Selection, Separation, and Unemployment

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Abstract

High-skill workers have a lower unemployment rate than their low-skill counterparts. This is because high-skill workers are less likely to become unemployed, not because they are more likely to find a job. This paper proposes an explanation for the skill discrepancy in likelihood of becoming unemployed: high-skill workers are less likely to become unemployed because they are selected more effectively during their hiring process. I use a labor search model with match specific quality to show that skill bias in employee selection practices can account for the differences in job separation probabilities and unemployment rates across skill groups.

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This paper explores the differences in job separation probabilities (probability of becoming unemployed) across skill groups. As is well documented, low-skill workers (workers without four years of a college degree) have a higher unemployment rate than high-skill workers. However, it is less well-known that the difference in unemployment rate by skill is due to the disparity in the job separation probabilities. This paper provides evidence and an explanation for this disparity.

I use the Current Population Survey (CPS) data to document the skill differences in unemployment experiences.[1] I find that low-skill workers have, on average, more than twice the unemployment rate of high-skill workers. The difference in the job finding probability between skill groups cannot cause the disparity in their unemployment rates as low skill workers have a higher average. Furthermore, low-skill workers are approximately three times more likely to become unemployed, compared to high-skill workers. Hence, the dispersion in the job separation probability by skill is the basis of the differences in unemployment rates these skill groups have.

Table 1: Data Summary

	Unemployment Rate	**Job Finding Probability**	**Separation Probability**
All	.048	.35	.021
	(.01)	(.06)	(.003)
Low-skill*	.058	.36	.026
	(.02)	(.06)	(.004)
High-Skill	.023	.32	.009
	(.005)	(.08)	(.002)

Monthly averages of prime age male data between 1976 and 2008.
* Workers with less than a college degree.

One should note that there are several possible reasons contributing to the gap. One such explanation is the following: firms with high-skill vacancies incur a higher opportunity cost as a result of failing to hire the best workers for the job. Consequently, firms who are looking for high-skill workers follow more effective hiring strategies and processes than they would have done when searching for

[1] The data is from January-1976 to December-2008. The details of the data and the calculations are discussed in more detail in the section 1.1.

low-skill workers. As a result of the more effective selection, high-skill vacancies get higher quality matches, which are less likely to be terminated.

I use a discrete time infinite horizon labor matching model with heterogenous agents to analyze the above hypothesis systematically. There are two different skill groups in the economy. High-skill workers can only produce at high-skill jobs while low-skill workers can only be hired by low-skill firms. Firms choose their skill type before entering the market and they can employ at most one worker. For each skill group, a match between a firm and a worker can be either of good or bad quality. The only difference between skill groups is that the difference between output produced in a good quality and a bad quality match is larger in high-skill matches than low-skill matches. The true quality of the match is revealed after the parties observe the output. Firms and workers learn about the probability of the match being good quality before they form the match, and they decide whether to form the match and produce or to continue searching. The probability of a good quality match is drawn from a distribution, which is the selection technology. There are two technologies available: a costless employee selection technology and a more effective but costly employee selection technology. The more effective technology has a higher likelihood of delivering high quality matches.

I calibrate the model to match data facts of the U.S. labor markets. In equilibrium, only high-skill firms employ the more effective selection technology. As a result, high-skill firms get many high quality matches, resulting in a lower job separation rate. The model also delivers, although not targeted directly, the observed magnitudes of the job separation probability disparities across skill groups. To illustrate that the job separation probability gap is a result of differences in selection technologies used, I compute an equilibrium in which there is only one type of selection technology In this equilibrium all skill groups use the same selection procedure and the difference between high- and low-skill job separation probabilities is tiny compared to the data.

Although there other studies that provide evidence for the job separation probability disparity across skill, papers with explanations for the disparity are scarce. The former are presented in section 1.1 whereas the latter are discussed now. Nagypal (2007) focuses on the difference in levels and the standard deviations of unemployment rate across skill groups. She explains that the existence of match specific capital (information about the quality of a match) for high-skill workers and the lack of such capital for low-skill workers make low-skill employment relations more vulnerable to adverse idiosyncratic shocks. Nagypal (2007) uses a matching model with firms employing both high- and low-skill workers, and with uncertainty regarding match quality for high-skill jobs. Firms' decisions to terminate a high-skill match, when faced with an adverse idiosyncratic shock, depends on the accumulated information about the quality of that match. Nagypal constructs a numerical example to show that differences in learning about the match quality can generate differences in unemployment and job separation rates across skill groups.

This paper is complementarity to Nagypal (2007) in the following way. Both papers agree that there is uncertainty regarding the match quality. There are two channels through which firms and workers can learn about the quality of their match. First, firms and workers can extract some information regarding the quality of the match before they form the employment relationship. Second, they can form the employment relationship and learn over time. This paper looks at the former channel of learning while Nagypal (2007) looks at the latter.[2]

The rest of the paper is organized as follows. The following section of the paper provides recent evidence on the unemployment experiences and hiring policies regarding different skill groups. Section 2 lays out the model. The equilibrium of the model is defined and analyzed in section 3. Section 4 presents the quantitative results of the model and is followed by concluding remarks.

1 Data On Unemployment and Hiring Processes

In this section, I present evidence for the disparity in job separation probabilities and hiring processes. First, I explain the data on unemployment experiences of skill groups in detail. Then, I discuss existing empirical evidence for the presence of differences in hiring processes by skill.

1.1 Unemployment Facts

To document the skill differences in unemployment rate, I use the CPS data from January 1976 to December 2008.[3] I focus on prime age (between 25 and 54)

[2]Relative importance of these forms of learning remains an open question.

[3]The CPS is a monthly survey of about 50,000 households. It is the primary source of information on the labor force characteristics of the U.S. population. The CPS is

males since they have the strongest labor market attachment among labor market participants. Same calculations for the overall labor force are displayed in the appendix. Following the standard definition for "skill" in the literature, I use education level as a proxy. Workers without a four-year college degree are low-skill workers while those with at least 16 years of education (a four-year college degree) are high-skill workers.

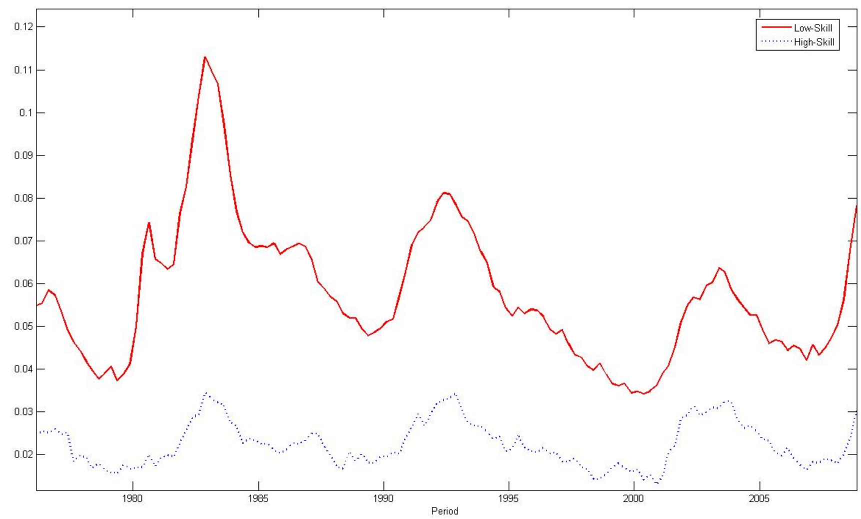

Figure 1: Unemployment Rates by Skill Groups

I calculate the unemployment rate as the ratio of the number of the unemployed in a particular month to the size of the labor force in that month for each skill group. Figure 1 illustrates the skill differentials in unemployment rates. All figures show quarterly averages of monthly values. Although unemployment rates of both skill groups follow a similar trend, the high-skill unemployment rate is always significantly lower than the low-skill unemployment rate.

There are two determinants of the unemployment rate. The first one is the job finding probability whereas the second one is the job separation probability. In computing those probabilities, I follow the methodology used in Shimer (2005) and Elsby, Michaels, and Solon (2009). I discuss this methodology in more detail

conducted by the Bureau of the Census for the Bureau of Labor Statistics. The web site for the Survey is: http://www.census.gov/cps. The data can be downloaded from http://www.nber.org/data/cps_basic.html.

in the appendix.

The job finding probability in a given month measures the likelihood of a worker finding a job within that month. Figure 2 shows the job finding probabilities of different skill groups. Although low-skill workers, on average, have slightly higher job finding probabilities than high-skill workers, the difference is not large enough for job finding probabilities to be the main reason for differences in unemployment rates between skill groups.

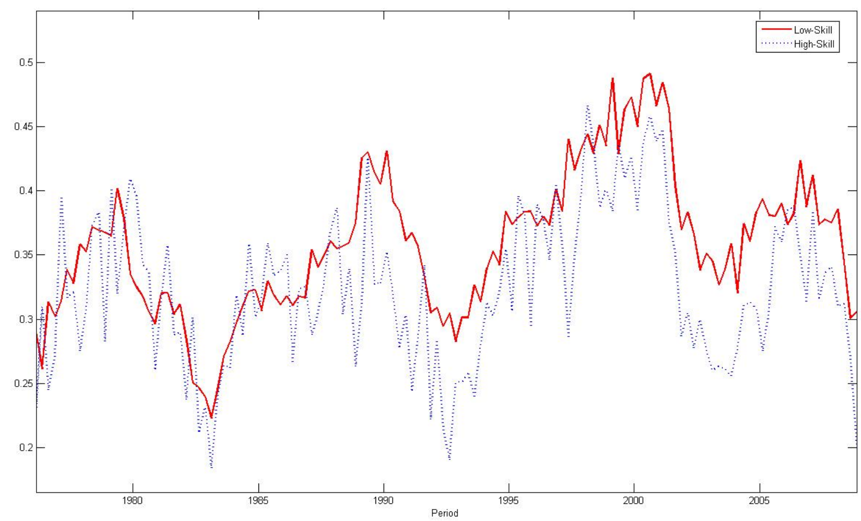

Figure 2: Job Finding Probabilities by Skill Groups

One can also look at the unemployment duration across skill groups. Low-skill workers have slightly longer unemployment durations, which implies a lower job finding probability for this group. However, the difference in unemployment duration is not large enough to generate the difference in unemployment rates either. Moreover, Tristao (2007) uses National Longitudinal Survey of Youth-79 data and looks at the unemployment duration for occupation groups.[4] Tristao (2007) finds that low-skill occupations have unemployment durations that are close to the overall average unemployment duration. Duration data on occupation groups also show that job finding probabilities are not different enough to cause the differences in unemployment rates across skill groups.

[4] For more information on National Longitudinal Survey of Youth see http://www.bls.gov/nls/nlsy79.htm.

The second determinant of unemployment rate, the job separation probability. It measures the likelihood of employed workers becoming unemployed within that month. Figure 3 illustrates the job separation probabilities of skill groups over time. High-skill workers have substantially lower job separation probabilities than low-skill workers.

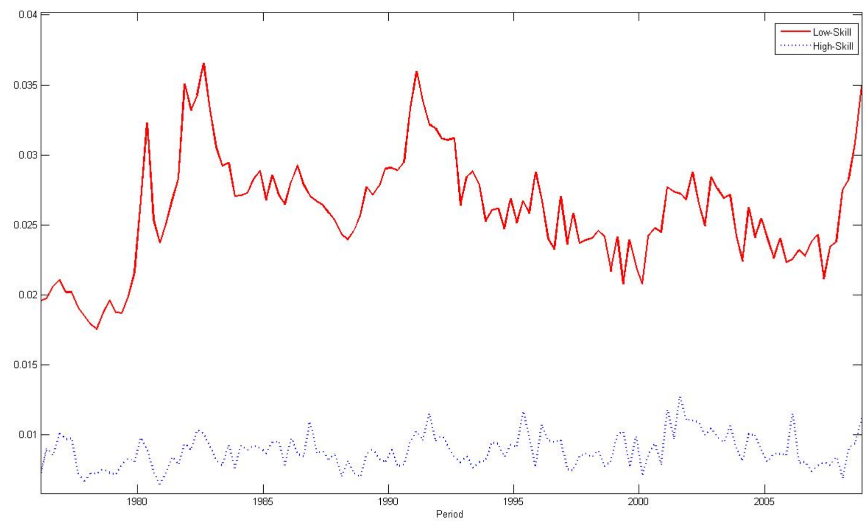

Figure 3: Job Separation Probabilities by Skill Groups

Table 1 summarizes unemployment experiences by skill groups. Between January 1976 and December 2008, low-skill workers experienced an average unemployment rate of five percent whereas high-skill workers faced a two percent unemployment rate. Moreover, low-skill workers had a slightly better chance of finding jobs than their high-skill counterparts. The data reveal that low-skill workers have higher unemployment rates due to higher job separation

These facts are robust to different specifications of the data. I analyze the unemployment rate difference between skill groups that is not due to lay-offs. The CPS collects data on the reason for unemployment after 1994. Over the period of 1994 to 2007, the average unemployment rate (counting those who are laid-off as unemployed) is 4.9 percent for low-skill workers, and 2.1 percent for high-skill workers. If we do not count laid-off workers as unemployed, look only at unemployment for reasons other than lay-offs, the average unemployment rate is 3.9 and 1.9 percent for low-skill and high-skill workers, respectively. Clearly, not counting laid-offs as unemployed reduces low-skill unemployment more. However, the relative number of lay-offs among low-skill workers, compared to high-skill workers, is not large enough to account for the differences in unemployment rates.

Data facts presented above are also robust to the different definitions of skill. Redefining high-skill workers as workers with at least some college education (instead of a four-year college degree) changes quantitative results slightly, but not enough for the gap to go away or decrease significantly. Figure (A.2) shows the unemployment rate and the job finding and separation probabilities when the cut-off for skill is some college degree. Moreover, the gap in the unemployment rate and the job separation probability is significant for finer categories of education.

The skill differences in unemployment experiences are also documented by other authors. Mincer (1991) uses the Panel Study of Income Dynamics data on male labor force participants and finds that higher levels of education reduce the risk of unemployment. Mincer (1991) also finds that the difference in the incidence of unemployment is more important than the difference in unemployment durations for educational differences in unemployment. Nagypal (2007) uses March Supplements and Displaced Worker Supplements of the CPS and finds differences in unemployment rate by education. Moreover, Nagypal (2007) documents that the differences in unemployment duration by education are not large enough to account for the differences in unemployment rates. Layard, Nickell, and Jackman (1991) use managerial vs. manual occupations, and Juhn, Murphy, and Topel (2002) use wages as a proxy for skill, and find a higher unemployment rate for low-skill workers.

1.2 Hiring Processes

This section presents facts on employee selection and its differentials by skill groups. Van Ours and Ridder (1992) analyze the employer search using Dutch establishment data. They show that 80 percent of vacancies are filled with applicants who applied for the job within the first two weeks of the vacancy opening. Hence, they conclude, vacancy durations should be interpreted as selection periods.

There are differences in the intensity of search by firms for vacancies with different skill requirements. Van Ours and Ridder (1993) report that the mean selection period increases with the required level of education and experience.

Moreover, Barron, Berger, and Black (1997) find that employers search more when hiring workers with more education and prior experience, and for jobs with higher training requirements. They show that as the education requirement of the vacancy increases, the number of interviews per offer and number of applicants per offer goes up as well as the number of hours per interview and per applicant. Barron, Bishop, and Dunkelberg (1987) report that total time spent on hiring is longer for high-skill occupations, in comparison to low-skill occupations.

There is also evidence on search for high-skill workers being more effective. Bagger and Henningsen (2008) use Danish and Norwegian data and look at the job ending hazard rates by skill. Bagger and Henningsen (2008) find that for all skill levels the likelihood of a job ending decreases with tenure.[5] At low levels of tenure, low-skill workers are more likely to separate from their jobs, and the difference diminishes as the tenure increases. The difference in levels of the hazards indicate that high-skill hires are more likely to be good matches.

Hiring policies are used to explain some other facts regarding labor markets. Pries and Rogerson (2005) analyze differences in labor and job turnovers between the US and Europe. The differences in labor market policies of these economies generate differences in hiring policies of firms. Tasci (2006) looks at firms' hiring policies over the business cycle, and proposes changes in hiring behavior over the cycle as another mechanism to increase the response of the key aggregate labor market variables to productivity shocks.

2 Model

This section introduces the model I use to analyze how selection technologies affect the job separation probability. The economy is inhabited by workers with two skill types; high-skill and low-skill. The skill distribution is exogenous and skill type of a worker is observable. There is also continuum of firms with heterogenous jobs. Upon entering the market, firms choose the skill type of the job they want to operate. A firm can employ at most one worker. Firms and workers that search for an employment relationship are brought together via a matching function. I assume no interaction between high- and low-skill sectors, in other words these

[5] It is well documented that job separation hazard is decreasing with tenure. See, for example, Farber (1999).

sectors are *segregated*.[6] Having no interaction between skill groups allows me to solve for the equilibrium for each skill group separately. In consequence, the model explained below applies to each of the skill groups.

There is a continuum of homogenous workers, with total mass equal to one. There is also a continuum of ex-ante identical firms. All agents are risk neutral, and they discount future at rate β. A worker can be either unemployed or employed. When unemployed, workers have unemployment income b.

The production unit in the economy is a firm-worker pair. The pair produces $y = y^i$ amount of output, where i is the quality of the match. The output y^i takes on the value $y^g(y^b)$ if the match is good (bad) quality, where $y^g > y^b$. I assume that the difference between outputs produced in a good quality and bad quality match is larger in high-skill matches than low-skill matches ($y^g_{hs} - y^b_{hs} > y^g_{ls} - y^b_{ls}$). This is the only exogenous difference between skill groups. This assumption and how the results relate to this assumption are discussed in Section 4. The wages are outcomes of Nash Bargaining, and μ is workers' bargaining power.

Production units that are active (that produce in the current period) are subject to an exogenous destruction at rate δ, which is the same across sectors. Bagger and Henningsen (2008) estimate the monthly job hazard function for different education groups using data on Danish and Norwegian workers. At low levels of tenure, less educated workers have higher hazard rates. However, the difference diminishes significantly after five years of tenure. Based on this evidence, it is plausible to assume a common exogenous destruction rate across these skill groups.

In the model, unemployment stems from frictions in the labor markets. These frictions are modeled via a matching function. The matching function provides a mapping from the number of vacancies (v) and the unemployed (u) to the number of total matches between firms and worker. Thus, the matching function

[6] Although there is empirical evidence in favor of interaction between labor markets, this simplifying assumption allows me to focus on the interaction between the unemployment rate and hiring policies of firms. Interactions across labor markets can take on many forms, affecting the unemployment rate disparity between sectors in different directions. One commonly modeled interaction across skill groups is that high-skill workers can look for both high- and low-skill jobs while unemployed and can continue to search for high-skill jobs if they exit from unemployment into a low-skill job. Granting such interaction will affect the unemployment rate of high-skill workers through affecting both their job finding and job separation probabilities in this model. Since this paper focuses on the causes of the job separation disparity between skill groups, which is the main factor of the unemployment rate gap, abstracting away from such interaction is plausible.

determines the matching probabilities of firms and workers endogenously.[7] The matching function, M, is constant returns to scale, therefore it only depends on the vacancy-unemployment ratio $v/u = \theta$, which is also known as the market tightness. A worker meets with a firm with probability $f(\theta) = M/u$, and a vacant firm meets with a worker with probability $q(\theta) = M/v$. The matching function also satisfies the following boundary conditions: $f \rightarrow 0$ and $q \rightarrow 1$ as $\theta \rightarrow 0$, and $f \rightarrow 1$ and $q \rightarrow 0$ as $\theta \rightarrow \infty$. Moreover, the probability that a worker meets with a vacancy, f, is an increasing function of θ while the probability of firms meeting with workers, q, decreases with θ ($f'(\theta) > 0$ and $q'(\theta) < 0$).

The quality of a match is ex-ante uncertain. The true quality of the match is revealed after the first period of production. Hence, a worker-firm pair does not know the quality of the match unless they produce. However, parties draw the probability of being in a good quality match, π, when they meet. After observing π, the worker and the firm decide whether to form a match. A higher value of π means that it is more likely that the match is good quality.

I model the employee selection procedure as a technology that randomly delivers a value of π to the firm-worker pair when they first match. Match quality π can be drawn from either a costless distribution Γ, or from a more effective but costly (κ) distribution Ω. The more effective the technology is, the more likely it is that the technology delivers higher values of π, i.e. Ω first order stochastically dominates Γ, $\Omega(\pi) \leq \Gamma(\pi)$, $\forall\ \pi$. One can think of more effective employee selection procedures being firms using more effective recruitment channels (such as advertising the job opening more extensively and intensively and using better employee assessment) that will result in a better quality match between firm and worker as opposed to the quality of a match had the firm not used a more effective technology. Another way of thinking of more effective selection is as if firms sample from the same distribution multiple times and choose the highest π level. In this case the empirical distribution of π for firms that have multiple draws will first order stochastically dominate the original distribution.

I assume that $y^b = b$ and $y^g > b$.[8] Under this assumption, bad quality matches

[7] The model does not have on-the-job search. Thus, all separations in the model result in unemployment. The CPS data reveal that both high- and low-skill workers experience job-to-job transitions. However, low-skill workers have substantially higher job separation rates even after accounting for job-to-job transitions. Hence, abstracting away from on the job search yields simplification without distorting the validity of the hypothesis of the model.

[8] Observe that under this assumption we have $y^b_{hs} = y^b_{ls}$ and $y^g_{hs}. > y^g_{ls}$

are undesirable in equilibrium. Firm and worker pairs terminate such matches. If all the separations in this economy were to be exogenous, then the job separation rates in the equilibrium would be the same across skill groups. However, undesirability of bad quality matches generates endogenous separations, which is the source of the difference in job separation rates across sectors.

Timing of Events

- A period begins with a number of unemployed workers, a number of worker-firm matches with known quality, and a number of worker-firm pairs that met in the previous period the first time and observed their π.
- All parties decide whether to produce or to separate.
- All workers that do not produce in the current period consume unemployment benefits.
- Worker-firm pairs that have decided to stay in the match produce, and workers consume their wages.
- All the matches with unknown quality learn the match quality (observe that separation decisions will be made at the beginning of the next period).
- Firms decide whether to post a vacancy or not.
- The vacant firms choose a selection technology to use.
- Job markets open; unemployed workers and vacant firms meet.
- Firm-worker pairs that has met learn the probability with which their match will be good quality. They will decide at the beginning of next period whether to form the match or not.
- Job markets close.
- Exogenous destructions occur; δ fraction of worker-firm pairs that produced in the current period are destroyed.
- New period begins.

2.1 Bellman Equations

Let λ be the probability that firms choose Ω as their employee selection technology. Moreover, let $U(\lambda)$ be the value of unemployment to a worker, and $W(\pi, \lambda)$ be the value of being in a match with a firm to a worker where π is the probability that the match is good quality. $U(\lambda)$ is

$$U(\lambda) = b + \beta(1 - f(\theta))U(\lambda) + \beta f(\theta)\left\{\lambda \int_0^1 W(\pi, \lambda)d\Omega + (1 - \lambda)\int_0^1 W(\pi, \lambda)d\Gamma\right\}. \quad (1)$$

If a worker is unemployed, she gets the unemployment income, b, in the current period. With probability $1 - f(\theta)$, the worker does not meet with any firms, thus continuing to be unemployed in the subsequent period. The worker meets with a firm with probability $f(\theta)$, and gets an expected value from being in a match with a firm. Note that the expected value the worker gets from being in a match with the firm depends on the selection technology the firm has chosen.

If a worker is in a match with a firm and with probability π the match is good quality, the worker decides whether to stay in the match or separate to unemployment. The value of match can be formalized as follows:

$$W(\pi, \lambda) = \max\left\{U(\lambda),\ w(\pi, \lambda) + \beta\Big[\pi\big\{\delta U(\lambda) + (1 - \delta)W(1, \lambda)\big\} + (1 - \pi)U(\lambda)\Big]\right\}. \quad (2)$$

If she stays in the match, the worker gets the wage in the current period. If the match survives to the next period and it is revealed to be good quality, the worker will get the value of being in a good quality match, $W(1, \lambda)$. If the match is revealed to be of bad quality, which will happen with probability $1 - \pi$, the worker will get the value of being in a bad quality match, $W(0, \lambda)$. Recall that, in equilibrium, a match will be terminated if it is bad quality, therefore $W(0, \lambda) = U(\lambda)$.

Let $V(\lambda)$ be the value of a firm with a vacancy, and $J(\pi, \lambda)$ be the value of a firm in a match. The value of a vacancy is the discounted value of expected profits, net of cost of the vacancy

$$V(\lambda) \quad = \quad -c + \max \left\{ \begin{array}{c} -\kappa + \beta(1-q(\theta))V(\lambda) + \beta q(\theta) \int_0^1 J(\pi,\lambda) d\Omega, \\ \beta(1-q(\theta))V(\lambda) + \beta q(\theta) \int_0^1 J(\pi,\lambda) d\Gamma \end{array} \right\}. \quad (3)$$

The first term of equation (3) is the per period cost of having a vacancy, c. The second term of (3) has a maximum operator since the firm will decide which selection procedure to implement. If the firm chooses the more effective distribution, then it pays the extra cost of using that distribution (κ). Regardless of the choice of the employee selection procedure, the firm will match with a worker with the probability $q(\theta)$. If the firm matches with a worker, then it will get the expected value of being in a match which is of good quality with probability π. If the firm does not match with any worker, which happens with probability $1 - q(\theta)$, then it will stay vacant and continue to get the value V.

Equation (4) formalizes the problem of a firm that is in a match with a worker:

$$J(\pi,\lambda) = \max \Big\{ V(\lambda),\ E(y|\pi) - w(\pi,\lambda) + \beta\delta V(\lambda) + \beta(1-\delta)\big(\pi J(1,\lambda) + (1-\pi) J(0,\lambda)\big) \Big\} \quad (4)$$

where $E(y|\pi) = \pi y^g + (1-\pi) y^b$. The firm compares the expected discounted value of profits from producing output with the current worker to the discounted present value of separating from the worker (being vacant). The value firm gets from producing is the sum of the current period profit, which is the expected value of output produced net of the wage paid to the worker, and the discounted value of being in a match with the same worker in the subsequent period, if the match survives. Observe that a surviving match is revealed to be good quality with probability π. In this case the firm will get the discounted present value of being in match with a worker with the match quality being good with probability one, $J(1,\lambda)$. However, with probability $(1-\pi)$ the match quality will be revealed to be bad, and the firm will get the value $J(0,\lambda)$. With the assumption that $y^b \leq b$, firms and workers will separate in equilibrium if the match is bad quality, thus $J(0,\lambda) = V(\lambda)$.

The probability that firms choose Ω selection technology, λ, appears as an argument in value functions of agents because beliefs about λ can potentially affect firms' decision whether to adopt the effective technology. The value of λ affects the value of unemployment for workers, since it affects the wages at other firms. Thus, a firm needs to take the value of λ into account. This point is discussed in more detail in section 3.1.

Wages are determined according to the Nash bargaining rule, where workers' bargaining power is μ. The wage rate that solves the bargaining problem is such that a worker gets a constant (μ) fraction of the net value generated by the worker-firm union. Let the second argument of the maximization operation in equation (2) and equation (4) be $\tilde{W}(\pi, \lambda)$ and $\tilde{J}(\pi, \lambda)$, respectively. Then, the wage is such that

$$\tilde{W}(\pi, \lambda) - U(\lambda) = \mu \Big\{ \tilde{W}(\pi, \lambda) - U(\lambda) + \tilde{J}(\pi, \lambda) - V(\lambda) \Big\}. \tag{5}$$

The Nash bargaining assumption guarantees the unanimity of the separation or match formation decision. That is because parties bargain over the net surplus of match surplus, and if the surplus is positive (negative) they decide to form the match (separate). Hence there is no inconsistency across parties in decision making.

2.2 Worker Flows Across Employment States

There are three labor market states in this economy. A worker can either be unemployed (u_t), or in a match with unknown quality (e_t^n), or in a match with good quality (e_t^g). Before I describe the flows across these states, I need to introduce some notation. Let $X(\pi) = 1(0)$ be the decision of a worker-firm pair who observe π (not) to form the match. Moreover, let $E(\pi | X(\pi) = 1)$ denote the expected probability of a good quality match, conditional on the match being formed, and $E(X(\pi) = 1)$ denote the probability of an acceptable match.

Since the match quality is revealed in the first period of employment, all unknown quality matches can only be one period old. After the first period, these workers either become unemployed or become employees with good quality matches. Hence, the number of unknown quality matches in any period are the same as the number of matches formed in that period. More formally:

$$e_{t+1}^n \quad = \quad u_t f_t E_\lambda (X(\pi) = 1). \tag{6}$$

Employed workers with a good quality match either survive to the subsequent period or get destructed. Thus, the number of good quality matches in a period is the sum of the surviving good quality matches of the previous period and the previous period unknown quality matches which are revealed to be good quality in the subsequent period that survive exogenous destruction.

$$e^g_{t+1} \quad = \quad (1-\delta)e^g_t + e^n_t(1-\delta)E_\lambda(\pi|X(\pi)=1),$$

where λ is the probability that Ω selection technology is chosen by firms.

Unemployment level of the current period is a result of three flows. The first flow is from the unemployed of the previous period who did not meet with a firm and who met with a firm but drew a low π (thus stayed unemployed). The second group consists of workers who were employed in the previous period but were hit by an exogenous destruction shock, thus become unemployed. The last group of workers to add to the pool of currently unemployed is the workers of the previous period with unknown match quality who survived exogenous destruction but learned to have a bad quality match. More formally:

$$u_{t+1} = u_t(1 - f_t E_\lambda(X(\pi)=1)) + \delta(e^g_t + e^n_t) + e^n_t(1-\delta)\big(1 - E_\lambda(\pi|X_\pi = 1)\big). \tag{7}$$

3 Equilibrium

The steady state equilibrium, for each sector, is a list $\{e^g, e^n, v, u, \lambda, \pi^*, w(\pi,\lambda), X(\pi), J(\pi,\lambda), V(\lambda), W(\pi,\lambda), U(\lambda)\}$ such that

- $\{J(\pi,\lambda), V(\lambda), W(\pi,\lambda), U(\lambda)\}$ satisfy equations (4), (3), (2), and (1).
- $V(\lambda) = 0$.
- $w(\pi,\lambda)$ is the solution to the Nash bargaining (it satisfies equation (5)).
- $X(\pi)$ is such that

$$X(\pi) = \begin{cases} 1 & \text{if} \quad \pi \geq \pi^* \\ 0 & \text{if} \quad \pi < \pi^* \end{cases}$$

where π^*, the reservation probability, satisfies two equations below: [9]

$$E(y|\pi^*) - w(\pi^*,\lambda) + \beta\delta V(\lambda) + \beta(1-\delta)\big(\pi^* J(1,\lambda) + (1-\pi^*)J(0,\lambda)\big) = V(\lambda),$$

$$w(\pi^*,\lambda) + \beta\delta U(\lambda) + \beta(1-\delta)\big(\pi^* W(1,\lambda) + (1-\pi^*)W(0,\lambda)\big) = U(\lambda).$$

[9]Because of the Nash bargaining rule these equations will both either be satisfied or not for the same π^* value.

- Selection technology is chosen optimally, i.e,

$$\lambda(\lambda^-) = \begin{cases} 1 & \text{if} \quad -\kappa + \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Omega \geq \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Gamma \\ 0 & \text{if} \quad -\kappa + \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Omega < \beta q(\theta) \int_0^1 J(\pi, \lambda^-) d\Gamma \end{cases},$$

  where λ^- is the decision rule for the rest of the firms.

- Selection technology choice of firms is consistent ($\lambda(\lambda^-) = \lambda^-$).

- The flows between employment and unemployment states are constant

$$u = 1 - e^g - e^n,$$

$$\delta e^g = e^n (1-\delta) E_\lambda(\pi | X(\pi) = 1),$$

$$e^n = uf E_\lambda(X(\pi) = 1).^{10}$$

The existence of equilibrium is discussed in the appendix. The following subsections discuss the employee selection technology in equilibrium and the model's implications for labor market outcomes.

3.1 Employee Selection in the Equilibrium

Firms choose the more effective selection technology, Ω, if

$$\beta q(\theta_\lambda) \Big[\int_{\pi_\lambda^*}^1 J(\pi_\lambda) d\Omega - \int_{\pi_\lambda^*}^1 J(\pi_\lambda) d\Gamma \Big] \geq \kappa.$$

One can rewrite a firm's expected profits as

$$\int_{\pi_\lambda^*}^1 J(\pi_\lambda) dF_\lambda = (1-\mu) \frac{(y^g - y^b)}{1 - \beta(1-\delta)(1-\pi_\lambda^*)} \int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] dF_\lambda,$$

where F_λ is the selection technology chosen. Note that $F_{\lambda=1} = \Omega$ and $F_{\lambda=0} = \Gamma$.

This equation uses the facts that the profit a firm gets is a constant fraction of the net match surplus and the match surplus depends on the gap between good

[10] There is also a flow equation that equates flows out of unemployment to flows into unemployemt:

$$uf E_\lambda(X(\pi) = 1) = \delta e^g + e^n \big[\delta + (1-\delta) \big(1 - E_\lambda(\pi | X(\pi) = 1) \big) \big].$$

Observe that given the two other flow equations, this equation is redundant.

and bad quality outputs as well as the reservation probability π^*.[11]

Substituting a firm's expected profits into the equation for selection technology yields

$$\frac{\beta q(\theta_\lambda)(1-\mu)(y^g - y^b)}{1-\beta(1-\delta)(1-\pi^*_\lambda)}\Big[\int_{\pi^*_\lambda}^{1}[\pi - \pi^*_\lambda]d\Omega - \int_{\pi^*_\lambda}^{1}[\pi - \pi^*_\lambda]d\Gamma\Big] \geq \kappa. \tag{8}$$

Observe that, firms are more likely to select Ω if, everything else being equal, $y^g - y^b$ is larger. Since high-skill firms have a higher uncertainty associated with the match quality, they are more likely to use the effective employee selection technology in equilibrium. Note that this is an ex-ante condition, it does not guarantee that high-skill firms use that technology.

Firms decide whether to use the effective technology by comparing the expected profits to the total vacancy cost, which includes the cost of using a selection technology. Although, profits from being in a match increases with λ, vacancy filling probability may increase or decrease with λ. As the fraction of the firms using the effective selection technology, λ, increases, reservation probabilities go up as workers get a higher probability of forming good quality matches elsewhere. Moreover, a higher reservation probability increases firms' expected profits. However, whether this increase is high enough to cover the higher expected cost of selection is ambiguous. Thus, how market tightness will react to changes in λ is ambiguous.

How the above equation reacts to changes in λ determines whether there are multiple equilibria. Suppose the left hand side of the equation (8) is decreasing in λ. If there are many (few) firms using the effective selection technique (λ is high [low]), then using the same technique, as opposed to the costless technique, is less (more) profitable. In other words, firms' actions are strategic substitutes. In this case the equilibrium is unique.

However, if the left hand side of equation (8) increases in λ, then firms' actions are strategic complements and it is possible to have multiple equilibria. Note that, if a higher fraction of firms use the effective selection technology, then using the effective technology, as opposed to the costless technology, yields higher profits.

Quantitative analysis reveals that market tightness, for these specific parameter values, decreases with λ. Thus there is a possibility of multiple equilibria.

[11] The derivation of this equation is in the appendix.

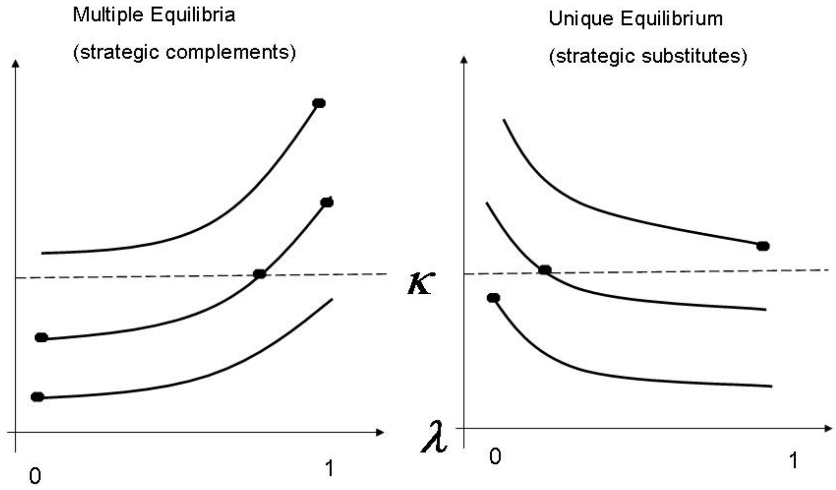

Figure 4: Illustration of Equation (8)

3.2 Implications for Labor Market Outcomes

An unemployed worker's probability of finding a job, which I denote by p, has two components. The first component is the probability that the worker matches with a firm, $f(\theta)$. This probability is endogenously determined in the model, and it is a function of the equilibrium vacancy to unemployment ratio, θ. The second component is the conditional expected probability of a worker-firm pair drawing a high enough probability of a match being good quality so that the parties decide to form the production unit. The reservation probability, together with the selection technology a firm chooses, determines the acceptance probability. Therefore, the job finding probability of a worker is

$$p = f(\theta)E_\lambda(X(\pi) = 1) = f(\theta)\int_{\pi^*_\lambda}^{1} dF_\lambda, \tag{9}$$

where F_λ is the chosen equilibrium distribution, i.e, $F_{\lambda=1} = \Omega$ and $F_{\lambda=0} = \Gamma$.

Note that the difference between the job finding probabilities of high- and low-skill workers can come from the differences in market tightness, the differences in the selection technologies implemented, or the differences in the reservation probabilities.

Similarly, the vacancy filling probability, denoted by h, is determined by market tightness, which determines the probability that the vacancy meets with a worker, and the acceptance probability of a match. Formally:

$$h = q(\theta)E_\lambda(X(\pi) = 1).$$

Another important moment is the total job separation rate. Total separation rate is determined by the exogenous and endogenous separations:

$$s(e^n + e^g) = (e^n + e^g)\delta + e^n(1 - \delta)(1 - E_\lambda(\pi|X(\pi) = 1)). \tag{10}$$

The first term in the above equation is the exogenous separations, which affects all of the matches. Endogenous separation, on the other hand, occurs only for matches with unknown match quality which have survived the exogenous destruction. Among those, the ones that are revealed to be bad quality get destroyed. Using the relationship between good (or known) quality matches and unknown quality matches, which is given via the equilibrium flow equation, I derive that

$$\frac{e^n}{e^n + e^g} = \frac{\delta}{\delta + (1 - \delta)E_\lambda(\pi|X(\pi) = 1)}.$$

Substituting for $\frac{e^n}{e^n+e^g}$ in equation (10), we get

$$s = \frac{\delta}{\delta + (1 - \delta)E_\lambda(\pi|X(\pi) = 1)}. \tag{11}$$

Observe that the separation rate is equal to the fraction of employees that have unknown quality matches. Every period, workers with unknown quality matches either separate to unemployment or replace the workers with good quality matches that separated to unemployment.

The total number of job separations depends on the exogenous destruction rate and the conditional expected probability of the match being good quality. Since the exogenous destruction rate δ is the same across sectors, the only source of a discrepancy between total job separation rates across skill groups is the differences in the conditional expected probability of a match being good quality. The higher the $E_\lambda(\pi|X_\pi = 1)$, the lower the total separations. Observe that, the assumption that Ω first order stochastically dominates Γ does not guarantee a higher separation rate, since the separation rate also depends on an endogenous value π^*.

Following simple algebra on the equilibrium flow equations, one can show that the unemployment rate is:

$$u = \frac{\delta}{\delta + p\big(\delta + E_\lambda(\pi|X(\pi) = 1)(1-\delta)\big)}.$$

Substituting δ from equation (11) and rearranging terms gives the familiar equation of

$$u = \frac{s}{s+p}.$$

The unemployment rate depends on the probability of workers finding a job and the probability of them separating from their jobs to unemployment.[12] To see whether the proposed model can generate labor market outcomes for high- and low-skill workers that are consistent with the data, I utilize the following quantitative exercise.

4 Quantitative Analysis

I assign values to the parameters of the model to match some facts of the U.S. labor markets. The time period of the model is a month. I set $\beta = 0.9967$, to get an annual interest rate of 4 percent. The bargaining power of the workers is generally set to a number between 0.3 and 0.5 in the literature.[13] I set the workers' bargaining power parameter (μ) to 0.5 as in Pries and Rogerson (2005). This also is the match elasticity of unemployment in the low-skill sector.

Observe that multiplying c, κ, y_g, y_b, and b by the same number does not change the solution to the equation system. Thus, I normalize b in each sector

[12] Although the unemployment series calculated using this approximation, i.e., using the equation $u = \frac{s}{s+p}$ with time series of probabilities, is highly correlated with the actual unemployment series, this approximation generates levels that are systematically higher than the data. The average approximated unemployment rates for low-skill and high-skill prime age males are 0.0695 and 0.0281, respectively, whereas their data counterparts are 0.0576 and 0.0225. Note that both skill groups are affected similarly. Low-skill unemployment is 2.56 times higher than the high-skill unemployment in the actual data and it is 2.4733 times higher if we use the approximated series. This issue has no significance on the main result. The level of unemployment neither appears in any of the equations that characterize the equilibrium, nor it is used in parameter estimation. The only implication of higher levels of unemployment rates is higher levels of vacancy rates.

[13] See Petrangola and Pissarides (2001) for a literature survey.

to 1. I also set $y_b = b$, which is sufficient for the bad quality matches to be terminated in the equilibrium. Moreover, this assumption implies that only the difference between high and low quality outputs, rather than their values, is what matters for the equilibrium.

Employee selection distributions are assumed to be Beta distributions. There is no direct evidence on the distributional form of the selection technologies. I choose Beta distribution because it has support in the unit interval. Moreover, I assume the first parameter of both Beta distributions is 1. I use the following functional form for the matching function: $M = \frac{uv}{(u^\alpha + v^\alpha)^{\frac{1}{\alpha}}}$. This functional form naturally bounds job finding and vacancy filling probabilities into unit interval.

Remaining parameters of the model are: the exogenous job destruction rate δ, matching function parameter α, the output of a good quality low-skill match y_{ls}^g, the cost of effective selection technology κ, and the parameter values of Γ and Ω distributions. I calibrate these parameters to match the model moments to some US data facts. In order to calculate the parameter values, I need an equilibrium in which skill groups use different selection technologies.[14] I search for an equilibrium in which low-skilled uses Γ and high-skilled uses Ω as their selection technology. Then, for given parameter values, I check for deviations from that equilibrium. The estimated parameter values are reported in Table 3 along with the other parameter values.

I use the following data moments (they are displayed on Table 4). I target a job separation probability of 0.026 (s_{ls}) for low-skill workers. Davis et al. (1996) report that 23 percent of all annual job destruction is due to the plant shutdowns in the manufacturing industry. I choose $\delta = 0.006$ so that 23 percent of all separations are exogenous in the low-skill sector in the steady state. Observe that $\delta / s_{ls} = 0.006/0.026 = 0.23$.

Recall from equation (11) that

$$s_{ls} = \frac{\delta}{\delta + (1-\delta) E_{\lambda_{ls}}(\pi | X_\pi = 1)}.$$

Since $s_{ls} = 0.026$ and $\delta = 0.006$, we have $E_{\lambda_{ls}}(\pi | X_\pi = 1) = 0.2254$. Since $\lambda_{ls} = 0$, if we know the value of π_{ls}^*, we can find the second parameter value of the Γ distribution.

[14] Otherwise value for κ cannot be determined.

Table 2: Targeted Moments

Moment	**Value**	**Source**
Low-skill job separation prob.	0.026	Monthly average, 76-07
Fraction of annual job destruction due to plant shutdowns	.23	Davis et al. (1996)
Low-skill job finding probability	0.323	Monthly average, 76-07
Highest to lowest wage ratio; low-skill	1.25	Juhn et al (2002)
Match elasticity of Unemployment, low-sk.	.36	Petrangolo et al(2001)
High-skill job finding probability	0.294	Monthly average, 76-07
Output difference across skill	1.79	Acemoglu et al (2001)
Expected wage gap across skill	1.9	Heatchote et al (2008)

One can rearrange equilibrium equations (see the appendix) to have

$$\pi^*_\lambda = \beta\mu p\Big[E_\lambda(\pi|X(\pi)=1) - \pi^*_\lambda\Big]. \tag{12}$$

Thus, given p_{ls} and δ; we can find the $\pi^*_{\lambda_{ls}}$. I target $p_{ls} = 0.323$, which is the average job finding probability among low-skill workers over the sample period. Solving equation (12) yields $\pi^*_{ls} = 0.0359$. For a Beta distribution with the first parameter normalized to 1, we need the second parameter to be 4.0883 to have $E_{\lambda_{ls}=0}(\pi|X_\pi = 1) = 0.2254$ for $\pi^*_{ls} = 0.0359$. Γ is a Beta distribution with parameter values of (1, 4.0883).

I target that the highest wage low-skill workers can earn is 25 percent higher than the lowest wage in the steady state to find y^g_{ls}. Topel and Ward (1992) find that the cumulative change in wages over the first 10 years of work history that is associated with job change is around 33 percent. I set the ratio of the highest wage to the lowest in the low-skill sector to be 25 percent, the number that is also used by Pries and Rogerson (2005).[15] I use the following equation to get the highest-to-lowest wage ratio in the low-skilled sector:[16]

$$w(\pi,\lambda) = \mu E(y|\pi) + (1-\mu)\Big(b + \pi^*_\lambda S'(\pi^*_\lambda,\lambda)\Big).$$

I solve for y^g_{ls} such that $w_{ls}(1,0)/w_{ls}(\pi^*_{ls},0) = 1.25$ where $w(\pi,\lambda)$ is defined above.

[15] If I target 33 percent instead of 25 percent, all the results follow through. I find that only the high-skill firms employ the effective selection technology and the job separation probability of high-skill workers is 0.009 instead of 0.008.

[16] Derivation of the wage equation is in the appendix.

Note that I can find the value of $f(\theta_{ls})$ using equation (9) since I know p_{ls} (targeted), π^*_{ls}, and the parameter values of the Γ distribution. I use this value to estimate the parameter value of the matching function, α. Match elasticity of unemployment, denote by ϵ, satisfies the following equation in equilibrium (see appendix for derivation):

$$\epsilon = f(\theta)^{\alpha}.$$

I assume the match elasticity of unemployment for low-skill workers is 50 percent in the steady state. This is the same number as workers' bargaining power in Nash Bargaining. Since we know the $f(\theta_{ls})$, we can find the α value that satisfies the equation above.

Observe that $f(\theta_{ls})$ and α values imply a unique θ_{ls} value. I target a vacancy cost value, c, such that this θ_{ls} value satisfies the equilibrium optimal match formation condition, which is: [17]

$$\theta_{ls}\mu\frac{c}{(1-\mu)} = (y^b - b) + \pi^*_{ls}S^{'}(\pi^*_{ls}, 0). \tag{13}$$

Remaining parameter values are the parameter value of Ω distribution, y^g_{hs}, and κ. To estimate these parameters, I use the following algorithm. I start with a guess of a value for the Ω distribution parameter. Then, I find π^*_{hs} to satisfy the equation (12) so that the job finding probability is 0.32 for high-skilled workers. Using equation (9) and the value of π^*_{hs}, I find $f(\theta_{hs})$, and thus θ_{hs}.

I solve for y^g_{hs} such that the ratio between total output produced by high-skilled and low-skilled workers is 1.79. Acemoglu and Zilibotti (2001) find a 79 percent difference between the value added in high- and low-skill sectors. Then, I solve for the cost of effective technology, κ, so that the equilibrium optimal match formation condition is satisfied:

$$\theta_{hs}\mu\frac{c+\kappa}{(1-\mu)} = (y^b - b) + \pi^*_{hs}S^{'}(\pi^*_{hs}, 1).^{18} \tag{14}$$

Note that values of κ and y^g_{hs} are contingent of the guess for the parameter value of Ω. Thus, I go back and update my guess for that parameter value until the expected wage ratio between high- and low-skilled workers is 1.9. I choose this moment value as Heathcote et al. (2008) report that the college premium is

[17]See appendix for the existence of equilibrium for derivation.

[18]See appendix for the existence of equilibrium for derivation.

Table 3: Parameter Values

Parameter	**Value**	**Source/Target**
β	0.996	4% interest rate
b	1	Normalization
y_b	b	Restriction
μ	0.5	Petrongolo et al. (2000)
α	0.787	Match elasticity of unemployment, low-skilled
δ	0.006	Exogenous fraction of job destructions, low-skilled
$c_{ls} = c_{hs}$	0.521	Low-skilled job finding probability
κ	2.02	High-skilled job finding probability
y^g_{ls}	1.658	Highest-to-lowest wage gap, low-skilled
y^g_{hs}	2.843	Output gap across skill
β_Γ	4.054	Job separation probability, low-skilled
β_Ω	0.401	Wage gap across skill

around 90 percent.

Note that, these parameter values are estimated under the assumption that low-skill firms use Γ while high-skilled firms use Ω. I check, for these parameter values, whether firms want to deviate from these technologies,a nd find that they do not. Thus, these parameter values yield an equilibrium with moment values displayed in Table (4) in which low- and high-skilled choose Γ and Ω technologies, respectively.

4.1 Results

Table 4 displays some key outcomes of the equilibrium for the parameter values given in Table 3. The job finding and the separation probabilities of low-skill workers are equal to their data values as they are targeted. Observe that the model has the following relationship between the unemployment rate and job finding and job separation probabilities: $u = \dfrac{s}{s+p}$. Thus, the unemployment rate of low-skill workers, although not directly targeted, is the same as in the data.

For high-skill workers, the model successfully delivers a job separation probability that is as low as it is in the data. As a result, the model replicates the discrepancy between high and low-skill unemployment rates. Observe that neither the unemployment rate nor the job separation probability of high-skill workers are targeted. Moreover, the difference in job separation probabilities across skill groups is due to the more effective selection procedure high-skill firms employ.

Table 4: Results

	High-Skill		**Low-Skill**	
	Data	**Model**	**Data**	**Model**
distribution		Ω		Γ
unemployment rate*	0.027	0.024	0.067	0.067
job finding probability	0.32	0.32^+	0.36	0.36^+
job separation probability	0.009	0.008	0.026	0.026^+
vacancy filling probability	-	0.48	-	0.36
θ	-	0.67	-	1
π^*	-	0.102	-	0.034
$E(\pi\|\pi^*)$	-	0.743	-	0.225
$w(1)/w(\pi^*)$	-	1.425	1.25	1.25^+

$^+$: Targeted moments.
*: Reported data unemployment rate is the rate implied by using $\frac{s}{s+p}$ equation. See footnote 12.

The market tightness skill groups experience is 0.69 and 1 for low- and high-skill workers, respectively. Although there is no data on market tightness of skill groups, these numbers imply an overall market tightness that is in between these two values. Market tightness for the US since 2001 is on average 0.46.[19] Although there is no data for vacancy duration by skill for the US, Danish data suggest that (Van Ours and Ridder (1993)) high-skill vacancies have higher durations. The model, however, predicts a lower duration for high-skill vacancies, compared to low-skill vacancies.

To clarify the effects of employee selection technology, I carry out the following exercises. In the first exercise (column three), firms can only use the less effective selection technology, Γ. In this case, the only difference between skill groups in generating the discrepancy in their equilibrium outcomes is the productivity differences. The third column of Table 5 presents the results of the model for parameter values given in Table 2. The unemployment rate of low-skill workers is 60 percent higher than the high-skill unemployment rate. Although the difference in unemployment rates is high, it is not as high as it is in the data. More importantly, the reason for the unemployment rate discrepancy is mainly the difference

[19]The data can be found at http://www.bls.gov/jlt/.

Table 5: Counterfactual Exercises

			Exercise	
High-Skill	**Data**	**Model**	**Only Γ**	**Only Ω**
unemployment rate	0.027	0.024	0.046	0.013
job finding probability	0.32	0.32	0.522	0.612
job separation probability	0.009	**0.008**	**0.025**	**0.008**
vacancy filling probability	-	0.478	0.174	0.181
θ	-	0.67	2.99	3.391
π^*	-	0.102	0.049	0.179
$E(\pi\|\pi^*)$	-	0.743	0.237	0.765
Low-Skill				
unemployment rate	0.067	0.067	0.067	0.018
job finding probability	0.36	0.36	0.36	0.426
job separation probability	0.026	**0.026**	**0.026**	**0.008**
vacancy filling probability	-	0.36	0.36	0.358
θ	-	1	1	1.19
π^*	-	0.034	0.034	0.132
$E(\pi\|\pi^*)$	-	0.225	0.225	0.751
Moments				
Ew_{hs}/Ew_{ls}	1.9	1.9	1.497	1.662
$w(1)/w(\pi^*)$, low-sk	1.25	1.25	1.250	1.211
Y_{hs}/Y_{ls}	1.79	1.79	1.748	1.724
exog. fr of destr., low-sk	0.23	0.23	0.230	0.753

in job finding probabilities between skill groups. High-skill workers experience a higher job finding probability because their labor market is too tight. Higher market tightness in the high-skill sector generates a longer vacancy filling duration for high-skill firms, compared to low-skill firms. High-skill firms have a higher market tightness because vacancy costs are the same for all firms while high-skill jobs are more productive. This exercise shows that if the only difference between high and low-skill jobs were the productivity differences, there still would be a gap between unemployment rates. However, this gap would have been mainly due to differences in job finding probabilities.

The last column of Table 5 shows the equilibrium of the model if Ω is the only selection technology that is available without any cost. The difference between high- and low-skill unemployment rates is close to 50 percent. Like the previous case, the difference in unemployment rates is due to the difference in job finding probabilities. The reason that columns two and four are not identical for high-skill workers is that in the case where Ω is the only selection technology (column four), there is no cost (κ) for using Ω. Observe that it is the high cost of using effective selection technology that draws market tightness for high-skill firms down.

The results are robust to different plausible values of μ. Re-estimating the model parameters with, for instance, $\mu = 0.4$ delivers targeted moments and slightly changes other moments of interest. However, the main result that the difference in unemployment across skill groups is due to the differences in job separations rates, which stems from the differences in employe selection technologies, pulls through.[20]

While searching for parameter values, I do not impose restriction that the more effective distribution, Ω, should first order stochastically dominate the less effective selection distribution, Γ. As Figure (5) shows, estimated values of Ω and Γ distributions are such that Ω first order stochastically dominates Γ. Numerical exercises show that the normalized value of the first parameter has no effect on the results. One can normalize it to some other value but still get the same results. One other normalization is with the unemployment incomes, b. Normalizing b to some other numbers (1.5 or 2) does not change the results.

5 Concluding Remarks

This paper proposes a new explanation for the unemployment rate disparity between skill groups. It is well documented that high-skill workers have lower unemployment rates. Data also show that the reason for the lower unemployment rate of high-skill workers is their lower probability of job separation. High-skill workers are less likely to separate from their jobs because they are selected more effectively. Firms do a more intensive and extensive employee search when hiring for high-skill vacancies in the data.

This paper uses a matching model with uncertainty about match quality to analyze the relationship between employee selection and job separation probabil-

[20]If one picks lower values for μ, then there may not be an equilibrium in which $c_{ls} = c_{hs}$ and high-skilled firms choose the effective technology. However, one can find a range of parameters for c_{hs} and κ such that high-skilled firms choose the effective technology while low-skilled firms do not.

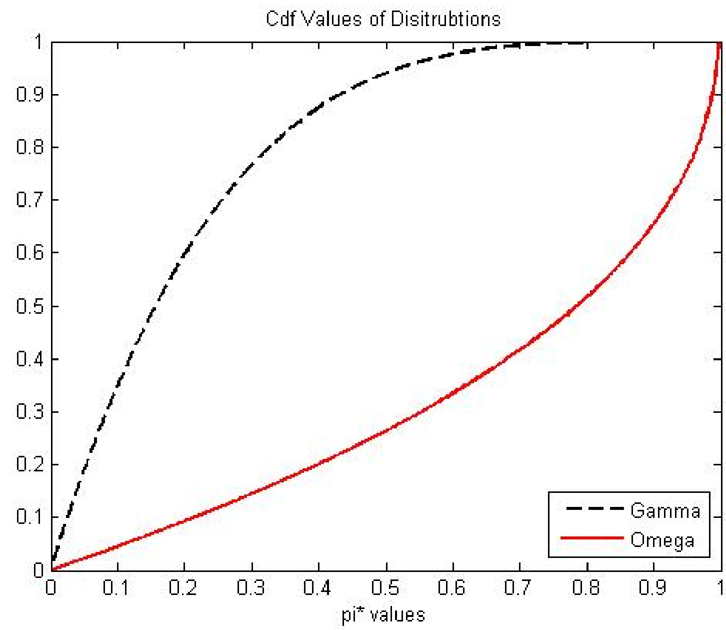

Figure 5: CDF Values of Employee Selection Distributions

ity. There are two employee selection technologies that differ in their cost and effectiveness. In the equilibrium, high-skill firms, which are the firms with higher productivity, selfselect into using more effective technology. As a result of the choice of more effective technology, a higher fraction of high-skill firms end up with good quality matches, thus a lower fraction of these firms experience endogenous match termination. Consequently, in equilibrium, high-skill workers have a lower unemployment rate compared to low-skill workers, as they have substantially low job separation rates.

High-skill firms choose the more effective technology as the output gap between good and bad quality matches for high-skill workers is high. It is not the output gap per se that makes firms choose effective technology, but the higher opportunity cost of hiring the wrong worker, which is the consequence of high output gap. One can also let the only ex-ante difference between high and low-skill workers be the vacancy cost with high-skill firms having a higher cost. Note that, a higher vacancy cost also means a higher opportunity cost of hiring the wrong worker, since the firm needs to incur high vacancy cost in the subsequent period if the match is of bad quality. In such an environment, we can have an equilibrium in which high-skill firms use the effective technology and thus have lower job separation rates. However, in such an equilibrium, the unemployment rate of high-skill workers will be higher, since they have a higher vacancy cost than but the same productivity as low-skill workers. Allowing for differences in both productivity and vacancy cost generates the same results as the model used in this paper does. Future research will identify the importance of these factors, as well as possible others, on employers' choice of selection techniques for jobs with different skill requirements.

There is more work that needs to be done to explore the skill bias in job separation probabilities. This paper focuses on the role of employee selection on job separation rates. Another possible reason is that learning about the match quality is slower for high-skill jobs. This will not only directly contribute to the lower job separation of high-skill, but also affect the employee selection procedures firms use.

The model abstracts from interaction across skill groups. For future work, it would be interesting to explore the effects of interactions across markets on employee selection technologies of firms. There are also other possible contributors to the bias, such as firm specific training, that should be explored.

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A Appendix

A.1 Data

I use the CPS Basic data from January 1976 to December 2008. My sample is male labor force participants who are older than 24 and younger than 55. Let (L_t^{nc}) L_t^c be the number of workers who (do not) have a four-year college degree in period t. Let (U_t^{nc}) U_t^c be the number of unemployed workers who (do not) have a four-year college degree in period t. Similarly, define U_t^{s-c} (U_t^{s-nc}) as the number of workers who are unemployed for less than 5 weeks, that is short-term unemployed, and who (do not have) have a four-year college degree in period t.

The CPS survey had a redesign in 1994, which affected the number of short term unemployed workers after the redesign. To make the pre and post redesign era data compatible, I do the following correction as suggested by Elsby, Michaels, and Solon (2009) on the post redesign data. For each skill group, I multiply the short-term unemployment numbers of post redesign era by the era's average of the ratio of short-term share for the first and fifth rotation groups to the full samples's short term share.

Let $S_t = \frac{U_t^s}{U_t}$ be the short-term share of the all sample. Moreover, let

$$S_t^{15} = \frac{u_t^{s15}}{u_t^{15}}$$

be the short-term share for 1st and 5th rotation groups. We want to find average $S_t^s = \frac{S_t^{15}}{S_t}$ and multiply overall short term series by this number. Then, I seasonally adjust

L_t, U_t, and U_t^s using Eviews' implementation of the Census Beraue's X-12 procedure.

Now, one can calculate job finding and separation probabilities using these time series data. Observe that the total number of unemployed in the current month is the sum of the number of workers who were unemployed in the previous month and didn't find a job and the number of workers who were employed last month and unemployed in the current month (short-term unemployed). More formally: $U_{t+1} = (1 - F_t)U_t + U_{t+1}^s$, where F_t is the probability of finding a job within a month. From this identity, one can work out the job finding probability as

$$F_t = 1 - \frac{U_{t+1} - U_{t+1}^s}{U_t}.$$

Then, then job finding rate will be $f_t = -log(1 - F_t)$.

Finding the job separation probability is not as straightforward. For this we need to use the following evolution equation for unemployment over time. Let s_t be the job separation rate for month t. Then, unemployment moves according to the following equation:

$$\frac{du_t}{dt} = (1 - u_t)s_t - f_t u_t = -(s_t + f_t)(u_t - u_t^*), \tag{A.1}$$

where $u_t^* = \dfrac{s_t}{s_t + f_t}$. If we solve the differential equation (A.1) for u_t under the assumption that s_t, f_t, and the labor force is constant across two consecutive surveys and forward it one month we get $u_{t+1} = u_t^* + (u_t - u_t^*)e^{-(s_t+f_t)}$. One can solve this equation for s_t and calculate the job separation probability S_t as $S_t = 1 - e^{-s_t}$.

A.2 Existence of the Equilibrium

It is easier to work with the match surplus to characterize the equilibrium in matching models. The match surplus is defined as the sum of the value of a match to the firm and to the worker, net of their outside options: $S(\pi, \lambda) = J(\pi, \lambda) - V(\lambda) + W(\pi, \lambda) - U(\lambda)$. Substituting in values of $J(\pi, \lambda)$ and $W(\pi, \lambda)$ from the Bellman equations result in

$$S(\pi, \lambda) = \max\Big\{E(y|\pi) + \beta(1 - \delta)\pi S(1, \lambda) - (1 - \beta)U(\lambda) - (1 - \beta)V(\lambda),\ 0\Big\}. \tag{A.2}$$

Note that the match surplus is linear in π for $\pi > \pi^*$, thus we can write the surplus as $S(\pi, \lambda) = (\pi - \pi^*)S^{'}(\pi^*, \lambda)$ where $S^{'}(\pi^*, \lambda)$ is [21]

$$S^{'}(\pi^*, \lambda) = \frac{y^g - y^b}{1 - \beta(1 - \delta)(1 - \pi^*)}. \tag{A.3}$$

[21] I take the derivative of $S(\pi, \lambda)$ with respect to π, substitute $S(1, \lambda) = (1 - \pi^*)S^{'}(\pi^*, \lambda)$ in, and rearrange the terms to get this expression.

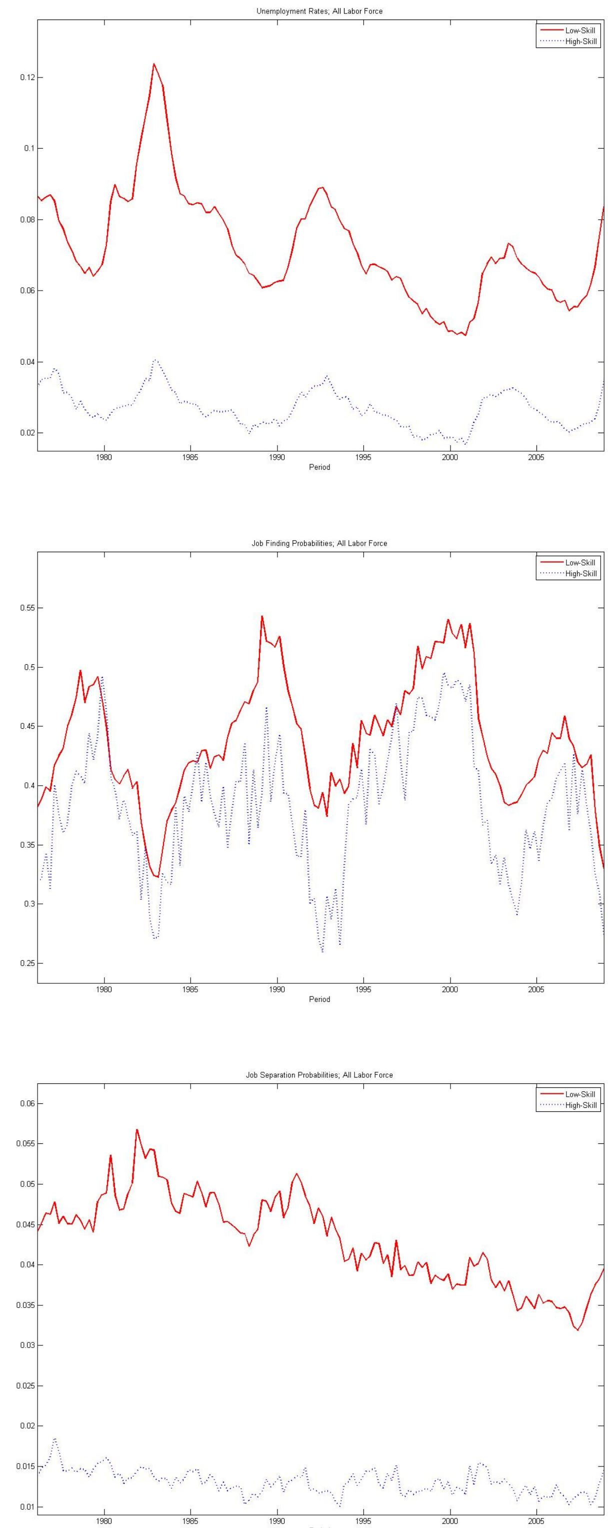

Figure A.1: Unemployment Data for All Labor Force

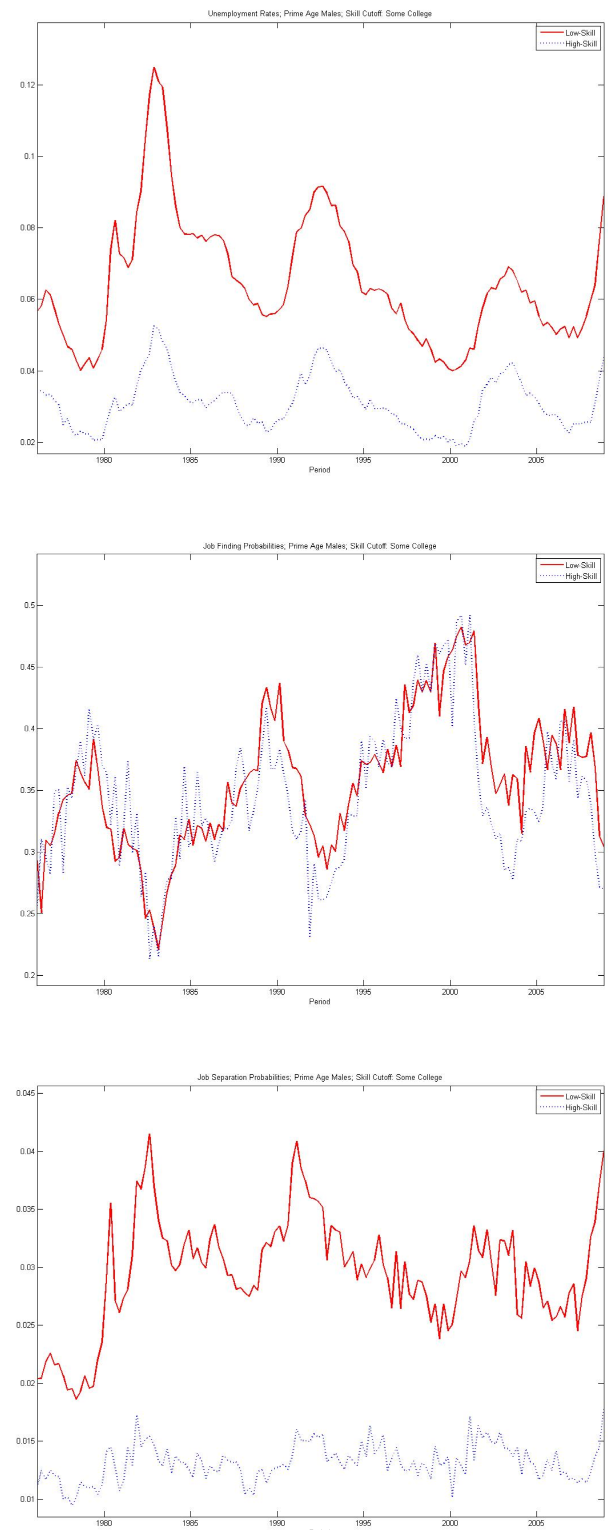

Figure A.2: Unemployment Data for Prime Age Males; Skill Cutoff: Some College Degree

The equilibrium can be characterized by three variables. These are the reservation probability, the market tightness, and the choice of the employee selection technology. For a given value of λ, the values of $(\theta_\lambda,\ \pi^*_\lambda)$ are determined by the intersection of two curves: the optimal match formation curve (OMF) and the free entry curve (FE).

The optimal match formation curve delivers the reservation probability for any given market tightness level. Recall that the π^*_λ leaves workers and firms indifferent between forming the production unit or staying unattached (it solves the equation $S(\pi^*_\lambda) = 0$). Using this condition, and substituting $S^{'}(\pi^*, \lambda)$ into $S(\pi^*_\lambda, \lambda) = 0$ equation yields $(1-\beta)U(\lambda) = y^b + \pi^*_\lambda S^{'}(\pi^*, \lambda)$. We can rewrite equation above as:[22]

$$\theta_\lambda \mu \frac{c + \lambda\kappa}{(1-\mu)} = (y^b - b) + \pi^*_\lambda S^{'}(\pi^*_\lambda, \lambda). \tag{A.4}$$

Note that $\pi^*_\lambda S^{'}(\pi^*_\lambda, \lambda)$ is increasing in π^*_λ. Thus, we have an upward sloping line in the $(\pi^*_\lambda,\ \theta_\lambda)$ space. The intuition for the upward sloping optimal match formation curve is as follows: At any $(\theta,\ \pi^*_\lambda)$ pair on the curve, we know that match surplus is zero. As market tightness increases, so does the worker's outside option (a firm's outside option is always zero in equilibrium). Since the worker's outside option is higher at the new market tightness, the match surplus at the new market tightness with the old reservation probability will be negative. Thus, the firm and the worker will decide to separate. They will be indifferent between being unattached and forming a production unit at a higher π^*_λ level, resulting in a positive slope.

The second curve that determines the equilibrium is the free entry (or vacancy creation) condition. I get this condition in the following way: The equilibrium value of the market tightness should be such that the vacancies earn zero profit in equilibrium, i.e., θ_λ solves $V(\lambda) = 0$; thus

$$c + \lambda\kappa = \beta q(\theta)(1-\mu)\Big(\lambda \int_{\pi^*_\lambda}^{1} S(\pi,\lambda) d\Omega + (1-\lambda)\int_{\pi^*_\lambda}^{1} S(\pi,\lambda) d\Gamma\Big) \tag{A.5}$$

[22] Recall that in equilibrium

$$(1-\beta)U(\lambda) = b + \beta f(\theta)\mu \left\{ \lambda \int_0^1 S(\pi,\lambda) d\Omega + (1-\lambda)\int_0^1 S(\pi,\lambda) d\Gamma \right\}, \quad \text{and}$$

$$(1-\beta)V(\lambda) = -c - \lambda\kappa + \beta q(\theta)(1-\mu) \left\{ \lambda \int_0^1 S(\pi,\lambda) d\Omega + (1-\lambda)\int_0^1 S(\pi,\lambda) d\Gamma \right\}.$$

Substituting for surplus values from $V(\lambda)$ equation into $U(\lambda)$ equation yields $(1-\beta)U(\lambda) = b + \theta\mu \dfrac{c+\lambda\kappa}{1-\mu}$.

Note that, since $S(\pi, \lambda)$ is linear in π, we can write the expected surplus as follows:

$$\int_0^1 S(\pi, \lambda) d\cdot = S^{'}(\pi_\lambda^*, \lambda) \int_{\pi_\lambda^*}^1 (\pi - \pi^*) d\cdot$$

Thus, the free entry condition can be written as

$$c + \lambda\kappa = \beta q(\theta)(1-\mu) S^{'}(\pi_\lambda^*, \lambda)\Big(\lambda \int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\Omega + (1-\lambda) \int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\Gamma\Big). \qquad \text{(A.6)}$$

Since both $S^{'}(\pi_\lambda^*, \lambda)$ and $\int_{\pi_\lambda^*}^1 [\pi - \pi_\lambda^*] d\cdot$ are decreasing in π_λ^*, this equation has negative slope in (π_λ^*, θ_λ) space. The intuition for the downward sloping free entry curve is as follows: As the π_λ^* increases, it gets harder for firms to find workers, which in turn makes firms incur vacancy costs longer (value of opening a vacancy decreases). Hence, the number of vacancies goes down resulting in a lower market tightness.

These two curves are required to intersect for the existence of an equilibrium. To get the conditions for the existence of an equilibrium, I need a functional form for the firms' matching probability. I assume the matching function to be $M = \dfrac{uv}{(u^\alpha + v^\alpha)^{\frac{1}{\alpha}}}$ to derive the condition below.

Proposition 1. *If* $\dfrac{(c+\lambda\kappa)(1-\beta(1-\delta))}{\beta(y_g - y_b)\Big(\lambda \int_0^1 \pi d\Omega + (1-\lambda)\int_0^1 \pi d\Gamma\Big)} < 1$, *then for any* $\lambda \in [0, 1]$ *there is a pair* (π_λ^*, θ_λ) *such that equations (6) and (8) are satisfied.*

Proof. Since the OMF and the FE curves have monotonically increasing and decreasing slopes, respectively, it is sufficient to show that there exists π_1, π_2 such that

$$\pi_1 < \pi_2, \quad \theta_{OMF}(\pi_1) < \theta_{FE}(\pi_1) \text{ and } \theta_{OMF}(\pi_2) > \theta_{FE}(\pi_2).$$

Then, by intermediate value theorem, there exists $\pi^* \in [\pi_1, \pi_2]$ such that $\theta_{OMF}(\pi^*) = \theta_{FE}(\pi^*)$.

Let π_2 be a number close to 1. As π goes to 1, $\theta_{FE}(\pi)$ goes to zero. Moreover, $\theta_{OMF}(1) = \dfrac{(1-\mu)}{\mu(c+\lambda\kappa)}(y^g - b) > 0$. Thus for a value of π_2 close to 1; $\theta_{OMF}(\pi_2) > \theta_{FE}(\pi_2)$.

Let $\pi_1 = 0$. Note that $\theta_{OMF}(\pi_1) = \dfrac{(1-\mu)}{\mu(c+\lambda\kappa)}(y^b - b) \leq 0$. Substituting this equation into $q(\theta) = \dfrac{1}{(1+\theta^\alpha)^{1/\alpha}}$ for $\theta_{OMF}(\pi_1)$ and rearranging terms yields

$$\theta_{FE}(\pi_1) = \Big[\frac{\beta(1-\mu)(y^g - y^b)\Big(\lambda \int_0^1 \pi d\Omega + (1-\lambda)\int_0^1 \pi d\Gamma\Big)^\alpha}{(c+\lambda\kappa)(1-\beta(1-\delta))} - 1\Big]^{1/\alpha}.$$

If, $\dfrac{\beta(1-\mu)(y^g-y^b)\Big(\lambda\int_0^1 \pi d\Omega + (1-\lambda)\int_0^1 \pi d\Gamma\Big)}{(c+\lambda\kappa)(1-\beta(1-\delta))} > 1$, then $\theta_{FE}(\pi_1) > 0$. Consequently, $\theta_{OMF}(\pi_1) < \theta_{FE}(\pi_1)$. □

The inequality will not hold if the means of distributions are sufficiently close to zero, the output difference by the quality of a match is implausibly small, or the total cost of opening a vacancy is implausibly high.[23] The condition does not bind for any plausible parameter values.

A.3 Algebra of the Quantitative Analysis

Equilibrium equation for π^*: for a clean display, let us denote the chosen equilibrium distribution as F_λ. Note that $F_{\lambda=1} = \Omega$ and $F_{\lambda=0} = \Gamma$.

Recall that equilibrium is defined by two equations: (A.4) and (A.6). Equation (A.4) can be rewritten as (note that I assume $y^b = b$ in the quantitative analysis)

$$S^{'}(\pi^*_\lambda, \lambda) = \theta_\lambda \mu \frac{c+\lambda\kappa}{\pi^*_\lambda(1-\mu)}. \tag{A.7}$$

Substituting the equation above into equation (A.6) for $S^{'}(\pi^*_\lambda, \lambda)$ yields:

$$c + \lambda\kappa = \beta q(\theta)(1-\mu)\theta_\lambda \mu \frac{c+\lambda\kappa}{\pi^*_\lambda(1-\mu)} \int_{\pi^*_\lambda}^1 [\pi - \pi^*_\lambda] dF_\lambda,$$

which can be simplified to (recalling that $f(\theta) = q(\theta)\theta$)

$$\pi^*_\lambda = \beta\mu f(\theta)\Big[\int_{\pi^*_\lambda}^1 \pi dF_\lambda - \pi^*_\lambda \int_{\pi^*_\lambda}^1 dF_\lambda\Big] = \beta\mu f(\theta)\int_{\pi^*_\lambda}^1 dF_\lambda \left(\frac{\int_{\pi^*_\lambda}^1 \pi dF_\lambda}{\int_{\pi^*_\lambda}^1 dF_\lambda} - \pi^*_\lambda\right). \tag{A.8}$$

Observe that $f(\theta)\displaystyle\int_{\pi^*_\lambda}^1 dF_\lambda = p$, and $\dfrac{\int_{\pi^*_\lambda}^1 \pi dF_\lambda}{\int_{\pi^*_\lambda}^1 dF_\lambda} = E_\lambda(\pi | X(\pi) = 1)$. Thus, the free entry condition (A.8) can be expressed as

$$\pi^*_\lambda = \beta p \Big[E_\lambda(\pi|X(\pi)=1) - \pi^*_\lambda\Big].$$

[23] One example for such implausible values would be the total vacancy cost to be 200 times higher than the low quality output, high quality output to be less than one percent higher than the low quality output, or the mean of the distributions to be 0.01.

Derivation of the wage equation: using the definition of match surplus and the Nash Bargaining rule, we have

$$S(\pi,\lambda) = (1-\mu)\Big(J(\pi,\lambda) - V(\lambda)\Big). \tag{A.9}$$

Using equation (4), one can write

$$J(\pi,\lambda) - V(\lambda) = \max\{0, \;\; E(y|\pi) - w(\pi,\lambda) + \beta\pi(1-\mu)(1-\delta)S(1,\lambda). \tag{A.10}$$

Substituting (A.2) and (A.10) into equation (A.9) for $S(\pi,\lambda)$ and $J(\pi,\lambda)-V(\lambda)$, respectively, yields (after some algebra) $w(\pi,\lambda) = \mu E(y|\pi) + (1-\mu)(1-\beta)U(\lambda)$. Moreover, rearranging equation 1 as a function of surplus and substituting surplus from equation (A.6), we get

$$(1-\beta)U(\lambda) = b + \beta\mu f(\theta)\int S(\pi,\lambda)dF_\lambda = b + \beta\mu\theta\frac{(c+\lambda\kappa)}{1-\mu}.$$

We can also use equation (A.7) to substitute in for $S^{'}(\pi^*_\lambda,\lambda)$ and get $(1-\beta)U(\lambda) = b + \pi^*_\lambda S^{'}(\pi^*_\lambda,\lambda)$. Hence, the wage can be rewritten as $w(\pi,\lambda) = \mu E(y|\pi) + (1-\mu)\Big(b + \pi^*_\lambda S^{'}(\pi^*_\lambda,\lambda)\Big)$.

Match elasticity of unemployment in equilibrium: the match elasticity of unemployment is

$$\epsilon = \frac{\partial M(u,v)}{\partial u}\frac{u}{M(u,v)}.$$

After some algebra, one can show that the above equation simplifies to $\epsilon = \frac{\theta^\alpha}{1+\theta^\alpha}$. Recall that $f(\theta) = \frac{M(u,v)}{u}$ and $\theta = \frac{v}{u}$. Thus $f(\theta)$ can be rewritten as $f(\theta) = \frac{\theta}{(1+\theta^\alpha)^{1/\alpha}}$. Note that we can rearrange this equation to get $(1+\theta^\alpha) = \frac{\theta^\alpha}{f(\theta)^\alpha}$. Then,w e can rewrite the elasticity as

$$\epsilon = \frac{\theta^\alpha}{\frac{\theta^\alpha}{f(\theta)^\alpha}} = f(\theta)^\alpha.$$