Bayesian DSGE Model Estimation

Lecture 1

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Course overview

Aim of week 1

A standard DSGE model

Gensys

General remarks

- Aim of the course: Develop your own Matlab code that estimates a DSGE model using Bayesian model estimation techniques.
- Homework consists of programming exercises.
- Grading: Final exam (50 %), Homework (30 %) and class participation (20 %).
- Contact
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- Course webpage: http://www.mwpweb.eu/AlexanderKriwoluzky/further_1.html

Overview I

- Week 1:
  - Introduction of the DSGE model used as a work horse throughout the course.
  - Derivation of its reduced form.
- Week 2
  - VAR representation of a DSGE model.
  - Derivation of the Kalman Filter.
- Week 3
  - The Likelihood of a DSGE model.
  - Finding the maximum of the likelihood and computation of the Hessian.

Overview II

- Week 4
  - Introduction to Bayesian estimation.
  - Specification of a prior distribution.
- Week 5
  - Sampling algorithm: Metropolis Hastings.
- Week 6
  - Identification of structural parameters.
  - Marginal data density and model comparison.
- Week 7
  - Diagnostics.
  - Wrap up.

Aim of week 1

Knowledge of the DSGE model used as workhorse and how to derive the reduced form.

Model choice

- DSGE model laid out by Ireland (2004): ’Technology Shocks In The New Keynesian Model’.
- Further described in chapter 5 of the course text book.
- As a starting point we distinguish from Ireland and the book by the following:
  - The technology shock is assumed to be transitory and not permanent.
  - We will first estimate the micro-founded reduced form and subsequently explore why Ireland deviated from estimating it.
- Notation: Stick to the notation of the text book with the exception that nominal variables will be denoted by capital letters.

Model overview

- Continuum of identical households.
- Two types of firms: intermediate- and final-good firms.
- Intermediate good firms:
  - Differentiated products, i.e. imperfectly competitive.
  - Face price setting frictions.
- Final-good firms combine intermediate products. Perfect competition.
- Monetary authority.

Households' problem

$$\max_{c_t,n_t,m_t} \quad U = E_0 \sum_{t=0}^{\infty} \beta^t \left(a_t log(c_t) + log\left(\frac{M_t}{P_t}\right) - \frac{n_t^{\xi}}{\xi} \right)$$

$$s.t. \quad P_t c_t + \frac{b_t P_t}{r_t} + M_t = M_{t-1} + b_{t-1} P_t + \tau_t + W_t n_t + D_t$$

FOC household

$$\frac{W_t}{P_t}\frac{a_t}{c_t} = n_t^{\xi-1} \tag{1}$$

$$\beta E_t\left[\frac{a_{t+1}}{P_{t+1}c_{t+1}}\right] = \frac{a_t}{r_t P_t c_t} \tag{2}$$

$$\frac{P_t}{M_t} + \beta E_t\left[\frac{a_{t+1}}{P_{t+1}c_{t+1}}\right] = \frac{a_t}{P_t c_t} \tag{3}$$

Final-good firm

$$\max_{y_{it}} \quad \Pi_t^F = P_t y_t - \int_0^1 P_{it} y_{it} di \tag{4}$$

$$s.t. \quad y_t = \left(\int_0^1 y_{it}^{\frac{\theta_t - 1}{\theta_t}} di \right)^{\frac{\theta_t}{\theta_t - 1}}$$

Solution to (4) and the non-profit condition yield:

$$y_{it} = y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta_t} \tag{5}$$

$$P_t = \left(\int_0^1 P_{it}^{1-\theta_t} di \right)^{\frac{1}{1-\theta_t}} \tag{6}$$

Intermediate-good firm

Define dividends d_t as:

$$\frac{D_{it}}{P_t} = \frac{P_{it}y_{it} - W_t n_{it}}{P_t} - \chi_t \tag{7}$$

The intermediate-good firm i solves:

$$\begin{aligned} \max_{P_{it}} \quad \Pi^I_{it} &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{a_t}{c_t}\right)\left(\frac{D_t}{P_t}\right) \qquad (8) \\ s.t. \quad y_{it} &= z_t n_t \\ y_{it} &= y_t \left(\frac{P_{it}}{P_t}\right)^{-\theta_t} \\ \chi_t &= \frac{\phi}{2}\left[\frac{P_{it}}{\bar{\pi} P_{it-1}} - 1\right]^2 y_t \end{aligned}$$

FOC Intermediate-good firm

$$
\begin{aligned}
(\theta_t - 1)\left(\frac{P_{it}}{P_t}\right)^{-\theta}\frac{y_t}{P_t} &= \\
\theta_t\left(\frac{P_{it}}{P_t}\right)^{-\theta-1}\frac{W_t}{P_t}\frac{y_t}{z_t}\frac{1}{P_t} - \phi\left[\frac{P_{it}}{\bar{\pi}P_{it-1}} - 1\right]\frac{y_t}{\bar{\pi}P_{it-1}} & \\
+\beta\phi E_t\left[\frac{a_{t+1}}{a_t}\frac{c_t}{c_{t+1}}\left(\frac{P_{it+1}}{\bar{\pi}P_{it}} - 1\right)\frac{y_{t+1}P_{it+1}}{\bar{\pi}P_{it}^2}\right] & \qquad (9)
\end{aligned}
$$

Monetary authority and specification of stochastic processes

The monetary authority follows a Taylor rule:

$$\frac{r_t}{\bar{r}} = \frac{r_{t-1}}{\bar{r}}^{\rho_r} \frac{\pi_t}{\bar{\pi}}^{\rho_\pi} \frac{y_t}{\bar{y}}^{\rho_y} \tag{10}$$

The shock processes evolve according to:

$$\begin{aligned} log(a_t) &= \rho_a log(a_{t-1}) + \epsilon_{a,t} & (11) \\ log(z_t) &= \rho_z log(z_{t-1}) + \epsilon_{z,t} & (12) \\ log(\theta_t) &= (1-\rho_\theta) log(\bar{\theta}) + \rho_\theta log(\theta_{t-1}) + \epsilon_{\theta,t} & (13) \end{aligned}$$

with

$$\epsilon_i \sim \mathcal{N}(0, \sigma_i), \quad i = a, z, \theta \tag{14}$$

Closing the DSGE model

To close the model:

- Assume a symmetric solution for the firm sector.
- Money market clearing: $M_t = M_{t-1} + \tau_t$.
- Bond market clearing: $b_t = b_{t-1} = 0$.

DSGE model solution I

Given the assumptions, the DSGE model is summarized by the following system of equations:

$$y_t = c_t + \frac{\phi}{2}\left[\frac{\pi_t}{\bar{\pi}} - 1\right]^2 y_t \tag{15}$$

$$1 = \beta r_t E_t\left[\frac{a_{t+1}}{a_t}\frac{c_t}{c_{t+1}}\frac{1}{\pi_{t+1}}\right] \tag{16}$$

$$\begin{aligned} 0 = {} & 1 - \theta_t + \theta_t \frac{w_t}{z_t} - \phi\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} + \\ & \beta\phi E_t\left[\frac{a_{t+1}}{a_t}\frac{c_t}{c_{t+1}}\left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{y_t}\right] \end{aligned} \tag{17}$$

DSGE model solution II

$$w_t = \frac{c_t}{a_t}\left(\frac{y_t}{z_t}\right)^{\xi-1} \tag{18}$$

plus the specification of the shock processes ((11) - (13)) and the Taylor rule (10).

The steady state

$$\begin{aligned}
\bar{c} &= \bar{y} \\
\bar{w} &= \bar{y}^{\xi} \\
\bar{\theta} &= \frac{1}{(1-\bar{y}^{\xi})}
\end{aligned}$$

The log-linearized DSGE model

$$\tilde{\pi}_t = \beta E_t[\tilde{\pi}_{t+1}] + \frac{\xi(\bar{\theta}-1)}{\phi}(\tilde{y}_t - \tilde{z}_t - \frac{1}{\xi}\tilde{a}_t) - \frac{1}{\phi}\tilde{\theta}_t \tag{19}$$

$$\tilde{w}_t = -\tilde{a}_t + \xi\tilde{y}_t - (\xi - 1)\tilde{z}_t \tag{20}$$

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \tilde{r}_t + E_t[\tilde{\pi}_{t+1}] + \tilde{a}_t - E_t[\tilde{a}_{t+1}] \tag{21}$$

$$\tilde{r}_t = \rho_r\tilde{r}_{t-1} + \rho_\pi\tilde{\pi}_t + \rho_y\tilde{y}_t \tag{22}$$

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Gensys

What is Gensys?

- Gensys is a Software written by Chris Sims. It solves the system of equation given as:

$$Ax_t = Bx_{t-1} + E + Cv_t + D\eta_t \tag{23}$$

  where x is the state vector of the DSGE model, E a constant, v the structural shocks, η the expectation error.

- It returns the reduced form as:

$$x_t = \Theta_E + \Theta_0 x_{t-1} + \Theta_1 v_t \tag{24}$$

  with $\Theta_E, \Theta_0, \Theta_1$ containing the recursive laws of motion.

- http://sims.princeton.edu/yftp/gensys/

Transformation of the DSGE model

- The vector expectation errors $\eta_t = [\eta_{\pi,t} \eta_{y,t} \eta_{a,t}]'$ is defined as:

$$\eta_{\pi,t} = \tilde{\pi}_t - E_{t-1}[\tilde{\pi}_t] \quad (25)$$
$$\eta_{y,t} = \tilde{y}_t - E_{t-1}[\tilde{y}_t] \quad (26)$$
$$\eta_{a,t} = \tilde{a}_t - E_{t-1}[\tilde{a}_t] \quad (27)$$

- Further define:

$$\varpi_{\pi,t} = E_t[\tilde{\pi}_{t+1}] \quad (28)$$
$$\varpi_{y,t} = E_t[\tilde{y}_{t+1}] \quad (29)$$
$$\varpi_{a,t} = E_t[\tilde{a}_{t+1}] \quad (30)$$

- Substitute $\varpi_{i,t}, \quad i = \pi, y, a$ into the system of equations and add the equations (25-27).

The DSGE model in Gensys form I

$$\tilde{\pi}_t - \beta \varpi_{\pi,t} - \frac{\xi(\bar{\theta}-1)}{\phi}\tilde{y}_t + \frac{\xi(\bar{\theta}-1)}{\phi}\tilde{z}_t + \frac{\bar{\theta}-1}{\phi}\tilde{a}_t + \frac{1}{\phi}\tilde{\theta}_t = 0 \quad (31)$$

$$\tilde{w}_t - \xi\tilde{y}_t + \tilde{a}_t + (\xi - 1)\tilde{z}_t = 0 \quad (32)$$

$$\tilde{y}_t - \varpi_{y,t} + \tilde{r}_t - \varpi_{\pi,t} - \tilde{a}_t + \varpi_{a,t} = 0 \quad (33)$$

$$\tilde{r}_t - \rho_\pi \tilde{\pi}_t - \rho_y \tilde{y}_t = \rho_r \tilde{r}_{t-1} \quad (34)$$

The DSGE model in Gensys form II

$$\tilde{\pi}_t = \varpi_{\pi,t-1} + \eta_{\pi,t} \tag{35}$$

$$\tilde{y}_t = \varpi_{y,t-1} + \eta_{y,t} \tag{36}$$

$$\tilde{a}_t = \varpi_{a,t-1} + \eta_{a,t} \tag{37}$$

$$\tilde{a}_t = \rho_a \tilde{a}_{t-1} + \epsilon_{a,t} \tag{38}$$

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \epsilon_{z,t} \tag{39}$$

$$\tilde{\theta}_t = \rho_\theta \tilde{\theta}_{t-1} + \epsilon_{\theta,t} \tag{40}$$

Homework I

1. Write a Matlab function that returns Θ_1, Θ_0, Θ_E and the shock covariance matrix, given the vector of deep parameters μ.
2. Write a routine that checks whether the solution returned by Gensys is unique and determined.