Inflation, Market Structure, and Innovation-Driven Growth with Various Cash Constraints

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December 1, 2015

Abstract

This paper explores the effects of monetary policy (or inflation) in a Schumpeterian growth model with an endogenous market structure and distinct cash-in-advance (CIA) constraints on consumption, production, and two distinct types of R&D investment – in-house R&D and entry investment. We show that the CIA constraints work through various channels and the effects of monetary policy depend on the strength of each channel. Although inflation seems like a uniform tax imposed on the whole economy, an identical monetary policy can render different distortions of inflation on the economy and give rise to different consequences. The steady-state effects provide a reconciliation to the empirical, mixed relationship between inflation and growth. Moreover, the market structure could exhibit either an intensive or extensive margin response to inflation. Our welfare effects show that Friedman's rule, in general, is not socially optimal, depending on the relative magnitude between the cash constraint on the quality-improved and variety-expanded R&D and the social return to product variety.

JEL classification: O30, O40, E41.

Keywords: CIA constraints on R&D, endogenous market structure, monetary policy, growth.

Acknowledgment: We would like to thank Guido Cozzi, Angus Chu, Ping Wang, John Seater, Ching-Chong Lai, Qiang Gong, Min Zhang, Yu Jin, Hui He, Lin Zhang, and all the participants in the seminars of Duke University, SINICA , SHUFE and SWUFE and the Southern Economic Association Meeting for valuable comments and suggestions. The usual disclaimer applies.

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1 Introduction

The relationship between economic growth and inflation is one of the most important macroeconomic issues. In the literature, there is no consistent theoretical prediction of the effects of monetary policy (inflation) on economic growth/output, nor is the empirical significance of these real effects agreed upon. The Mundell (1963)-Tobin (1965) effect indicates that if money is regarded as an asset substitute for capital, higher inflation is in favor of capital accumulation and hence economic growth. By contrast, if money is treated as a factor of production (as in Stockman 1981), then the opposite result emerges. Moreover, the real business cycle models refer to a super-neutrality of money. Empirical studies confirm the existence of either a positive or negative relationship between inflation and growth, indicating that the relationship remains inconclusive. Conventional evidence studies (e.g., Fisher 1983 and Cooley and Hansen 1989) report a negative steady inflation-growth (output) relationship. Recently, Bullard and Keating (1995), Bruno and Easterly (1998), Ahmed and Rogers (2000) have referred to a non-monotonic relationship.

In this paper we argue that an identical monetary policy may render distinct inflation distortions to the economy, resulting in different growth consequences, although inflation seems to act as a uniform tax - the inflation tax - imposed on the entire economy. To reconcile the empirically bivariate inflation-growth relationship, we develop a monetary model of fully endogenous Schumpeterian growth in which the role of (i) a variety of cash constraints, particularly on R&D activities, and (ii) an endogenous market structure are highlighted.

First, we consider four different types of cash-in-advance (CIA) constraints which influence in-house R&D, entry investment, manufacturing production, and consumption. The cash constraint on R&D is particularly important, given the fact that there is a strong R&D-cash flow sensitivity for firms (Hall 1992 and Himmelberg and Petersen 1994). The main reason, pointed out by Hall and Lerner (2010, p. 612), is that over 50 percent of R&D spending is paid as wages and salaries for highly skilled scientists and engineers whose efforts form the firm's knowledge base and are crucial to generate firm's profit. Therefore, R&D-intensive firms are required to hold cash to smooth their R&D operation. Brown and Petersen (2009) offer more direct evidence that US firms relied heavily on cash reserves as a buffer to smooth R&D spending during the 1998-2002 boom. The evidence has shown that R&D exhibits a stronger investment-cash flow sensitivity than physical investment.[1] Given that the conventional monetary model only focuses on the CIA constraint on consumption and physical investment (e.g., Stockman 1981 and Wang and Yip 1992), the lack of an appropriate consideration of a CIA constraint on R&D would fail not only to reflect reality, but also to provide a complete picture for the implications

[1]For recent observations, the reader can refer to Harhoff (1998), Hall et al. (1999), Mulkay et al. (2001), Brown et al. (2009, 2012), and Brown and Petersen (2009, 2011), among others.

of monetary policy.

The second salient feature of our model is the endogeneity of the market structure, measured by the number of firms and firm market share/size. It is important to consider endogenous firm entry and exit, because in the US manufacturing industry 25% of annual gross job destruction can be attributed to establishment deaths and 20% of annual gross job creation to new establishment births (Davis and Haltiwanger 1990). The comovement between the number of new incorporations and GDP is pretty obvious, with correlation as high as 0.73 (Bergin and Crosetti 2008). It is also important to consider endogenous firm market size, because a large body of empirical research has indicated that larger size can foster productivity growth because it allows firms to take advantage of the increasing returns associated with R&D (see, Cohen and Klepper 1996 and more recently Pagano and Schivardi 2003).

The model consists of two-dimensional innovations: the vertical (quality improvement) and horizontal (variety expansion) innovations. In the vertical dimension, incumbents engage in in-house R&D in order to reduce their production costs and earn higher profits. In the horizontal dimension, entrepreneurs enter the market by conducting variety-expanded R&D and the new products compete with those of the incumbents for market share. The interaction between the quality-improved and variety-expanded R&D endogenously determines the market structure.[2] In particular, the process of development of new product lines can effectively fragment the aggregate market into sub-markets whose size does not increase with total R&D labor or population. That is, the endogeneity of the market structure allows the proliferation of product varieties to reduce the effectiveness of R&D aimed at quality improvement, by causing it to be spread more thinly over a larger number of different products. Thus, the *scale effect* is eliminated with the IO foundation.[3] The hypothesis of product proliferation is not only supported by the US data (see Laincz and Peretto 2006), but is also consistent with the fact that the in-house R&D activities depend on firm market size which is endogenously determined through the market structure.

Given the competition between incumbents (old firms) and entrants (new firms), one may expect that a uniform inflation tax will give rise to non-uniform distortions to the quality-improved and variety-expanded R&Ds, if these firms' R&D activities are subject to distinctive strengths of CIA constraints. Once the distortionary effect of inflation depends upon the strength of each cash constraint, an identical monetary policy then has different impacts on the market structure and R&D allocation which ends up with distinctive growth consequences.

[2]The two-dimensional R&D competition appropriately captures the industrial organization (IO) evidence in the sense that R&D is conducted predominately by incumbent firms (NSF 2010); at the industry level 75% of the average total factor productivity growth is accounted for by incumbents and the remaining 25% is accounted for by new establishments (Bartelsman and Doms 2000 and Foster and Krizan 2000).

[3]Backus et al. (1992) and Jones (1995) have shown that the scale effect is inconsistent with the actual data.

Therefore, the endogeneity of the market structure provides a reconciliation for the opposite empirical findings and, accordingly, a new insight into the literature.

In this paper we *analytically* explore not only the long-run steady-state, but also the short-run transition effects of monetary policy. *Quantitatively*, we also perform a normative welfare analysis by examining the optimal monetary policy assignment. In terms of the steady-state effects, we show that in a unified model, the growth-inflation relationship can be either negative (in the case of the CIA constraint on in-house R&D), positive (the CIA constraint on entry investment), or independent (the CIA constraints on production and consumption). The Mundell-Tobin effect in the sense that high inflation is associated with high growth is empirically plausible, as the evidence indicates that R&D is more likely to be liquidity constrained for young and new firms (see Brown and Petersen 2009 and 2011 and Janiak and Monteiro 2011).[4]

By highlighting the variety of CIA constraints, we also find that inflation leads to quite different market structures. In the case with CIA constraints on entry and consumption, inflation results in an "intensive margin," in the sense that the product market is characterized by a small number of large-sized firms. By contrast, in the case with CIA constraints on in-house R&D and production, inflation results in an "extensive margin," whereby there exists a large number of small-sized firms in the economy. This result differs from Wu and Zhang's (2001) prediction which indicates that at higher rates of inflation firms become fewer and smaller in size. However, based on the generalized cash constraint, our ambiguous result provides a plausible explanation to the empirical findings concerning the mixed responses of firm entry to monetary shocks (see the recent studies of Bergin and Corsetti 2008, Lewis and Poilly 2012, and Lewis and Stevens 2015), which the model with a fixed market structure cannot. Monetary policy also leads to rich transitional dynamics when the market structure is endogenized. In particular, we find that under a convincing parameterization, growth may exhibit a mis-adjustment: along the transition path, the TFP growth rate mis-adjusts from its long-run steady state. This implies that in response to higher inflation, economic growth may fall in the short run, but will rise in the long run.

In terms of welfare analysis, we show that Friedman's rule, in general, is not socially optimal, crucially depending on (i) the differences among each cash constraint and (ii) the extent of the social return to product variety. Friedman's rule is invalid, when the liquidity constraint on entry is stronger and on in-house R&D, production and consumption is weaker. Under our parametrization, a higher nominal interest rate results in an extensive margin, and as a result, the Friedman rule is not optimal, if the social return to product variety is relatively low.

[4]A particular emphasis is that our result is obtained via the liquidity constraints and market-structure adjustments, rather than the conventional asset-substitution effect, as stressed by Mundell (1963) and Tobin (1965). Recently, Wang and Xie (2013) pointed to the positive steady inflation-growth relationship by shedding light on the labor market friction.

Related Literature

This paper is related to two strands of the literature. First, in the well-established tradition of cash constraints and growth, the vast majority of studies focus on the CIA constraints on consumption and capital investment (see, e.g., Wang and Yip 1992). The relative scarcity of research explores the impacts of steady inflation on innovation-driven growth via cash constraints on R&D. Recently, Funk and Kromen (2010) and Chu and Lai (2013) investigate the growth effects of inflation by incorporating a money-in-utility specification into a canonical quality-ladder model. In spite their valuable contribution, our results suggest rather different growth implications for inflation from theirs, when we consider various CIA constraints on R&D.

Chu and Cozzi's (2014) study is a rare exception. They develop a two-sector, scale-invariant growth model to reveal an ambiguous effect of inflation on growth, which crucially depends on whether manufacturing production or quality-improved R&D is subject to a cash constraint. Our study differs from theirs in two significant respects. In their model, R&D has only a vertical dimension, whereas our paper highlights the endogeneity of the market structure, which consists of both vertical and horizontal R&D. The two-dimensional innovations generates different monetary implications. For example, our study refers to the money superneutrality of growth when manufacturing production is subject to the CIA constraint, but they obtain a positive growth effect under the same constraint. With an endogenous market structure, we find that the CIA constraint on innovation does not uniformly deter all firms' R&D activities; the R&D allocation between old and new firms is a key determinant, governing economic growth. Second, their model with a fixed market structure is unable to explore the impacts of inflation on the market concentration and firm size. By contrast, our model enables us to to investigate the responses of the market structure to the monetary policy.

Second, in the literature on market structure, Janiak and Monteiro (2011) numerically show that market structure plays a crucial role in terms of enhancing the welfare cost of inflation, although their analysis lacks an R&D sector. Chu and Ji (2014) examine the growth effect in a model with an endogenous market structure, but their analysis is confined to a Lucasian CIA economy without any cash constraint on R&D activities. Recently, a small but growing strand of the literature (e.g., Bergin and Corsetti 2008, Lewis and Poilly 2012, and Lewis and Stevens 2015) has studied how the extensive margin of firm entry and product variety can contribute to the understanding of business cycles in a monetary model. These studies show that firm entry ambiguously responds to monetary shocks, due to the distinctions in the degree of wage rigidity, the congestion externality in entry, and the various measures of monetary policy. Instead, our study emphasizes that, due to various cash constraints, a uniform inflation tax gives rise to non-uniform distortions to the incumbents (quality-improved R&D) and entrants (variety-expanded R&D), resulting in different consequences on the market structure.

2 The Model

There is a monetary variant of the Peretto (1998a) model with various cash constraints on in-house R&D, entry, production and consumption. The economy consists of households, firms (incumbents and entrants), and a government (solely represented by the monetary authority). Time t is continuous. For compact notation, the time index is suppressed throughout the paper.

2.1 Households

Consider an economy with a population growth rate λ, which is associated with a population size L. Each household chooses consumption C and leisure $(1-l)$ (where 1 is the normalized time and l are working hours) to maximize the following discounted sum of future instantaneous utilities:

$$U = \int_0^\infty e^{-(\rho-\lambda)}[lnC + \gamma ln(1-l)]dt, \tag{1}$$

subject to the budget constraint

$$\dot{A} + \dot{M}_m = (r-\lambda)A + l + iB + T - (\pi+\lambda)M_m - E, \tag{2}$$

and the CIA constraint

$$\xi_C E \leq M_m - B, \tag{3}$$

where γ measures the relative weight of leisure in the utility and $M_m = \frac{M_L}{P_m L}$. In line with Peretto (1998a), the price of labor (i.e., the wage rate) is a numéraire and normalized to 1. Thus, by defining M_L as the nominal money balances, P_m can then be viewed as the price of money in terms of labor and, accordingly, M_m represents the real money balances per capita.[5] Thus, all quantity (non-price) variables are real and in per capita terms: A is real asset holdings, M_m is real money holdings, B is the real loans for R&D and production activities, E is the real consumption expenditure per capita, and T is the real lump-sum transfer from the government. Moreover, r is the real interest rate, i is the nominal interest rate, π is the inflation rate, and ρ is a constant time preference rate.

The CIA constraint (3) indicates that the real money balances M_m held by the households are required not only to purchase consumption goods E, but also to finance the firms' investment B. The amount $M_m - B$ is available for transactions for purchasing consumption goods and ξ_C is the weight of consumption on the cash constraint. The term B can be simply thought

[5] The choice of deflator does not alter our main results.

of as one-period loans, which are used to finance either the incumbent firms' in-house (quality improvement-type) R&D, new firms' entry investment (variety expansion-type R&D), or their production. As will be clear in Subsection 2.2, the amount of B crucially depends on how much these R&D and production activities are restricted by the cash (liquidity) constraint. The specification of one-period loans is similar to Williamson (1987) and, accordingly, iB is then the interest rate payment on the loan for R&D and production activities. In Subsection 2.4, when deriving the no-arbitrage condition between this loan and other assets (i.e., Fisher's equation $i = r + \pi$), we can see that the loan rate i is also the nominal interest rate.

Households consume all differentiated intermediate goods. Let c_i be the consumption of the intermediate good j and ϵ be the elasticity of substitution. Thus, the bundle of consumption C is set as the following CES combination of N types of intermediate goods:

$$C = [\int_o^N c_j^{(\epsilon-1)/\epsilon} dj]^{\frac{\epsilon}{\epsilon-1}}. \tag{4}$$

By denoting p_j as the price for intermediate good j in terms of labor, the expenditure per capita then is:

$$E = \int_o^N p_j c_j dj. \tag{5}$$

Define η and ψ as the multipliers associated with (2) and (3). Thus, the first-order conditions necessary for the household's optimization problem are given by:

$$\frac{1}{C} = P(\eta + \psi \xi_C),$$

$$\frac{\gamma}{1-l} = \eta,$$

$$\psi = \eta i,$$

$$-\eta(\pi + \lambda) + \psi = -\dot{\eta} + \eta(\rho - \lambda),$$

$$\eta(r - \lambda) = -\dot{\eta} + \eta(\rho - \lambda),$$

where $P \equiv [\int_0^N p_j^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$. The first two equations are the optimal conditions for consumption and leisure, respectively. The latter three equations are the optimal conditions for three distinct types of assets. Furthermore, a simple two-stage budgeting procedure yields the demand for the consumption good j:

$$c_j = E[\frac{p_j^{-\varepsilon}}{\int_0^N p_k^{1-\varepsilon} dk}]$$

As a result, the total demand for all goods, X_j, is given by:

$$X_j = Lc_j = LE\,[\frac{p_j^{-\epsilon}}{\int_0^N p_k^{1-\epsilon} dk}]. \tag{6}$$

Accordingly, we can have the market share of firm j, κ_j, as follows:

$$\kappa_j = \frac{p_j^{1-\epsilon}}{\int_0^N p_k^{1-\epsilon} dk} = \frac{p_j X_j}{LE}. \tag{7}$$

Since there is a continuum of goods and each firm is atomistic, taking X_j as given, monopolistic competition then prevails and individual firms face isoelastic demand curves.

2.2 Firms

The interaction between incumbents and entrants is the core of the model. There are two dimensions of technology change in this sector – production cost reduction (the vertical dimension) and variety expansion (the horizontal dimension). In the vertical dimension, incumbents engage in in-house R&D in order to reduce the production costs and earn higher profits.[6] In the horizontal dimension, entrepreneurs make entry decisions and compete with incumbents for market share. Through firm entry, the number of firms N and the individual firm's market $\frac{LE}{N}$ are endogenously determined.[7] In what follows, we first focus on the determination of the price and investment in R&D of incumbents given the existing market structure and then turn to the endogeneity of the market structure, which is related to the entry decisions of entrepreneurs.

2.2.1 Incumbents

The goods sector comprises a continuum of monopolistically competitive incumbents, each of which produces a single intermediate good X_j with the following technology:

$$L_{X_j} = h(Z_j)X_j, \tag{8}$$

[6]Cost reducing technological progress is equivalent to quality improvement progress. See Tirole (1988).

[7]In line with the common specification in the literature on macroeconomics and growth, we assume that each firm produces only one product. This rules out the possibility whereby an incumbent firm simultaneously engages in quality-improved and variety-expanded R&D. In terms of the data, Dunn et al. (1988) indicate that 93.4% of firms are single-product firms by using 4-digit SIC data. By employing 5-digit SIC data, Bernard et al. (2010) show that 61% of firms are single-product firms. This assumption then does not lead our model to significantly lose its generality. Moreover, as shown in Smulders and Van de Kumdert (1995) and Minniti (2006), if a firm can produce multiple products, this will not alter the main results.

where $h(Z_j) = Z_j^{-\theta}$, with $0 < \theta < 1$. Each incumbent j undertakes R&D to increase the knowledge Z_j. An increase in knowledge decreases the cost of production L_{X_j}. Thus, (8) can be rewritten as $X_j = Z_j^{\theta} L_{X_j}$, which indicates that an increase in knowledge improves the productivity of production labor. The firms accumulate knowledge according to:

$$\dot{Z}_j = \alpha K \, L_{Z_j}. \tag{9}$$

The flow of knowledge $\dot{Z}_j$ depends on R&D productivity α, the employment in the R&D sector of firm j, L_{Z_j}, and the stock of public knowledge:

$$K \equiv \int_0^N \kappa_j Z_j dj.$$

Note that the knowledge is non-rival within a firm, and augments labor at the firm level.

Assume that the proportion ξ_Z of the in-house R&D investment and the proportion ξ_X of the production cost are subject to the CIA constraint. Thus, incumbents have to borrow $\xi_Z L_{Z_j} + \xi_X L_{X_j}$ at the rate i from households to finance their R&D investment and production. The net profit of an individual firm j can then be expressed as:

$$\Pi_j = p_j X_j - (1-\xi_X)L_{X_j} - (1-\xi_Z)L_{Z_j} - (1+i)(\xi_Z L_{Z_j} + \xi_X L_{X_j}). \tag{10}$$

Accordingly, the present discounted value $V_j(t)$ of net profit is:

$$V_j(t) = \int_t^{\infty} \Pi_j \, e^{-\int_t^{\tau} r(s)ds} d\tau. \tag{11}$$

The incumbent firm chooses the paths of its product price p_j and its R&D expenditure L_{Z_j} to maximize (11), subject to the demand function (6), production cost (8), and the R&D production function (9).

2.2.2 Entry

Entrepreneurs create new varieties to compete with incumbents for market share. By following Peretto (1998a), we assume that entrepreneurs have to pay a sunk cost of $\frac{1}{\beta}$ units of labor hours in order to enter the market. In the presence of the cash constraint, they have to borrow money to finance the ξ_N proportion of entry cost, i.e., the $\xi_N \frac{1}{\beta}$ units of labor hours. Therefore, the total entry cost measured in terms of labor hours is:

$$(1-\xi_N)\frac{1}{\beta} + (1+i)\xi_N \frac{1}{\beta} = (1+\xi_N i)\frac{1}{\beta}. \tag{12}$$

The free entry condition requires the value of the firm, shown in (11), to be equal to the entry cost, i.e.:

$$V_j = (1 + \xi_N i)\frac{1}{\beta}. \tag{13}$$

By combining the labor requirement for entry $L_N = V_j \dot{N}$ with (13), we further have:

$$\dot{N} = \frac{\beta}{1 + \xi_N i} L_N. \tag{14}$$

From (10) and (13), we can define the loan B for the firms' R&D and production activities (which appears in the household budget constraint (2)) as $B = \frac{\xi_Z L_Z + \xi_X L_X + \xi_N/(1+\xi_N i) L_N}{lL}$. Hall and Lerner (2010, p. 612) report that in practice more than 50 percent of R&D spending is wage payments to highly skilled technology workers – highly educated scientists and engineers. The R&D-intensive firms need to hold cash to smooth their R&D spending over time, because most of the resource base of the firms will disappear when such workers leave. Thus, as shown in Brown and Petersen (2009), the US firms relied heavily on cash reserves to smooth their R&D spending during the 1998-2002 boom. Our specification exactly captures this observation.

2.3 Monetary Authority

The monetary authority implements a nominal interest rate peg by targeting i. Let the growth rate of the nominal money supply be $\mu = \frac{\dot{M}_L}{M_L}$. By recalling that $M_m = \frac{M_L}{P_m L}$, the evolution of money real balances is: $\frac{\dot{M}_m}{M_m} = \mu - \pi - \lambda$. The monetary authority adjusts the money growth rate μ to whatever level is needed for the targeted interest rate i to prevail.

To balance its budget, the government simply returns the seigniorage revenues to households as a lump-sum transfer T. Thus, the government budget constraint is:

$$T = \frac{\dot{M}_L}{P_m L} = \mu M_m = \dot{M}_m + (\pi + \lambda) M_m. \tag{15}$$

2.4 Competitive Equilibrium

In a competitive equilibrium, households choose a tuple of paths $\{C(t), l(t), B(t), M_m(t)\}_{t=0}^{\infty}$ to maximize utility (1), subject to (2) and (3), given prices $\{r(t), w(t)\}$ and policy $\{i\}$. The first-order conditions of the household's maximization problem, reported in Section 2.1, can be summarized as follows:

$$i = r + \pi, \tag{16}$$

$$l = 1 - \gamma E(1 + \xi_C i), \tag{17}$$

$$\frac{\dot{E}}{E} = r - \rho. \tag{18}$$

In (16), the no-arbitrage condition between assets and money implies the Fisher equation. Equation (17) indicates a trade-off between labor supply and consumption expenditure. Equation (18) is the standard Euler equation of consumption.

Incumbents choose a tuple of paths $\{p_j(t), L_{Z_j}(t)\}_{t=0}^{\infty}$ to maximize the present value of profits (11), subject to (6) and (9), given the liquidity constraints $\{\xi_Z, \xi_X\}$ and policy $\{i\}$. Entrants make entry decisions, given $\{V_j(t)\}$, entry cost (12), the liquidity constraint $\{\xi_N\}$ and policy $\{i\}$. In the study, we focus on a symmetric equilibrium and hence the index j can be suppressed in equilibrium.

Accordingly, we have:

Proposition 1. *Assuming $\theta(\epsilon - 1) < 1$, the Nash Equilibrium is symmetric, under which the goods prices, returns to in-house R&D, and returns to entry, respectively, are:*

$$p = (1 + \xi_X i) h(Z) \frac{\epsilon}{\epsilon - 1}, \tag{19}$$

$$r_Z = \frac{\alpha}{1 + \xi_Z i} [\frac{\theta(\epsilon - 1) LE}{\epsilon N} - (1 + \xi_Z i) \frac{L_Z}{N}], \tag{20}$$

$$r_N = \frac{\pi}{V} + \frac{\dot{V}}{V} = \frac{\beta}{1 + \xi_N i} [\frac{LE}{\epsilon N} - (1 + \xi_Z i) \frac{L_Z}{N}], \tag{21}$$

where L_Z is the aggregate employment in the R&D sector.

Proof All proofs are relegated to the Appendix. ■

Recall that θ measures the degree of diminishing returns of R&D to production and ϵ is the elasticity of substitution of intermediates. Thus, the condition $\theta(\epsilon - 1) < 1$ guarantees that the diminishing returns to R&D are high enough so that no firms have the incentive to engage in more R&D than others (see Peretto (1998b) for the details).

The model generates three different growth regimes: the regime with only in-house R&D, the regime with only firm entry, and the regime with both. This study focuses on the regime with both in-house R&D and entry, because in practice both incumbents and new establishments make a contribution to the TFP growth. Bartelsman and Doms (2000) and Foster and Krizan

(2000) document that incumbents account for about 75% of average TFP growth at the industry level, with the remaining productivity improvements being accounted for by the entry of new establishments. To this end, we impose the following parameter restrictions:

$$\alpha > \alpha\theta(\epsilon - 1) > \frac{1 + \xi_Z i}{1 + \xi_N i}\beta. \tag{22}$$

Two complements are worth noting here. In the Peretto (1998a, b) models, $\alpha > \beta$ is a necessary condition to ensure that the regime with both types of R&D is a *stable* Nash equilibrium. This condition requires that the productivity of labor in the quality-improved R&D (incumbents) be larger than that in the variety-expanded R&D (entrants). This requirement seems to be strong; in reality it is not necessary for the productivity of an incumbent to be larger than that of an entrant. In our monetary model, the corresponding stability condition is $\alpha > \frac{1+\xi_Z i}{1+\xi_N i}\beta$, implying that the productivity of an entrant can be larger than that of an incumbent, provided that the entrant incurs a higher cash constraint ($\xi_N > \xi_Z$).[8]

To ensure the market-clearing condition of the goods market, the total supply of goods measured by labor cost is equal to the total demand measured by household expenditure based on (6) and (19), i.e.,

$$L_X = NL_{X_j} = \frac{\epsilon - 1}{\epsilon}\frac{LE}{1 + \xi_X i}. \tag{23}$$

Moreover, the market-clearing condition of the financial market leads to $r = r_Z = r_N$. Accordingly, setting (20)=(21) yields:

$$L_Z = \frac{LE}{\epsilon}\frac{\left[\frac{\alpha}{1+\xi_Z i}\theta(\epsilon - 1) - \frac{\beta}{1+\xi_N i}\right]}{\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta}. \tag{24}$$

Finally, the labor market clears implying that

$$Ll = L_N + L_Z + L_X, \tag{25}$$

where $L_Z = \int_0^N L_{Zj}dj = NL_{Zj}$, $L_X = \int_0^N L_{Xj}dj = NL_{Xj}$, and l is reported in (17).

[8]Although our analysis focuses on the regime in which both in-house R&D and entry are active, the monetary innovation may lead to a switch from the regime to another one, due to various cash constraints. For example, given the symmetric Nash equilibrium condition $\theta(\epsilon - 1) < 1$, if $\xi_Z > \xi_N$, a sufficiently large increase in the interest rate could make the in-house R&D become more expansive than the entry, which discourages the firm from devoting resources to in-house R&D. Thus, the regime with both R&D and entry switches to the regime with entry only, as in a variety-expansion model.

3 Monetary Policy (Inflation) and Economic Growth

In this section, we solve the dynamic system and then analyze both the steady-state and transition effects of an increase in the nominal interest rate under the symmetric equilibrium. Define firm size as $s = \frac{L}{N}$, the TFP growth as $g = \theta\frac{\dot{Z}}{Z}$, and the consumption growth as $g_C = \frac{\epsilon}{\epsilon-1}\frac{\dot{N}}{N} + \frac{\dot{c}}{c}$. Combining (24) with (9), we have:

$$g = \alpha\theta\frac{LE}{\epsilon N}\frac{[\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}]}{\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta}. \tag{26}$$

Equation (26) indicates that the TFP growth is related to the firm's market size (i.e., $\frac{LE}{N}$), the competition parameters between incumbents and entrants (α, β, θ, $\epsilon-1$), and with a particular emphasis, to the distinct liquidity constraints (ξ_Z and ξ_N) on the R&D activities. With an endogenously-determined N, the TFP growth depends on the firm's market size $\frac{LE}{N}$, rather than the aggregate market size LE. Other things being equal, g negatively depends on entry productivity β, implying that the proliferation of product varieties reduces the effectiveness of R&D aimed at quality improvement, by causing it to be spread thinly over a larger number of products. This, on the one hand, endogenizes the market structure (the firm and market sizes) and, on the other hand, eliminates the scale effect. Laincz and Peretto (2006) have shown that data on US employment, R&D personnel and production establishments support the idea that the scale effect is sterilized by product proliferation.

Given (26), by using the optimal labor supply (17), labor requirement for production (23), labor clearing condition (25), free entry condition (14), no-arbitrage condition that (21)=(20), and Euler equation (18), we can reduce the whole dynamic system to the following two differential equations in terms of g and s:

$$\frac{\dot{s}}{s} = \lambda - \frac{\beta}{1+\xi_N i}(s - \frac{g}{\alpha\theta}\Omega), \tag{27}$$

$$\frac{\dot{g}}{g} = \frac{g}{\alpha\theta}\frac{\beta}{1+\xi_N i}\{\frac{\alpha[1-\theta(\epsilon-1)]}{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}} + \Omega\} - \rho + \lambda - \frac{\beta}{1+\xi_N i}s, \tag{28}$$

where $\Omega = \frac{\gamma(1+\xi_c i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}} + \frac{[\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_Z i}](1+\xi_X i) + (\epsilon-1)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)}{[\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}](1+\xi_X i)}$. Accordingly, the loci of the system are given by:

$$\dot{s} = 0 \Rightarrow g = Q_1(s - \frac{1+\xi_N i}{\beta}\lambda), \tag{29}$$

$$\dot{g} = 0 \Rightarrow g = Q_2[s + \frac{1+\xi_N i}{\beta}(\rho - \lambda)]. \tag{30}$$

Note that Q_1 and Q_2 are functions of i and $\xi_q, q = (Z, N, X, C)$, which are relegated to the Appendix. The corresponding phase diagram is depicted in Figure 1.

3.1 Steady-State Effects

In the steady state, the TFP growth rate $g^\star$ and the ratio of the labor force to the number of firms $s^\star$ are solved by setting $\dot{s} = 0$ and $\dot{g} = 0$. Given these, the steady-state inflation rate $\pi^\star$ is determined by (16) and (18) with $\dot{E} = 0$, while the consumption expenditure per capita $E^\star$ is determined by the market-clearing condition of the financial market (20)=(21) with (9). With the steady-state $s^\star$ and $E^\star$, we can use (17) to pin down the steady-state employment rate $l^\star$. Finally, given that $g_C = \frac{\epsilon}{\epsilon-1}\frac{\dot{N}}{N} + \frac{\dot{c}}{c}$, the steady-state growth rates of entry $(\frac{\dot{N}}{N})^\star$ and consumption $g_C^\star$ can further be determined by using (14), (23), (24), and (25).

There is a nondegenerate, competitive equilibrium of growth, which is stable and unique. On the growth path,

$$g^\star = \theta\rho\frac{1}{\beta}[\frac{\alpha\frac{1+\xi_N i}{1+\xi_Z i}\theta(\epsilon-1) - \beta}{1-\theta(\epsilon-1)}], \tag{31}$$

$$s^\star \equiv (\frac{L}{N})^\star = \frac{1+\xi_N i}{\beta}\{\rho\frac{[V_1+V_2](1+\xi_X i) + V_3}{\alpha[1-\theta(\epsilon-1)](1+\xi_X i)} + \lambda\}, \tag{32}$$

$$l^\star = \frac{(1+\xi_X i)V_2 + V_3 + \alpha[1-\theta(\epsilon-1)]\frac{\lambda}{\rho}(1+\xi_X)}{(1+\xi_X i)[V_1+V_2+\alpha[1-\theta(\epsilon-1)]\frac{\lambda}{\rho}] + V_3}, \tag{33}$$

$$(\frac{\dot{N}}{N})^\star = \lambda, \tag{34}$$

$$g_C^\star = \frac{\dot{C}}{C} = \frac{1}{\epsilon-1}\lambda + \theta\rho\frac{1+\xi_N i}{\beta}[\frac{V_2}{1-\theta(\epsilon-1)}]. \tag{35}$$

$$E^\star = \frac{\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)(1+\xi_X i)}{(1+\xi_X i)[V_1+V_2+\alpha[1-\theta(\epsilon-1)]\frac{\lambda}{\rho}] + V_3} \tag{36}$$

$$\pi^\star = i - \rho, \tag{37}$$

where $V_1 \equiv \gamma(1+\xi_C i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)$, $V_2 \equiv \frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\epsilon_N i}$, and $V_3 \equiv (\epsilon-1)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)$.

We are ready to investigate how an increase in the nominal interest rate i (or inflation) can have different long-run consequences through various CIA constraints.

Proposition 2. *With a CIA constraint on in-house R&D ($\xi_Z > 0$, $\xi_N = \xi_X = \xi_C = 0$), a higher nominal interest rate i decreases the steady-state firm size, TFP growth rate and consumption growth rate, while it increases the inflation rate. It has an ambiguous effect on employment and consumption expenditure per capita.*

The Fisher equation (37) indicates that in the long run a higher nominal interest rate i is associated with a higher inflation rate. A higher inflation rate raises the cost of holding money and hence reduces real money balances in the economy. If the money balances are required to engage in in-house R&D and entry investment is not restricted by such a constraint, in-house R&D becomes more expensive compared to firm entry. Thus, (21) and (20) indicate that a higher i makes both r_Z and r_N decrease, but r_Z decreases more than r_N (i.e., $r_Z < r_N$). Therefore, the economic resource (labor) shifts away from the quality improvement-type to the variety expansion-type innovation. This implies that the number of firms expands faster than the population ($\dot{N}/N > \lambda$) and the firm size ($s = L/N$) and its market size (LE/N) thereby shrink. In the steady state, small-sized firms engage in less in-house R&D, which decreases the rate of innovation growth, as shown in (9).[9] Since the consumption growth g_C, as indicated in (36), is a weighted sum of the TFP growth g and the growth rate of entry $\dot{N}/N$ (which is equal to λ in the steady state), the consumption growth rate decreases as well.

The response of equilibrium employment $l^{\star}$ could be negative or positive. A higher i restricts the in-house innovation, which favors the entrants. While labor shifts away from R&D and production to entry, this *labor reallocation* has an ambiguous impact on the *total employment*, as shown in (25). Moreover, from (17), there is a trade-off between consumption expenditure and labor supply. Thus, the steady-state consumption expenditure changes in the opposite direction to employment and has an uncertain response to a higher nominal interest rate or inflation. Interestingly, a higher rate of inflation may increase the consumption expenditure per capita, even though it has a negative effect on the long-run consumption growth rate. When higher inflation discourages in-house R&D activities, the decrease in the quality-adjusted price of goods becomes slower. Households thus consume less, but pay more for relatively low-quality goods.

Proposition 3. *With a CIA constraint on firm entry investment ($\xi_N > 0$, $\xi_Z = 0 = \xi_X = \xi_C$), a higher nominal interest rate i increases the steady-state firm size, inflation, employment, and the growth rates of TFP and consumption, but decreases the consumption expenditure per capita.*

By contrast, if entry investment is subject to the CIA constraint, a decrease in the real money balances, followed by a higher nominal interest rate, restricts the variety-expanded

[9] This result is supported by the empirical studies. See, for example, Cohen and Klepper (1996) and more recently Pagano and Schivardi (2003).

innovations. As shown in (21) and (20), r_N declines and r_Z remains unchanged. Because $r_N < r_Z$, the resource shifts from entry to R&D and production, which leads to $\dot{N}/N < \lambda$ and increases the firm's size (s) and its market scale (LE/N). The expansion in firm size leads to more in-house R&D, resulting in higher growth rates of TFP and of consumption expenditure. This implies that to gain a higher growth rate, some degree of monopoly power is needed to act as the reward accruing to the successful firms from their innovations. Our monetary model suggests that if entry investment is subject to a higher degree of cash constraint, a rise in the nominal interest rate renders the existing innovators with a larger degree of monopoly power by increasing entry costs to potential competitors. Thus, high inflation can be associated with higher growth and the Mundell-Tobin effect occurs. Such a case could be empirically plausible, as the evidence shows that R&D is more likely to be liquidity constrained for young or new firms.[10] This result is in contrast to the findings of Funk and Kromen (2010), who predict a negative growth effect of inflation.[11] In a money-in-utility-function model, Chu and Lai (2013) also find that the growth-inflation relationship could be positive, provided that the elasticity of substitution between money and consumption is larger than one.

The equilibrium employment increases with the nominal interest rate. Equation (14) indicates that a higher i decreases the productivity of entry. Thus, more labor resources are required for entry to maintain the original entry rate, since the steady-state entry growth must be fixed at the population rate (i.e., $(\frac{\dot{N}}{N})^\star = \lambda$). This gives rise to a positive direct effect, increasing entry labor and hence total employment. On the other hand, in the face of a higher i entry becomes unfavorable, owing to $r_N < r_Z$. Therefore, labor resources move away from entry to R&D and production. Although this *labor reallocation effect* may have a negative impact on the total employment, it is dominated by the direct entry effect. As a result, the steady-state total employment $l^\star$ unambiguously increases in response to a higher interest rate. In addition, as mentioned above, the consumption expenditure per capita changes in the opposite direction of employment and hence decreases in the steady state. While consumption expenditure decreases, the consumption growth rate increases. When entry is restricted by the CIA constraint, a higher i renders the incumbents with an effective shield against potential competition, which motivates them to engage in more in-house R&D. The expansion in the R&D decreases the quality-adjusted price of goods and therefore households can enjoy more

[10] See Janiak and Monteiro (2011) for a discussion on the CIA constraint on new establishments.

[11] In addition to the cash/liquidity constraint, the evidence (e.g., Harhoff, 1988) also shows that younger firms (or entrants) are more likely to be constrained by the availability of finance (i.e., the borrowing/financial constraint) due to the moral hazard problem and uncertainty. To capture this observation, a simple way is to modify equation (12) as: $(1 - \xi_N)\frac{1}{\beta} + [1 + i(1 + \phi)]\xi_N \frac{1}{\beta}$, where ϕ is a risk premium, reflecting the difficulty of obtaining finance for an entrant in a financial market with friction. Thus, there are two different interest rates, which reflect lending to incumbents and entrants. Given a constant rate of risk premium, our main results are robust to this modification.

consumption by incurring less expenditure.

Proposition 4. *With a CIA constraint on manufacturing production ($\xi_X > 0$, $\xi_Z = \xi_N = \xi_C = 0$), a higher nominal interest rate i decreases the steady-state firm size and employment, but increases the inflation rate and consumption expenditure per capita. It has no effect on the TFP and consumption growth rates.*

If manufacturing production is subject to the CIA constraint, a higher nominal interest rate raises the inflation rate and decreases the real money balances, which leads to higher production costs. As shown in (19), this further raises the good price p. In the face of a higher price, the households are inclined to decrease their consumption and increase their leisure time. Thus, employment falls as a response.[12] Note that due to a higher price, consumption decreases, but the aggregate consumption expenditure increases.[13] When the aggregate consumption expenditure (the aggregate market size) rises, entry becomes profitable, which attracts more new firms to enter the market, expanding the product variety. Thus, entry, on the one hand, erodes the incumbents' profits and, on the other hand, decreases the firm size $s = \frac{L}{N}$. As it turns out, the expansion in consumption expenditure E is eroded by entry, leading the firm's market size $\frac{LE}{N}$ to remain constant. As a result, (26) indicates that the TFP growth rate (and hence the consumption growth rate) is irresponsive to the increase in the nominal interest rate.

Even though both models are scale free, the result of Proposition 4 contradicts that of Chu and Cozzi (2014), which predicts that raising the nominal interest rate permanently increases the growth rate, if manufacturing production is subject to the CIA constraint. Our model with the endogenous market structure predicts that a higher interest rate only has a positive transitional effect (this will be shown in the next subsection), but no long-run steady-state effect on growth. The Chu and Cozzi (2014) model eliminates the scale effect by re-scaling the firm level innovation arrival rate by population size and normalizing the number of firms to unity. When manufacturing production is subject to the CIA constraint, a higher interest rate shifts labor from the manufacturing to the R&D sector. Since the number of firms is fixed at unity, the rise in the R&D labor share directly increases the arrival rate of the new firms to replace the old firms in the same product line, which in turn stimulates economic growth. In our model with an endogenous number of firms, the firm's profitability stemming from the expansionary consumption expenditure (caused by the CIA constraint on production) will attract new firms to enter the market. Since entry erodes the profitability, the monetary shock has no effect on growth.

[12] Equation (17) shows that a rise in consumption expenditure E results in a decrease in employment l.

[13] This implies that the household's demand is relatively inelastic in the model.

	$g^\star$	$s^\star$	$g_C^\star$	$l^\star$	$(\frac{\dot{N}}{N})^\star$	π	$E^\star$
CIA constraint on in-house R&D	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow\downarrow$	$-$	$\uparrow$	$\uparrow\downarrow$
CIA constraint on entry	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$-$	$\uparrow$	$\downarrow$
CIA constraint on production	$-$	$\downarrow$	$-$	$\downarrow$	$-$	$\uparrow$	$\uparrow$
CIA constraint on consumption	$-$	$\uparrow$	$-$	$\downarrow$	$-$	$\uparrow$	$\downarrow$

Table 1: Comparative Statics Results

Proposition 5. *With a CIA constraint on consumption ($\xi_C > 0$, $\xi_Z = \xi_N = \xi_X = 0$), a higher nominal interest rate i increases the steady-state inflation rate and firm size, but decreases employment and the consumption expenditure per capita. It has no effect on the TFP and consumption growth rates.*

Proposition 5 restates Chu and Ji's (2014) finding. One may note that the money superneutrality on growth contradicts the traditional CIA growth model with flexible labor (e.g., Gomme 1993 and Wang and Yip 1992), which refers to a negative effect of inflation on growth.[14]

For ease of comparison among the cases, we summarize the comparative statics above in Table 1. First, a higher inflation rate is monotonically followed by a higher nominal interest rate in all cases, but economic growth can increase, decrease or be neutral in relation to the nominal interest rate, depending on the strength of distinct cash constraints. Our model gives rise to a mixed long-run relationship between growth and inflation. This non-monotony reconciles the finding in the output/growth-inflation relationship based on recent empirical evidence, such as Bullard and Keating (1995), Bruno and Easterly (1998), and Ahmed and Rogers (2000).

In addition, in response to a unified increase in inflation, the relationship between employment and growth is also non-monotonic. Table 1 shows that the employment-growth relationship could be either positively related (the CIA constraint on in-house R&D or entry), negatively related (the CIA constraint on in-house R&D), or independent (the CIA constraint on production and consumption). This outcome differs from the conventional wisdom, which refers to a positive employment-growth relationship. Nonetheless, our results seem to be consistent with the findings of empirical studies. Bean and Pissarides (1993) find that there is little evidence of a robust bivariate relationship between the employment and growth rates during the period of the 1950s-1980s. Gordon (1997) offers an empirical possibility of a negative employment-growth relationship.

How does the monetary policy (or inflation) affect the market structure? Table 1 shows that the various CIA constraints end up with very different market structures. Targeting a higher nominal interest rate is unfavorable to the variety expansion-type R&D, if entry investment

[14] Fisher and Seater's (1993) empirical study supports the money superneutrality on growth, but not on output.

is subject to a relatively high cash constraint. Thus, the market is characterized by a small number of large-sized firms (the intensive margin). By contrast, a higher i is unfavorable to the quality improvement-type R&D and production, if in-house R&D or production is restricted by a larger cash constraint. As a result, the market is characterized by a large number of small-sized firms (the extensive margin).[15] These outcomes are different from Wu and Zhang's (2001) results. By linking inflation to the firm's markup, they find that at higher rates of inflation firms are fewer and smaller in size. In the real data, the response of firm entry (measured by the net business formation and the number of new incorporations) to monetary expansions appears to be positive or hump-shaped, depending on the degree of wage rigidity, the congestion externality in entry, and the various measures of monetary policy (see the recent findings in Bergin and Corsetti 2008, Lewis and Poilly 2012, and Lewis and Stevens 2015). Based on the generalized cash constraint, our model generates such a ambiguity, which the growth model with a fixed market structure cannot.

3.2 Transition Effects

We now turn to the transition effects of inflation. Figures 2-7 are used to present our results. From (29) and (30), we have:

$$\frac{\partial Q_1}{\partial i} = [-\frac{\partial(\frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\partial i}A - B] \cdot D_1^2; \tag{38}$$

$$\frac{\partial Q_2}{\partial i} = [-\frac{\partial(\frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\partial i}(A + A') - (B + B')] \cdot D_2^2; \tag{39}$$

where A, B, A', B',D_1 and D_2 are all positive and $D_1 > D_2$ (the exact expressions are relegated to the Appendix). It is easy to derive that $\frac{\partial Q_1}{\partial i} < 0, \frac{\partial Q_2}{\partial i} < 0$ under the case with the CIA constraint on in-house R&D ($\xi_Z > 0$ and $\xi_X = \xi_C = \xi_N = 0$), while $\frac{\partial Q_1}{\partial i} > 0, \frac{\partial Q_2}{\partial i} > 0$ under the case with the CIA constraint on either entry ($\xi_N > 0$ and $\xi_X = \xi_C = \xi_Z = 0$) or production ($\xi_X > 0$ and $\xi_N = \xi_C = \xi_Z = 0$). When either R&D or entry is subject to the cash constraint, the condition $|\frac{\partial Q_1}{\partial i}| < |\frac{\partial Q_2}{\partial i}|$ is true, while when production is subject to the CIA constraint, $|\frac{\partial Q_1}{\partial i}| > |\frac{\partial Q_2}{\partial i}|$ holds true. These imply that in response to a rise in i both the $\dot{g} = 0$ and $\dot{s} = 0$ loci shift downwards with the former shifting more than the latter under the case with the CIA constraint on R&D only (see Figure 2). By contrast, both the $\dot{g} = 0$ and $\dot{s} = 0$ loci shift

[15] This prediction is not a guide for policy-makers to block all new firms and new products to maximize per capita growth. In effect, Table 1 has potentially pointed out that in the presence of various cash constraints raising the nominal interest rate to block all potential entrants cannot maximize growth. In this model, we need the structure of monopolistic competition in order to maintain both horizontal and vertical dimensions of technology space, which remove the scale effect. If there is just one variety, the model structure will break down to a monopoly, which is abstracted from our analysis.

upwards with the former shifting more (less), if entry investment (production) is subject to the cash constraint, as shown in Figure 3 (Figure 4). For the three cases, the dynamic adjustments of all variables are depicted in Figures 5-7. To avoid repetition, we abstract the case where only consumption is subject to the CIA constraint (while the dynamic adjustments can refer to Figure 8).[16]

CIA constraint on in-house R&D

An increase in i creates a wedge between the returns to R&D and to entry. It is favorable to entry, i.e., $r_Z < r_N$, if the in-house R&D is restricted by the CIA constraint. Given a predetermined N, economic resources shift out from in-house R&D/production to entry, leading TFP growth g to jump down on impact (referring to (9)) and the entry rate $\dot{N}/N$ to jump up (referring to (14)), as shown in Figures 2 and 5. Since the number of firms expands faster than the population, the firm size (s) decreases along the transitional path. Given that small-sized firms engage in less R&D, this implies that TFP growth g gradually declines to a lower steady-state rate until the growth rate of the population returns to the steady-state value λ.

As noted previously, the consumption growth rate is a combination of TFP and population growth. Thus, Figure 5 indicates that the growth rate of consumption may jump up or down on impact, since the population growth rate jumps up, while the TFP growth rate initially jumps down. Afterwards, the consumption growth rate gradually converges to a lower value of the steady state, given that the TFP growth and population growth both gradually decline in transition.

Proposition 2 indicates that in the face of a higher nominal interest rate i the resource reallocation effect has a mixed effect on the steady-state employment rate $l^\star$. This resource reallocation effect governs employment not only in the long-run steady state, but also in the short-run transition. At the moment of the policy change, the predetermined N is given. Thus, a higher i shifts the labor resource away from R&D and production (a decrease in L_Z and L_X) to entry (an increase in L_N). Since the labor reallocation is not symmetric in terms of affecting incumbents and entrants, on impact, employment could either jump down or up, and afterwards it monotonically converges to a higher (or lower) steady state, as shown in Figure 5. With regard to the transition of consumption expenditure, (17) demonstrates that its trajectory is opposite to that of employment.

[16] See Chu and Ji (2014) for a detailed discussion on this case.

CIA constraint on entry

If entry, instead of R&D, is subject to the CIA constraint, a higher i leads the entry investment to become more expensive, relative to the in-house R&D. When the resources move away from entry to in-house R&D/production, Figures 3 and 6 show that g jumps up, but $\dot{N}/N$ jumps down at the moment of the policy change. As $\dot{N}/N$ grows more slowly than the population λ, the firm size s goes up, leading to higher values of r_Z and r_N. Therefore, on the one hand, the TFP growth goes up further and gradually converges to a new and higher steady-state value. On the other hand, the entry growth also gradually increases until it returns to the steady state λ.

As a result of the adjustments of g and $\dot{N}/N$, the consumption growth rate, with a jump on impact, gradually increases to a higher steady-state value. Of particular interest, because more intensive R&D activities decrease the prices of products, households can increase their consumption while incurring less expenditure. That is why the consumption expenditure E exhibits a transitional trajectory, which is just the opposite of that of the consumption growth rate, as shown in Figure 6. In terms of the adjustment of l, on impact employment could either jump up or jump down, while in transition it gradually converges to a higher steady state. The reason is that given the predetermined number of firms N, at the moment of policy change the positive direct effect is inactive, but the labor reallocation gives rise to a mixed effect on the total employment.

CIA constraint on production

If manufacturing production is subject to the CIA constraint, a higher i leads to a higher unit cost of production relative to that of R&D activities. When the resources move away from production to R&D (including both quality-improved and variety-expanded R&D), Figures 4 and 7 show that both the TFP growth g and the entry growth $\dot{N}/N$ jump up at the moment of the policy change. Afterwards, since the firm entry grows faster than the population growth λ, the firm size s shrinks, leading a lower value of r_Z and r_N. Lower returns to R&D lead the TFP growth and entry to both slow down and gradually converge to the original steady state levels. Along such transitional adjustments, the growth rate of consumption, which is a linear combination of g and $\dot{N}/N$, jumps up and then gradually goes back to the original steady state. Besides, as the increase in i raises the production costs, the firm is inclined to raise the good price p by using its monopoly power. In the face of a higher price, on the one hand, the households' expenditure E increases and, on the other hand, they substitute consumption for leisure, decreasing working hours. As shown in Figure 7, followed by an increase in the nominal interest rate, consumption expenditure monotonically increases to a new steady state, while employment monotonically decreases to a lower level of steady state.

It is worth noting that the growth effect of inflation in the case with the CIA constraint on production is sharply different from the conventional result, such as in Stockman (1981) and Wang and Yip (1992), who refer to a negative long-run growth effect. By contrast, we find that there is a positive transitional effect, while inflation has no impact on the steady-state growth. As mentioned previously, this long-run super-neutrality of money with only an effect on the level of output also contrasts with the recent finding of Chu and Cozzi (2014), who refer to a positive long-run growth effect.

The transition effects above are summarized in the following proposition:

Proposition 6. *In response to an increase in the nominal interest rate i, the transitional adjustments of the firm size s and the TFP growth g are monotone: both monotonically decrease (resp. increase) in relation to the steady-state value, if in-house R&D (resp. entry) is subject to the CIA constraint. In either case, along the transition path the consumption growth rate g_C, the employment rate l, and the consumption expenditure E may mis-adjust from their long-run steady states. If manufacturing production is subject to the CIA constraint, the transitional adjustments are not so rich: the TFP and consumption growth rates jump up and then gradually revert to their original values. While the firm size and employment monotonically decrease to new steady states, consumption expenditure monotonically expands to a higher level.*

4 Quantitative Analysis

In this section, we perform a simple quantitative study. The purpose is twofold. First, in order to examine the effects of monetary policy in a more realistic scenario in which all cash constraints coexist, we perform the quantitative effects of the policy under different magnitudes of cash constraints. Second, our model with endogenous market structure provides different welfare implications for monetary policy and thus, the optimal design of monetary policy is in contrast to the model with a fixed market structure. We thus provide a quantitative analysis to evaluate the optimality of the Friedman's rule.

4.1 Calibration

Our model features the set of parameters $\{\rho,\epsilon,\lambda,\alpha,\beta,\theta,\gamma,\xi_C,\xi_X,\xi_Z,\xi_N\}$. To match our quantitative analysis with reality, we set the time preference rate as $\rho = 0.05$, as in Acemoglu and Akcigit (2012). We set the population growth rate as $\lambda = 1.5\%$, which is a compromise between the average population growth rate (around 1%) and the average net entry rate (around 2%) in the US, given that the steady-state entry rate must be pinned down by the population growth

ρ	ϵ	λ	α	β	θ	γ	ξ_C	ξ_X	ξ_Z	ξ_N
0.05	4.33	0.015	0.1714	0.0404	0.15	1.7691	0.4	0.01	0.35	0.45

Table 2: Benchmark Parameters

rate in our model (equation (34)).[17] For the elasticity of substitution between two products, we set $\epsilon = 4.33$, corresponding to an estimate of the markup of 1.3.[18] With $\epsilon = 4.33$ and given that θ must fall in $[0, \frac{1}{\epsilon-1}]$ under the symmetric equilibrium, we follow Ferraro and Peretto (2015) and set the diminishing rate of return to R&D as $\theta = 0.15$, which is the middle value of the feasible range. Accordingly, we can jointly calibrate the in-house R&D productivity $\alpha = 0.1714$, the entry cost parameter $\beta = 0.0404$, and the weight on the utility of leisure $\gamma = 1.7691$ to match the TFP growth rate $g = 1.7\%$, the firm size $s = 22.8$, and the working hours $l = 0.33$. The TFP growth is consistent with the US long-run average rate (see, Shackleton 2013) and the value of the market size meets OECD firm-level project data (see, Laincz and Peretto 2006).

Next, we calibrate the degree of CIA on consumption $\xi_C = 0.4$, which is within the reasonable range of the M1-consumption ratios (see, Dotsey and Sarte 2000). We set the degree of CIA on production $\xi_X = 0.01$, corresponding to the fact that for the US and most OECD countries the investment(production)-cash flow sensitivity has declined sharply and is probably close to zero (Dotsey and Sarte 2000). We set the degree of CIA on in-house R&D $\xi_Z = 0.35$, which is located within the reasonable range of estimates in Chu and Cozzi (2014).[19] As for the parameter of CIA on entry, we have no direct data to follow. Nonetheless, since newer firms face a larger cash constraint than older firms (see, Janiak and Monteiro 2011), we set a higher degree of CIA on entrants $\xi_N = 0.45$ than that of CIA on in-house R&D, distinguishing between the two liquidity constraints on R&D. The nominal interest rate is set as $i = 0.06$, which is around the average ratio of nominal interest payments on government bonds and Treasury debt during 1915-2010. This enables us to match the inflation rate $\pi = 2\%$ and the average rate of return to R&D $r = 0.04$ in the US, as in Peretto (2007). The benchmark parameter values are summarized in Table 2.

4.2 A Complete Scenario with Various Cash Constraints

Based on reasonable parametrization of the model economy delineated above, we now examine both the long-run and short-run effects of raising the nominal interest rate i from 6% to 7%.

[17]Yu et al. (2009) estimate that the US average net entry rate is around 2% over the 1977-2005 period.

[18]Based on the US value-added data, the markup lies in the range from 1.2 to 1.4. See, for example, Basu and Fernald (1997).

[19]They indicate that a lower bound of 0.33 matches the US data, while an upper bound of 0.56 matches the Euro Area.

First, Figure 9 shows that under our parametrization, in the face of a higher nominal interest rate the market structure exhibits an intensive margin response, i.e., the number of firms decreases, but the firm size expands (the steady-state s increases by 0.84%). This result is consistent with Bergin and Corsetti's (2008) finding whereby a rise in the federal funds rate discourages entry in terms of a decrease in the net business formation.

Second, the TFP growth rate exhibits an interesting mis-adjustment in the sense that along the *transition* path g mis-adjusts from its *long-run steady state*. As shown in Figure 9, in response a higher nominal interest rate, g decreases on impact (by 0.31%), but increases in the steady state (by 0.35%). Raising the nominal interest rate decreases the real money balances, which in turn lower consumption and employment, due to the cash constraint on consumption. The decrease in employment leads the growth rate to jump down at the moment of policy implementation. Moreover, since newer firms face a higher degree of cash constraint than older firms ($\xi_N > \xi_Z$), inflation generates a positive growth effect in the long run, as in our analytical prediction in Proposition 3.

Third, Figure 9 also shows that in response to a rise in the nominal interest rate, the steady-state growth rate g increases, but the steady-state employment l decreases (by 0.28%). This implies that employment and growth can be negatively related, which confirms Gordon's (1997) empirical observation. In addition, under our parametrization, a higher nominal interest rate i renders the incumbents with an effective shield against potential competition, which motivates them to engage in more in-house R&D. Since the expansion in the R&D decreases the quality-adjusted price of goods, in equilibrium households can enjoy more consumption (an increase in the steady-state g_C by 0.28%) by incurring less expenditure (a decrease in the steady-state E of 0.25%).

Finally, we should note that if incumbents and entrants are subject to the same degree of cash constraint ($\xi_N = \xi_Z$), the two conflicting growth effects cancel each other out and as a result, the super-neutrality of money holds. Thus, our result is reduced to that of Chu and Ji (2014).

4.3 Welfare and Friedman's rule

Based on the competitive equilibrium, we derive the life-time welfare function from (1) as follows:

$$W = \int_0^\infty [lnE_t + \omega\, lnN_t + \theta lnZ_t + ln(\frac{\epsilon - 1}{\epsilon}) - ln(1 + \xi_x i) + \gamma\, ln(1 - l_t)]e^{-(\rho-\lambda)t}dt,$$

where $lnN_t = lnN_0 + \int_0^t n_s ds$ and $lnZ_t = lnZ_0 + \int_0^t g_s ds$. ω measures the social return to variety. To internalize the welfare gain from variety, we additionally consider a positive value of

ω in our welfare analysis.[20] Considering the social return to variety is not specific in the model with an endogenous market structure. For convenience, we set $\omega = 0.1$ in the benchmark, which is similar to that of Iacopetta et al. (2015). As mentioned previously, the endogenous market structure (the extensive or intensive margin) will play a crucial role in affecting the welfare effects of monetary policy. Thus, we also consider a variety of values of the social return to variety, say, $\{0.09, 0.095, 0.1, 0.105, 0.11\}$. For simplicity, we normalize N_0 and Z_0 to unity. Accordingly, Figures 10-12 show the optimal nominal interest rate in a variety of the degrees of social return to variety ω and of the liquidity constraint ξ_N, ξ_Z, ξ_X, and ξ_C. In these figures, the levels of welfare W are normalized at the initial value W_0, for the sake of exposure.

Figure 10 shows that in the benchmark ($\omega = 0.1$), to achieve the social optimum, the welfare-maximizing nominal interest rate is $i_{max} = 4.04\%$, implying that Friedman's rule is not socially optimal. Given that newer firms face a higher degree of cash constraint than older firms ($\xi_N > \xi_Z$) in the benchmark, a higher nominal interest rate enhances the steady-state TFP growth g, but reduces the number of firms N and employment l (see, Figure 9). On the one hand, a higher g implies a higher quality of product Z and a greater level of output income which are associated with higher consumption C, while consumption expenditure E is lower, due to a lower product price. A lower l is associated with higher leisure $(1 - l)$, increasing social welfare. On the other hand, a reduction in N implies less product variety, decreasing the welfare gain from variety. To balance these welfare effects, a positive nominal interest rate is needed for the calibrated economy to achieve the social optimum. Given that ($\xi_N > \xi_Z$), a higher nominal interest rate results in an intensive margin of market. Since the product variety negatively responds to the nominal interest rate, the optimal i_{max} decreases with the measurement of ω, as shown in Figure 10.

Next, we examine how the optimal nominal interest rate is sensitive to cash constraints. First of all, Figures 11-a and 11-b show that the optimal i_{max} increases with the degree of the liquidity constraint on entry ξ_N, but decreases with the degree of the liquidity constraint on in-house R&D ξ_Z. The intuition is straightforward. When the liquidity constraint on entry is stronger (the difference between $\xi_N - \xi_Z$ is more significant), the positive growth effect is more pronounced. As a result, the optimal monetary policy calls for a higher nominal interest rate to maximize welfare. By contrast, when the liquidity constraint on in-house R&D is stronger, the adverse result appears. Once the liquidity constraint on entry becomes relatively low ($\xi_N = 0.25$ or $\xi_N = 0.35$ under our parameterization), the optimal nominal interest rate turns out to be zero, $i_{max} = 0$, and Friedman's rule is valid.

Figures 12-a and 12-b indicate that the optimal i_{max} is decreasing in both ξ_C and ξ_X.

[20] Specifically, we rewrite (4) as $C = N^{\omega - \frac{1}{\epsilon - 1}} [\int_o^N c_j^{(\epsilon-1)/\epsilon} dj]^{\frac{\epsilon}{\epsilon-1}}$. Note that given this modification, all the results of our positive analysis still remain.

By referring to Figures 8 and 9, we learn that in response to a stronger cash constraint on consumption ξ_C, a higher nominal interest rate decreases not only the product variety in the steady state, but also the growth in transition. These negative effects on welfare lead to a lower optimal nominal interest rate. In addition, a liquidity constraint on production ξ_X gives rise to a directly distortionary effect on the product price and hence the real consumption (see the welfare function). Thus, a greater ξ_X also lowers the optimal nominal interest rate.

5 Concluding Remarks

By considering a variety of cash constraints, we have constructed a monetary version of the Schumpeterian growth model with an endogenous market structure to explore the long-run steady-state and the short-run transition effects of monetary policy on the number of firms, firm market size, labor employed, and economic growth. We have shown that these CIA constraints work through various channels and the effects of monetary policy depend on the strength of each channel. Inflation, which seems like a uniform tax, can give rise to different consequences because of different CIA constraints in the economy. Thus, an identical monetary policy may end up with very different market structures, employment and growth consequences in the presence of distinct cash constraints.

Our results have provided a couple of new implications of relevance to the literature or policymakers. First, the case with the CIA constraint on entry identifies a new channel for the Mundell-Tobin effect. Second, the mixed long-run relationship between growth and inflation, as well as employment can reconcile the recent empirical findings. Third, under a convincing parameterization, growth may exhibit a mis-adjustment in the sense that along the transition path, the TFP growth rate mis-adjusts from its long-run steady state. This implies that in response to higher inflation, economic growth may fall in the short run, but rise in the long run. Fourth, the market structure exhibits an intensive margin response (the number of firms decreases, but each firm's size become larger) to a higher nominal interest rate. This is in accordance with the finding of Bergin and Corsetti (2008), who show that a rise in the federal funds rate decreases the net business formation. Finally, our welfare analysis has shown that Friedman's rule, in general, is not socially optimal, depending on the difference between each cash constraint and the extent of the social return to product variety. Specifically, the optimal nominal interest rate increases with the degree of the liquidity constraint on entry, but decreases with the degree of the liquidity constraints on in-house R&D, production and consumption, as well as the social return to variety.

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Appendix: (A major portion of the Appendix is not intended for publication.)

Proof of Proposition 1:

For the proof of the symmetric condition $\theta(\epsilon - 1) < 1$ the reader can refer to Peretto's (1998b) Proposition 1. Under this condition, the incumbent chooses the paths of its product price P_j and its R&D expenditure L_{Z_j} to maximize (11) subject to the demand function (6) and the R&D production function (9). By defining q_j as the costate variable, which is the value of the marginal unit of knowledge, this optimization problem is to maximize the following current-value Hamiltonian

$$CVH_j = [p_j - h(Z_j)]X_j - (1 + i\xi_X)L_{X_j} - (1 + i\xi_Z)L_{Z_j} + q_j,$$

s.t. (6) and (9). The firm's knowledge stock Z_j is the state variable, while the in-house R&D resource L_{Z_j} and the product price p_j are the control variables. By taking the first-order derivative with respect to p_j, we can obtain the optimal price, reported in (19). Moreover, the linear Hamiltonian yields

$$L_{Z_j} = \begin{cases} 0 & for\, 1 + \xi_Z i > q_j \alpha K \\ L_Z/N & for\, 1 + \xi_Z i = q_j \alpha K \\ \infty & for\, 1 + \xi_Z i < q_j \alpha K \end{cases},$$

where $1 + \xi_Z i$ is the marginal cost of R&D and $q_j \alpha K$ is the value of the marginal unit of knowledge. The interior solution is determined under the condition that the marginal cost of R&D equals its marginal benefit. The differential equation for the costate variable gives:

$$r_j = \frac{\dot{q}_j}{q_j} - \frac{h'(Z_j)X_j}{q_j}, \tag{40}$$

indicating that the return to R&D is the ratio of the revenue from the innovation to its shadow price $(-h'(Z_j)X_j/q_j)$ plus the change in the value of the knowledge stock $(\dot{q}_j/q_j)$. Consider the interior solution and let $g_K = \dot{K}/K$ be the growth rate of public knowledge. Taking logs and time derivatives of $1+\xi_Z i = q_j \alpha K$, (6), (7), (8), (9), (19) and $h(Z_j) = Z_j^{-\theta}$ allow us to reduce (40) to (20) under the symmetric equilibrium.

Given entry costs $(1+\xi_N i)\frac{1}{\beta}$ and the value produced V_j, taking logs and time derivatives of the free entry condition (13) yields:

$$r_j = \frac{\pi_j}{V_j} + \frac{\dot{V}_j}{V_j}. \tag{41}$$

This implies that the rate of return on the firm ownership equals the rate of return on the riskless loan of V_j. By using (13), (6), (7), (8), (19), and (10), and imposing symmetry, we can reduce (41) to (21).

Proof of Proposition 2:

From (29) and (30), it is easy to derive the steady state values of g and s, as reported in (31) and (32). From the arbitrage condition (21)=(20), we obtain (24). By recalling that $g = \theta\frac{\dot{Z}}{Z}$ and $\dot{Z} = \alpha K \frac{L_Z}{N}$, we then have $g = \theta\alpha\frac{L_Z}{N}$ under symmetry (i.e., $Z_j = K$). Using (31), (32) and (24), one can solve the steady state value of E through solving $g = \theta\alpha\frac{L_Z}{N}$. By plugging the steady state value of E into the optimal labor supply (17), the steady state value of l can be derived, as shown in (33). The steady-state entry rate (34) can be solved through $\frac{\dot{L}}{L} - \frac{\dot{N}}{N} = \frac{\dot{s}}{s}$, given that $\frac{\dot{L}}{L} = \lambda$ and $\frac{\dot{s}}{s} = 0$ in the steady state.

From (4), we have:

$$g_C = \frac{\epsilon}{1-\epsilon}\frac{\dot{N}}{N} + \frac{\dot{c}_j}{c_j}.$$

From (6), (8), (23), and (18) and by imposing the symmetric condition $L_X = NL_{X_j}$, we further obtain:

$$g_C = \frac{1}{\epsilon-1}\frac{\dot{N}}{N} + g\{1 + \frac{\beta}{(1+\xi_N i)\theta}[\frac{1-\theta(\epsilon-1)}{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}}]\} - \rho. \tag{42}$$

The steady state value of g_C, reported in (35), is then solved by using (42), (31) and (34). Finally, the steady-state inflation rate is pinned down by the Fisher equation (16).

The Derivatives of Q_1, Q_2, A, B, $A^{'}$, $B^{'}$,D_1 and D_2:

It is easy to obtain these derivatives, which are expressed as follows:

$$Q_1 = \frac{(1+\xi_X i)\alpha\theta V_4}{(1+\xi_X i)[(1+\xi_Z i)V_1 + V_4] + (1+\xi_Z i)V_3},$$

$$Q_2 = \frac{(1+\xi_X i)\alpha\theta V_4}{(1+\xi_X i)[(1+\xi_Z i)V_1 + V_4] + \alpha(1+\xi_Z i)[1-\theta(\epsilon-1)]\} + (1+\xi_Z i)V_3},$$

$$A = [\gamma(1+\xi_C i)\epsilon + \frac{\epsilon-1}{1+\xi_X i}](1+\xi_Z i)\alpha[1-\theta(\epsilon-1)],$$

$$B = [\gamma\xi_Z(1+\xi_C i)\epsilon + (\epsilon-1)\xi_Z + \frac{\gamma(1+\xi_Z i)\xi_C\epsilon}{1+\xi_X} - \frac{(\epsilon-1)(1+\xi_Z i)}{(1+\xi_X)^2}]\cdot(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)\cdot V_4,$$

$$A^{'} = (1+\xi_Z i)\alpha[1-\theta(\epsilon-1)],$$

$$B^{'} = \alpha\xi_Z[1-\theta(\epsilon-1)]V_4,$$

$$D_1 = [(1+\xi_Z i)V_1 + V_4 + \frac{(1+\xi_Z i)}{1+\xi_X i}V_4]^2,$$

$$D_2 = \left\{(1+\xi_Z i)V_1 + V_4 + \alpha(1+\xi_Z i)[1-\theta(\epsilon-1)] + \frac{(1+\xi_Z i)}{1+\xi_X i}V_3\right\}^2,$$

where $V_4 \equiv \alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta$.

The Threshold of ξ_C:

If the cash constraints on in-house R&D, entry investment, and consumption exist simultaneously, in response to a higher nominal interest rate the TFP growth rate could jump down on impact, provided that ξ_C is higher than a threshold such that $\frac{\partial Q_1}{\partial i} < 0, \frac{\partial Q_2}{\partial i} < 0$:

$$\xi_C > \frac{(1+\xi_Z i)\alpha[1-\theta(\epsilon-1)](\gamma+\epsilon-1)\frac{\xi_N-\xi_Z}{(1+\xi_Z i)^2} - (\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)V_4\xi_Z(\gamma\epsilon+\epsilon-1) + A' + B'}{(\gamma\epsilon+2\gamma\xi_Z i\epsilon)(\alpha-\frac{1+\xi_Z i}{1+\xi_N i}\beta)V_4 - \gamma(1+\xi_Z i^2)\alpha[1-\theta(\epsilon-1)]\frac{\xi_N-\xi_Z}{(1+\xi_Z i)^2}}.$$

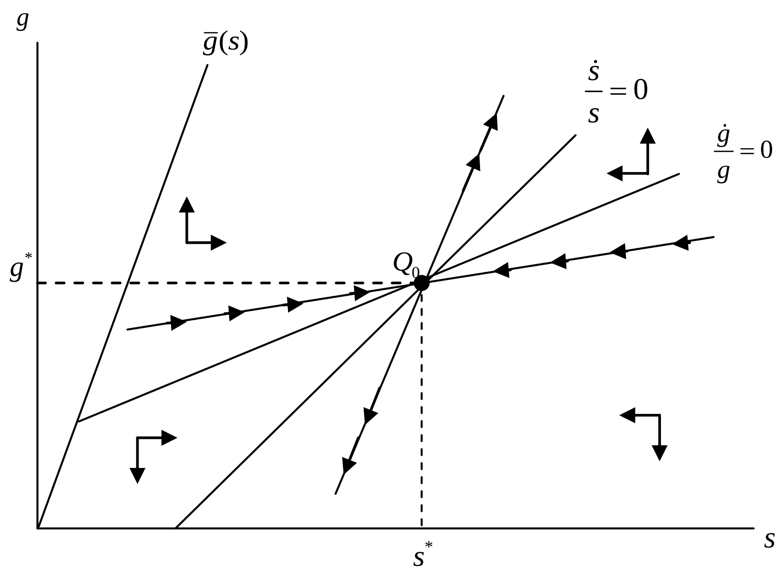

Figure 1. Phase Diagram

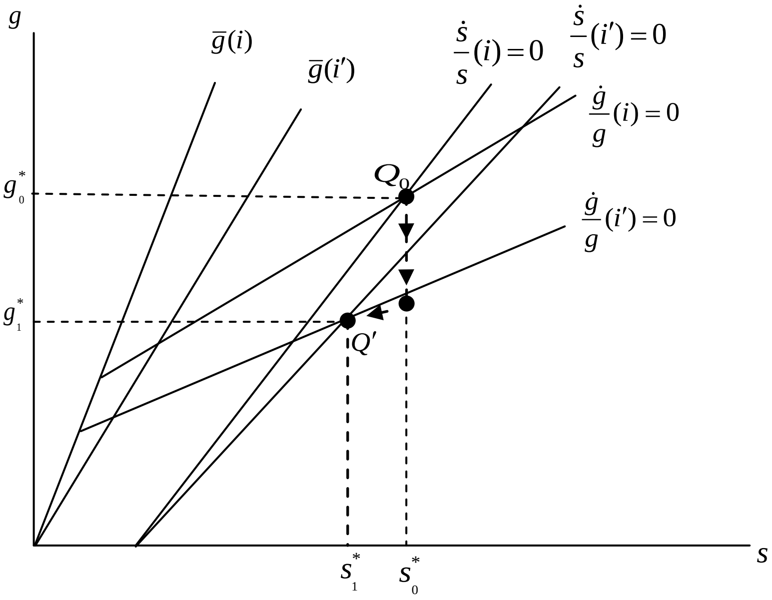

Figure 2. An Increase in i : CIA Constraint on In-House R&D

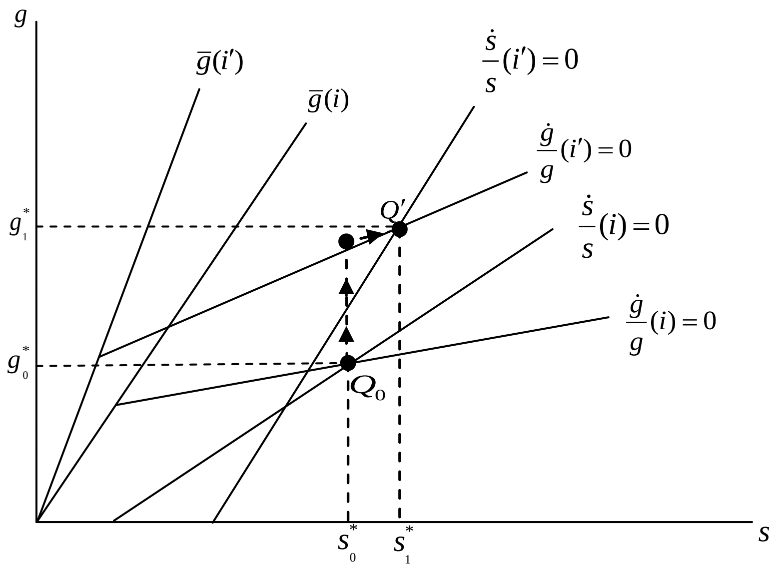

Figure 3. An Increase in i : CIA Constraint on Entry Investment

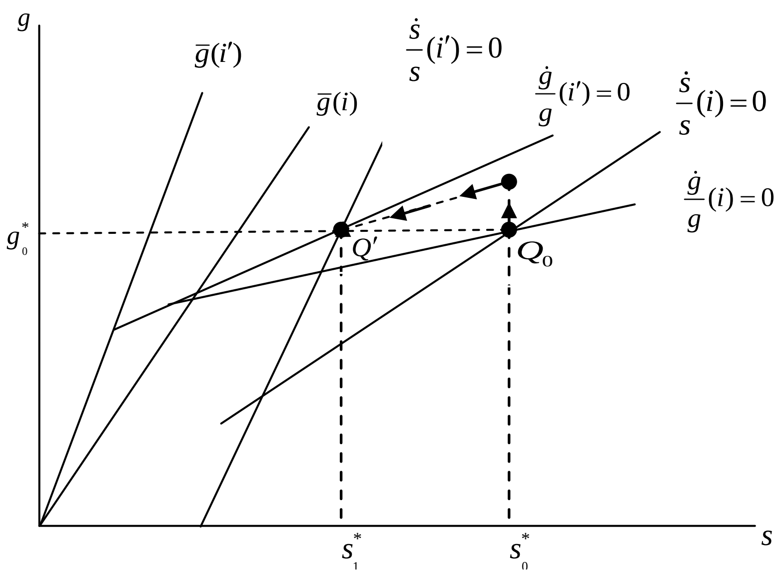

Figure 4. An Increase in i : CIA Constraint on Production

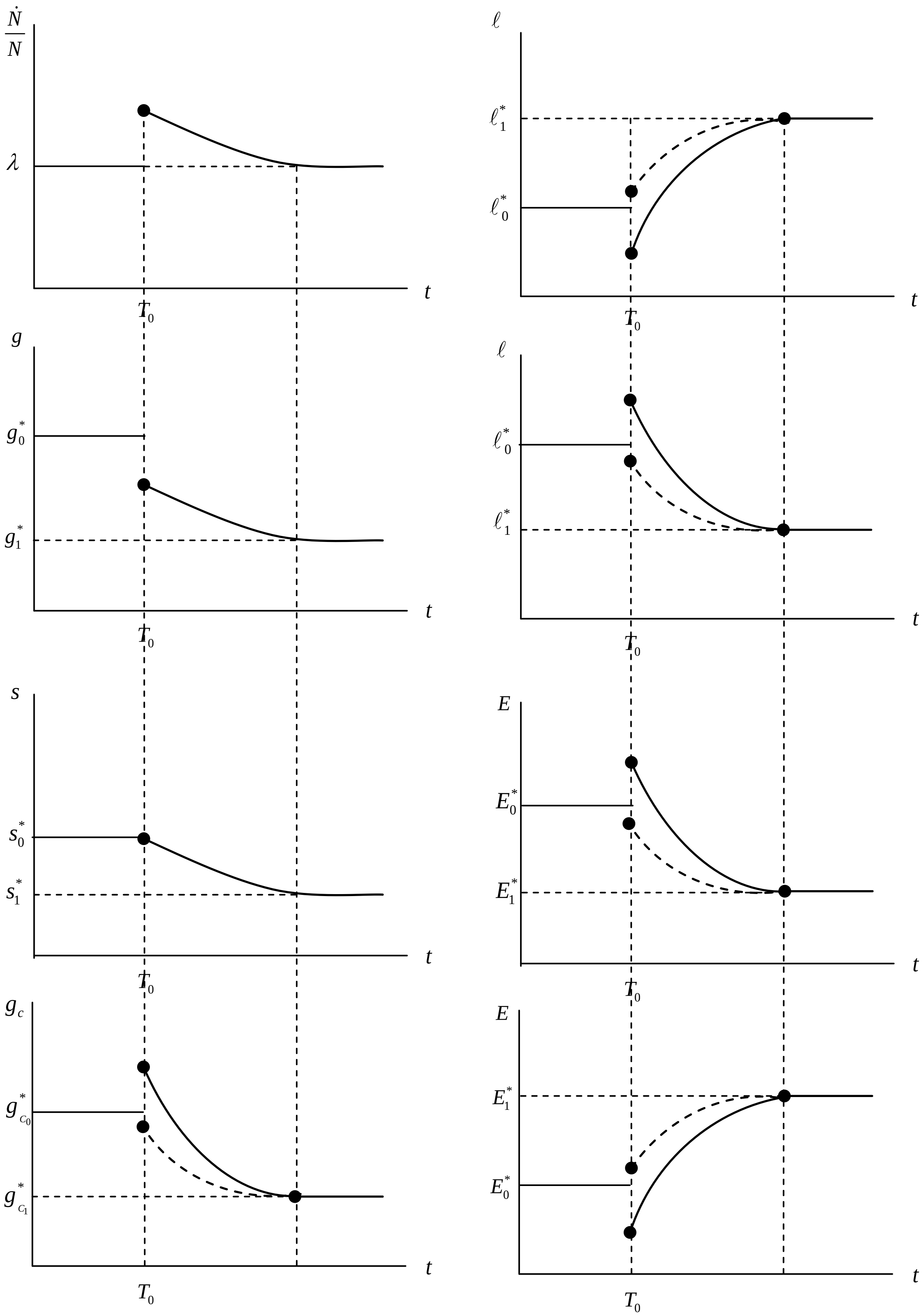

Figure 5. Time Path

An Increase in i : CIA Constraint on In-House R&D

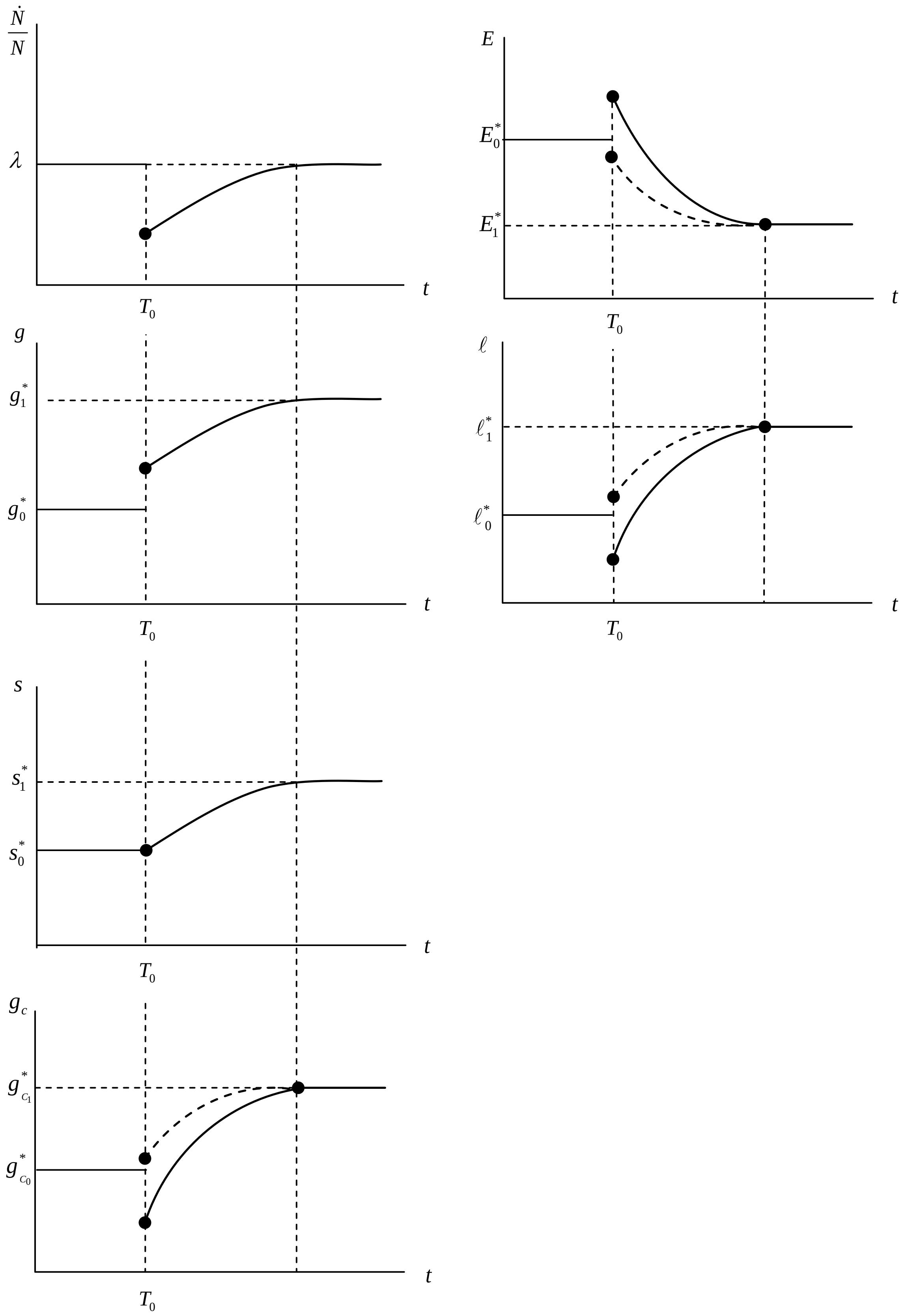

Figure 6. Time Path

An Increase in i: CIA Constraint on Entry Investment

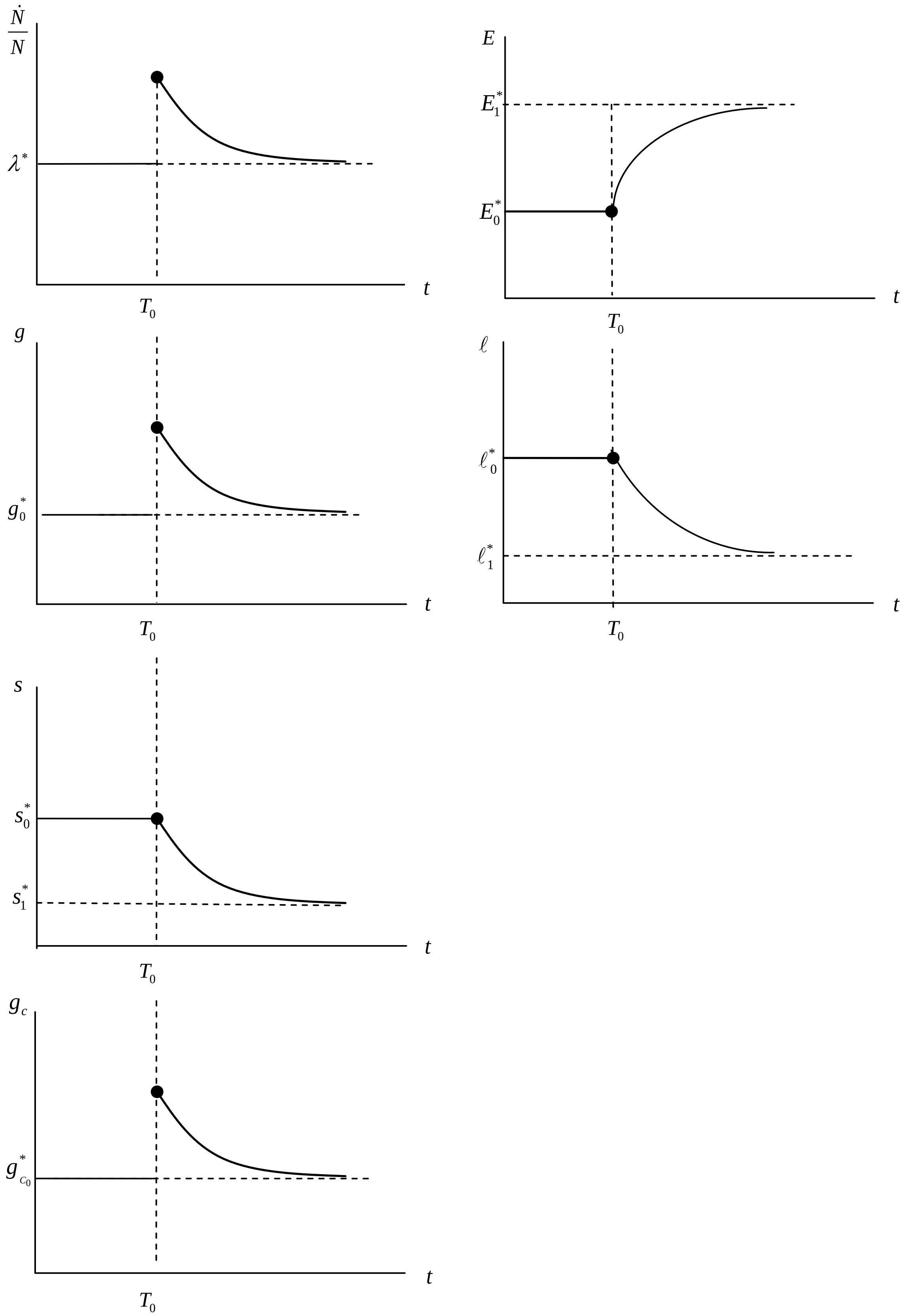

Figure 7. Time Path

An Increase in i : CIA Constraint on Production

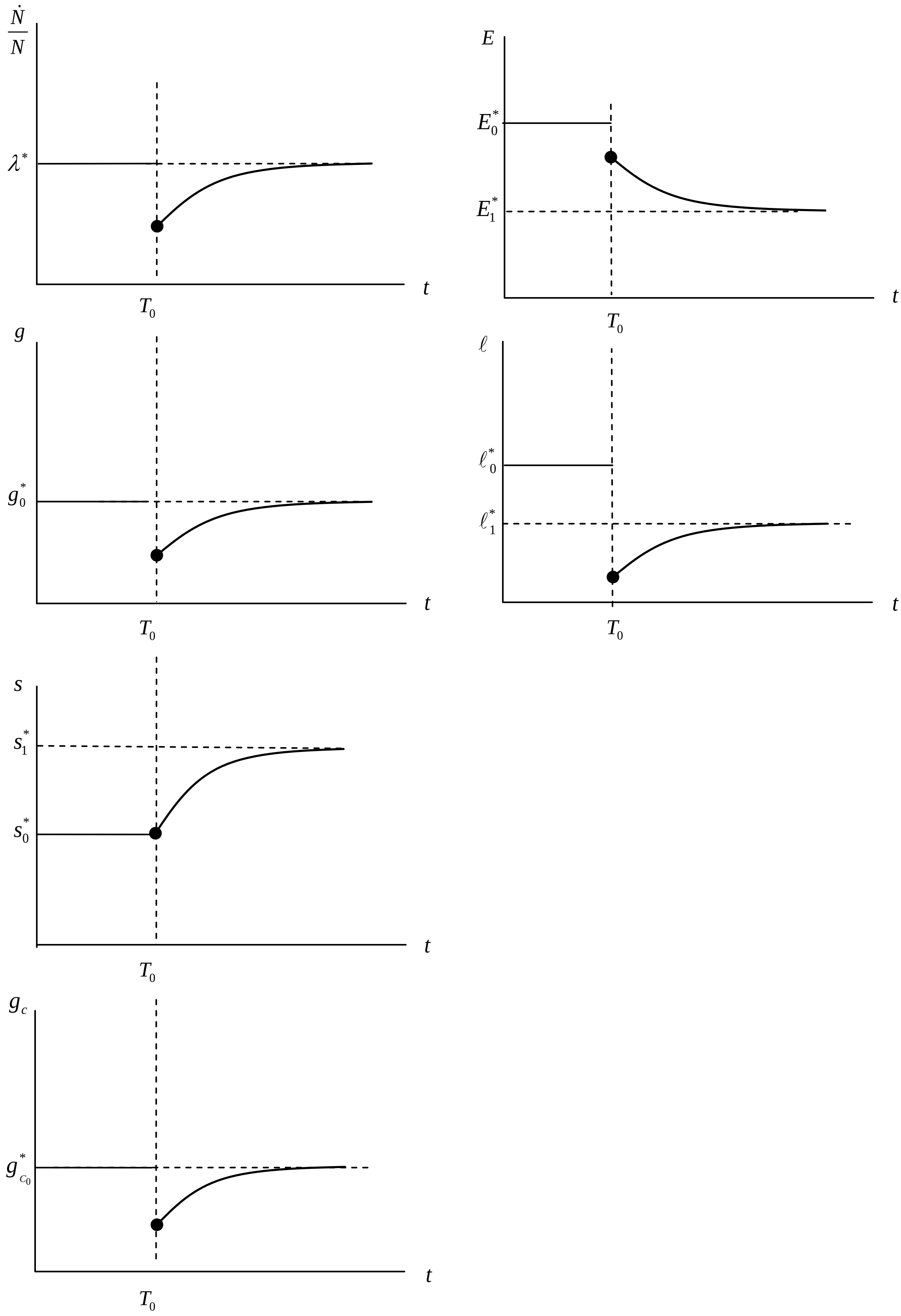

Figure 8. Time Path

An Increase in i: CIA Constraint on Consumption

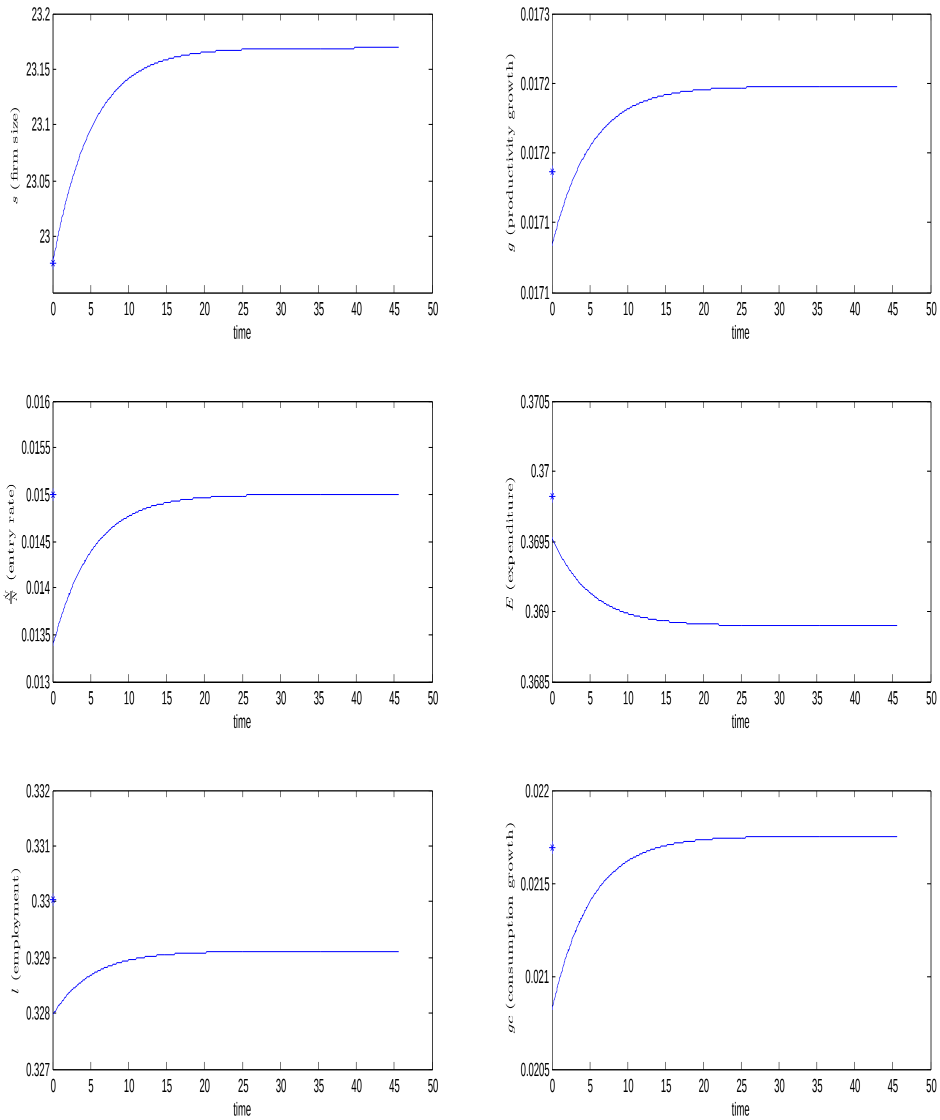

Figure 9. Effects of Raising Nominal Interest Rate

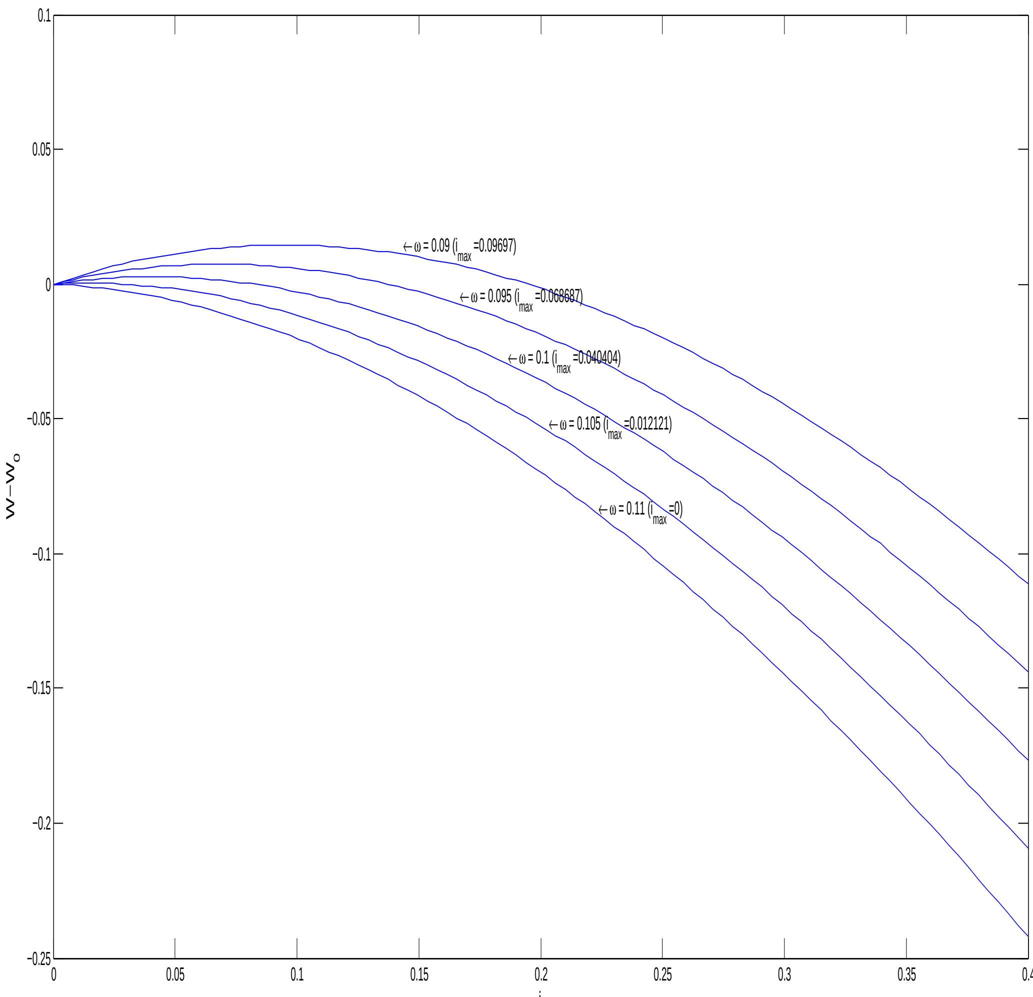

Figure 10. Optimal Nominal Interest Rate with Various ω

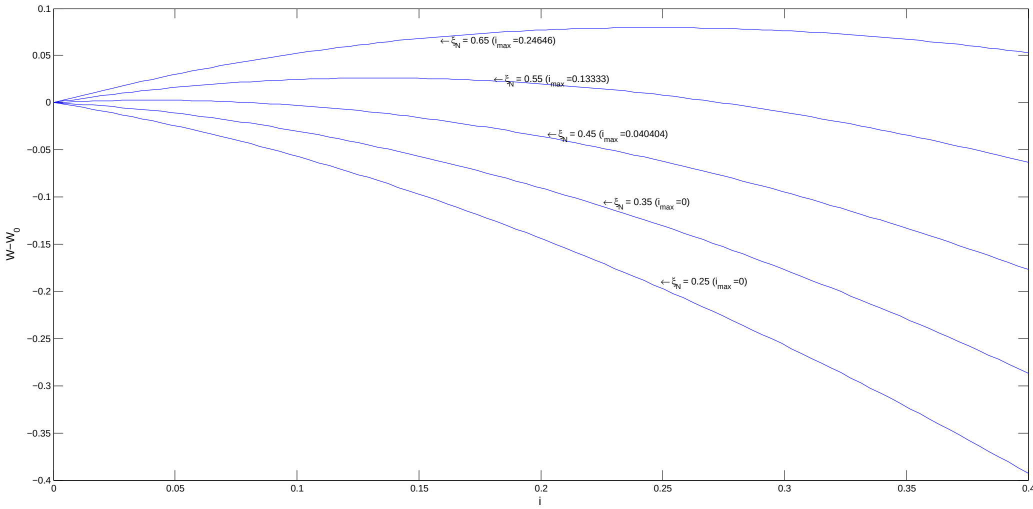

Figure 11-a. Optimal Nominal Interest Rate with Various ξ_N

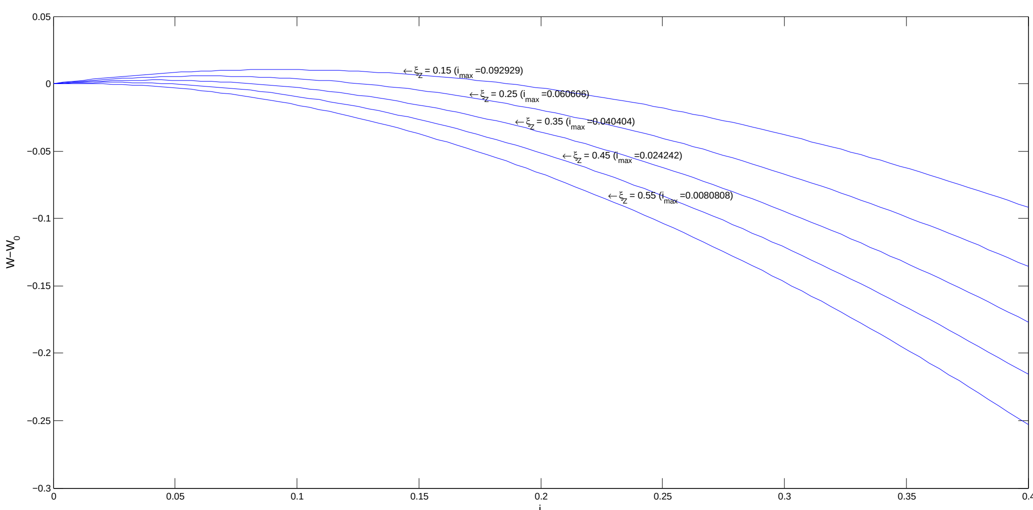

Figure 11-b. Optimal Nominal Interest Rate with Various ξ_Z

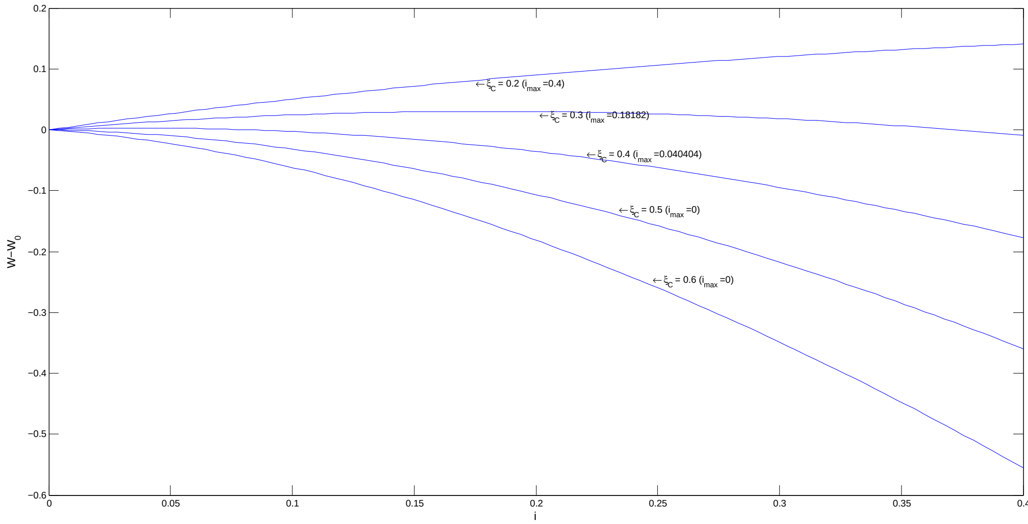

Figure 12-a. Optimal Nominal Interest Rate with Various ξ_C

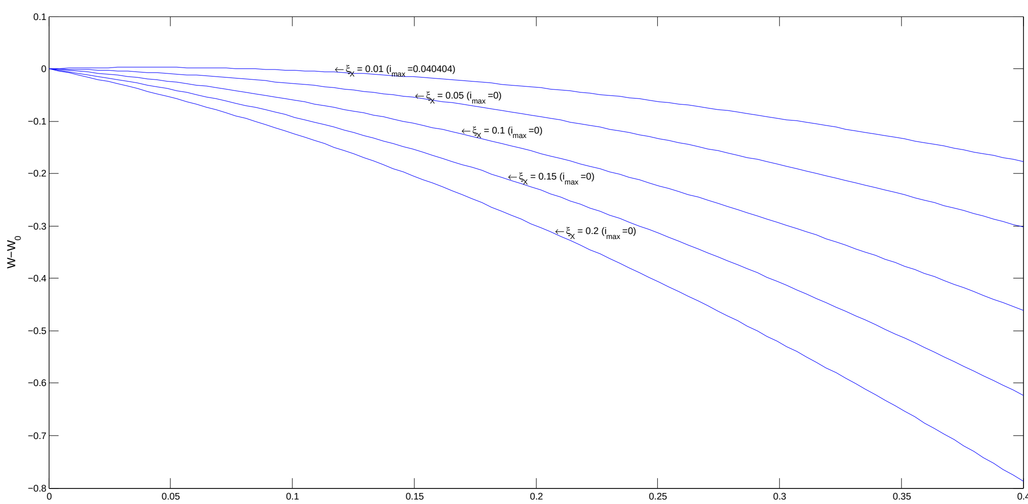

Figure 12-b. Optimal Nominal Interest Rate with Various ξ_X