Bayesian DSGE Model Estimation

Lecture three

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Aim of week three

Likelihood function

The Hessian

Issues and Pitfalls

Aim of the week

Evaluate and maximize the likelihood function. Compute the Hessian at the maximum.

Aim of week three

Likelihood function

The Hessian

Issues and Pitfalls

Definition

We want to derive a function that tells us how likely the time series is, i.e. the realization of the data, given a vector of deep parameters (and a DSGE model structure).

Formal derivation

- Assume: The initial state and the sequence of innovations $\{v_t, u_t\}_{t=1}^{T}$ are multivariate Gaussian.
- The distribution of X_t conditional on $\mathscr{I}_{t-1} = X_{t-1}, \cdots, X_1$ is given by:

$$X_t | \mathscr{I}_{t-1}, \mu \sim \mathcal{N}\left(H(\mu)\hat{x}_t, (H(\mu)\Sigma_{x,t}H(\mu)' + \Sigma_u(\mu))\right) \tag{1}$$

- Recall that: $\Sigma_{st} = H(\mu)\Sigma_{xt}H(\mu)' + \Sigma_u(\mu)$ and X is of size $n \times 1$. Then:

$$\mathscr{L}(X_t|\mathscr{I}_{t-1}, \mu) = (2\pi)^{-n/2}|\Sigma_{st}|^{-0.5} \times exp\{-0.5(X_t - H\hat{x}_t)'\Sigma_{st}^{-1}(X_t - H\hat{x}_t)\} \tag{2}$$

Conditional and unconditional likelihood function

- Equation (2) denotes a conditional likelihood function.
- The likelihood associated with the sample X is given by the product of the conditional likelihood functions:

$$\mathscr{L}(X|\mu) = \Pi_{t=1}^{T} \mathscr{L}(X_t|\mathscr{I}_{t-1}\mu) \quad (3)$$

- Correspondingly, the log likelihood $l = log(\mathscr{L})$is given by:

$$l(X|\mu) = \sum_{t=1}^{T} l(X_t|\mathscr{I}_{t-1}\mu) \quad (4)$$

Evaluation of the likelihood function

(4) is evaluated the following way.

- Compute the constant:

$$\mathscr{C} = -(T * n/2) * log(2 * pi) \tag{5}$$

- At every iteration step i, given s_i and $\Sigma_{s,i}$ compute:

$$l(X_i|\mathscr{I}\mu) = -0.5 * log|\Sigma_{s,i}| - 0.5 * (s_i'\Sigma_{s,i}^{-1}s_i) \tag{6}$$

- Add up (5) and the i-values computed according to (6).

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Issues and Pitfalls

Definition

- The Hessian $\mathscr{H}$ as a matrix, which $i,j-th$ element is defined as:

$$\mathscr{H}_{i,j} = \frac{\partial^2 l(X|\mu)}{\partial \mu_i \partial \mu_j} \tag{7}$$

- The inverse of the negative Hessian is a consistent estimator of the variance covariance matrix of a maximum likelihood estimator.

Hints on numerically evaluating a Hessian

- Consider the $i, j-th$ entry (μ_i picks the $i-th$ entry):
- Define $f = f(\mu), f_i = f(\mu_i + dx), f_j = f(\mu_j - dx),$ $f_{i,j} = f(\mu_{ij} + / - dx).$
- The $i, j-th$ of the Hessian is defined as:

$$\mathcal{H}_{i,j} = \frac{(f_i - f + f_j - f_{i,j})}{dx^2}$$

- Remember that the Hessian is a symmetric(!) matrix.
- It could be useful to consider different numerical values of dx and compute the average of the resulting $\mathcal{H}_{i,j}$.

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Likelihood function

The Hessian

Issues and Pitfalls

Issues and Pitfalls

- Singularity.
- Maximization.
- Different local maxima.
- Cliffs, spikes and walls.
- Identification.

Singularity

- Number of structural shocks has to equal the number of observable variables.
- If not, observable variables are assumed to be linear-combinations.
- Possible way out: Add measurement errors in the observation equation.

Maximization of the likelihood

- We will use a software by Chris Sims: csminwel
- It is a minimization routine, so we will minimize the negative likelihood.
- It is based on a derivative-based minimization routine. If the likelihood surface displays discontinuities it employs a simplex algorithm.
- Define a starting value x_0, a corresponding Hessian h_0,precision p,max number iteration *maxiter*
- Call: $[fh, xh, gh, Hhr, itct, fcount, retcodeh] = csminwel('functionname', x_0, h_0, [], p, maxiter)$

Issues related to numerical routines

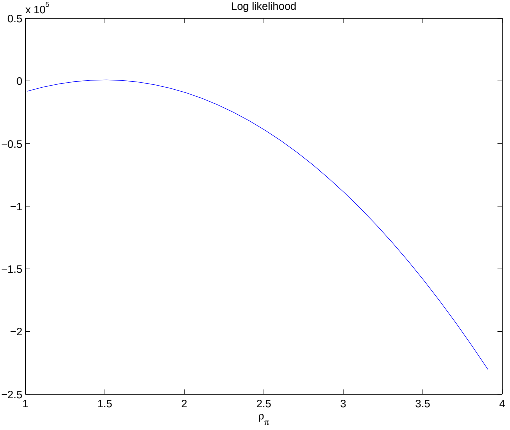
- To account for different locale maxima: use different starting values.
- Restart csminwel with the optimized value but a 'reset' Hessian.
- Splitting up the vector of parameters.

Artificial time series

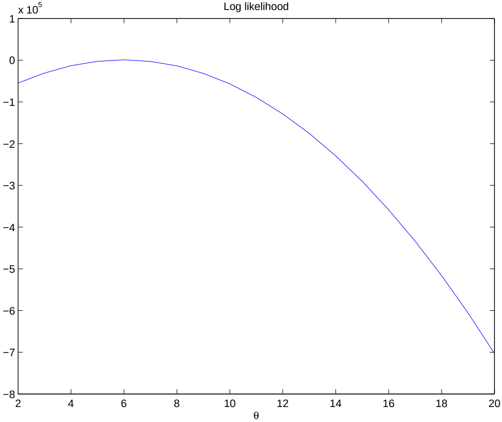
Table: Parametrization of the DSGE model

Parameter	distribution
β	0.99
ξ	1/0.06
ϕ	10
$\bar{\theta}$	6
ρ_r	0.5
ρ_π	1.5
ρ_y	0.3
$\rho_a, \rho_z, \rho_\theta$	0.5
$\sigma_a, \sigma_z, \sigma_\theta$	1

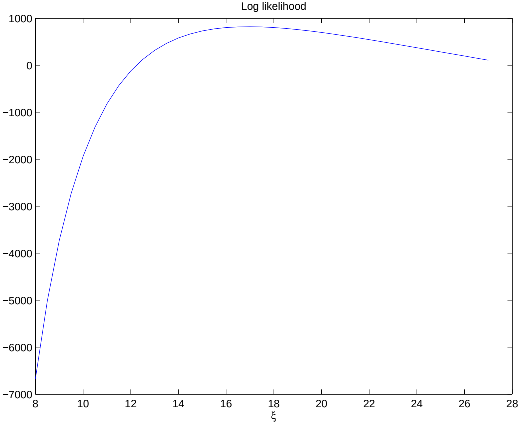
Log likelihood plot I



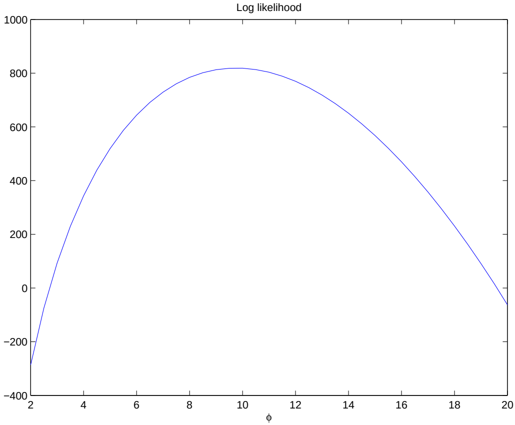
Log likelihood plot II



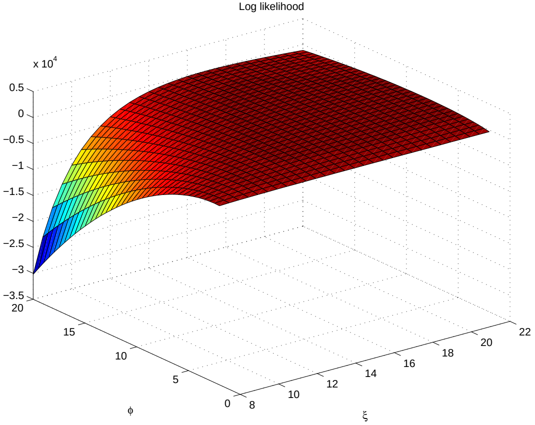
Log likelihood plot III



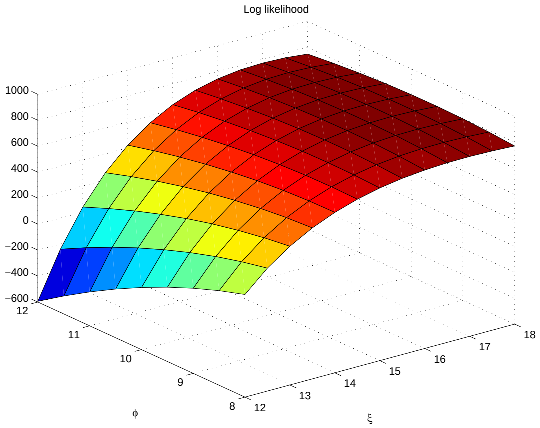
Log likelihood plot IV



Log likelihood plot V



Log likelihood plot VI



Cliffs

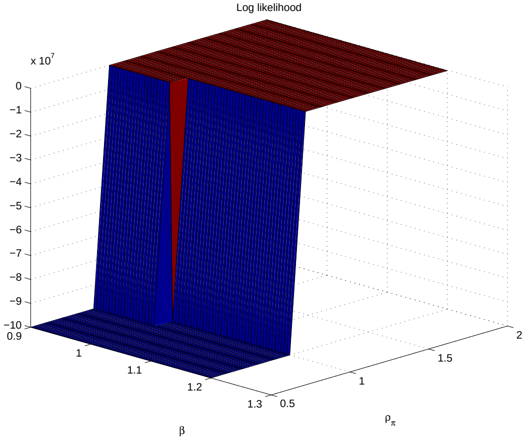


Figure: Data was simulated with $\rho_r = \rho_y = 0$, $\rho_\pi = 1.05$.

Homework

- Simulate artificial data from the DSGE model.
- Add to the Kalman-Filter function the computation of the likelihood.
- Write a code that maximizes the likelihood of a DSGE model.
- Write a function that computes the Hessian at the maximum of the likelihood.
- Check your code by estimating some parameters of the DSGE model employing the artificial time series.