq q$.

If this is an equilibrium strategy, then from (18), $P_M = 1$. Furthermore, $\gamma = 0$ implies $(1-q)P_M - \phi = 1 - q - \phi \leq 0$, that is, $\phi \geq 1 - q$. Finally, using (16), $\overline{\lambda} > 0$ implies $\phi < 1$. Conversely, suppose $1 - q \leq \phi < 1$, then $\overline{\lambda} = 1 - \phi$, $\gamma = 0$, and $P_M = 1$ satisfy (16), (17) and (18).

3. $\overline{\lambda} > 0$ and $\gamma \in (0, 1)$.

   If this is an equilibrium strategy, then from (17), $(1-q)P_M - \phi = 0$, and from (16), $\overline{\lambda} = q$. Using (18), this implies $P_M = \frac{q}{q+(1-q)\gamma} = \frac{\phi}{1-q}$. Since $\gamma \in (0,1)$, one must have $q < P_M < 1$, hence $\phi \in (q(1-q), 1-q)$. Conversely, if $\phi \in (q(1-q), 1-q)$ then $\overline{\lambda} = q$, $\gamma = \frac{q(1-q-\phi)}{(1-q)\phi} \in (0,1)$, and $P_M = \frac{\phi}{1-q}$ satisfy (16), (17) and (18).

4. $\overline{\lambda} > 0$ and $\gamma = 1$.

   If this is an equilibrium strategy, then from (18), $P_M = \overline{\lambda}$. Using (17), $(1-q)P_M - \phi \geq 0$ which implies, using (16), $\overline{\lambda} = q$, hence $P_M = q$, and therefore $\phi \leq (1-q)q$. Conversely, if $\phi \leq q(1-q)$ then $\overline{\lambda} = q$, $\gamma = 1$, and $P_M = q$ satisfy (16), (17) and (18).

Overall, there is a unique equilibrium for all ϕ. □

Proof of Proposition 1

Since $\omega \leq \frac{1}{2}$, it is a weakly dominant strategy to set $\phi = \omega$. The expected profit of the certifier is as follows:
If $q \leq \frac{1}{2}$,

$$\pi(q) = \int_0^{q(1-q)} \omega f(\omega) d\omega + \int_{q(1-q)}^{\frac{1}{2}} q(1-q) f(\omega) d\omega.$$

If $q > \frac{1}{2}$,

$$\pi(q) = \int_0^{q(1-q)} \omega f(\omega) d\omega + \int_{q(1-q)}^{1-q} q(1-q) f(\omega) d\omega + \int_{1-q}^{\frac{1}{2}} \omega(1-\omega) f(\omega) d\omega.$$

$\pi(.)$ is continuous on $[0,1]$. In addition,

$$\pi'(q) = \int_{q(1-q)}^{\frac{1}{2}} (1-2q) f(\omega) d\omega \text{ if } q \leq \frac{1}{2}$$

and

$$\pi'(q) = \int_{q(1-q)}^{q} (1-2q) f(\omega) d\omega \text{ if } q > \frac{1}{2}.$$

Therefore, $\pi'(\frac{1}{2}^+) = \pi'(\frac{1}{2}^-) = 0$, $\pi'(q) > 0$ if $q < \frac{1}{2}$, and $\pi'(q) < 0$ if $q > \frac{1}{2}$. $\pi(.)$ is therefore continuously differentiable on $[0,1]$ and single-peaked with a maximum reached at $q = \frac{1}{2}$.

Proof of Proposition 2

Let

$$\hat{\pi}^*(\rho) \equiv \pi^* \left[q(\rho, 0)\right] \tag{20}$$

denote the period-2 profit of the certifier if it enters period 2 with a reputation ρ. Because q is strictly increasing in ρ, $q(0,0) = \alpha_L < \frac{1}{2}$ and $q(1,0) = \alpha_H > \frac{1}{2}$, $\hat{\pi}^*(.)$ inherits the properties of $\pi^*(.)$. In particular, it is hump-shaped and maximum for $\rho = \frac{1}{2}$ (since $\alpha_L = 1 - \alpha_H$, $q\left(\frac{1}{2}, 0\right) = \frac{1}{2}$).

The marginal impact of period-1 effort on the certifier's expected profit writes

$$L(\rho_1, e_1) \equiv \hat{\pi}^*[\rho^+(\rho_1, e_1)] - \hat{\pi}^*[\rho^-(\rho_1, e_1)] - ce_1, \tag{21}$$

where the dependence of $\rho^+(\rho_1, e_1)$ and $\rho^-(\rho_1, e_1)$ on ρ_1 is made explicit (see Eq. (9) and (10)). We let $L_i(.,.)$ denote the partial derivative of L with respect to the i-th variable.

Note that $\frac{\partial \rho^+}{\partial e_1}$, $\frac{\partial \rho^-}{\partial e_1}$ and $\frac{\partial \hat{\pi}^*}{\partial \rho}$ are bounded. Hence, we can make the following assumption:

Assumption A.1.

$$c > \sup_{\rho_1, e_1} \frac{\partial}{\partial e_1} \left\{ \hat{\pi}^*[\rho^+(\rho_1, e_1)] - \hat{\pi}^*[\rho^-(\rho_1, e_1)] \right\}.$$

Assumption A.1 guarantees that the certifier's optimization problem admits a unique solution. This solution is either $e_1^* = -\epsilon$ if $L(\rho_1, -\epsilon) < 0$, $e_1^* = \epsilon$ if $L(\rho_1, \epsilon) > 0$, or e_1^* such that $L(\rho_1, e_1^*) = 0$. Furthermore, since L is continuously differentiable in each argument, $e_1^*(\rho_1)$ is continuous in ρ_1.

Note that, for all $e_1, \rho^+(0, e_1) = \rho^-(0, e_1) = 0$ and $\rho^+(1, e_1) = \rho^-(1, e_1) = 1$. It follows that $e_1^*(0) = e_1^*(1) = 0$. Furthermore, when the solution to (11) is interior, we derive from the implicit function theorem:

$$\begin{aligned} \frac{\partial e_1^*}{\partial \rho_1}(\rho_1) &= -\frac{L_1[\rho_1, e_1^*(\rho_1)]}{L_2[\rho_1, e_1^*(\rho_1)]} \\ &= -\frac{\frac{\partial \hat{\pi}^*}{\partial \rho}[\rho^+(\rho_1, e_1^*(\rho_1))]\frac{\partial \rho^+}{\partial \rho_1}[\rho_1, e_1^*(\rho_1)] - \frac{\partial \hat{\pi}^*}{\partial \rho}[\rho^-(\rho_1, e_1^*(\rho_1))]\frac{\partial \rho^-}{\partial \rho_1}[\rho_1, e_1^*(\rho_1)]}{L_2(\rho_1, e_1^*)} \end{aligned}$$

Note that

$$\frac{\partial \rho^+}{\partial \rho_1}(\rho_1, e_1) = \frac{(\alpha_H + e_1)(\alpha_L + e_1)}{[\rho_1 \alpha_H + (1 - \rho_1)\alpha_L + e_1]^2} \geq 0$$

and

$$\frac{\partial \rho^-}{\partial \rho_1}(\rho_1, e_1) = \frac{(1-\alpha_H - e_1)(1-\alpha_L - e_1)}{[\rho_1(1-\alpha_H) + (1-\rho_1)(1-\alpha_L) - e_1]^2} \geq 0.$$

Since $L_2[\rho_1, e_1^*(\rho_1)] < 0$ (from Assumption A.1),

- $\frac{\partial e_1^*}{\partial \rho_1}(0)$ has the sign of $\frac{\partial \hat{\pi}^*}{\partial \rho}(0)\left[\frac{\partial \rho^+}{\partial \rho_1}(0,0) - \frac{\partial \rho^-}{\partial \rho_1}(0,0)\right] = \frac{\partial \hat{\pi}^*}{\partial \rho}(0)\left[\frac{\alpha_H}{\alpha_L} - \frac{1-\alpha_H}{1-\alpha_L}\right] \geq 0,$
- $\frac{\partial e_1^*}{\partial \rho_1}(1)$ has the sign of $\frac{\partial \hat{\pi}^*}{\partial \rho}(1)\left[\frac{\partial \rho^+}{\partial \rho_1}(1,0) - \frac{\partial \rho^-}{\partial \rho_1}(1,0)\right] = \frac{\partial \hat{\pi}^*}{\partial \rho}(1)\left[\frac{\alpha_L}{\alpha_H} - \frac{1-\alpha_L}{1-\alpha_H}\right] \geq 0.$

Since $e_1^*(\rho_1)$ is continuous in ρ_1, and $e_1^*(0) = e_1^*(1) = 0$, we derive that there exists at least one $\overline{\rho}$ such that $e_1^*(\overline{\rho}) = 0$.

Now, $L(\overline{\rho}, 0) = 0$ is equivalent to

$$\hat{\pi}^*[\rho^+(\overline{\rho}, 0)] = \hat{\pi}^*[\rho^-(\overline{\rho}, 0)],$$

with $\rho^+(\overline{\rho}, 0) > \rho^-(\overline{\rho}, 0)$. In turn, the single-peakedness of $\hat{\pi}^*(.)$ implies

$$\frac{\partial \hat{\pi}^*}{\partial \rho}[\rho^+(\overline{\rho}, 0)] < 0 < \frac{\partial \hat{\pi}^*}{\partial \rho}[\rho^-(\overline{\rho}, 0)],$$

and therefore

$$\frac{\partial e_1^*}{\partial \rho_1}(\overline{\rho}) = -\frac{\frac{\partial \hat{\pi}^*}{\partial \rho}[\rho^+(\overline{\rho}, 0)]\frac{\partial \rho^+}{\partial \rho_1}(\overline{\rho}, 0) - \frac{\partial \hat{\pi}^*}{\partial \rho}[\rho^-(\overline{\rho}, 0)]\frac{\partial \rho^-}{\partial \rho_1}(\overline{\rho}, 0)}{L_2(\overline{\rho}, 0)} < 0.$$

Therefore, by continuity of $e_1^*(\rho_1)$, $\overline{\rho}$ must be unique. Finally, the uniqueness of $\overline{\rho}$ and $\frac{\partial e^*}{\partial \rho_1}(\overline{\rho}) < 0$ imply $e_1^*(\rho_1) > 0$ for $\rho_1 \in (0, \overline{\rho})$ and $e_1^*(\rho_1) < 0$ for $\rho_1 \in (\overline{\rho}, 1)$. $\square$

Appendix B: Multiple Certifiers

Preliminary results

We start with definitions and auxiliary results which we use throughout Appendix B.

In this section, a seller can buy certification from certifier A, B or both A and B. We use the notation "AB" to refer to the latter case and let

$$q_{AB} \equiv q_A + q_B(1 - q_A) \text{ and } \phi^{AB} \equiv \phi^A + \phi^B$$

denote the aggregate precision and cost of the certification process. Let P_M^z denote the product price following certificate $\sigma^z = M$ issued by certifier $z \in \{A, B, AB\}$. Since a single signal H or L perfectly identifies product quality, $\sigma^{AB} = M$ means $\sigma^A = M$ *and* $\sigma^B = M$, while $\sigma^{AB} \in \{H, L\}$ means $\sigma^A \in \{H, L\}$ *or* $\sigma^B \in \{H, L\}$.

Let us first prove that the equilibrium features a cutoff type $\overline{\lambda}$ such that all types below $\overline{\lambda}$ choose high quality.

Consider a seller with cost $\tilde{\lambda}$ who invests in quality. Since it is pointless to invest in quality without soliciting certification, $\tilde{\lambda}$ must go to a certifier, say $i \in \{A, B, AB\}$, with positive probability. One must have therefore

$$q_i + (1 - q_i)P_M^i - \phi^i - \tilde{\lambda} \geq \max\left\{(1 - q_i)P_M^i - \phi^i, (1 - q_j)P_M^j - \phi^j, q_j + (1 - q_j)P_M^j - \phi^j - \tilde{\lambda}, 0\right\}$$

for all $j \in \{A, B, AB\} \setminus \{i\}$.

This implies $q_i + (1 - q_i)P_M^i - \phi^i \geq q_j + (1 - q_j)P_M^j - \phi^j$, which implies in turn

$$q_i + (1 - q_i)P_M^i - \phi^i - \lambda \geq \max\left\{(1 - q_i)P_M^i - \phi^i, (1 - q_j)P_M^j - \phi^j, q_j + (1 - q_j)P_M^j - \phi^j - \lambda, 0\right\}$$

for all $\lambda \leq \tilde{\lambda}$ and $j \in \{A, B, AB\} \setminus \{i\}$.

Accordingly, all types $\lambda < \tilde{\lambda}$ also prefer to invest in quality. This proves the existence of a cutoff type $\overline{\lambda}$ such that all $\lambda < \overline{\lambda}$ hold high-quality products.

We now show how D1 restricts out-of-equilibrium beliefs.

Lemma B.1. *Consider an equilibrium characterized by a cutoff type $\overline{\lambda}$ such that all types below $\overline{\lambda}$ hold high-quality products (regardless of their certification strategies), and consider a certifier*

$j \in \{A, B, AB\}$ such that no seller solicits j. D1 imposes $P_M^j = 1$ if $q_j > \overline{\lambda}$ and $P_M^j = 0$ if $q_j < \overline{\lambda}$.

Proof. Conditional on deviating to j, types $\lambda \leq q_j$ choose high quality, while types $\lambda > q_j$ choose low quality. Accordingly, the deviation to j generates a payoff of

$$\max\{q_j - \lambda, 0\} + (1 - q_j)P_M^j - \phi^j \tag{22}$$

The equilibrium payoff of a type $\lambda \leq \overline{\lambda}$ is (say that he goes to $i \in \{A, B, AB\} \setminus \{j\}$ with positive probability):

$$q_i + (1 - q_i)P_M^i - \phi^i - \lambda.$$

The equilibrium payoff of a type $\lambda \geq \overline{\lambda}$ is

$$\underline{W} \equiv \max_{z \in \{A,B,AB,\varnothing\}} (1 - q_z)P_M^z - \phi^z,$$

where by convention $q_\varnothing = P_M^\varnothing = \phi^\varnothing = 0$.

By definition of $\overline{\lambda}$, this equilibrium payoff is continuous at $\lambda = \overline{\lambda}$, i.e. $q_i + (1 - q_i)P_M^i - \phi^i - \overline{\lambda} = \underline{W}$.

1. Suppose $\overline{\lambda} < q_j$. The benefit from the deviation reads

   (a) $q_j + (1 - q_j)P_M^j - \phi^j - q_i - (1 - q_i)P_M^i + \phi^i$ for $\lambda < \overline{\lambda}$;

   (b) $q_j - \lambda + (1 - q_j)P_M^j - \phi^j - \underline{W}$ for $\lambda \in (\overline{\lambda}, q_j)$;

   (c) $(1 - q_j)P_M^j - \phi^j - \underline{W}$ for $\lambda \geq q_j$.

   This benefit from the deviation is continuous and nonincreasing in λ, and decreasing over the range $(\overline{\lambda}, q_j)$, so it is (strictly) highest for $\lambda \leq \overline{\lambda}$. Therefore, D1 imposes to consider that the deviation comes from a low-cost type holding a good product: $P_M^j = 1$.

2. Suppose $\overline{\lambda} > q_j$. The benefit from the deviation reads

   (a) $q_j + (1 - q_j)P_M^j - \phi^j - q_i - (1 - q_i)P_M^i + \phi^i$ for $\lambda < q_j$;

   (b) $(1 - q_j)P_M^j - \phi^j - q_i - (1 - q_i)P_M^i + \phi^i + \lambda$ for $\lambda \in (q_j, \overline{\lambda})$;

   (c) $(1 - q_j)P_M^j - \phi^j - \max\{(1 - q_i)P_M^i - \phi^i, 0\}$ for $\lambda \geq \overline{\lambda}$.

   In that case, the benefit from the deviation is continuous and nondecreasing in λ, and increasing over the range $(q_j, \overline{\lambda})$, so it is (strictly) highest for $\lambda \geq \overline{\lambda}$. Therefore, D1 imposes to consider that the deviation comes from a high-cost type holding a bad product: $P_M^j = 0$. □

Lemma B.2. *If certifier $i \in \{A, B\}$ is more precise than certifier j ($q_i > q_j$), then certifier i makes a positive profit in equilibrium.*

Proof. To show that i can always secure a positive profit, we show that there is no equilibrium such that i is inactive when $0 < \phi^i < (1-q_j)^2$.

Suppose that $0 < \phi^i < (1-q_j)^2$ and there exists an equilibrium where i is inactive. This implies $\overline{\lambda} \leq q_j < q_i$ and therefore, from Lemma B.1, $P^i_M = 1$.

There are two cases:

1. $\phi^j < (1-q_j)q_j$. Then the equilibrium payoff of types $\lambda \leq \overline{\lambda}$ is $q_j + (1-q_j)q_j - \phi^j - \lambda$. By deviating to i, they would get $1 - \phi^i - \lambda$. This is strictly profitable if $\phi^i < \phi^j + (1-q_j)^2$, which is always true when $0 < \phi^i < (1-q_j)^2$.

2. $\phi^j \geq (1-q_j)q_j$. Then the equilibrium payoff of types $\lambda \leq \overline{\lambda}$ is $\overline{\lambda} - \lambda$. The payoff from deviating to i is $1 - \phi^i - \lambda$. This is a strictly profitable deviation if $\phi^i < 1 - \overline{\lambda}$. Since $\phi^i < (1-q_j)^2 < 1 - q_j \leq 1 - \overline{\lambda}$, this is always the case as well. □

Finally, the next result illustrates how competition constrains pricing.

Lemma B.3. *If certifier $i \in \{A, B\}$ is more precise than certifier j ($q_i > q_j$), then $\phi^i \leq 1 - q_j$ in equilibrium.*

Proof. Suppose $\phi^i > 1 - q_j$ in equilibrium. First of all, if i gets 0 profit by charging $\phi^i > 1 - q_j$, then such pricing cannot be an equilibrium. Indeed, from Lemma B.2, i can always secure a positive profit. Let us then suppose that there is an equilibrium where i charges $\phi^i > 1 - q_j$ and is active, i.e., some sellers apply to $k \in \{i, ij\}$. Note that $\phi^k \geq \phi^i > 1 - q_j > 1 - q_k$, so that a seller who applies to k holds a high-quality product, and therefore $P^k_M = 1$.
Let us examine the possible configurations:

1. Some sellers with a high-quality product solicit j only.

   (a) Some seller types with a high-quality product also solicit i. The indifference condition for these high-quality sellers reads $1 - \phi^i - \lambda = q_j + (1-q_j)P^j_M - \phi^j - \lambda \Leftrightarrow \phi^i = \phi^j + (1-q_j)(1-P^j_m)$. This cannot be an equilibrium in the fee-setting game, as i would have an incentive to undercut and capture j's share of the market.

(b) Some seller types with a high-quality product multihome, i.e., solicit i and j. The indifference condition reads $1-\phi^{ij}-\lambda = q_j+(1-q_j)P_M^j-\phi^j-\lambda \Leftrightarrow \phi^i = 1-q_j-(1-q_j)P_M^j$, which contradicts $\phi^i > 1-q_j$.

2. No seller with a good product applies to j only in equilibrium. There are two subcases:

   (a) Some sellers go to j only, but they hold low-quality products. This implies $P_M^j = 0$, and $\phi^j = 0$ is a necessary condition. However, $\phi^j = 0$ cannot be a best response, as sellers with $\lambda \in (1-\phi^k, q_j)$ would be willing to pay a positive price for certification from j only, as it is then optimal for them to invest in quality.

   (b) No seller ever goes to j only.

   Since $\overline{\lambda} = 1-\phi^k \leq 1-\phi^i < q_j$, one has $P_M^j = 1$ out of equilibrium. For any $\lambda \leq \overline{\lambda}$, the net benefit of deviating to j is $1-\phi^j-(1-\phi^k) = \phi^k-\phi^j$. If $k = ij$, the benefit from deviating is $\phi^i > 0$. If $k = i$, it is $\phi^i - \phi^j$, so j can secure a positive profit by setting $\phi^j \in (0, \phi^i)$. In either case, there is a profitable deviation, so this cannot be an equilibrium. □

Proof of Lemma 2

Assume A is the more precise certifier, $q_A > q_B$ (the results are symmetric if $q_B > q_A$). From Lemma B.2, if a singlehoming equilibrium exists, then A is active. There are two cases:

1. $\omega < 1 - q_B$

Suppose that A charges the monopoly price $\phi^A = \omega$, and B charges a positive price $\phi^B > 0$.

If A is the only active certifier in equilibrium, then $\overline{\lambda} = \min\{q_A, 1-\omega\} > q_B$, which, from Lemma B.1, implies $P_M^B = 0$. Furthermore, by definition, $\overline{\lambda} = \min\left\{q_A + (1-q_A)P_M^A - \phi^A, q_A\right\} \leq q_A + (1-q_A)P_M^A - \phi^A$.

Using $P_M^B = 0$, the payoff of a seller $\lambda > q_B$ who deviates to B (with a low-quality product) is $0 - \phi^B < 0$. For types $\lambda \leq q_B < \overline{\lambda}$, the deviation to certifier B yields a net benefit of $q_B - \phi^B - [q_A + (1-q_A)P_M^A - \phi^A] \leq q_B - \phi^B - \overline{\lambda} < 0$. Therefore, there is no profitable deviation to B. Finally, because $\phi^A = \omega$ and $\phi^B > 0$, a seller cannot apply to both A and B, which rules out a deviation to AB. Overall, there exists an equilibrium of the subgame in which $\phi^A = \omega$ and $\phi^B > 0$ such that A earns its monopoly profit. Since this is the highest profit which A (i.e., the more precise certifier) can reach under singlehoming, and since multihoming cannot be an equilibrium ($\phi^A + \phi^B > \omega$), this is the equilibrium we select.

It follows that $\phi^A = \omega$ and $\phi^B > 0$ is also an equilibrium of the full game including the fee-setting stage, as neither of the two certifiers has a strictly profitable deviation. Finally, suppose there exists a singlehoming equilibrium in which A does not earn its monopoly profit. It has to be such that $\phi^B > 0$ or $P_M^A = 1$. Indeed, if $\phi^B = 0$ and $P_M^A < 1$ since $\overline{\lambda} \leq q_A < q_{AB}$, then $p_M^{AB} = 1$ so that then AB (multihoming) is a strictly profitable deviation for any $\lambda < \overline{\lambda}$. But, if $\phi^B > 0$, A can reach its monopoly profit by setting $\phi^A = \omega$ (see above). Likewise, if there exists a singlehoming equilibrium such that $P_M^A = 1$, then $1 - q_A \leq \phi^A \leq \omega$ and $A's$ profit is $\phi^A(1 - \phi^A)$. In this case, A could increase its profit by setting $\phi^A = \omega \leq \frac{1}{2}$, which yields the monopoly profit.

2. $\omega \geq 1 - q_B$

Since $\omega \leq \frac{1}{2}$, $\omega \geq 1 - q_B$ implies that q_B and therefore q_A are larger than $\frac{1}{2}$. From the analysis of the monopoly case, the highest revenue that certifiers can jointly achieve when A is active and the precision of the certification process is higher than $\frac{1}{2}$ is $\phi^k(1 - \phi^k)$ where $k \in \{A, AB\}$ is the active certifier(s). In this case, A's profit is $\phi^A(1 - \phi^k)$. Since, from Lemma B.3, $\phi^A \leq 1 - q_B \leq \frac{1}{2}$, A's profit is bounded above by $(1 - q_B)q_B$.

Consider a subgame where $\phi^A = 1 - q_B > 1 - q_A$. If A is the only active certifier, then $\overline{\lambda} = 1 - \phi^A = q_B$. It follows that $P_M^B = 0$ and for any $\lambda \leq q_B$, the net benefit of deviating to B is $q_B - \phi^B - \overline{\lambda} = -\phi^B \leq 0$.[30] In addition, $\overline{\lambda} = 1 - \phi^A < q_{AB}$, so that $P_M^{AB} = 1$ and for any $\lambda \leq \overline{\lambda}$, the net benefit of deviating to AB is $1-(\phi^A+\phi^B)-(1-\phi^A) \leq 0$. Therefore, if $\phi^A = 1-q_B$, there exists a singlehoming equilibrium such that A earns its highest possible profit $q_B(1-q_B)$. Therefore, this is the equilibrium of this subgame which we select. Finally, since A (the more precise certifier) can reach its maximim profit by setting $\phi^A = 1 - q_B$ irrespective of ϕ^B, this is the also the unique equilibrium in fee-setting game.

Summarizing points 1. and 2. above, if $q_A > q_B$, singlehoming is an equilibrium, and certifiers' payoffs are uniquely defined: B earns 0 and

- if $\omega < 1 - q_B$, A earns its monopoly profit $\pi^*(q_A)$;
- if $\omega \geq 1 - q_B$, A earns $q_B(1 - q_B) < \omega(1 - \omega)$: A's profit is lower than the monopoly profit.

[30] Note that Lemma B.1 above imposes $P_M^B = 0$ if $\overline{\lambda} < q_B$ and we assume here that this out-of-equilibrium belief extends to the broader case where $\overline{\lambda} \leq q_B$. This only simplifies the exposition: we could indeed consider $\phi^A = 1-q_B-\varepsilon$ for ε arbitrarily small, in which case that $\overline{\lambda} < q_B$ is satisfied.

Integrating over ω on $[0, \frac{1}{2}]$, one has, for all $q_B \geq \frac{1}{2}$,

$$\delta(q_B) \equiv \int_{1-q_B}^{\frac{1}{2}} [\omega(1-\omega) - q_B(1-q_B)] f(w) dw, \tag{23}$$

which is increasing in q_B. □

Proof of Proposition 3

In the same way we defined $\hat{\pi}^*(\rho)$ in the monopoly case (see Eq. (20)), let

$$\hat{\pi}^{sh}(\rho, q_B) \equiv \pi^{sh}\left[q(\rho, 0), q_B\right] \tag{24}$$

denote the period-2 profit of the incumbent A if it enters period 2 with a reputation ρ and the entrant has a reputation q_B.

The incumbent certifier A with reputation ρ_A selects e_1^{sh} which maximizes

$$\max_{e_1 \in [-\varepsilon, \varepsilon]} q(\rho_A, e_1) \int_0^1 \hat{\pi}^{sh}(\rho^+(\rho_A, e_1^{sh}), q_B)\, dG(q_B) + [1 - q(\rho_A, e_1)] \int_0^1 \hat{\pi}^{sh}(\rho^-(\rho_A, e_1^{sh}), q_B)\, dG(q_B) - c\frac{e_1^2}{2}. \tag{25}$$

Since G is continuous, this objective function is continuous in e_1, so the problem admits a solution.

Suppose ρ_A is such that $e_1^*(\rho_A) < 0$, and consider $e_1 \leq e_1^*(\rho_A)$.

1. Suppose $q_B > q[\rho^-(\rho_A, e_1), 0]$. In that case, A's period-2 profit is $\hat{\pi}^{sh}[\rho^-(\rho_A, e_1), q_B] = 0$ if $\sigma^A = M$. Since $e_1 < 0$, the certifier marginal benefit of effort is

$$\hat{\pi}^{sh}[\rho^+(\rho_A, e_1), q_B] - \hat{\pi}^{sh}[\rho^-(\rho_A, e_1), q_B] - ce_1 > 0.$$

2. Suppose $q_B < \min\{\frac{1}{2}, q[\rho^-(\rho_A, e_1), 0]\}$. Then

$$\begin{aligned}
& \hat{\pi}^{sh}[\rho^+(\rho_A, e_1), q_B] - \hat{\pi}^{sh}[\rho^-(\rho_A, e_1), q_B] - ce_1 \\
= \;& \hat{\pi}^*[\rho^+(\rho_A, e_1)] - \hat{\pi}^*[\rho^-(\rho_A, e_1)] - ce_1 \\
= \;& L(\rho_A, e_1) \\
\geq \;& 0, \text{ using } e_1 \leq e_1^*(\rho_A) < 0.
\end{aligned}$$

3. Finally, suppose $\frac{1}{2} < q_B < q[\rho^-(\rho_A, e_1), 0]$. Then

$$\begin{aligned}
& \hat{\pi}^{sh}[\rho^+(\rho_A, e_1), q_B] - \hat{\pi}^{sh}[\rho^-(\rho_A, e_1), q_B] - ce_1 \\
= \; & \left\{\hat{\pi}^*[\rho^+(\rho_A, e_1), q_B] - \delta(q_B)\right\} - \left\{\hat{\pi}^*[\rho^-(\rho_A, e_1), q_B] - \delta(q_B)\right\} - ce_1 \\
= \; & \hat{\pi}^*[\rho^+(\rho_A, e_1)] - \hat{\pi}^*[\rho^-(\rho_A, e_1)] - ce_1 \\
\geq \; & 0.
\end{aligned}$$

It follows that, if $e_1 \leq e_1^*(\rho_A) < 0$,

$$\int_0^1 \left\{\hat{\pi}^{sh}[\rho^+(\rho, e_1), q_B] - \hat{\pi}^{sh}[\rho^-(\rho, e_1), q_B]\right\} dG(q_B) - ce_1 > 0,$$

that is, the marginal incentive to exert effort is strictly positive for any $e_1 \leq e_1^*(\rho_A)$. This, in turn, implies that the equilibrium effort $e_1^{sh}(\rho_A)$ is strictly higher than $e_1^*(\rho_A)$.

Proof of Lemma 3

Suppose $q_A > q_B$. We show here that there always exists a multihoming equilibrium such that $\phi^A = \min\{\omega, 1 - q_B\}$ and $\phi^B = 0$.[31] When the arguments are similar to those used in the derivation of the singlehoming equilibrium (see Lemma 2), we omit some details.

1. $\omega < 1 - q_A$

Consider a subgame in which $\phi^A \leq \omega$ and $\phi^B = 0$.

Suppose any seller who applies for certification solicits A and B. Then $\overline{\lambda} = \min\{q_{AB}, 1 - \phi^A\} > q_A > q_B$, which implies $P_M^B = P_M^A = 0$, using Lemma B.1. This, in turn, makes a deviation to singlehoming with A or B unprofitable for any λ. It follows that multihoming is an equilibrium of the subgame.

Suppose there exists an equilibrium in which some types with high-quality products apply to $i \in \{A, B\}$ only. There are three possibilities.

1. Some seller types with a high-quality product apply to A and B, i.e., multihome. These sellers have to be indifferent between i and AB, i.e., $q_i + (1 - q_i)P_M^i = q_{AB} + (1 - q_{AB})P_M^{AB}$, which implies $P_M^i = P_M^{AB} = 1$ or $P_M^i > P_M^{AB}$. If $P_M^i > P_M^{AB}$, then any seller with a low-quality

[31] Remember that we defined in the text a "multihoming" equilibrium as one in which *every* seller who applies for certification solicits both certifiers.

product is better off applying to i only. This, in turn, implies $P_M^{AB} = 1$, a contradiction. If $P_M^i = P_M^{AB} = 1$, then since $\phi^i < 1 - q_i$, applying to i is always profitable for a low-quality seller, hence $P_M^i < 1$, a contradiction.

2. Some seller types apply to A and B, i.e., multihome, but they hold low-quality products. This implies $P_M^{AB} = 0$, so a necessary condition to have this is $\phi^A = \phi^B = 0$. If this is an equilibrium, it is dominated by the multihoming equilibrium for A and therefore ruled out.

3. No seller applies to A and B and the equilibrium therefore involves singlehoming:
   - Singlehoming with A is such that $\phi^A < 1 - q_A$, so that $P_M^A < 1$, and $\overline{\lambda} = q_A < q_{AB}$, so that $P_M^{AB} = 1$. This, in turn, makes a deviation to AB profitable for any $\lambda \leq q_A$.
   - Singlehoming with B would generate zero profit for A, which is dominated by the multihoming equilibrium for A and therefore ruled out.

In sum, if $\phi^A < 1 - q_A$ and $\phi^B = 0$, the equilibrium we select in the subgame must involve multihoming. Taking $\phi^B = 0$ as given, A's problem is then similar to monopoly pricing but with a precision q_{AB} instead of q_A. Therefore, A's best response to $\phi^B = 0$ in the range $[0, \omega] \subset [0, \frac{1}{2}]$ is to raise its price as much as possible, i.e., set $\phi^A = \omega$, which generates a profit $\pi^{sh}(q_{AB}, q_B) = \pi^*(q_{AB})$ (see Lemma 2). Conversely, suppose $\phi^A = \omega$ and B deviates by posting $\phi^B > 0$, then $\phi^A + \phi^B > \omega$ implies that the equilibrium has to be singlehoming, and using Lemma 2, the revenue of certifier B is 0. Hence B has no strictly profitable deviation.

This proves the existence of a multihoming equilibrium such that $\phi^A = \omega$ and $\phi^B = 0$.

2. $\omega > 1 - q_A$

From Lemma B.3, $\phi^A \leq 1 - q_B$. Since we know from the analysis of the case $\omega < 1 - q_A$ what the equilibrium is in subgames where $\phi^A < 1 - q_A$ and $\phi^B = 0$, we directly analyze subgames where $1 - q_A \leq \phi^A \leq 1 - q_B$ and $\phi^B = 0$.

Suppose a seller who applies for certification always solicits A and B. Then $\overline{\lambda} = 1 - \phi^A > q_B$ which, from Lemma B.1 implies $P_M^B = 0$. This, in turn, makes a deviation to singlehoming with B unprofitable for any λ. On the other hand, for any P_M^A, a deviation to A generates a profit $q_A + (1 - q_A)P_M^A - \min\{q_A, \lambda\} - \phi^A$ which is weakly lower than sticking to AB (and get $1 - \phi^A$) if $\lambda \leq 1 - \phi^A$, and weakly negative if $\lambda > 1 - \phi^A$. It follows that multihoming is an equilibrium of the subgame, such that A's profit is $\phi^A(1 - \phi^A)$ and B's profit is 0.

As above there cannot be an equilibrium of the subgame in which some sellers apply to B only. There does exist an equilibrium such that sellers apply to A only but this equilibrium generates the same outcome as the multihoming equilibrium: $\overline{\lambda} = 1 - \phi^A$, types above $\overline{\lambda}$ never apply for certification, A's profit is $\phi^A(1-\phi^A)$ and B's profit is 0.

In sum, if $1 - q_A \leq \phi^A \leq 1 - q_B$ and $\phi^B = 0$, the equilibrium of the subgame is (equivalent to) multihoming. It follows from the analysis of the monopoly case that A's best response to $\phi^B = 0$ in the $[0, \frac{1}{2}]$ range is to raise its price as much as possible, i.e., set $\phi^A = \min\{\omega, 1 - q_B\}$, which generates the revenue $\pi^{sh}(q_{AB}, q_B)$ (see Lemma 2). Conversely, if $\phi^A = \min\{\omega, 1 - q_B\}$ and B deviates by quoting $\phi^B > 0$, then, from the proof of Lemma 2, the revenue of certifier B is 0. It follows that B does not have a profitable deviation.

This proves the existence of a multihoming equilibrium such that $\phi^A = \min\{\omega, 1 - q_B\}$ and $\phi^B = 0$.

Proof of Proposition 4

Under Assumption 2, we obtain analytical expressions for $\pi^{mh}(q_A, q_B)$ and for its partial derivatives.

The monopoly profit reads:

If $q \geq \frac{1}{2}$,

$$\begin{aligned}
\pi^*(q) &= \int_0^{q(1-q)} 2\omega d\omega + \int_{q(1-q)}^{1-q} 2q(1-q) d\omega + \int_{1-q}^{\frac{1}{2}} 2\omega(1-\omega) d\omega \\
&= -(1-q)^4 + \frac{2(1-q)^3}{3} + \frac{1}{6}
\end{aligned}$$

If $q < \frac{1}{2}$,

$$\begin{aligned}
\pi^*(q) &= \int_0^{q(1-q)} 2\omega d\omega + \int_{q(1-q)}^{1/2} 2q(1-q) d\omega \\
&= q(1-q)\left(1 - q(1-q)\right).
\end{aligned}$$

In addition,

$$\delta(q_B) = 2 \int_{1-q_B}^{\frac{1}{2}} \left[\omega(1-\omega) - q_B(1-q_B)\right] d\omega = -(2q_B - 1)q_B(1-q_B) - (1-q_B)^2 + \frac{2}{3}(1-q_B)^3 + \frac{1}{6}.$$

Given the definition of π^{mh}, one can check that

$$\frac{\partial^2 \pi^{mh}}{\partial q_A \partial q_B} = \begin{cases} 2(1-q_A)^2(1-q_B)^2(3-8(1-q_A)(1-q_B)) & \text{if } q_{AB} < \frac{1}{2}, \\ -16(1-q_A)^3(1-q_B)^3 + 18(1-q_A)^2(1-q_B)^2 - 8(1-q_A)(1-q_B) + 1 & \text{if } q_{AB} \geq \frac{1}{2}. \end{cases}$$

This implies that

$$\frac{\partial^2 \pi^{mh}}{\partial q_A \partial q_B} < 0 \Leftrightarrow q_{AB} = q_A + q_B - q_A q_B < 5/8, \tag{26}$$

which, recalling that in period 2 $q_A = \rho_A \alpha_H + (1-\rho_A)\alpha_L$, is true for any $\rho_A \in [0,1]$ and any $q_B \in [\underline{q}_B, \overline{q}_B]$ under Assumption 3.

As usual, let

$$\hat{\pi}^{mh}(\rho, q_B) \equiv \pi^{mh}\left[q(\rho, 0), q_B\right] \tag{27}$$

denote the period-2 profit of the incumbent A if it enters period 2 with a reputation ρ and the entrant has a reputation q_B.

The incumbent certifier A with reputation ρ_A selects e_1^{mh} which maximizes

$$\max_{e_1 \in [-\varepsilon, \varepsilon]} q(\rho_A, e_1) \int_0^1 \hat{\pi}^{mh}(\rho^+(\rho_A, e_1^{mh}), q_B)\, dG(q_B) + [1 - q(\rho_A, e_1)] \int_0^1 \hat{\pi}^{mh}(\rho^-(\rho_A, e_1^{mh}), q_B)\, dG(q_B) - c\frac{e_1^2}{2}. \tag{28}$$

G is continuous, so this objective function is continuous in e_1, and the problem admits a solution.

Certifier A's marginal benefit of effort is

$$\tilde{L}(\rho_A, e, q_B) \equiv \pi^{mh}(q(\rho^+(\rho_A, e)), q_B) - \pi^{mh}(q(\rho^-(\rho_A, e)), q_B) - ce. \tag{29}$$

If $q_B = 0$, $\tilde{L}(\rho_A, e, 0)$ is the marginal benefit of effort under monopoly, $L(\rho_A, e)$ (see Eq. (21)). Hence, the difference between marginal benefits of effort under multihoming and monopoly is

$$\begin{aligned} \tilde{L}(\rho_A, e, q_B) - \tilde{L}(\rho_A, e, 0) &= \pi^{mh}(q(\rho^+(\rho_A, e)), q_B) - \pi^{mh}(q(\rho^-(\rho_A, e)), q_B) \\ &- \left[\pi^{mh}(q(\rho^+(\rho_A, e)), 0) - \pi^{mh}(q(\rho^-(\rho_A, e)), 0)\right] \\ &= \int_{q(\rho^-(\rho_A, e))}^{q(\rho^+(\rho_A, e))} \int_0^{q_B} \frac{\partial^2 \pi^{mh}}{\partial q_A \partial q_B}(r, s)\, dr\, ds \end{aligned} \tag{30}$$

which is strictly negative since (26) is verified. Since this holds pointwise for any $q_B \in [\underline{q}_B, \overline{q}_B]$, it is also true integrating over q_B. Accordingly, the marginal incentive to provide effort is lower under multihoming than monopoly, which proves the result. □