Research Note

Deliberation Rules and Voting*

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ABSTRACT

Most models of pre-vote deliberation assume that voters send messages simultaneously. In practice, however, communication is almost always sequential. This review shows that sequential communication makes it even more difficult to induce truthful communication. Specifically, I show that for any voting rule truthful communication can only be achieved if all voters will agree on the preferred outcome in any state.

When a committee makes a decision there is usually an opportunity for the members to communicate with each other before the voting takes place. This communication stage is typically modeled as a simultaneous move cheap talk game. This may be a reasonable model of some voting environments, for example when pre-vote communication takes the form of a straw poll. However, in practice, communication is usually sequential with the members of the committee being asked to speak to the issue in turn.

The importance of pre-vote communication is highlighted in Coughlan (2000), which shows that when there is an underlying consensus, so that once all private information has been revealed all voters would agree on the preferred outcome, it is an equilibrium for all voters to truthfully reveal their private information. Once all information has been revealed the resulting vote will be unanimous so the voting rule is irrelevant. Unfortunately, information is only fully revealed if such an underlying consensus exists. Austen-Smith and Feddersen (2006) consider the role of bias uncertainty, and provide some examples where voters truthfully reveal their private information when they are uncertain of the other voters’ preferences. However, they show that truthful

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communication is never possible without an underlying consensus if the voting rule is unanimity.

The current review extends the framework of Austen-Smith and Feddersen (2006) to sequential communication and shows that truthful communication is even more difficult to achieve: no matter what voting rule is used truthful communication is an equilibrium only if it is common knowledge that there is always an underlying consensus. To see why, consider a committee that must decide whether to convict a defendant, where unanimity is required to convict. Suppose all voters communicate simultaneously and consider a voter who has information indicating that the defendant is innocent. Since this voter can always veto conviction there is no risk in withholding this information and reporting that she believes the defendant to be guilty. If, after all other voters have revealed their information, she prefers to acquit she can always ensure that the defendant is acquitted. However, she might benefit from the mis-report if she prefers to convict and her report convinces other voters, more reticent about convicting, to convict. If the voting rule is not unanimity, however, voters do not have a veto and so mis-reporting is not without risk. Reporting that she thinks the defendant is guilty can cause those with the same bias as her (or more willing to convict) to vote to convict when she would prefer the defendant be acquitted. With bias uncertainty the voter does not know the fraction of the committee that is more cautious about convicting than she is. So while a mis-report may still cause the committee to convict when she prefers conviction it is also possible that mis-reporting causes the committee to convict when she wants to acquit. Depending on the relative probabilities of these two events it may be an equilibrium for all voters to reveal their information truthfully.

Now suppose the communication is sequential. If all previous voters are reporting truthfully then the last voter to speak knows all the other voters’ private information and so knows which alternative she prefers. Consequently there is no risk in reporting her private information as more favorable to her preferred alternative, and so truthful communication cannot be an equilibrium.

There are three reasons why we might be concerned with sequential communication. From a purely theoretical perspective, Dekel and Piccione (2000) show that in a symmetric environment the equilibria with simultaneous voting are also equilibria with sequential voting. Here I show that an analogous result does not hold for sequential and simultaneous communication: in fact, simultaneous communication facilitates truthful communication more often. Second, Austen-Smith and Feddersen’s results contribute to a large literature on the inferiority of unanimity to other voting rules in strategic voting environments (e.g., Feddersen and Pesendorfer (1998), Gerardi and Yariv (2007)), while this review shows that by slightly changing the communication procedure the failure to achieve truthful communication is no longer specific to unanimity voting. Finally, from a practical perspective most communication is sequential, and in fact since it is not possible for all voters to speak at the same time simultaneous communication may not always be feasible. It may be possible to convey simple information, such as which outcome is preferred or the realization of a binary signal, through a straw poll but more

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1 Dewatripont and Tirole (2005) formalizes the idea that in order for information to be communicated there must be both a speaker and a listener.
detailed information will require more complicated messages than can be communicated simultaneously.\textsuperscript{2} The justification for considering a specific mechanism as opposed to taking a more general mechanism design approach\textsuperscript{3} is that the model is a reasonable approximation of actual voting procedures. The results of this review show that once the sequential nature of communication is incorporated into the model truthful communication becomes even more difficult to achieve.

**MODEL**

Suppose there is a committee of \( n \) voters, \( N = \{1, \ldots, n\} \) that must decide between two alternatives \( X \) and \( Y \). Each voter \( i \) has private information \((b_i, s_i) \in B \times S\), regarding her bias and signal. Assume \( B \) and \( S \) are finite. Let \( B = B^n \) and \( S = S^n \). Let \( p(b, s) \) be the probability of bias profile \( b = (b_1, \ldots, b_n) \) and state \( s = (s_1, \ldots, s_n) \), where \((b, s) \in B \times S\). Let \( u(a, b, s) \) be the utility of a voter with bias \( b \) in state \( s \) if \( a \in \{X, Y\} \) is selected. Define \( S_b = \{s \in S : u(Y, b, s) > u(X, b, s)\} \) as the set of states where an individual with bias \( b \) prefers alternative \( Y \). Assume that individuals are never indifferent in any state so \( s \notin S_b \) implies that \( u(Y, b, s) < u(X, b, s) \).

This review considers an environment where communication is sequential, so voter \( i \) will have observed the first \( i - 1 \) messages sent. Let \( M \supseteq S \) be the set of possible messages, then voter \( i \) sends message \( m_i = \sigma_i(b_i, s_i, \ldots, m_{i-1}) \) where \( \sigma_i : B \times S \times M^{i-1} \rightarrow M \). Notice the distinction with simultaneous communication where \( m_i = \sigma_i(b_i, s_i) \). After communication, the voting takes place simultaneously, with each voter \( i \) casting vote \( v_i(b_i, s_i, m) \), where \( m = (m_1, \ldots, m_n) \). Outcome \( Y \) is selected if and only if at least \( q \) votes are in favor of \( Y \).

Following Austen-Smith and Feddersen (2006), it is assumed that the state and bias profiles have full support, that the state affects which alternative voters prefer, and that signals are monotonic.

**Axiom 1** (Full Support) For all \((b, s) \in B \times S\), \( p(b, s) > 0 \).

**Axiom 2** (No Dogmatic Types)\textsuperscript{4} For all \( b \in B, \emptyset \neq S_b \neq S \).

The full support assumption is standard, and the second axiom rules out the case where enough voters are determined to vote in one direction that the state is irrelevant for the voting outcome. Notice that in any interesting situation dogmatic types would make communication impossible since they would always have an incentive to lie.

\textsuperscript{2} It would always technically be possible to design ways for communication to be simultaneous, for example by having the members write down their information and then have all statements read. In practice this is not done, and it is questionable whether this can be done costlessly.

\textsuperscript{3} See Gerardi and Yariv (2007) for a mechanism design approach to this problem.

\textsuperscript{4} Austen-Smith and Feddersen use a slightly weaker condition that guarantees a potential consensus since they are only concerned with unanimity rule. Combining this axiom with the monotonicity axiom to be stated guarantees that there is a state where all biases agree on the preferred alternative.
Finally, I impose a monotonicity assumption that all biases agree whether a change in each voter’s private information supports $X$ or $Y$. Different biases can still have widely different views on how important the change is and about which outcome is preferable.

**Axiom 3 (Monotonicity)** $S$ can be ordered with a binary relation $\succ$ such that the following holds: For any $s, s' \in S$ such that $s \succ s'$ and any $s_- \in S^{n-1}$, let $s = (s_-, s)$ and $s' = (s_-, s')$, then $u(Y, b, s) > u(Y, b, s')$ and $u(X, b, s) < u(X, b, s')$ for all $b \in B$.

So if $s \succ s'$ then any voter who preferred outcome $Y$ when voter $i$’s signal is $s'$ would also prefer $Y$ if $i$ had signal $s$. I conclude this section with some definitions. The first definition, minimally diverse, says that there is some state where some types disagree on the preferred alternative.

**Definition 1 (Minimally Diverse)** A committee is minimally diverse if there exist $b, b' \in B$ such that $S_b \neq S_{b'}$.

Finally I define the solution concept.

**Definition 2 (Fully Revealing Debate Equilibrium)** An equilibrium is a fully revealing debate equilibrium if

1. $\sigma_i(b_i, s_i, m_1, \ldots, m_{i-1}) = s_i$ for all $b_i, s_i, (m_1, \ldots, m_{i-1})$, and
2. Voting is in weakly undominated strategies.

If all voters reveal their private information in the communication stage then their vote provides no additional information and so voters have a weakly dominant strategy in the voting stage. If the communication stage is to be effective it must be that the voters use the information gained from communication in their votes. A fully revealing debate equilibrium has three benefits. It guarantees full information equivalence, so the decision that the committee makes is the same as would have been made had all private information been public. Truth-telling is also a focal equilibrium and voting games typically admit many equilibria. Finally, the strategies both in the communication and voting stage are simple enough to reasonably expect voters to play.

**RESULTS**

Before presenting the new result of this review it is useful to review what has been established for simultaneous communication. Coughlan (2000) showed that with known biases it is only an equilibrium for voters to truthfully reveal their signals if preferences are not minimally diverse. Austen-Smith and Feddersen show that the same is true with unanimity voting even if there is uncertainty about the biases.\(^5\)

\(^5\) This impossibility result for simultaneous communication only holds, in general, for unanimity rule. See Austen-Smith and Feddersen (2006) for an example where truth-telling is an equilibrium when the voting rule is not unanimity.
Proposition 1 (Austen-Smith and Feddersen, 2006) Suppose $A1, A2, A3$, and $q = n$ (unanimity rule). Then a Fully Revealing Debate Equilibrium with simultaneous communication exists if and only if preferences are not minimally diverse.

I now state the new result of this review, that with sequential communication and bias uncertainty truthful communication is only an equilibrium if preferences are not minimally diverse.

Proposition 2 Suppose $A1, A2, A3$ hold and that communication is sequential. Then for any $q$-rule a Fully Revealing Debate Equilibrium exists if and only if preferences are not minimally diverse.

Proof: The if direction is trivial, so it only remains to show that a Fully Revealing Debate Equilibrium cannot exist when preferences are minimally diverse.

By Austen–Smith and Feddersen’s result it is possible to restrict attention to the case where $q \not\in \{1, n\}$. For there to be an incentive to mis-report with simultaneous communication there must be some state where some voter benefits from mis-reporting. This voter would have the same incentive to mis-report when that state is realized in a sequential environment.

Suppose the committee is minimally diverse. That means that there exists state $s = (s_{-n}, s_n)$, and biases $b, b' \in B$ such that $s \in S_b, s \not\in S_{b'}$. A fully revealing debate equilibrium requires truth-telling for every signal and bias profile, so it is sufficient to show that there is a bias profile for which truth-telling is not an equilibrium in state $s$. Define $B_Y = \{b \in B : s \in S_b\}$ to be the set of bias types that prefer alternative $Y$ in state $s$.

There are then three possible cases to consider:

1. There exists $s > s_n$ and $b' \not\in B_Y$ such that $(s_{-n}, s) \in S_{b'}$.
2. There exists $s' < s_n$ and $b \in B_Y$ such that $(s_{-n}, s') \not\in S_b$.
3. Given $s_{-n}$, we have that $(s_{-n}, s) \in S_b$ if and only if $(s_{-n}, s') \in S_b$ for all $b \in B$ and $s, s' \in S$.

If the first case holds, and the first $n - 1$ voters are reporting truthfully ($m_{-n} = s_{-n}$) then if $b_n \in B_Y$ voter $n$ would prefer alternative $Y$. By monotonicity any voter who would vote for $Y$ when $s_n$ is reported would also vote for $Y$ when $s$ is reported. If all other voters have bias $b'$ then $X$ will be chosen if voter $n$ reports truthfully and $Y$ will be chosen if she reports $s$. The full support assumption guarantees that the probability that $b_i = b'$ for all $i = 1, \ldots, n - 1$ is strictly positive. So a mis-report of $m_n = s$ instead of $s_n$ is strictly beneficial when $b_n \in B_Y$.

If the second case holds, and the first $n - 1$ voters are reporting truthfully ($m_{-n} = s_{-n}$) then an identical argument shows that a mis-report of $m_n = s'$ instead of $s_n$ is strictly beneficial when $b_n \not\in B_Y$.

Finally, in the third case $s_n$ is irrelevant to all bias types given $s_{-n}$, so voter $n - 1$ knows which alternative she prefers before hearing voter $n$ speak. It is then possible to repeat the argument with voter $n - 1$ and proceed recursively until reaching a voter such that some bias type has not decided on their preferred alternative before the voter speaks.
Since no information has been revealed before the first voter has spoken such a voter must exist.

Notice from the proof that the voter does not always have an incentive to mis-report, but the situations where there is an incentive to mis-report correspond to the situations where the report is relevant.

CONCLUSION

The purpose of this review was to consider how established results on pre-vote deliberation extend to the situation where communication takes place sequentially. In practice most communication is sequential, and indeed simultaneous communication may not be feasible. Unlike sequential and simultaneous voting, equilibria with simultaneous communication are not necessarily equilibria with sequential communication. It is more difficult to ensure truthful communication when the communication is sequential. Since the most frequently used voting mechanism is sequential communication followed by voting, this indicates that either simultaneous communication is infeasible or the commonly used mechanism is sub-optimal, at least assuming the objective is to elicit truthful communication. Based on these results unanimity rule is not the impediment to truthful communication, but rather full truth-telling may be too much to hope for in committee environments regardless of the voting rule used. How the efficiency of different communication protocols and voting rules compare in partially revealing equilibria remains an open question.

REFERENCES