Lexicographic Voting: Holding Parties Accountable in the Presence of Downsian Competition

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Abstract

This paper combines ideas from models of electoral competition with forward-looking voters and models of electoral competition with backward-looking voters. Two political parties can commit in advance to policy platforms, but not to a maximum level of rent extraction. In the case without uncertainty, the electorate can limit rents to the same extent as in a purely backward-looking model of accountability and the policy preferred by the voter who represents the median preferences of the electorate is implemented. In the case with uncertainty about the bliss point of the representative voter, the electorate has to accept higher rent seeking by the incumbent politician, but nonetheless retains some control over rent extraction. The policy positions of the two competing parties do not converge as they do in the case without uncertainty. I show in an example

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that this nonconvergence can increase the welfare of the representative voter.

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1 Introduction

Do voters reward incumbents for past success and honesty or do they disregard the past and only consider future policies when they vote? This is one of the most fundamental questions for a positive theory of electoral competition. In models of pre-election politics, candidates commit to their post-election actions before the election takes place. In contrast, in models of postelection politics, politicians are free to decide about their policies when they are in office. However, in the successive election, the voters can condition their vote on the performance of the *incumbent* party.[1]

In this paper, I combine a simple prospective model of Downsian spatial electoral competition on an ideological policy dimension and a simple retrospective model of electoral accountability with rent extraction. Specifically, parties commit to a policy position before an election takes place, as in Downs (1957), but decide on the level of rent extraction once they are in office, as in Barro (1973) and the simplified model of political accountability discussed in Persson and Tabellini (2000).

In the baseline model in Section 2, I show that, as long as there is certainty about the position of the representative voter, having voters with divergent policy prefer-

[1] Retrospective and prospective voting seem to be self-explanatory terms. Either voters consider past performance or expectations about future performance when they make their voting decision. However, as soon as we use game theoretic models of elections, the distinction turns out to be far from trivial. By the definition of Nash-equilibrium, every voting strategy that is part of an equilibrium must be prospective in the sense that it maximizes the (expected) utility of the voter. Nonetheless, it seems reasonable to call strategies that can be completely described by past utility levels as retrospective and strategies that depend only on variables that influence a voter's utility in the future as prospective. A formal definition along these lines is provided by Duggan (2000).

ences does not restrict the possibility of holding politicians accountable. The possible equilibrium rent levels are the same as in a model without the additional ideological policy dimension. The representative voter achieves this by following a straightforward and intuitive lexicographic voting strategy. More specifically, if the parties commit to policy positions that differ in attractiveness to a voter, the voter casts her vote for the party that minimizes her disutility on the policy dimension. Only when she is indifferent with respect to the parties' policy platforms, she conditions her vote on the degree of rent extraction of the *incumbent* party. She supports the *incumbent* party only if the rents have not exceeded a maximum acceptable level. I call this voting strategy "lexicographic" because voters cast their votes as if they had lexicographic preferences over policy and rents.[2] My model is the first to show that lexicographic voting reconciles backward-looking and forward-looking voting when the identity of the decisive voter is known. The lowest possible rent level that is sustainable in equilibrium is positive, but smaller than the maximum rent the *incumbent* party could extract. Moreover, it is the same as in a model without ideological policy dimension.

The lexicographic voting strategy forces the parties to converge on the policy dimension, but also allows for control of the *incumbent* party's rent extraction. It is intuitive that a voter who is indifferent will take past actions of the parties into account, whereas it is impossible for a rational forward-looking voter to consider the past when she is not indifferent with respect to the future.

Generally, the equilibria in backward-looking models hinge on the fact that voters are indifferent between the *incumbent* party and the *opposition* and can therefore

[2] The term lexicographic voting has been used before to describe similar voting strategies, for example in Dutter (1981) and Soberman and Sadoulet (2007). However, in these papers, lexicographic voting follows directly from lexicographic preferences. In my model, lexicographic voting is part of an equilibrium of the voting game, although the voters' preferences are not lexicographic.

reward or punish past actions while playing undominated strategies. The fact that a simple strategy can solve the accountability problem in a model combining rent extraction with Downsian competition can be explained by the fact that competition on the ideological policy dimension forces both parties to choose the same platform so that voters are indeed indifferent between the parties in equilibrium. Full convergence of policy is a result of the lack of uncertainty over the preferences of the representative voter in the baseline model.

Section 3 provides the most interesting results. It shows that with uncertainty over voters' preferences, the minimum equilibrium rent extraction by the *incumbent* party increases. Because the parties do not know the position of the representative voter's bliss point with certainty, the *opposition* party now has a chance of winning office by offering a different policy position than the *incumbent* party. Nonetheless, the *incumbent* party has an incentive to accept rents below the maximum level in return for being re-elected whenever the voters are indifferent between the policy positions of the two parties as this increases its chances of being reelected. Now, the parties play mixed strategies and choose different policy positions with positive probability in equilibrium. Consequently, convergence of policy platforms becomes random and the exception rather than the rule. In equilibrium the *incumbent* party wins with a probability that is larger than 50%, but not with certainty.

Platform divergence is an interesting result because it is usually observed only in models where uncertainty over the preferences of the representative voter is combined with ideological politicians, and not in models in which parties have no policy preferences (Persson and Tabellini 2000). The reason is simply that in standard models without rent-seeking there is one platform that maximizes the probability of winning for both parties, but this is not longer the case when voters decide according to past behavior when they are indifferent. In addition, the minimum amount of rent-seeking is now partly determined by the additional ideological dimension of policy. Consequently, relying on a separate analysis for accountability and competition on the policy dimension could not capture this important interaction between accountability and policy choice, although backward-looking and forward-looking voting motives are combined in the strategy of the voter in the same way as in the case with certainty.

1.1 Related literature

Models of preelection politics are especially popular for modeling spatial policy choices in the tradition of Downs (1957), where voters decide between announced policy positions, while models of postelection politics are often, but not exclusively, applied to accountability issues. In models of postelection politics, politicians are induced to put in more effort or to limit rent extraction due to the possibility of losing the successive election and office if they do not comply (Barro 1973). Essentially, these accountability models apply a principal-agent framework to elections with the politicians as agents and the voters as their principals.[3]

Van Weelden (2013) and a follow up paper that provides some additional results, Van Weelden (2015), provide, to the best of my knowledge, the only other model that combines rent-seeking and competition on policy positions. A major difference to my model is that, instead of parties, Van Weelden assumes a continuum of possible candidates who are ideological and cannot commit to policy platforms. As a consequence, *incumbent* parties can be held accountable by the threat of the election of an alternative candidate who implements policies they do not like. This reduces the minimum rents sustainable in equilibrium, but comes at the cost of policies that diverge from the bliss point of the representative voter. It remains open how the

[3]For an overview of both types of model, see Persson and Tabellini (2000). For an overview of models of accountability, see Ashworth (2012) and Besley (2006).

model could deal with uncertainty over the preferences of the representative voter.

There is a growing subbranch of the literature on political accountability that is dealing with the question how elections can incentivize politicians to exert effort when effort is not directly observable. This literature is related to my model because the rents in my setup could alternatively be interpreted as shirking by the party of politician in office (that is, the politician does not provide the optimal amount of unobservable effort).[4] The major difference to my approach is that while my model researches the interaction with competiton (and in Section 3 also uncertainty) on a Downsian policy Dimension, the literature so far focuses on heterogeneity (and uncertainty over) the candidates' types. Many ideas in this literature go back to a seminal theoretical paper by Holmström (1982) and were applied to elections by Banks and Sundaram (1993). The main difference to a standard principal-agent problem is that the payoff of the agent cannot be directly linked to the output produced. However, output can still influence future wage (Holmström) respectively the reelection prospects of a politician (Banks and Sundaram). A more recent paper in this vein is Schwabe (2011). He presents a model closely related to Banks and Sundaram, but while Banks and Sundaram present only equilibria in which voters use simple retrospective voting rules with a performance threshold that is the same in all periods (as in the model presented here), Schwabe shows that in his slightly less general setup equilibria exist in which the performance threshold is not constant over time, voters are better off and the equilibrium is renegotiation proof. Moreover, contrary to the results in Banks and Sundaram, voters are in equilibrium indifferent between high quality and low quality politicians. Ashworth, de Mesquita, and Friedenberg (2010) combine models of selecting high-quality politicians with rewarding effort of

[4] For a discussion of this possible reinterpretation of the assumptions, see Martinez (2009). Persson and Tabellini (2000) also choose a simple model with rent-extraction instead of unobservable effort to introduce the literature.

politicians and asks whether it is possible to incentivize politicians even when it is known that not all politicians have the same quality. Moreover, this paper contains a very useful discussion about the distinction between standard-setting for creating incentives (as in the model presented here) and standard-setting for the purpose of selecting good types and shows that the two purposes can be consistent with each other.

Several papers on Downsian competition are related to the model I present. Aragones and Palfrey (2002) model Downsian competition in a model where one of the candidates is of higher quality. As a consequence, just as in the model with uncertainty presented below in Section 3, there is only an equilibrium in mixed strategies. While Aragones and Palfrey (2002) present a one-shot model, the model I present is a contribution to the literature that models electoral competition on an ideological policy dimension as a repeated game. Important papers in this literature are Duggan (2000), Duggan and Fey (2006), Aragones, Palfrey, and Postlewaite (2007) and Banks and Duggan (2008). However, none of these papers considers the rent-seeking issue. Duggan (2000) and Banks and Duggan (2008) model repeated elections when the policy preferences of the candidates are private information. The first paper shows that there is an equilibrium that is consistent with prospective and retrospective voting at the same time, while the second paper extends the model to multiple policy dimensions. Aragones, Palfrey, and Postlewaite (2007) address the credibility of policy announcements when politicians have policy preferences that are known to the voters, a problem that makes a dynamic model necessary. Not surprisingly, what kind of promises politicians can make in a credible way depends on their policy preferences. Similar to Duggan (2000) and Banks and Duggan (2008), it is shown that and how policy compromise is possible in repeated games with ideological candidates.

Duggan and Fey (2006) assume parties that care only about winning office. Their paper shows what kind of equilibria are possible in an infinitely repeated Downsian model of political competition. As the folk theorem suggests, many equilibria can be supported in a model of repeated elections. Duggan and Fey (2006) make some additional restrictions that are standard in game theoretic models of elections and show that arbitrary paths of policies can be supported in equilibrium if some conditions on discount factors hold.

1.2 Organization of the paper

The paper proceeds as follows. Section 2 develops the model with certainty about the position of the representative voter and discusses its equilibrium. Section 3 shows that uncertainty over the position of the representative voter leads to less electoral accountability and higher minimum rents in equilibrium. The paper ends with a conclusion and an example for an equilibrium with uncertainty is provided in the Appendix.

2 The model

I consider a polity with two parties interested in winning office only for rent-seeking purposes, and an odd number N of voters $i = 1, 2, ..., n$ interested in policy as well as rent reduction. The ideological policy space is the interval $[0, 1]$. Party $j \in \{x, y\}$ maximizes its expected payoff:

$$U_p^j = E_0 \sum_{t=0}^{\infty} \beta^t r_t^j, \tag{1}$$

where rents in future periods are discounted by the factor $\beta \in (0, 1)$. r_t^j is the rent extracted by party j in period t. The party in government (also called the *incumbent* party) in period t is denoted by $I_t \in \{x, y\}$. The *opposition* party in period t is denoted by $O_t \in \{x, y\}$, $O_t \neq I_t$. Parties decide how much rent $r_t \in [0, R]$ they extract in a period in which they are in office. R is the total amount of available public funds and constitutes the maximum per period rent. Parties out of office cannot acquire any rents. Hence, $r_t^{O_t} = 0$ in all periods.

The representative (median) voter's utility is given by:

$$U_v = E_0 \sum_{t=0}^{\infty} \beta^t (- |p_t - b|^{\lambda} + R - r_t)), \tag{2}$$

where $\lambda > 0$, b is the policy bliss point of the representative voter, $r_t = r_t^x + r_t^y$ the rent extraction and p_t the policy implemented in period t. Because the policy platform announced by the *incumbent* is always implemented and $r_t^{O_t} = 0$ in all periods t, $p_t = p_t^{I_t}$ and $r_t = r_t^{I_t}$. I abstract from any details on how rents are extracted and assume that rent payments reduce a given amount of public funds, which reduces every voter's utility in the same way. Hence, $R - r_t^{I_t}$ gives the amount of public funds that are used in the voters' interest. For simplicity, I assume that the utility from public good spending is uncorrelated with the ideological policy position. $p_t \in P = \{\hat{p}_1, \hat{p}_2, ..., \hat{p}_K\}$ denotes the policy in period t. P is a set containing the finite number K of possible policy positions. The set is ordered so that policy positions further to the left are denoted with lower subscripts: $\hat{p}_1 < \hat{p}_2 < ... < \hat{p}_K$. Restricting the possible policies to a finite number plays no role for the results presented in Section 2 and all of them would hold without any restrictions on possible policy positions. However, this assumption will be important to ensure the existence of an equilibrium once we allow for uncertainty over the preferences of the representative voter in Section 3. By assumption, the policy bliss point of the representative voter $b \in P$ and thus parties can choose to offer the policy platform favoured by the representative voter. Disutility in policy is concave as often assumed if and only if $\lambda > 1$. However, for the results presented here the assumption of concavity is not necessary and increasing disutility in increasing distance from the policy bliss point as implied by $\lambda > 0$ is sufficient. The exact value of λ matters only for welfare, not for equilibrium.

2.1 The order of moves

The order of moves is the following: In any period t, the policy position p_t^I of the *incumbent* party $I_t \in \{x, y\}$ is implemented (thus, $p_t = p_t^I$), then the rent level $r_t^{I_t}$ is chosen by the *incumbent* party. Next, after $r_t^{I_t}$ is observed by all players, both parties simultaneously choose a policy position contained in the set of possible platforms P. The policy chosen by the *incumbent* is called $p_{t+1}^{I_t}$and the policy chosen by the *challenger* is called $p_{t+1}^{O_t}$. After policy positions are chosen, an election takes place and the representative voter casts her vote and thus decides which party wins the election and becomes the *incumbent* in the next period. In period 0, the identity and the policy positions of the *incumbent* party and the *opposition* are exogenously given.

2.2 Strategies

To denote the entire history of a variable z_t up to period t, I use a superscript t to denote an ordered vector $z^t = \{z_0, z_1, z_2, ..., z_t\}$. Then $h_t = \{p^{y,t}, p^{x,t}, I^t, r^{t-1}\}$ denotes the complete history of the game up to the beginning of period t and contains all past values of all variables. A strategy for a party j consists of the rent payment $r_t^j(h_t)$ for all histories with $j = I_t$ and the decision about a policy platform $p_{t+1}^j(h_t, r_t)$ for all histories up to the beginning of period t and the rent extraction in that period. A strategy for the representative voter is an $I_{t+1}(h_t, r_t, p_{t+1}^y, p_{t+1}^x) \in \{y, x\}$ for every period $t+1$ and every possible history up to the time of her voting decision. However, in all equilibria discussed in the paper the vote that decides over the next incumbent depends only on $p^y_{t+1}, p^x_{t+1}, r_t$ and I_t. This is discussed in more detail in the following Section.

2.3 Stationarity

In the analysis, we consider only subgame perfect equilibria with strategies that are stationary and symmetric according to the following definition:[5]

Definition 1 *Stationary symmetric strategies*

Strategies are stationary if and only if:

1. The voting decision of the representative voter depends only on announced policy positions and rent-seeking by the *incumbent* party in the previous period.

2. The parties' rent-seeking and policy platforms are not influenced by the history of the game.

   The parties' strategies are symmetric if and only if:

3. Both parties play the same strategy.

   Strategies that are stationary and symmetric are called stationary symmetric strategies.

Discussion of stationarity and symmetry In a stationary equilibrium in which both parties follow the same strategy, the representative voter is not able to influence the future policy announcements or rents. As a consequence, when she maximizes her

[5] See Van Weelden (2013) for a related definition and a discussion of the advantages of focusing on stationary equilibria.

utility with respect to policy in the next period, she also maximizes her total utility. This has the advantage that the results are robust to changes in voters preferences and changes in the electorate when we interpret the representative voter as the median voter of the electorate whose identity can change over time. In addition, our definition of stationarity implies that on the policy dimension the analysis essentially boils down to the analysis of a one-shot game as in standard models of Downsian competition and the fact that the election game is infinitely repeated allows us to deal with issues of rent-seeking and accountability in the way that is standard in the accountability literature. Consequently, the differences to the results in the literature that we will find in Section 3 are due to the interaction of accountability and Downsian competition, and not to the additional dynamics of the game.

2.4 An equilibrium with lexicographic voting

The strategies formulated in Proposition 1 below constitute an equilibrium which has all the essential features of a backward-looking model in the tradition of Barro (1973) and Ferejohn (1986) as well as those of a forward-looking model in the tradition of Downs (1957). Parties converge on the ideological dimension and rents are at the lowest level sustainable in the purely backward-looking model without policy dimension. This is the result of the intuitive lexicographic voting strategy. The representative voter casts her ballot in favor of her preferred policy position. Only when she is indifferent in this respect does she decide according to past rent extraction by the *incumbent* party. With this a strategy, she encounters no credibility or time-inconsistency problem.

Proposition 1 *An equilibrium of the game is constituted by the following strategies:*

The parties play:

$$p_{t+1}^{j} = b \text{ for } j = y, x \text{ in all } t \text{ in all histories}, \quad (3)$$
$$r_t^{I_t} = \bar{r} \text{ in all } t \text{ in all histories},$$

where $\bar{r} = (1-\beta)R$.

The representative voter plays:

$$I_{t+1} = \begin{cases} y \text{ if } \left|p_{t+1}^{y} - b\right|^{\lambda} - \left|p_{t+1}^{x} - b\right|^{\lambda} < 0 \\ x \text{ if } \left|p_{t+1}^{y} - b\right|^{\lambda} - \left|p_{t+1}^{x} - b\right|^{\lambda} > 0 \\ I_t \text{ if } \left|p_{t+1}^{y} - b\right|^{\lambda} - \left|p_{t+1}^{x} - b\right|^{\lambda} = 0 \text{ and } r_t \leq \bar{r} \\ O_t \text{ if } \left|p_{t+1}^{y} - b\right|^{\lambda} - \left|p_{t+1}^{x} - b\right|^{\lambda} = 0 \text{ and } r_t > \bar{r} \end{cases} \text{ in all } t \text{ in all histories.} \quad (4)$$

Given the strategies, it follows that:

$$I_t = I_0 \text{ in all } t, \quad (5)$$
$$p_t = b \text{ in all } t \geq 1,$$
$$r_t = \bar{r} \text{ in all } t.$$

The party with the support of the representative voter wins. Given the equilibrium strategies of the parties, $|p_t^y - b|^{\lambda} = |p_t^x - b|^{\lambda}$ *in all periods. Because* $r_t = \bar{r}$ *in all periods, the representative voter votes for the incumbent party, which remains in office and implements* $p_{t+1}^{I} = b$.

Proof. Given the strategies of the parties, the representative voter in period t neither influences future rent payments nor future policy platforms (any p_s^j with $s > t+1$) with her vote. Therefore, the voter has no utility-increasing deviation from voting for the party that offers the policy closest to her bliss point in period $t+1$. In case she is indifferent between the candidates' policy platforms in period $t+1$, there is no utility-increasing deviation from voting according to the past performance of the *incumbent* because, again, it does not influence future policy or rent payments.

The fact that the *opposition* party cannot be better off by deviating follows from the fact that given the position and rent extraction of the *incumbent* party and the strategy of the representative voter, it either wins with certainty or has no possibility of winning office. Moreover, it cannot influence any election results or rent payments in the future with its choice of policy position. For the *incumbent* party, any policy position different from $p_{t+1}^I = b$ leads to a loss of office (and therefore rent payments) forever because given the reply of the *opposition*, the latter is preferred by the representative voter. The same is true for the combination of any policy position p_{t+1}^I with any rent $r_t > \bar{r}$. Therefore, re-election is only possible with $r \leq \bar{r}$. Hence, there is no possibility for the *incumbent* party of increasing its utility by deviating with a strategy that leads to its re-election. If it accepts defeat by deviating in an arbitrary period s, the *incumbent* party can, at most, achieve a rent of R in the period in which it deviates and then lose office and rents forever. This gives the same utility level that the *incumbent* party achieves by not deviating and receiving a rent of $r_t = \bar{r} = (1-\beta)R$ forever, because the present discounted value of future rent payments in period s is the same:

$$\sum_{t=0}^{\infty} \beta^t \bar{r} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \sum_{t=s}^{\infty} \beta^t \bar{r} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \sum_{t=s}^{\infty} \beta^s \frac{\bar{r}}{1-\beta} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \beta^s R.$$

Therefore, no deviation from the given strategy increases the utility of the *incumbent* party. ■

The identity of the *incumbent* party in period 0 is exogenously given. This party remains in office forever, as in the standard case of backward-looking models without uncertainty. However, this will no longer be the case when I introduce some uncertainty in Section 3.

Corollary 1 *There is no equilibrium with a present discounted value of future rent payments in any period s of the game that is lower than the maximum per-period rent extraction R.*

Proof. Suppose that there is an equilibrium with $\sum_{t=s}^{\infty} \beta^{t+s} r_t < R$ in any period s. Then, the *incumbent* party in period s is better off by deviating and taking a rent of $r_s = R$. This is a contradiction. ■

Therefore, the equilibrium rent level in Proposition 1 gives a lower bound for rents in equilibrium.[6] The rent level is identical to the lower bound on rent extraction in a model without a policy dimension.[7]

As is also common in models of political accountability, the given equilibrium is not unique and other equilibria with larger rent payments exist. However, the existence of the equilibrium presented above is sufficient to establish that retrospective and prospective motives in voting are not inconsistent with each other. The voter who represents the median preferences of the electorate has only one instrument, namely her single vote, but this is sufficient to control policy as well as to hold politicians accountable to a certain degree.

[6] There are equilibria with a lower rent payment $r_t < \bar{r}$ in period t that are sustainable because the incumbent party expects higher rent payments in the future. However, from Corollary 1, we know that the present discounted value of rent extraction cannot be smaller than R. Equilibria with increasing rent payments over time seem rather implausible because the opposition party could try to convince the voters that it would only demand a constant rent payment of $\bar{r}$ once in office.

[7] This can easily be established following the same line of reasoning as in the proof of Corollary 1.

While certainty over the preferences of the median voter and our restriction to symmetric stationary strategies seems to imply convergence on the policy dimension, there are equilibria without full policy convergence because a party that never wins in equilibrium can choose any policy position. However, there are no equilibria with symmetric stationary strategies that lead to policies different from the median bliss point.

2.5 Discussion of the different treatment of rents and policy

I assume that commitments to electoral platforms are credible on the policy dimension, but lack credibility on the rent dimension. These are widely accepted standard assumptions for both types of models and thus it is important to explore whether combining them leads to results that cannot be found by looking at the models separately. A justification for the different treatment can be seen in the fact that parties have no reason to break their electoral promises with regard to policy because it does not enter their utility function.

3 Uncertainty over the electorate's preferences

So far, I have assumed that the identity of the representative voter is known when parties decide on their policy platforms. How robust are the results to relaxing this assumption? This section shows that voters retain some control over rent extraction in a straightforward and plausible equilibrium where they follow again a lexicographic voting strategy as in Section 2. However, now parties do not converge on the policy dimension and the minimum rent sustainable in equilibrium is larger.

The assumptions and the order of moves are the same as in Section 2. The only difference is that the favorite position of the representative voter is now uncertain.[8] Voters keep some control over rent extraction, but the control is limited because sometimes the *incumbent* party loses office even when it does not deviate and therefore can demand higher rents in equilibrium.

As before, there is one representative voter. Her expected utility is now given by:

$$U_v = E_0 \sum_{t=0}^{\infty} \beta^t (- |p_t - b_t|^\lambda + R - r_t), \tag{2'}$$

where $b_t \in P$ is her bliss point in period t that is now a random variable determined right after the policy positions of the parties are announced.[9] The value of b_t is distributed identically and independently of past bliss points. Let $q_k \geq 0$ be the probability that the representative voter in period t turns out to have the bliss point $\hat{p}_k \in P$, with $\sum_k^K q_k = 1$ and $q_k > 0$ for at least 2 different policy positions contained in P to ensure that there is some uncertainty over the preferences of the median voter. We use $Q = \{q_1, q_{2,...,}q_K\}$ to denote the ordered set of the probabilities. By assumption, the probabilities contained in Q are constant over time and independent of the outcomes in previous periods. The set $B = \{\hat{p}_k : \hat{p}_k \in P,\ q_k > 0\}$, a subset of P, contains all potential bliss points of the median voter.

The expected utility function of the parties $j = y, x$ is identical to the expected utility function in Section 2:

$$U_p^j = E_0 \sum_{t=0}^{\infty} \beta^t r_t^j. \tag{1}$$

[8]It does not matter for the equilibrium presented here if the parties observe the bliss point of the representative voter after the elections. Because it seems more realistic I assume that they do not. In addition, this assumption makes commitment to policy platforms ahead of the elections more plausible.

[9]Consequently, the history of the game up to period t now includes the bliss points of the representative voter and is now denoted by $h_t = \{p^{y,t}, p^{x,t}, b^t, I^t, r^{t-1}\}$ and includes the bliss point of the representative voter. However, because politicians do not observe the bliss point of the representative voter, their strategies can only depend on the other variables.

3.1 The simultaneous policy choice game

We now analyze a simultaneous policy choice of the parties, taking as given that a representative voter plays a lexicographic strategy similar to the one introduced in Section 2.4. Consequently, the representative voter elects the party she prefers with respect to the policy position whenever one of the two parties is closer to her policy bliss point, but votes for her *favorite* party F and against the *challenger* party C whenever she is indifferent with respect to the policy positions. The aim of both parties, the *favorite* as well as the *challenger*, is to maximize the probability of winning the election.[10] When presenting the equilibrium of the complete model, we will endogenize the identity of the *favorite* party by making it dependent on the level of rent extraction of the *incumbent* party, but for the moment it significantly simplifies the analysis that we first ignore the fact that the simultaneous policy choice is part of a larger game.[11] This allows us to apply some standard result for simultaneous move zero-sum games.

In the game at hand, for a party to maximize its expected payoff is the same as maximizing its probability of winning the elections. A strategy in the simultaneous policy choice game simply assigns a nonnegative probability σ_p^J to every policy position that can be chosen by a party with the constraint that $\sum_1^K \sigma_k^J = 1$ for both parties $J \in \{F, C\}$. We have a zero-sum game because the payoffs, in our case the probabilities for winning, sum up to 1. The loss of one of the players is the gain of the other player. Because both players have a finite number of strategies, an

[10] A somewhat related simultaneous policy choice game also resulting in mixed strategy equilibria is presented in Aragones and Palfrey (2002). However, instead of only allowing for a discrete number of possible policy bliss points, in their model voters prefer one of the candidates whenever the distance between the policy positions does not exceed a certain distance.

[11] The simultaneous policy choice game is not a subgame of the larger game. There are subgames that begin with a simultaneous policy choice, but these subgames also contain all decisions made at later points of time.

equilibrium in mixed strategies exists (see for example Mas-Colell, Whinston, Green, et al. (1995)). We denote the equilibrium strategies, containing the probabilities with which every policy position is played in equilibrium, with $\sigma_F^*(P, Q)$ or short σ_F^* (for the *favorite* party) and $\sigma_C^*(P, Q)$ or short σ_C^* (for the *challenger* party). In all equilibria in a two-player zero-sum game a player achieves the same expected payoff and is thus indifferent between all possible equilibria if more than one exists (This result is stated in Myerson (1991)). Thus, while it is conceivable that there exists more than one equilibrium of the policy choice game, both parties would be indifferent between them because in all equilibria they must have the same expected payoff and the expected payoff is the probability of winning the election. We denote this equilibrium probability of the *favorite* winning depending on the possible policy bliss points of the representative voter and their probabilities as $\pi_F^*(P, Q)$ or short π_F^*. The probability of the *challenger* winning is denoted by $\pi_C^*(P, Q)$ or short π_C^*.

Without analyzing a specific simultaneous policy choice game described by two sets P and Q, we can still make some general statements that do hold for arbitrary sets P and Q. $F(\hat{p}_k) = \sum_{l=1}^{l=k} q_l$ is the cumulative distribution function of the possible bliss points of the representative voter b_t in any period t. I define:

$$\begin{aligned} b^m &= \min\{\hat{p}_k : \ \hat{p}_k \in B, \ F(\hat{p}_k) \geq 0.5\}, \\ b^{m-1} &= \max\ \{\hat{p}_k : \ \hat{p}_k \in B, \ \hat{p}_k < b^m\}, \\ b^{m+1} &= \min\ \{\hat{p}_k : \ \hat{p}_k \in B, \ \hat{p}_k > b^m\}, \end{aligned} \tag{6}$$

so that b^m is the median of the possible bliss points of the representative voter and b^{m-1} and b^{m+1} are the potential bliss points of the representative that are situated closest to it on its left and on its right in the policy space. Because at least two elements are contained in B, either b^{m-1} or b^{m+1} or both exist. The *favorite* party wins with a chance of at least 50% given any P and Q by choosing $p_f = b^m$ with certainty. Against $p_f = b^m$, a best reply of the *challenger* is given by $p_c = b^{m-1}$ or by $p_c = b^{m+1}$. With these replies, the *challenger* wins either whenever the bliss point of the representative voter is smaller or larger than b^m. For all cases except when $F(b^m) = 0.5$, this leads to a chance of winning of more than 0.5 for the *favorite*. Only in the case $F(b^m) = 0.5$, the *challenger* achieves a chance of winning of 0.5 by choosing b^{m+1}, but in this case the *favorite* can randomize between b^m and b^{m+1} and nonetheless ensure victory with a probability larger than 0.5. Consequently, in any equilibrium the *favorite* party wins with a probability that is larger than 0.5, otherwise it had a deviation that would make it better off. Moreover, an equilibrium in pure strategies cannot exist. The reason is that if the *challenger* plays a pure strategy, the best reply of the *favorite* is to choose the same policy position and win with certainty. But by randomizing between at least two policy positions that are the bliss point of the representative voter with positive probability, the *challenger* has always a chance of winning the elections. Consequently, in equilibrium the *favorite* party is not elected with certainty.

3.2 The equilibrium with uncertainty

Now we put together our analysis of the simultaneous move policy choice with the complete model. The following Proposition gives the equilibrium with the lowest possible rent level that can be achieved with lexicographic voting.

Proposition 2 *The equilibrium of the game with uncertainty over the bliss point of the representative voter with lexicographic voting and lowest possible rent extraction: π^*_F, π^*_C, σ^*_F and σ^*_C are defined in Section 3.1. The incumbent party always takes a rent of $r^* = (1 - \beta(2\pi^*_F - 1))R$ in all periods and in all histories, the probabilities for* *the policy positions announced for period $t+1$ are given by $\sigma_I = \sigma_F^*$ and $\sigma_O = \sigma_C^*$ when the incumbent party has chosen a rent level $r_t \leq r^*$ in period t and by $\sigma_I = \sigma_C^*$ and $\sigma_O = \sigma_F^*$ when the incumbent party has chosen a rent level $r_t > r^*$ in period t. The representative voter plays the lexicographic strategy given by:*

$$I_{t+1} = \begin{cases} y \text{ if } \left|p_{t+1}^y - b_{t+1}\right|^\lambda - \left|p_{t+1}^x - b_{t+1}\right|^\lambda < 0 \\ x \text{ if } \left|p_{t+1}^y - b_{t+1}\right|^\lambda - \left|p_{t+1}^x - b_{t+1}\right|^\lambda > 0 \\ I_t \text{ if } \left|p_{t+1}^y - b_{t+1}\right|^\lambda - \left|p_{t+1}^x - b_{t+1}\right|^\lambda = 0 \text{ and } r_t \leq r^* \\ O_t \text{ if } \left|p_{t+1}^y - b_{t+1}\right|^\lambda - \left|p_{t+1}^x - b_{t+1}\right|^\lambda = 0 \text{ and } r_t > r^* \end{cases} \quad \text{in all } t \text{ and in all histories.} \tag{7}$$

Given the strategies of the players, in every period the probability that the incumbent party wins is given by $\pi_I^* = \pi_F^*$.

Proof. Again, we can apply the single deviation principle. In Section 3.1, we have defined σ_F^* and σ_C^* as the equilibrium strategies of a game in which two parties try to maximize their chances of reelection given a lexicographic strategy of a representative voter. Here, for given history of the game and given the stationary nature of the strategies of all players, maximizing the probability of reelection when choosing a policy position is utility maximizing given the strategies of the other players and thus playing σ_F^* respectively σ_C^* when choosing policy platforms is consistent with equilibrium. The representative voter maximizes her utility by electing a party with the policy closest to her bliss point available in every period. This is optimal because otherwise her utility is not influenced by her voting decision and consistent with her lexicographic strategy.

The third possible deviation we have to check is the rent level chosen by the *incumbent*. Because the chances of reelection depend only on the threshold r^*, it is sufficient to check if the *incumbent* party would not be better off by taking R instead of r^*. If the *incumbent* is not better of with taking R, any rent-seeking between R and r^* cannot make the incumbent better off because the *incumbent* party could keep a higher rent without reducing its chances of being reelected further. Similar, any rent-seeking $r < r^*$ cannot make the incumbent better off than taking r^* because again, the *incumbent* party could keep a higher rent without reducing its chances of being reelected.

It remains to show that a rent threshold of $r^* = (1 - \beta(2\pi_F^* - 1))R$ is a sufficient incentive for the *incumbent* party not to take R. Let $I(r)$, with slight abuse of notation, denote the value of being in office for a constant rent level r with a probability of being reelected of π_F^* for an *incumbent* who does not take more than r and $1 - \pi_F^*$ the probability of being reelected for an *incumbent* who takes more. $O(r)$ denotes the value of being out of office. The minimum rent level r^* that is consistent with equilibrium makes the *incumbent* indifferent between cheating and taking R and playing equilibrium and taking r^*. Consequently, for the rent level r^* the value of deviating and not deviating is the same and the following three equations hold for the minimum achievable rent level:

$$I(r^*) = r^* + \beta(\pi_F^* I(r^*) + (1 - \pi_F^*)O(r^*)),$$

$$I(r^*) = R + \beta(\pi_F^* O(r^*) + (1 - \pi_F^*)I(r^*)),$$

$$O(r^*) = \beta(\pi_F^* O(r^*) + (1 - \pi_F^*)I(r^*)).$$

The first equation gives the value of being in office and taking the equilibrium value of rent r^*. In this case, the *incumbent* party has a chance of π_F^* of being reelected and having the same value $I(r^*)$ again in the next period. If the party loses office the value in the next period is given by $O(r^*)$. The second equation holds because the *incumbent* party is indifferent between taking r^* and R and consequently taking R gives the *incumbent* party the same value $I(r^*)$ as playing equilibrium and taking r^*. The last equation calculates the value of being out of office given that the chance of winning office in this case is $(1 - \pi_F^*)$.

The solution of the system of three equations is given by:

$$\begin{aligned} r^* &= (1 - \beta(2\pi_F^* - 1))R, \\ I(r^*) &= \frac{1 - \pi_F^*\beta}{1 - \beta}R, \\ O(r^*) &= \frac{(1 - \pi_F^*)\beta}{1 - \beta}R. \end{aligned}$$

Consequently, given the strategies of the player an *incumbent* is indifferent between taking $r^* = (1 - \beta(2\pi_F^* - 1))R$ and R when in office and thus the *incumbent* has no reason to deviate when deciding over rent-extraction. ■

3.3 Interpretation

Essentially, just in Section 2, due to stationarity of all strategies the parties play a one-shot game on the policy dimension. However, due to the uncertainty over the bliss point of the representative voter we no longer find policy platform convergence in all periods. This leads a major difference to most other models that combine prospective and retrospective voting motives (and Section 2): The representative voter is not always (in equilibrium) indifferent between the parties when she votes as in many papers mentioned in the introduction. This makes the intuition for and plausibility of the lexicographic voting strategy of the representative voter rather stronger. After all, in elections we often observe that some parties seem genuinely to offer policy positions that are more attractive to a majority of voters than other parties in an election. However, whenever they are indifferent voters can reward or punish past behavior of the politicians in power. The more likely this is to be the case in equilibrium, the higher are the performance thresholds the voters can set. $\pi_F^* - \pi_C^* = \pi_F^* - (1 - \pi_F^*) = 2\pi_F^* - 1$ is the difference in probability between the *incumbent* party and the *challenger* party being elected in equilibrium. Consequently, the smaller $2\pi_F^* - 1$, the larger the electoral uncertainty and larger the probability that an *incumbent* party that restricts itself to a rent level at or below the threshold nonetheless loses office. As a result, the equilibrium rent level $r^* = (1 - \beta(2\pi_F^* - 1))R$ is increasing in electoral uncertainty. When the *incumbent* faces a larger chance of losing office, a higher level of acceptable rent-seeking is necessary to make the *incumbent* party willing to accept a limit on rent extraction. Moreover, with uncertainty an *incumbent* party which loses office can regain office later, which also reduces the incentives of the *incumbent*.

3.4 Minimum equilibrium rent level

Given symmetric stationary strategies as defined in Section 2.3, the party that offers the policy closest to the bliss point of the representative voter wins. This implies that there is no equilibrium with stationary strategy of the representative voter that would not give a party at least a chance of victory of at least π_C^* in equilibrium. This follows directly from the analysis in Section 3.1. A party has a chance of at least π_C^* to win the election by choosing the different policy position with the probabilities given by σ_C^* because in this case it is preferred by the representative voter with a chance of at least π_C^* for any strategy of the other party. Consequently, an equilibrium with lower rents than r^* in stationary strategies cannot exist, because for any lower rent level the *incumbent* would rather take R and have nonetheless a chance of reelection of at least π_C^*. As a consequence, the equilibrium stated in Proposition 2 is the one with the lowest rent payments that the voter can achieve with stationary strategies.

3.5 An example

To provide an example for the equilibrium with uncertainty, we focus on a situation with just three possible policy positions. These are simultaneously the three possible bliss points of the representative voter. The three possible bliss points are denoted b^l, b^m and b^r with $b^l < b^m < b^r$. By assumption, the distance between b^l and b^m and between b^r and b^m is the same distance d, and consequently the *favorite* party wins if the representative voter turns out to have the bliss point $b_t = b^m$ while either the *favorite* chooses b^l and the *challenger* b^r, or the other way around. For simplicity, I assume the parties can only choose the potential policy bliss points as platforms ($B = \{b^l, b^m, b^r\} = \{\hat{p}_1, \hat{p}_2, \hat{p}_3\} = P$). The representative voter has the bliss point $b = b^l$ with probability $q_l = \alpha \in (0, 0.5)$, the bliss point $b = b^m$ with probability $q_m = 1 - 2\alpha$ and the bliss point $b = b^r$ with probability $q_r = \alpha$. Given the probabilities, b^m is the median bliss point as defined in Equation 6. In the Appendix, it is shown that:

$$
\begin{aligned}
\sigma_F^*(\{b^l, b^m, b^r\}, \{\alpha, 1-2\alpha, \alpha\}) &= (\sigma_F^{l*}, \sigma_F^{m*}, \sigma_F^{r*}) = \left(\frac{\alpha}{2-\alpha}, \frac{2-3\alpha}{2-\alpha}, \frac{\alpha}{2-\alpha}\right), \\
\sigma_C^*(\{b^l, b^m, b^r\}, \{\alpha, 1-2\alpha, \alpha\}) &= (\sigma_C^{l*}, \sigma_C^{m*}, \sigma_C^{r*}) = \left(\frac{1-\alpha}{2-\alpha}, \frac{\alpha}{2-\alpha}, \frac{1-\alpha}{2-\alpha}\right), \\
\pi_F^*(\{b^l, b^m, b^r\}, \{\alpha, 1-2\alpha, \alpha\}) &= \frac{2\alpha^2 - 3\alpha + 2}{2-\alpha}, \\
\pi_C^*(\{b^l, b^m, b^r\}, \{\alpha, 1-2\alpha, \alpha\}) &= 2\alpha\frac{1-\alpha}{2-\alpha}.
\end{aligned}
$$

And from Proposition 2 we know that in equilibrium $\pi_I^* = \pi_F^*(\{b^l, b^m, b^r\}, \{\alpha, 1 - 2\alpha, \alpha\})$ in all periods and for all histories. The smaller α is within the relevant interval

$\alpha \in (0, 0.5)$, the larger is the electoral certainty $2\pi_I^* - 1$, and the larger the probability that the *incumbent* party wins in equilibrium. Consequently, the minimum rent level consistent with equilibrium $r^* = (1 - \beta(2\pi_F^* - 1))R = (1 - \beta\frac{4\alpha^2 - 5\alpha + 2}{2-\alpha})R$ is decreasing in α in the relevant interval $\alpha \in (0, 0.5)$.

3.5.1 Welfare

Lower rents make the representative voter better off for given policy. However, we now also compare the result of lexicographic voting to an equilibrium in which past behavior of the parties play no role for the decision of the representative voter as in a model of policy determination in which rent-seeking plays no role. If the representative voter always votes for the policy positions that makes her better off, but tosses a coin to determine her vote whenever she is indifferent instead of considering the past, the equilibrium strategy of both parties will be to always choose a rent R and the policy position b^m, while in the equilibrium with lexicographic voting we have nonconvergence on the policy dimension.[12] Bernhardt, Duggan, and Squintani (2009) show that some divergence makes all voters ex ante better off, which is not surprising in light of the literature on spatial competition (Hotelling 1929). For the representative voter, lack of policy convergence has the advantage that her policy bliss point is implemented more often, while it as the disadvantage that sometimes both parties choose b^l as policy platform when she has the bliss point b^r and vice versa. In the Appendix it is shown that in the example the expected loss of the representative voter before either her preferences or policy platforms are determined is, given the

[12] Tossing a coin when being indifferent is a quite standard assumptions in models of electoral policy determination.

lexicographic voting strategy:

$$\begin{aligned} L_P^* &= E(p-b_t)^2 = \alpha E\left|p-b^l\right|^\lambda + \alpha E\left|p-b^r\right|^\lambda + (1-2\alpha)E\left|p-b^m\right|^\lambda \\ &= 2d^\lambda \alpha \frac{1-\alpha}{(2-\alpha)^2}\left(2^\lambda \alpha - 5\alpha + 4\right). \end{aligned}$$

Thus, the expected loss on the policy dimension is increasing in α.

We want to contrast the welfare in a lexicographic equilibrium with a purely forward looking model of electoral competition. In the purely forward looking setup an indifferent representative voter does not consider the past, but throws a fair coin before making her voting decision. In such a setup both parties converge on the median bliss point even in the case of uncertainty as long as parties have no policy preferences. Consequently, the expected loss of the representative voter on the policy dimension is:

$$L_P^F = E\left|p-b^m\right|^\lambda = 2\alpha d^\lambda$$

Now we can compare L_P^* and L_P^F. The difference is:

$$L_P^* - L_P^F = \frac{2d^\lambda \alpha^2(4\alpha - 5 + 2^\lambda(1-\alpha))}{(2-\alpha)^2}.$$

Solving for λ, we see that $L_P^* \leq L_P^F$ as long as $\lambda \leq \frac{\ln\left(\frac{5-4\alpha}{1-\alpha}\right)}{\ln(2)}$ and the representative voter is in expectations better off with lexicographic voting not only on the rent dimension but also on the policy dimension. Because $\frac{\ln\left(\frac{5-4\alpha}{1-\alpha}\right)}{\ln(2)} > 2$ for all α in the relevant range $\alpha \in (0, 0.5)$, this would always be the case for the commonly used utility function with quadratic disutility. However, when $\lambda > \frac{\ln\left(\frac{5-4\alpha}{1-\alpha}\right)}{\ln(2)}$, lexicographic voting leads to larger expected losses on the policy dimension. The reason is that when voters are very risk averse (large λ), even a small chance of having extreme policy of the kind not preferred implemented. In this case, the representative voters prefers a larger likelihood of a small deviation from their policy bliss point in return for the certainty that the worst case never happens. Nonetheless, lexicographic voting can still make the voter better off as long as the decrease in rents is sufficient to compensate her for the loss on the policy dimension. The difference in total expected loss from lexicographic voting compared to the purely forward looking model is:

$$L^* - L^F = L_P^* + r^* - L_P^F - R = (2d^\lambda \frac{\alpha^2}{(2-\alpha)^2})(4\alpha - 5 + 2^\lambda(1-\alpha)) - \beta\frac{4\alpha^2 - 5\alpha + 2}{2-\alpha}R.$$

For $\lambda \leq \frac{\ln\left(\frac{5-4\alpha}{1-\alpha}\right)}{\ln(2)}$, $L^F > L^*$ because $R > r^*$. Moreover, the difference between L^* and L^F has no upper bound and is strictly increasing and continuous in λ for $\lambda \geq \frac{\ln\left(\frac{5-4\alpha}{1-\alpha}\right)}{\ln(2)}$. $L^* - L^F = 0$ for $\lambda = \frac{\ln\left(\frac{5-4\alpha}{1-\alpha}\right)}{\ln(2)}$ and consequently, there is a unique $\lambda^* > \frac{\ln\left(\frac{5-4\alpha}{1-\alpha}\right)}{\ln(2)}$ for which $L^* - L^F = 0$ and the voter is indifferent between the two equilibria. For all $\lambda < \lambda^*$, $L^* < L^F$ and the representative voter is in expectations better off with lexicographic voting, while in the case $\lambda > \lambda^*$ the representative voter is so risk averse that she is worse off with lexicographic voting ($L^* > L^F$).

Thus, with lexicographic voting the representative voter achieves not only a lower rent level than in an equilibrium in which the past is not considered by the electorate, but is also made better off (compared to a model with policy platform convergence on the median bliss point) by the policy divergence of the parties as long as she is not to risk averse. A risk averse representative voter with large λ suffers more from policies that are very distant from her bliss point. And because such policies are sometimes implemented in equilibrium with lexicographic voting, for a very risk averse voter the lexicographic equilibrium gives the representative voter a lower expected utility than an equilibrium with policy convergence. I do not find the result that a bit of divergence is always good as Bernhardt, Duggan, and Squintani (2009) do. This is due to the fact that the distribution of policy bliss points and platform divergence can only be changed simultaneously in my example. Consequently, wider divergence is associated with more uncertainty over the representative voter's bliss point, and the welfare effects of both cannot be shown separately.

4 Conclusion

It is surprising that until now, there seem to have been no attempts to combine models of retrospective voting with aspects of Downsian competition. My model shows that forward-looking and backward-looking motives can be reconciled in a single model. This should be considered in future empirical research because so far, the question seems to have been if voters vote retrospectively or prospectively. If there is not necessarily a contradiction, some empirical results might have to be re-evaluated.

As long as there is certainty about the position of the voter who represents the median preferences of the electorate, I find that on the policy dimension where commitment is possible, the usual median voter results apply, while rent extraction by politicians is limited to the same degree as in a standard model without a policy dimension. If there is uncertainty over the position of the representative voter, voters cannot limit rent extraction to the same degree as in the case with certainty about the preference of the representative voter, but accountability is not completely lost either. The reason is that even when the *incumbent* party complies with the voters demands for limited rent extraction, it will still lose office if the *opposition* party commits to a policy that is more attractive to the majority of voters. Models of political accountability can explain the often observed incumbency advantage, as is pointed out by Austen-Smith and Banks (1989). Models in the Downsian tradition, on the other hand, provide no explanation for an incumbency advantage. My basic model in Section 2 leads to the implausible result that in equilibrium, the *incumbent* party is always re-elected. In the extended model with uncertainty over the exact position of the representative voter in Section 3, I find that the *incumbent* party always has a chance of winning the elections that is larger than 50%, but does not win with certainty. This result is consistent with election results in many countries. Incumbent parties win more often than not, but their victory is far from certain. Moreover, some divergence on the policy dimension between parties is usually observed.

A The example

We use σ_J^i to denote the probabilities with which party $J = C, F$ chooses a policy position b^i with $i = l, m, r$ equilibrium. π_J^i denotes the expected payoff for party $J = C, F$ from choosing position b^i with $i = l, m, r$.

Given the structure of the game the expected payoffs are:

$$
\begin{aligned}
\pi_F^l &= \sigma_C^l + \sigma_C^r(1-\alpha) + \sigma_C^m\alpha = \sigma_C^l + \sigma_C^r(1-\alpha) + (1-\sigma_C^l-\sigma_C^r)\alpha, \\
\pi_F^m &= \sigma_C^l(1-\alpha) + \sigma_C^r(1-\alpha) + \sigma_C^m = \sigma_C^l(1-\alpha) + \sigma_C^r(1-\alpha) + (1-\sigma_C^l-\sigma_C^r), \\
\pi_F^r &= \sigma_C^l(1-\alpha) + \sigma_C^r + \sigma_C^m\alpha = \sigma_C^l(1-\alpha) + \sigma_C^r + (1-\sigma_C^l-\sigma_C^r)\alpha \\
\pi_C^l &= (1-\sigma_F^l)\alpha, \\
\pi_C^m &= \sigma_F^l(1-\alpha) + \sigma_F^r(1-\alpha), \\
\pi_C^r &= (1-\sigma_F^r)\alpha.
\end{aligned}
$$

$\sigma_F^m = 1$ (and thus $\sigma_F^l = 0$ and $\sigma_F^r = 0$) is not consistent with equilibrium because in any best response the *challenger party* chooses only b^r and b^l and $\sigma_C^m = 0$. But against a strategy with $\sigma_C^m = 0$, $\sigma_F^m = 1$ is not part of a best reply. Against any strategy with $\sigma_F^l = 0$ and $\sigma_F^r > 0$, choosing b^l gives the *challenger* a probability of victory of α, while choosing b^r gives the *challenger* $(1-\sigma_F^r)\alpha < \alpha$. Consequently, the *challenger* would not choose b^r with positive probability in its best response. But if $\sigma_C^r = 0$, the *favorite* has a higher expected probability of victory from choosing b^l than from choosing b^r. Thus, $\sigma_F^r = 0$ in any best response of the *favorite* against the *challengers* best response to a strategy with $\sigma_F^l = 0$ and $\sigma_F^r > 0$, what is a contradiction. A symmetric argument rules out $\sigma_F^l > 0$ and $\sigma_F^r = 0$ to be both true in equilibrium. Together, the three discussed cases imply that $\sigma_F^l > 0$ and $\sigma_F^r > 0$. $\sigma_F^m = 0$ is not consistent with equilibrium because the *challenger* has an expected payoff of $1-\alpha > 0.5$ from choosing b^m as response. This is inconsistent with equilibrium because the *favorite* party can always achieve a payoff of $1-\alpha > 0.5$ by choosing b_m with probability $\sigma_F^m = 1$. Thus, in equilibrium the *favorite* plays a totally mixed strategy.

Next, suppose the *challenger* party would not play a totally mixed strategy. Then, playing a best reply against such a strategy, the *favorite* party would only play the positions played by the *challenger* with positive probability because this always gives a higher expected payoff than choosing any position that is never chosen by the *challenger*. But this is a contradiction because we have already established that the *favorite* plays a totally mixed strategy. Thus, both parties play totally mixed strategies in equilibrium.

A party is only willing to play a totally mixed strategy when it has the same expected payoff from all three possible policy positions, and we solve for the unique equilibrium using $\pi_F^l = \pi_F^m = \pi_F^r$ and $\pi_C^l = \pi_C^m = \pi_C^r$:

From $\pi_F^l = \pi_F^r$, it follows that $\sigma_C^l = \sigma_C^r$ and thus:

$$\begin{aligned}\pi_F^l &= \pi_F^r = \sigma_C^l(2-\alpha) + (1-2\sigma_C^l)\alpha, \\ \pi_F^m &= 2(1-\alpha)\sigma_C^l + (1-2\sigma_C^l)\end{aligned}$$

Using $\pi_F^l = \pi_F^m$, it follows that: $\sigma_C^l = \sigma_C^r = \frac{1-\alpha}{2-\alpha}$ and $\sigma_C^m = \frac{\alpha}{2-\alpha}$. Consequently, the *favorite* party wins in equilibrium with probability $\pi_F^* = \pi_F^l = \frac{1-\alpha}{2-\alpha}(2-\alpha) + (1-2\frac{1-\alpha}{2-\alpha})\alpha = \frac{2\alpha^2-3\alpha+2}{2-\alpha}$.

From $\pi_C^l = \pi_C^r$, follows that $\sigma_F^l = \sigma_F^r$ and thus:

$$\begin{aligned}\pi_C^l &= \pi_C^r = (1-\sigma_F^l)\alpha, \\ \pi_C^m &= 2\sigma_F^l(1-\alpha).\end{aligned}$$

Using $\pi_C^l = \pi_C^m$, the equilibrium probabilities are given by: $\sigma_F^l = \sigma_F^r = \frac{\alpha}{2-\alpha}$ and $\sigma_F^m = \frac{2-3\alpha}{2-\alpha}$ and the *challenger* wins in equilibrium with probability $\pi_C^* = \pi_C^l = (1-\sigma_F^l)\alpha = (1-\frac{\alpha}{2-\alpha})\alpha = 2\alpha\frac{1-\alpha}{2-\alpha} = 1-\pi_F^*$. Summarizing the results:

$$\begin{aligned}
\sigma_F^*(\{b^l, b^m, b^r\}, \{\alpha, 1-2\alpha, \alpha\}) &= (\sigma_F^{l*}, \sigma_F^{m*}, \sigma_F^{r*}) = (\frac{\alpha}{2-\alpha}, \frac{2-3\alpha}{2-\alpha}, \frac{\alpha}{2-\alpha}), \\
\sigma_C^*(\{b^l, b^m, b^r\}, \{\alpha, 1-2\alpha, \alpha\}) &= (\sigma_C^{l*}, \sigma_C^{m*}, \sigma_C^{r*}) = (\frac{1-\alpha}{2-\alpha}, \frac{\alpha}{2-\alpha}, \frac{1-\alpha}{2-\alpha}), \\
\pi_F^*(\{b^l, b^m, b^r\}, \{\alpha, 1-2\alpha, \alpha\}) &= \frac{2\alpha^2-3\alpha+2}{2-\alpha}, \\
\pi_C^*(\{b^l, b^m, b^r\}, \{\alpha, 1-2\alpha, \alpha\}) &= 2\alpha\frac{1-\alpha}{2-\alpha}.
\end{aligned}$$

Welfare in the example Given the equilibrium strategies of both players we can calculate the welfare implications of the equilibrium for the voters and contrast it with other models.

The probability that none of the parties chooses b^m is given by:

$$\Pr(p_F \neq b^m \wedge p_C \neq b^m) = (1-\sigma_F^{m*})(1-\sigma_C^{m*}) = \frac{4\alpha(1-\alpha)}{(2-\alpha)^2}$$

Consequently, in any period a representative voter with bliss point b^m has an expected loss of:

$$E\left|p-b^m\right|^\lambda = 4\alpha\frac{1-\alpha}{(2-\alpha)^2}d^2,$$

from policy because the distance between b^m and either b^l or b^r is d.

Given the equilibrium strategies of the parties, the probability that both parties choose b^r and thus a voter with bliss point b_l suffers a disutility of $(2d)^\lambda$ on the policy dimension is given by:

$$\Pr(p_F = p_C = b^r) = \sigma_F^{r*}\sigma_C^{r*} = \frac{\alpha(1-\alpha)}{(2-\alpha)^2}.$$

The probability that at least one party chooses b^m while none chooses b^l is given by:

$$\Pr((p_F = b^m \ \wedge \ p_C \neq b^l) \vee (p_F \neq b^l \ \wedge \ p_C = b^m)) = \sigma_F^{m*}\left(1-\sigma_C^{l*}\right) + \sigma_C^{m*}\sigma_F^{r*} = \frac{1-\alpha}{2-\alpha}$$

Using the probability that both parties choose b^r and the probability that at least one party chooses b^m while none chooses b^l, we can calculate the expected disutility of a representative voter with bliss point b^l on the policy dimension. Because of the symmetry of the example, this is also the expected disutility of a representative voter with bliss point b^r:

$$E\left|p-b^l\right|^\lambda = E\left|p-b^r\right|^\lambda = \frac{\alpha(1-\alpha)}{(2-\alpha)^2}(2d)^\lambda + \frac{1-\alpha}{2-\alpha}d^\lambda = d^\lambda\frac{1-\alpha}{(2-\alpha)^2}\left(\alpha(2^\lambda-1)+2\right)$$

We denote the expected loss of the representative voter on the policy dimension with L_P. The expected loss of the representative voter before either her preferences or policy platforms are determined is, given the lexicographic voting strategy:

$$\begin{aligned} L_P^* &= E(p-b_t)^2 = \alpha E\left|p-b^l\right|^\lambda + \alpha E\left|p-b^r\right|^\lambda + (1-2\alpha)E\left|p-b^m\right|^\lambda \\ &= 2d^\lambda\alpha\frac{1-\alpha}{(2-\alpha)^2}\left(2^\lambda\alpha-5\alpha+4\right). \end{aligned}$$

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