On Measuring Welfare 'Behind a Veil of Ignorance'

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Abstract

How should we rank different income distributions? Should we adopt the Rawlsian criterion that focuses on the minimal income in the distribution? Or should we rather maximize the geometric mean, a criterion advocated by gamblers and welfarists alike? This paper microfounds these two criteria by showing that each of them can be obtained by granting veto rights to the members of society 'behind a veil of ignorance', where society is represented by the set of Regular Utilities (Hart, 2011). The Rawlsian maximin criterion is obtained by granting each member of society a right to veto *acceptance* of any candidate income distribution, while the geometric-mean criterion is obtained by granting instead a right to veto *rejection* of income distributions. This way, the proposed method circumvents the need to arbitrarily choose a representative agent when ranking income distributions. The Rawlsian maximin criterion is further shown to be robust to extending the set of utilities that constitutes "society" to all risk-averse utilities, while the geometric-mean criterion is not as robust.

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1 Introduction

Rawls (1971) raises the fundamental question of how we should choose between different income distributions, or how we should rank these distributions, 'behind a veil of ignorance.' The idea of choosing 'behind a veil of ignorance' is that of a two-stage process, where in stage 2 an income would be *randomly assigned* to the decision maker according to the distribution of incomes she had chosen in stage 1. Rawls then offers an answer to his own question and promotes ranking according to the *maximin criterion*, i.e., distribution x would be ranked higher than distribution y if and only if $\min x > \min y$.[1] However, an economist would probably approach the question quite differently than Rawls. In economic terms, the question of ranking income distributions 'behind a veil of ignorance' is similar to the question of ranking lotteries, i.e., distributions over final outcomes (see e.g. Harsanyi 1953, 1955 and Atkinson 1970). Therefore, a decision maker with vN-M utility function u would rank distribution x higher than distribution y if and only if $E[u(x)] > E[u(y)]$. But how should we choose the "right" utility function according to which to rank the distributions?

In this paper I present two complementary methods for ranking income distributions. Importantly, these ranking methods circumvent the need to arbitrarily choose a representative agent. Instead, the idea is to use the tendency of *a broad set of utilities* to either unanimously accept or unanimously reject the lotteries corresponding to the income distributions. This broad set of utilities is thought of as representing the "members of society" needing to reach a decision 'behind a veil of ignorance.' Two *complete* rankings emerge as a result of applying the two methods. One ranking coincides with the Rawlsian principle of maximin, while the other ranks income distributions according to their geometric means.

[1] I assume throughout that income distributions have finite support in $\Re^+$, and therefore the maximum and minimum incomes exist and are well defined. For ease of exposition I denote by x (and y) both the distribution itself and the corresponding random variable.

The question of how to reconcile considerations of income inequality with considerations of aggregate or average income goes back at least to Sheshinski (1972). Sheshinski develops a set of welfare functions from the Gini Index, and many would follow his footsteps and develop sets of welfare functions out of measures of inequality aversion (e.g., Shorrocks 1983, Ebert 1987 and Safra and Segal 1998). The drawback of this approach is that it often leads to a whole family of welfare functions and eventually to partial orders, while a unique and complete ranking is usually desirable. Similarly, choosing a set of utilities and looking for a complete agreement among the members of the set on how to rank distributions over outcomes results in a partial order. The most notable examples for this approach are the (partial) first- and second-order stochastic-dominance relations, which correspond to the rankings agreed upon by all utilities with $u' > 0$ and with $u'' < 0$ respectively (see Hadar and Russell 1969, Hanoch and Levy 1969 and Rothschild and Stiglitz 1970).[2] Thus, in order to develop complete rather than only partial rankings, a different approach is needed. To this end, the current paper partly borrows a technique from Hart (2011), which eventually leads to the two aforementioned complete rankings.[3] These rankings take into account both the average income and income inequality in a manner that is determined by the preferences of the set of utilities that represents "society." The main contribution of the paper is thus in providing an economic microfoundation to the Rawlsian principle as well as the increasingly popular criterion of maximal geometric mean (which indeed, starting from 2010, replaces the arithmetic mean in the calculation of the United Nations Human Development Index). An extension of the main model to include a larger set of utilities (Section 3) demonstrates that the Rawlsian maximin criterion is robust to the set of utilities used while the geometric-mean criterion is not as robust.

[2]Levy (1992) adds further the third-order stochastic-dominance, which corresponds to the partial ranking agreed upon by all utilities with $u' > 0$, $u'' < 0$ and $u''' > 0$.

[3]Hart (2011) uses his technique to rationalize two riskiness indices applicable to net gambles: The A-S Index (Aumann and Serrano 2008) and the F-H Measure (Foster and Hart 2009).

2 The Main Model

An *income distribution*, the item to be ranked, is represented by a random variable with finite support in $\Re^+$. Generally speaking, the preference of society over these distributions is a function of the preferences of the individuals it comprises. However, this paper aims to establish principles that are not dependent on the particularities of a specific society. Rather, the idea is to use a broad set of utilities U^*(see below) as representing a "benchmark" society whose members employ unanimity considerations in order to produce a societal ranking of income distributions.

2.1 Utilities

Who are the members of the benchmark society that will rank income distributions 'behind a veil of ignorance'? I do not take here a strong stance but do place some very standard requirements on the set of utilities $U^* \subseteq \mathcal{C}^2$ that represents society.[4] Later on, in Section 3, I study the effect of removing some of these requirements.

(R1) "More is (always) better": $\forall u \in U^*$, $u' > 0$.

(R2) Risk aversion: $\forall u \in U^*$, $u'' < 0$. When considering income distributions, this implies some basic form of inequality aversion: if distribution y is a mean-preserving spread of distribution x, each member of U^* prefers x over y.[5]

(R3) Weakly increasing relative risk aversion (weak IRRA): $\forall u \in U^*$, $\forall w > 0$, $\gamma_u(w) \equiv -\frac{wu''(w)}{u'(w)}$ (≥ 0) is weakly increasing in w.[6] The set of utilities that respect this

[4]The differentiability assumption ($U^* \subseteq \mathcal{C}^2$) is without consequences for the resulting rankings. My set U^* is very similar to the set of *regular utilities* of Hart (2011) but is a bit less restrictive because my last requirement (R4) replaces two requirements that Hart sets: decreasing absolute risk aversion and "some rejection". These two requirements of Hart are, taken together, more demanding than my own substitute requirement.

[5]Whenever I write that u prefers distribution x over distribution y, I mean that $E[u(x)] \geq E[u(y)]$.

[6]This property, advocated by Arrow (1965, 1971), says that acceptance of risky prospects is weakly decreasing with *relative* wealth. Informally, this means that the inclination to invest one's total wealth in a risky prospect whose returns are given in *proportional terms* (like a stock) weakly declines with the amount of one's wealth. There exists empirical evidence (albeit being inconclusive) in favor of IRRA, see e.g. Siegel and Hoban (1982), Morin and Suarez (1983), Halek and Eisenhauer (2001), and, more

property includes (but is not restricted to) all HARA utilities, of which CRRA utilities (where relative risk aversion is constant) and CARA utilities (where relative risk aversion is linearly increasing with wealth) are special cases.

(R4) "strong aversion to bankruptcy": $\forall u \in U^*, \lim_{w \to 0^+} u(w) = -\infty$. This requirement captures the notion that being left with absolutely nothing is considered by u to be intolerable.

2.2 The rankings

Each $u \in U^*$ judges a given income distribution by treating the random variable associated with it as a lottery. In particular, the members of the benchmark society (all $u \in U^*$) consider income distributions by comparing them to an outside option w_0. I say that u *accepts* income distribution x "at wealth w_0" if $E[u(x)] > u(w_0)$.[7]

I consider two criteria for ranking income distributions. The first is the standard veto principle, i.e. requiring unanimous acceptance. An income distribution is *unanimously accepted* at income level w_0 if all $u \in U^*$ accept it at w_0. I then say that income distribution x *acceptance-based dominates* income distribution y, which I write $x \geq_{AD} y$, if, $\forall w > 0$, $[E[u(y)] > u(w_0)\ \forall u \in U^*] \Rightarrow [E[u(x)] > u(w_0)\ \forall u \in U^*]$.

In words, income distribution x acceptance-based dominates income distribution y if members are given the right to veto acceptance (so that acceptance must be unanimous), and whenever y is unanimously accepted at some wealth w, also x is unanimously accepted at wealth w, implying x is "(weakly) more acceptable."

The second criterion of ranking is that of unanimous rejection: an income distribution is said to be *unanimously rejected* by society "at income level w_0" if all $u \in U^*$

recently, Paravisini et al (2016).

[7]One may replace the strong inequality with a weak one without changing the results of the model. Note that I do not assume that everybody has the same wealth – below I use uniformity over w. The comparison to an outside option w_0 is just a part of the proposed mechanism for judging the attractiveness of various income distributions 'behind a veil of ignorance'.

reject it when comparing it to an outside option w_0. I say that income distribution x *rejection-based dominates* income distribution y, which I write $x \geq_{RD} y$, if, $\forall w > 0$, $[E[u(x)] \leq u(w_0)\ \forall u \in U^*] \Rightarrow [E[u(y)] \leq u(w_0)\ \forall u \in U^*]$.

That is, an income distribution x rejection-based dominates income distribution y if whenever x is unanimously rejected at some wealth w also y is unanimously rejected at wealth w, implying y is "(weakly) *less* acceptable."

As it turns out, each of these two domination relations leads to a *complete* ranking of income distributions, and each of these rankings coincides with a familiar measure of welfare. In particular, the first domination relation coincides with the Rawlsian maximin criterion and the second coincides with maximization of the geometric mean of the income distribution, known also as the Kelly criterion.[8] The following proposition states this result.

Proposition 1. *Let x and y be two random variables with finite support in $\Re^+$. Then:*

1. $x \geq_{AD} y$ *if and only if* $\min x \geq \min y$.
2. $x \geq_{RD} y$ *if and only if* $E[\log(x)] \geq E[\log(y)]$.

Part (1) of Proposition 1 invokes the Rawlsian maximin rule because, by requiring unanimous acceptance when the extent of risk aversion is not bounded, we guarantee that for any wealth level $w > \min x$, there would be $u \in U^*$ who is sufficiently risk averse to reject income distribution x at w, fearing she might end up getting $\min x$.

Noting that maximizing $E[\log(x)]$ is equivalent to maximizing the geometric mean of x, part (2) of Proposition 1 invokes the usage of the geometric mean of the income distribution as a measure of welfare. The result in this part of the proposition stems from the fact that the log utility is the least risk averse among all the utilities that respect

[8]Not to be confused with the Nash social welfare function, which is achieved by multiplying the individual (expected) utilities of all members of society from a given income distribution.

property (R4), i.e., those that always reject a distribution whose support includes the zero outcome.

3 Extending the set of utilities

One might wonder what changes in the results if we extend U^*, the set of utilities that represent society, by eliminating some of the restrictions listed in Section 2.1. As it turns out, the fourth requirement (R4: "strong aversion to bankruptcy") plays an important role, but only for the unanimous-rejection method, while the unanimous-acceptance method would still yield the Rawlsian maximin criterion.

Formally, let U_2 denote the set of utilities that satisfy only requirements R1 and R2 (i.e. U_2 contains all increasing and concave utility functions), and let U_3 denote the set of utilities that satisfy requirements R1-R3 (i.e. U_3 contains all increasing and concave utility functions that also satisfy weak IRRA). We can then denote by $x \geq_{AD_2} y$ and by $x \geq_{RD_2} y$ the acceptance-based domination and rejection-based domination relations (respectively) when they are applied to the set U_2, and similarly for $x \geq_{AD_3} y$ and $x \geq_{RD_3} y$ when applied to the set U_3. Then:

Proposition 2. *Let x and y be two random variables with finite support in $\Re^+$. Then:*

1. (a) $x \geq_{AD_2} y$ *if and only if* $\min x \geq \min y$; similarly (b) $x \geq_{AD_3} y$ *if and only if* $\min x \geq \min y$.
2. (a) $x \geq_{RD_2} y$ *if and only if* $E[x] \geq E[y]$; similarly (b) $x \geq_{RD_3} y$ *if and only if* $E[x] \geq E[y]$.

The lesson learned from part (1) of Proposition 2 is that the Rawlsian maximin criterion is robust to an extension of society. This is quite intuitive: like in Proposition 1, we require here acceptance of the proposed income distribution by all members of society, including the most prudent individuals, and asking for the acceptance of *more* individuals – in particular those who are added by lifting constraints – does not affect the pivotal role of these most prudent individuals.

The lesson learned from part (2) of the proposition is a mirror image of the first: the maximal-geometric-mean result turns out to be highly dependent on the set of utilities that constitutes society. In fact, any result built on unanimous rejection is bound to be fragile to an extension of society by removal of constraints on the set of utilities. The intuition for this result is that the pivotal individual under the unanimous-rejection method is the one who tends *not to* reject the proposed income distributions, hence any extension of society by removing requirements is likely to bring in new utility functions that are more inclined to accept (= not reject) the proposed income distributions. In particular, removing requirement R4 brings in the risk-neutral utility function, leading to the result stated in part (2) of the proposition.[9] One might even argue that U_2 is a more reasonable characterization of society than U^*. The unanimous-rejection method then yields a preference that, to a large extent, completely abstracts from utilities, by ranking income distributions simply by their expected value.

4 Discussion

The literature that aims to define an optimal income distribution is normative in its nature. It usually takes an axiomatic approach, where the axioms (or principles) are chosen to capture the desirable properties of the allocation. This literature contains already papers that have singled out the Ralwsian maximin criterion (e.g. Gotoh and Yoshihara 2003 and Lombardi et al. 2016) or its leximin variant (Mariotti and Veneziani

[9] Note that the inclusion of R3 on top of R1 and R2 (i.e. using U_3 rather than U_2) does not affect the outcome, as stated in part (2) of Proposition 2. The reason for this is that, as mentioned, the least risk averse utility under requirements R1 and R2 is the risk-neutral utility, and this utility has a fixed relative risk aversion coefficient of 0, hence satisfies weak IRRA (R3) too while excluding the existence of a more risk-loving utility in U_3.

2009 and Sprumont 2013) as desirable, as well as papers that have singled out a product property as desirable (e.g. Mariotti and Veneziani 2018).[10]

This paper takes a different approach. It doesn't use an axiomatic approach and in fact it mixes normative and positive considerations. On the one hand, I deal here with the normative question of what constitutes higher welfare, and what should be considered as more desirable 'behind a veil of ignorance'. On the other hand, I construct the set of utilities that represent the benchmark society 'behind a veil of ignorance' in a way that aims to reflect descriptive characteristics of human behavior, such as risk aversion. This approach results in a special role for the log utility: the criterion of unanimous rejection implies via Proposition 1 that the choices of the log utility should guide the choices of society.[11] Note however that, in my model, the log utility is not chosen in order to represent a typical agent, but is rather the result of using the preferences of a broad set of utilities that satisfy some plausible properties. Thus, the obtained rule can be thought of as a "positively-driven normative rule", where the positive aspect is captured by the set of utilities considered – which could also be extended as demonstrated in Section 3 – and the normative aspect is captured by, e.g., a decision to use the unanimous-rejection criterion rather than unanimous acceptance. Similarly, the Rawlsian criterion of maximin is shown here to *reflect* the choices of society, hence has a positive aspect as well, as opposed to its purely normative appeal in Rawls (1971) and its axiomatizations.

As mentioned in the introduction, the technique used in the current paper heavily

[10]There are of course also papers that axiomatize the Nash social welfare function, i.e. the product of individual *utilities* (see e.g. Luce and Raiffa 1957 and Kaneko and Nakamura 1979 for classical treatments and Sprumont 2020 for a recent axoimatization). Like noted in footnote 8, this is *not* the same as multiplying the payoffs themselves (as done when calculating the geometric mean of the income distribution). In fact, the Nash social welfare relates to the max-geometric-mean criterion that results from applying rejection-based domination to the set U^*, just like Utilitarianism (the sum of individual utilities, see Harsanyi 1955) relates to the max-expected-value criterion that results from applying rejection-based domination to the set U_2 or U_3 (Section 3).

[11]Indeed, Arrow (1971) advocates the usage of the log utility in economic models because he finds this utility function to be the most reasonable one from a descriptive point of view (see also Blanchard and Fischer 1989).

borrows from Hart (2011). Hart (2011) explores two types of unanimous rejection, one that is called utility-uniform rejection, which is the one adapted in the current paper with some adjustments, and another one called wealth-uniform rejection, which is not applied here.[12] The notion of unanimous acceptance is new to the current paper.[13] At the technical level, the concept of unanimous rejection was originally introduced by Hart (2011) for ranking *gambles*, where gambles are stochastic net changes in wealth. The primitives of the model presented in this paper – income distributions – are instead distributions over final outcomes. This difference is crucial for the implementation of the ranking method, and therefore it was not a-priori clear that a complete ranking of income distributions was indeed achievable. But more importantly, even if the adaptation of the technique of Hart (2011) was more straightforward, the main contribution of the current paper is in its application to the domain of welfare analysis, which results in a novel microfoundation for the Rawlsian maximin criterion and for the max-geometric-mean criterion.

It is worth noting that the geometric mean of the income distribution has some properties that make it appropriate as a measure of welfare. First, it is homogeneous (of degree 1): if all incomes are multiplied by some positive factor λ, the geometric mean is also multiplied by that factor λ. Second, it is invariant to replication of the population (which is an important feature for a measure of welfare, see e.g. Sen 1976 and Shorrocks 1983). Third, like the arithmetic mean, it captures the central tendency of the incomes in the distribution, but unlike the arithmetic mean it decreases under mean-preserving spreads, thus capturing the negative impact of inequality on welfare.

[12] The reason why wealth-uniform rejection is not applied here is that when the primitives of the model are distributions over final outcomes rather than over net outcomes (see more below), this notion has no bite, as it refers to rejection at all wealth levels, while its clear that no decision maker will reject an income distribution over strictly positive values at *all* wealth levels.

[13] Hart (2011) does have a different notion of acceptance domination, which does not produce a complete ranking and is not applied here either.

Appendix: proofs

Proof of part 1 of Proposition 1

Proof. For any income distribution (random variable) z, let $L(z) \equiv \min z$ and let $M(z) \equiv \max z$. Notice that for any income distribution z, and every $u \in U^*$, u accepts z at any $w \leq L(z)$. Now let $w_0 = L(z) + \epsilon$ for some arbitrary small and positive ϵ, and let p_L denote the non-zero probability of $L(z) \in \text{supp}(z)$. Now let $\hat{u}_\alpha(w) := (\log(\alpha w) - 1)/e$ for $w \leq \frac{1}{\alpha}$ and $\hat{u}_\alpha(w) := -\exp(-\alpha w)$ for $w \geq \frac{1}{\alpha}$; then $\gamma_{\hat{u}_\alpha}(w) = 1$ for $w \leq \frac{1}{\alpha}$ and $\gamma_{\hat{u}_\alpha}(w) = \alpha w$ for $w \geq \frac{1}{\alpha}$ and so $\hat{u}_\alpha(w) \in U^*$ for each $\alpha > 0$.[14] For every $\alpha > \frac{1}{L(z)}$, we get that $\hat{u}_\alpha(w)$ rejects distribution z at w_0 if and only if $\hat{u}_\alpha(w_0) \geq E[\hat{u}_\alpha(z)]$. A sufficient condition for a rejection of z by $\hat{u}_\alpha(w)$ at w_0 is then

$$-\exp(-\alpha w_0) = -\exp(-\alpha(L(z)+\epsilon)) \geq p_L(-\exp(-\alpha L(z))) + (1-p_L)(-\exp(-\alpha M(z))).$$

Dividing by $-\exp(-\alpha L(z))$ and setting $\alpha(\epsilon) = -\frac{\log(p_L)}{\epsilon}$, we get that $\hat{u}_{\alpha(\epsilon)}(w)$ rejects distribution z at w_0, because $p_L \leq p_L + (1-p_L)\, p_L^{\frac{M(z)-L(z)}{\epsilon}}$. Letting $\epsilon \to 0$, we get that $\forall \epsilon > 0$, $\exists \alpha(\epsilon)$ such that every $\hat{u}_\alpha(w) \in U^*$ with $\alpha > \alpha(\epsilon)$ rejects z at wealth level $L(z) + \epsilon$. Consequentially, income distribution x is unanimously accepted at w if and only if $w \leq L(x)$, and income distribution y is unanimously accepted at w if and only if $w \leq L(y)$, and so x unanimously-acceptance dominates y if and only if $L(x) \geq L(y)$. ■

Lemma 1. *Let $u_1, u_2 \in U_2$ be two utility functions with absolute risk aversion coefficients ρ_1 and ρ_2 respectively, where $\rho_1(w) \geq \rho_2(w)$ for every $w > 0$. Then for every income distribution z with finite support in $\Re^+$ and every $w > 0$, if u_2 rejects z at w then u_1 rejects z at w too.*[15]

Proof. Let ψ be such that $u_1 = \psi \circ u_2$; then ψ is strictly increasing (since u_1 and u_2 are such), and concave (since for every $w > 0$ we have $\psi\prime(u_2(w)) = u_1'(w)/u_2'(w)$, hence $(\log \psi\prime(u_2(w)))' = (\log u_1'(w))' - (\log u_2'(w))' = -\rho_1(w) + \rho_2(w) \leq 0$, and so $\psi'' \leq 0$). Therefore $E[u_2(z)] \leq u_2(w)$ implies by Jensen inequality and by the monotonicity of ψ that $E[u_1(z)] = E[\psi(u_2(z))] \leq \psi(E[u_2(z)]) \leq \psi(u_2(w)) = u_1(w)$. ■

Proof of part 2 of Proposition 1

Proof. For any income distribution z and any wealth level $w > 0$, I will show that z is unanimously rejected at w if and only if $\log(w) \geq E[\log(z)]$. This means that

[14] $\hat{u}_\alpha(w)$ was originally proposed by Hart (2011).

[15] Lemma 1 and its proof and the proof to the second part of Proposition 1 borrow from Hart (2011).

unanimous rejection by all $u \in U^*$ implies and is implied by rejection by the log utility, hence rejection-based domination boils down to the preferences of the log utility (i.e., x rejection-based dominates y if and only if $E[\log(x)] > E[\log(y)]$). The "only if" direction is immediate: if $\log(w) < E[\log(z)]$, then $u_l \equiv \log(w) \in U^*$ accepts z at w, thus z is not unanimously rejected at w. The proof for the "if" direction is less straightforward. Lemma 8 in Hart (2011) implies that if u is increasing and strictly concave, and $\lim_{w \to 0^+} u(w) = -\infty$ (as implied by property (R4) of U^*), then $\lim_{w \to 0^+} \gamma_u(w) \geq 1$. From property (R3) of U^* (weakly increasing $\gamma_u(w)$), it follows that $\forall u \in U^*$, $\gamma_u(w) \geq 1$ for all $w > 0$. Thus, Lemma 1 implies that $\forall u \in U^* \subset U_2$, if $u_2 = u_l$ rejects z at w then $u_1 = u$ rejects z at w too (because $\gamma_u(w) \geq 1 = \gamma_{u_l}(w) \Rightarrow \rho_u(w) \geq \rho_{u_l}(w)$), i.e., $\log(w) \geq E[\log(z)]$ implies that z is unanimously rejected at w. ■

Proof of part 1 of Proposition 2

Proof. The proof of part (1) of Proposition 1 applies here too, for both statements (a) and (b), because $\hat{u}_\alpha(w) \in U^* \subset U_3 \subset U_2$, hence the fact that every $\hat{u}_\alpha(w)$ with $\alpha > \alpha(\epsilon)$ rejects z at wealth level $L(z) + \epsilon$ (see the proof of part (1) of Proposition 1) is relevant here too, implying that income distribution x is not unanimously accepted at w whenever $w > L(x)$; and at the same time it still holds that for any income distribution z, and every $u \in U_2$ (hence also every $u \in U_3 \subset U_2$), u accepts z at any $w \leq L(z)$. ■

Proof of part 2 of Proposition 2

Proof. (a) For any income distribution z and any wealth level $w > 0$, I will show that income distribution z is unanimously rejected at w if and only if $w \geq E[z]$ (this would then imply that $x \geq_{RD_2} y$ *if and only if* $E[x] \geq E[y]$). Let $u_{neut} \in U_2$ denote the risk-neutral utility function. The "only if" direction: if $w < E[z]$, then u_{neut} accepts z at w, thus z is not unanimously rejected at w. The "if" direction: Lemma 1 implies that $\forall u \in U_2$, if $u_2 = u_{neut}$ rejects z at w then $u_1 = u$ rejects z at w too (because $\gamma_u(w) \geq 0 = \gamma_{u_{neut}}(w) \Rightarrow \rho_u(w) \geq \rho_{u_{neut}}(w)$), i.e., $w \geq E[z]$ implies not only that u_{neut} rejects z at w, but also that z is unanimously rejected at w.

(b) The same proof of (a) holds, substituting U_2 with U_3 and $\geq_{RD_2}$ with $\geq_{RD_3}$. ■

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