Motivating employees through career paths*

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Abstract

Firms have discretion over task allocations, which may dampen employees' career prospects, and, hence, their motivation. Task assignments and worker motivation interact through the extent of labor market competition – that is, the possibility of moving to another firm. More competition enhances motivation but decreases firms' incentives to assign workers to informative tasks. One consequence is that competitive firms sometimes choose strategies that lead to intermediate competition. When the employee pool is heterogeneous, firms might choose different human resources practices that attract different kinds of workers, and differentiate themselves through the career opportunities that they offer within and beyond the firm.

1 Introduction

When evaluating different job offers, young graduates consider not only the salaries offered, but also the career trajectories that these jobs may bring. In fact, surveys suggest that career paths rank among the top five factors when considering a job opportunity.[1]

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[1]See, for instance, `https://content.linkedin.com/content/dam/business/talent-solutions/global/en_us/blog/2015/07/what-students-want-in-a-job.jpg` or `http://asia.fnst.org/content/top-5-things-graduates-look-their-first-job`

Career prospects are particularly relevant in professional services such as management consulting, investment banking and accounting, where firms often sell their jobs to candidates as springboards to a great career. For instance, Bidwell, Won, Barbulescu, and Mollick (2015) find that “students applying to investment banks consistently rated the extent to which the firm's reputation would help with future employability as the most important factor shaping their decisions” (p. 1170). This preference for firms that enhance employability points to an intrinsic conflict between two major goals of human resources management: attracting and motivating talented workers on the one hand, and retaining them on the other. In the face of this dilemma, employers will often market themselves to potential recruits not only by describing potential careers within the firm, but also by emphasizing how they promote potential career development beyond the firm.[2]

Workers recognize that career trajectories reflect the evolution of their reputations in the labor market and, accordingly, will depend on the opportunities to develop and showcase their talents. These opportunities, however, are controlled by the employer, who decides on task assignments.[3] An efficient task assignment is critical for workers along two dimensions: first, *productive* efficiency requires that more-talented workers perform more-challenging tasks; second, these tasks are often more *informative* about the worker's talent than more-routine tasks. As a consequence, such tasks are also valuable to the extent that they generate information about the worker, which improves the efficiency of future task allocations. In addition to these well-studied effects, one contribution of this paper is its emphasis on how such learning fosters career-concerns incentives, which ultimately increase productive efficiency. Accordingly, part of the welfare gain arises from the enhanced worker motivation that efficient task allocation yields.

We assume that workers undertake efforts that are non-contractable and are, accordingly, governed by career-concerns incentives. As noted, we also assume that firms have

[2]As a prominent example, Bain & Company states at the beginning of its careers homepage: “We want our employees to thrive at Bain, regardless of what their future plans are. Our dedicated career teams provide guidance and support at all stages (...). Just two or three years with us will offer you incredible opportunities, both at Bain and beyond – from becoming a Bain partner to starting your own business, stepping into a senior role at a top tech company, joining a private equity firm or making a meaningful social impact at a nonprofit you love.” See `https://www.bain.com/careers/`, accessed last on May 22, 2019.

[3]As McKenna and Maister (2010) point out, “Junior staff, by definition, are at the early stages of a career, and need one thing above all: the chance to develop and build their skills. How well skills are built depends upon (...) the work assignment system that decides what projects they get to work on” (page 221).

discretion over the nature of the work. Given this lack of commitment on both sides of the employment relationship, both parties will take costly short-run actions – effort for workers or an assignment for the purpose of learning for employers – only if they expect sufficient future rents. In this context, labor market competition has a potentially ambiguous impact: although workers have stronger motivation when they are more likely to receive an outside job offer, they worry that their employer might (inefficiently) assign tasks that deprive them of the opportunity to prove their value.

It follows that total welfare can be non-monotonic in the extent of labor market competition. In particular, welfare may be larger in a monopsony than in a competitive labor market. This happens when performance is a poor indicator of talent, so that effort incentives are relatively weak. In this case, the firm assigns the worker to an informative task only if it can privately appropriate enough of the value of information, which is impossible when the labor market is perfectly competitive. Instead, when performance is a more reliable signal of talent – that is, when career-concerns incentives are stronger – welfare increases with labor market competition, as the higher worker efforts resulting from more competition outweigh the loss in the private value of experimentation.

In such an environment, we analyze how firms can set up human resources practices that respond to workers' career aspirations. Specifically, we allow firms to commit to the likelihood with which their employees receive outside offers and, hence, to choose how much they expose themselves to labor market competition. For example, consulting firms can choose the extent to which junior associates work directly with clients or, instead, force all contact through the partner; they can also engage in more or less active outplacement policies;[4] technology firms can choose whether or not to allow their workers to spend time on open-source projects (Lerner and Tirole, 2005; Blatter and Niedermayer, 2008) or, more generally, on individual research agendas (Stern, 2004); finally, firms can also directly impose non-compete clauses, or other contractual limitations that give them some monopsony power with respect to their employees. To the extent that it is easier to commit to such policies (indeed, some of these may be explicit contractual terms), they may imperfectly mitigate the lack of commitment on task allocation.

[4]As mentioned in Footnote 2, some professional service firms extensively advertise their outplacement effort – i.e., how they provide support to employees denied partnership by helping them land a high-quality job outside the firm. See, also, Section 5 for more details.

We assume that firms are *ex ante* competitive, so, in equilibrium, firms endogenously choose the efficient exposure to labor market competition. When the pool of potential workers is heterogeneous, we show that *ex ante* identical firms choose to offer different career trajectories to appeal to different types of workers when performance is a poor indicator of talent. In particular, by committing to allowing workers opportunities to leave, some firms can actually attract, hold on to, and motivate the most talented workers. Further, this equilibrium outcome features a "Matthew effect" (Merton, 1968), in that workers who enter the workforce with a better reputation obtain jobs that not only pay better, but also are more motivating and provide better opportunities for career advancement.

Finally, we relate our findings to the human resources practices set up by professional services firms. In particular, we discuss the "underdelegation problem" associated with task assignment, the importance of outplacement activities, and the evolution of careers in this industry. While the universal model of careers in professional services was historically based on the up-or-out system, there is evidence that these firms now offer differentiated career tracks. We document how this evolution can be related to the change in the nature of the tasks performed in those firms (they have become more technical) and to the increase in the demand for their services, which led them to recruit associates with lower education levels.

Our paper is related to a literature that focuses on learning in employment relationships (Holmström, 1999; Harris and Holmström, 1982) – in particular, studies in which the extent of learning can be manipulated through task assignments (MacDonald, 1982; Gibbons and Waldman, 1999a,b; Waldman, 2012; Pastorino, 2013).[5] As we do, Antonovics and Golan (2012) and Canidio and Legros (2017) stress that different jobs vary in the information they generate about ability but they focus, instead, on workers' incentives to learn through their occupational choice.[6]

In these papers, because the information that various tasks generate about workers is observed to the same extent by everyone in the labor market, rents accrue to workers. Another stream of the literature has focused, instead, on situations in which the information

[5]Waldman (2017) presents an excellent overview of this literature.

[6]In Canidio and Legros (2017), workers become entrepreneurs precisely to choose the tasks they perform and, in that way, learn efficiently.

created on the job is private to the employer, which generates inefficiencies (Greenwald, 1986). As a response to asymmetric learning, employers may have strategic incentives to distort task assignments. For instance, in Waldman (1984), employers can decide to make their employees *visible* by offering them a promotion, which then acts as a signal; and in Milgrom and Oster (1987), firms might deny promotions to some workers to keep them *invisible*, so as to lower their rents. In a similar vein, Picariello (2018a,b) shows that task allocation or promotion decisions are distorted because some tasks generate human capital that is *portable* to other firms and, thus, hampers worker retention. As in these papers, we allow firms to strategically choose the *visibility* of individual performance. However, we suppose that firms can choose the visibility of performance independently of the task chosen. Mukherjee (2008) and Bar-Isaac, Jewitt, and Leaver (2018) also endogenize the information observed by the labor market and consider a richer set of information structures. These papers feature match-specific ability, so that too little disclosure may impair efficient turnover. However, ability is not match-specific in our framework, and firms, instead, strategically design their disclosure policies to remedy their lack of commitment on task assignments.

Our focus on how firms commit to the career paths that they offer to workers also relates this paper to the literature on up-or-out contracts.[7] Kahn and Huberman (1988) consider a two-sided moral hazard problem, as we do, but they abstract from learning about workers' capabilities. Instead, Barlevy and Neal (2019) argue that up-or-out is efficient when there are valuable but limited opportunities for learning about young workers, but they do not consider workers' incentives. Similar to our paper, Waldman (1990) and Ghosh and Waldman (2010) consider both incentive provision and learning, but, in these works, outside firms observe task allocations (promotions), so that wages are determined after task assignments. Instead, in our setup, task-allocation decisions are unobservable to the market and take place once wages are set and workers hired. One consequence is that our model features inefficient task assignments for young recruits, while inefficiencies arise at the promotion stage in Waldman (1990) and Ghosh and Waldman (2010).

The two-sided moral hazard problem we study also relates our analysis to the literature on human capital, which, following Becker (1962), has extensively discussed how limits to

[7] In an up-or-out promotion system, employees who are not promoted within a given period of time have to leave the firm.

commitment make it difficult to incentivize investments in human capital, whether general or firm-specific.[8] In our model, though, the returns to a worker's investment (effort) arise through his reputation. In this context, we emphasize the role of the noise-to-signal ratio that measures the sensitivity of reputation to performance. Indeed, the results and much of the applied insight of the paper are based on the precision of learning, a channel absent from theories based on human capital investments.

Finally, our result that homogeneous firms may endogenously choose different human resources practices in equilibrium relates our paper to Bond (2017), who establishes that some firms decide to lock in their employees by choosing a technology that precludes external hiring, while others choose to keep open access to the outside labor market.

The paper is organized as follows. We describe the model in Section 2, and study the impact of labor market competition on welfare in Section 3. In Section 4, we examine firms' human resources strategies when they can endogenously choose their exposure to labor market competition. Finally, in Section 5, we discuss the connection of our results to various applications, notably human resources practices in professional firms, non-compete clauses, and antitrust matters related to labor markets.

2 Model

2.1 Setup

We consider a two-period game where, in each period, firms allocate tasks to their employees. The labor market consists of a mass 1 of workers and a mass $M >> 1$ of firms. Workers and firms are risk-neutral and have a common discount factor $\delta = 1$.[9] While firms are perfectly competitive in the first period, we assume that frictions may arise within the employment relationship that could give employers some monopsony power vis-à-vis their own employees in period 2, which we describe in Section 2.2.

Within firms, workers may be assigned one of two tasks: a routine task that generates a certain output s, and a risky task that yields a random output y_t. We assume that workers

[8] See Acemoglu and Pischke (1999) for a useful survey.

[9] Since the discount factor plays no qualitative role in the analysis, nothing would change if we assumed discounting, or if workers and firms had different discount factors.

can produce the safe output s through self-employment, which implies that firms generate no surplus from the routine task. The output y_t produced by an employee performing the risky task at period $t \in \{1, 2\}$ is given by

$$y_t = \theta + e_t + \varepsilon_t,$$

where θ is the (invariant) intrinsic productivity of the worker; e_t is the effort that the worker exerts at date t; and ε_t is a noise term following a Normal distribution with zero mean and variance σ^2, where ε_t are i.i.d. across periods and orthogonal to θ. Effort involves a cost $C(e_t) = \frac{1}{2}e_t^2$ to workers and is non-contractable. Furthermore, we assume symmetric information on θ : both workers and employers believe at the outset that the talent θ is Normally distributed with mean μ_1 and variance 1. We focus on the case in which $\mu_1 < s$ – i.e., the routine task is optimal in the static game with no effort.[10] Finally, we assume that wages in each period can be contingent neither on performance nor on task allocation, and must be paid upfront. Given these assumptions, our environment is similar to the career-concerns setup pioneered by Holmström (1999), which we augment with a strategic task-allocation problem. Note that the employer has discretion on task assignments – i.e., cannot commit to a task-allocation policy. Therefore, our model features a two-sided commitment problem, on the worker side (effort) and on the firm side (task allocation).

On the one hand, because effort is non-contractable, workers will tend to undersupply effort, as we will later see. Let us remark for now that, if they could commit, they would choose their effort levels in each period to maximize

$$E(\theta) + e - \frac{1}{2}e^2,$$

and select the efficient level of effort $e^{FB} = 1$. Note that this efficient level of effort is independent of talent, or beliefs about talent. This is an artefact of the additive separability of talent and effort in the production function, a standard assumption in the career-concerns literature following Holmström (1999).[11]

[10] We will later see that the case $\mu_1 \geq s$ is uninteresting.

[11] This assumption does not drive our results, and we stick to it for simplicity. In applications, it might be reasonable to suppose that effort and ability are complements rather than substitutes. Bar-Isaac and Ganuza (2008) present a career-concerns model that allows for flexibility in the extent to which effort and ability are substitutes or complements in production. As in the standard model, the strength of

On the other hand, the absence of commitment on task assignments generates an inefficiency related to learning. In the repeated environment that we consider, the social value of the risky task features two dynamic components: first, it generates learning about the worker's talent, which improves the efficiency of future task allocations (there is a positive value of information);[12] second, precisely because it conveys information about talent, the risky task induces career-concerns incentives: workers are willing to exert effort in order to jam the signal observed on performance, which also boosts welfare by increasing output. In this context, an efficiency concern arises because firms can appropriate only a fraction of the value of information generated by the risky task (unless they are fully monopsonistic), and, therefore, will not assign it efficiently to its employees. This inefficiency compounds the inefficiency created by non-contractable effort.

2.2 Division of surplus

The extent of these inefficiencies will depend on the firms' and workers' incentives in the first period. In the absence of commitment, these are driven by the expectation of future rents, which, in turn, result from the *ex post* (i.e., second-period) division of surplus between firms and workers. If the worker captures all this surplus, the firm does not benefit from any information generated, which may lead to inefficiently assigning the worker to the uninformative task; conversely, if the worker gets no surplus, his reputation has no value, and this shuts down his effort incentives.[13]

For simplicity, we assume that either the firm or the worker has all the bargaining power *ex post*, but that there is *ex ante* uncertainty as to which party does. We let the parameter α denote the prior probability that a worker has bargaining power. Therefore, α is a reduced-form parameter that captures the extent of labor market competition. It it is

career-concerns-based incentives is shown to increase in the extent of uncertainty about the worker's type and the related signal-to-noise ratio, which is the key driving force in our results and the aspect which we extensively relate to applications.

[12]The risky task could, for example, be understood as granting the worker greater autonomy, which allows firms to learn about his ability. Alternatively, one may interpret the risky task as an exploration of new ideas or processes, and the safe task as an exploitation of well-known actions (see, e.g., Manso (2011)).

[13]This is akin to the well-known argument that firms must earn sufficient returns to invest in general human capital training, following Becker (1962). Note that our model also features workers' investments (in their future career); in addition, there is a strong form of complementarity between firms' and workers' investments, as the employer can fully deprive the worker of his investment opportunity by assigning him the routine task.

easy to see how one could micro-found α as the likelihood that a worker currently employed at a firm is in a position to receive an outside job offer in period 2. For instance, suppose that α measures the probability that competitors are able to observe the performance of a given firm's employee. Whenever they do observe the performance, Bertrand competition on wages delivers all the expected surplus to the employee. However, if competing firms do not observe the employee's performance, they will never try to poach an employed worker due to a lemons problem. Indeed, if the current employer, who holds private information on the worker's past performance, does not match the outside offer, the offered wage must be too high.[14]

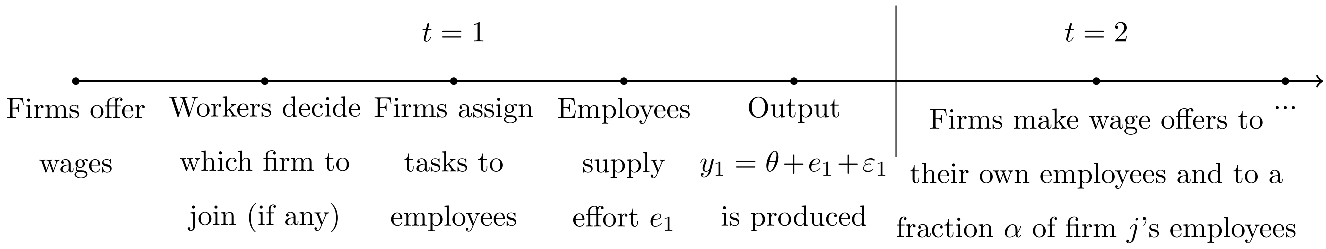
Two related remarks are in order. First, α governs the division of surplus in period 2, but not in period 1. That is, it measures the bargaining power of workers already employed. This captures the idea that, even if firms are *ex ante* competitive, monopsony power (e.g., barriers to exit) may arise within the employment relationship. Second, we view α as having both an exogenous and an endogenous dimension: workers' performance may be intrinsically more or less observable, but firms can also decide to make it more or less visible. In the same spirit, different labor markets may vary in how competitive they are, but firms can still try to reduce turnover through active retention policies.

In the next section, we first analyze the impact of α (treated as an exogenous parameter) on the equilibrium task allocation and welfare. Then, in Section 4, we explore how firms endogenously set α.

3 Exogenous competition

Let us start by solving for the equilibrium effort and task allocation for an exogenous $\alpha \in [0, 1]$. The timing of events for this case is summarized as follows:

[14]See, for instance, Greenwald (1986). Adverse selection is reinforced here by the fact that the worker's talent has the same value in all firms.



We solve the game backwards and first derive the equilibrium behavior in period 2. If the worker has executed the risky task in period 1 and has produced an output y_1, players believe at the outset of period 2 that θ is Normally distributed with mean μ_2 given by

$$\mu_2 = E(\theta|y_1) = \lambda\mu_1 + (1-\lambda)(y_1 - e_1^*), \tag{1}$$

where $\lambda \equiv \frac{\sigma^2}{1+\sigma^2}$ is the noise-to-signal ratio and e_1^* is the equilibrium (expected) effort.

When facing the risky task in period 2 (the final period), the worker never exerts effort ($e_2^* = 0$) because his wage has been paid upfront. Therefore, the surplus-maximizing policy is to allocate the risky task in period 2 if and only if $\mu_2 \geq s$; then, the expected surplus to be split in period 2 is worth $E\max(\mu_2 - s, 0)$. Note that, since we focus on the case $\mu_1 < s$, the firm cannot generate any surplus in period 2 unless it learns something in period 1, so the surplus $E\max(\mu_2 - s, 0)$ also corresponds to the value of information that the risky task generates.

3.1 Career-concerns incentives

We now turn to the effort choice in period 1. The worker obtains a share α of the future surplus generated by the employment relationship in addition to the outside option, s. His continuation expected payoff is, thus, given by

$$s + \alpha E\max(\mu_2 - s, 0).$$

We derive the following lemma:

Lemma 1. *The equilibrium level of effort is given by*

$$e_1^*(\alpha) = \alpha(1-\lambda)\left(1 - \Phi\left(\frac{s-\mu_1}{\sqrt{1-\lambda}}\right)\right), \tag{2}$$

where Φ denotes the cdf of a Normal distribution with mean 0 and variance 1.

Proof. In the Appendix. □

As in Holmström (1999), effort incentives stem from the desire to influence employer beliefs about θ. Since higher talent translates into higher output, employers believe that employees who perform better are more talented, which, in turn, provides incentives to employees to exert effort in order to increase the likelihood of higher outputs. In equilibrium, it is impossible to fool the market, but positive effort is still supplied.[15] A key determinant of effort is the sensitivity of future beliefs to current output $(1-\lambda)$. When the noise-to-signal ratio λ is high, the output y_t primarily reflects noise, so that a worker's efforts have only a limited impact on the posterior beliefs (see (1)), and incentives to exert effort are dull.

In addition, effort incentives depend on the marginal value of a better reputation $\alpha\left(1-\Phi\left(\frac{s-\mu_1}{\sqrt{1-\lambda}}\right)\right)$. Indeed, a better reputation at date 2 is valuable only when the worker has bargaining power and when his reputation μ_2 is sufficiently large to justify adoption of the risky task. The probability of being assigned the risky task at date 2 $\Pr(\mu_2 \geq s)$ can be written as $1-\Phi\left(\frac{s-\mu_1}{\sqrt{1-\lambda}}\right)$.[16] One can check that it is both decreasing and convex in λ. Indeed, because $\mu_1 < s$, the worker will be assigned the routine task absent further information. To be assigned the risky task, the posterior belief μ_2 has to increase sufficiently compared to the prior μ_1, which is more likely to happen as production gets more informative about talent, i.e., λ decreases. Since the worker cares only about the upside risk of his future reputation, he is effectively risk-loving; that is, the marginal value he derives from an increase in the variance of μ_2 is larger when performance is more informative about talent. Overall, this discussion implies that the equilibrium effort e_1^* is linearly increasing in α and that it is decreasing and convex in λ.

Finally, note that the equilibrium level of effort e_1^* is lower than the efficient one:

$$e_1^*(\alpha) = \alpha(1-\lambda)\left(1-\Phi\left(\frac{s-\mu_1}{\sqrt{1-\lambda}}\right)\right) < 1 = e^{FB}.$$

That is, reputation provides incentives to supply positive effort, but the equilibrium

[15] That it is impossible to influence employers' beliefs in equilibrium implies that the value of information $E\max(\mu_2 - s, 0)$ is independent of e_1^*.

[16] See the proof of Lemma 1 in the Appendix for details.

level of effort is always below the first best. Therefore, effort becomes more efficient as α increases. More generally, this implies that a worker who is assigned the risky task benefits from more labor market competition. Let

$$u_r(\alpha) \equiv s + \alpha E \max(\mu_2 - s, 0) - \frac{1}{2} e_1^*(\alpha)^2 \quad (3)$$

denote the expected intertemporal utility (gross from his period 1 wage) that a worker derives from being assigned the risky task (the dependence on μ_1 is kept implicit).

Lemma 2. *$u_r(\alpha)$ is increasing in α.*

Proof. In the Appendix. □

3.2 Task allocation in period 1

A firm gets a share $1 - \alpha$ of the future expected surplus, so its expected profit (gross of the period 1 wage) from choosing the risky task in period 1 is equal to

$$\pi_r(\alpha) \equiv \mu_1 + e_1^*(\alpha) + (1 - \alpha) E \max(\mu_2 - s, 0), \quad (4)$$

whereas the profit from choosing the safe task is

$$\pi_s(\alpha) \equiv s + (1 - \alpha) \max(\mu_1 - s, 0). \quad (5)$$

Since the period 1 wage is sunk at the time when the firm allocates a task to the worker, the firm chooses the risky task if and only if

$$\pi_r(\alpha) \geq \pi_s(\alpha). \quad (6)$$

Note that, when $\mu_1 \geq s$, we have $\pi_r(\alpha) \geq \pi_s(\alpha)$ for all α. This can be seen by noting that $e_1^*(\alpha) \geq 0$ and using Jensen's inequality. This implies that the firm always (efficiently) chooses the risky task in period 1, so that its lack of commitment has no bite.

Consequently, as noted above, we focus on the more interesting case $\mu_1 < s$, which implies that $\pi_s(\alpha) = s$, and we examine the conditions under which a firm is willing to experiment – i.e., to choose the risky task even though the routine task yields a higher static profit. We can now derive the equilibrium task allocation.

Proposition 1. *There exists a cutoff* $\overline{\mu} < s$ *such that a firm assigns the risky task if and only if the worker's initial reputation is large enough,* $\mu_1 > \overline{\mu}$*. The cutoff* $\overline{\mu}$ *is increasing in the noise-to-signal ratio* λ*. Finally, there exists a cutoff value* $\lambda^* \in (0, 1)$ *such that: the threshold value for assigning the risky task* $\overline{\mu}$ *is increasing in the extent of competition,* α*, if* $\lambda > \lambda^*$*;* $\overline{\mu}$ *is decreasing in* α *if* $\lambda < \lambda^*$*; and,* $\overline{\mu}$ *is independent of* α *if* $\lambda = \lambda^*$*.*

Proof. In the Appendix. □

A firm is less willing to experiment by choosing the risky task when performance is noisier – i.e., when λ increases. In this case, performance is less informative about talent, which *both* reduces incentives to exert effort and lowers the value of information. Thus, a higher λ unambiguously reduces the firm's benefit from inducing learning.

The impact of labor market competition, α, is more nuanced. On the one hand, a higher α boosts effort incentives but, on the other hand, reduces how much the firm privately benefits from the information that the risky task generates. Since both the effort level $e_1^*(\alpha)$ and the private value of information $(1-\alpha)E\max(\mu_2 - s, 0)$ are linear in α, the relative magnitude of these two countervailing effects is independent of α.[17] The former effect dominates (more labor market competition generates more experimentation) when λ is small so that career concerns are strong, and the latter effect dominates when λ is large and career concerns are mild.

The intuition relates to the observation that the value of information exhibits diminishing returns. That is, $E\max(\mu_2 - s, 0)$ decreases in λ, but at an increasing rate. When the noise-to-signal ratio λ is small, the value of information is large, and a marginal increase in λ has a small (negative) impact on the value of information generated by the risky task. Instead, when λ is large, the value of information is small, but further improvements in the quality of information are highly valuable at the margin.

However, as we saw above, more-precise information boosts the level of effort e_1^* at an increasing rate. Consequently, when λ is small, and information precise, more competition

[17]This is an artefact of having a quadratic cost. With a more convex cost function, one easily checks that increases in α would have a marginal impact on effort that decreases in α, which would further stack the deck towards a negative impact of more competition on the efficiency of task assignments.

decreases the private value of information by little compared to the extra effort incentives induced for workers. Thus, overall, more competition provides firms with stronger incentives to assign the risky task. However, when λ is large, the impact of more competition on effort is comparatively too weak to induce firms to learn through the risky task.

3.3 Welfare

The total welfare $W(\alpha)$ depends on the initial task assignment to the safe (s) or risky task (r) and can be written as follows:

$$W(\alpha) = \begin{cases} W_s = 2s & \text{if } \mu_1 < \overline{\mu}(\alpha) \\ W_r(\alpha) = \mu_1 + e_1^*(\alpha) + E\max(\mu_2, s) - \frac{1}{2}e_1^*(\alpha)^2 & \text{if } \mu_1 \geq \overline{\mu}(\alpha) \end{cases}$$

Welfare is independent of labor market competition, α, if the worker is assigned to the safe task. Besides, as we have seen, a higher α, by boosting effort, improves welfare as long as the risky task is chosen, but since it might lead firms to under-assign the risky task, its overall impact is ambiguous, as the next Proposition shows.

Proposition 2. *Labor market competition may have an ambiguous impact on welfare:*

1. *if workers' expected ability is sufficiently low,* $\mu_1 < \min(\overline{\mu}(0), \overline{\mu}(1))$, *then the safe task is chosen for any level of competition, and total welfare does not depend on* α;

2. *if workers' expected ability is sufficiently high,* $\mu_1 > \max(\overline{\mu}(0), \overline{\mu}(1))$, *then the risky task is chosen for any* α, *and total welfare increases in* α*;*

3. *otherwise, there exists a threshold* $\overline{\alpha} \in (0, 1)$ *such that* $\overline{\mu}(\overline{\alpha}) = \mu_1$; *there are then two subcases:*

   (a) *if* $\lambda < \lambda^*$, $\overline{\mu}(\alpha)$ *is decreasing: the risky task is chosen if and only if* $\alpha > \overline{\alpha}$. *Then, welfare weakly increases with* α*;*

   (b) *if* $\lambda > \lambda^*$, $\overline{\mu}(\alpha)$ *is increasing: the risky task is chosen if and only if* $\alpha < \overline{\alpha}$. *Then, welfare is single-peaked and reaches its maximum at* $\alpha = \overline{\alpha}$.

The first two parts of Proposition 2 are entirely straightforward: if workers' expected ability is so low that the firm always assigns a worker to the safe task, then welfare does not depend on the extent of labor market competition; instead, if the worker is always assigned to the risky task, more labor market competition increases workers' efforts and, hence, welfare. In the third case, firms assign the risky task only for some values of α. From Proposition 1, whether the risky task is assigned for high enough or low enough values of α depends on the value of λ. This is illustrated in Figure 1, where we depict welfare and how it is split between employees and firms (abstracting from the wage paid to workers in period 1).[18]

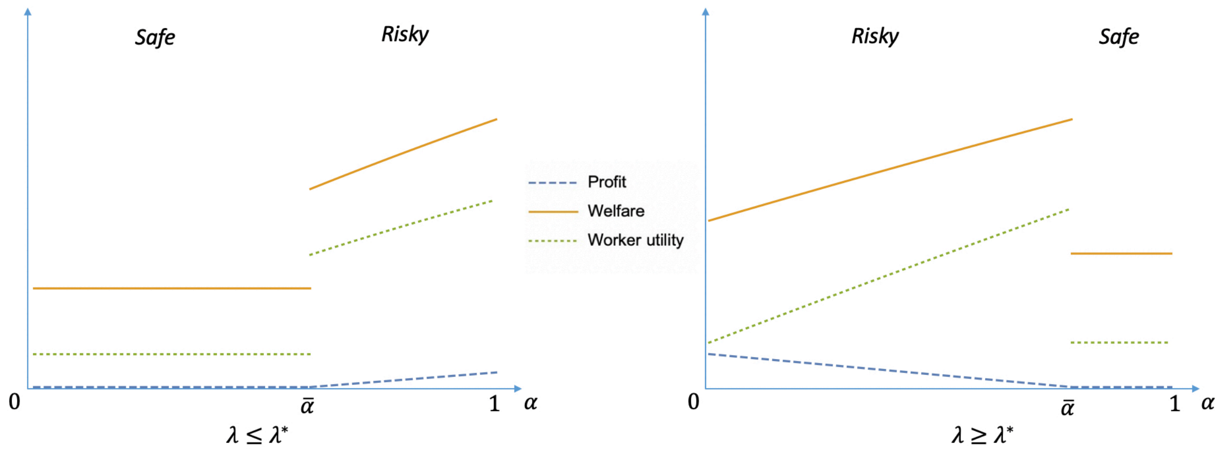


Figure 1: Expected welfare, profit and worker utility as a function of α.
Parameter specification: $s = 0.1$ *(both panels);*
$\mu_1 = -0.15, \lambda = 0.3$ *(left panel);* $\mu_1 = -0.03, \lambda = 0.7$ *(right panel)*

When learning is precise, or, equivalently, the noise-to-signal ratio λ is low, greater competition boosts effort incentives sufficiently to make firms more willing to assign the risky task, and, hence, it unambiguously increases welfare (left panel). Instead, when λ is large (right panel), more competition makes it less likely that the firm will assign the risky task in spite of heightened effort incentives. Since effort is relevant only to the extent that the risky task is assigned, more competition might reduce welfare. Indeed, welfare is then single-peaked in α, with a maximum attained at the value at which a firm is exactly indifferent between both tasks – that is, $\overline{\alpha}$. Because firms internalize only a fraction of the social value of the risky task, the risky task is socially optimal at $\alpha = \overline{\alpha}$. This implies that an increase in α at $\alpha = \overline{\alpha}$, by triggering a switch in the task allocation, generates a downward jump in welfare. We can derive the following corollary.

[18]The profit curve is simply the difference between total welfare and workers' intertemporal utility.

Corollary 1. *If $\lambda > \lambda^*$ and $\overline{\mu}(0) < \mu_1 < \overline{\mu}(1)$, then $W(0) > W(1)$: Total surplus is larger with a monopsonistic firm than with competitive firms.*

Proof. In the Appendix. □

When performance is a poor indicator of talent, it is better to give all bargaining power to firms than to workers. This shuts down effort incentives but restores firms' incentives to assign the task that generates more intertemporal surplus. An important normative implication of Proposition 2 and Corollary 1 is that public policies designed to foster competition, promote transparency, or reduce frictions in the labor market may actually backfire in environments in which performance is more volatile or more difficult to observe. In particular, this implies that, in these situations, no-poaching agreements may actually be warranted from a welfare perspective.[19]

3.4 Commitment benchmarks

As mentioned above, given the two-sided commitment problem, there are two sources of inefficiency: first, for an employer's given task-allocation policy, the employee's effort is inefficiently low; second, for a given effort, the employer assigns the routine task too often, as the firm can appropriate only a fraction of the value of information generated by the risky task. In the benchmark case in which there is no commitment problem whatsoever – i.e., workers can commit to effort, and firms can commit to task allocations – it is easy to see that α affects only the way that firms and workers share surplus and has no efficiency impact.[20] In the benchmark case in which workers can commit to effort, but firms cannot commit to tasks, an increase in α has no impact on effort but at some point may generate a switch in task assignment, so that welfare is non-increasing in α.[21]

[19] In the US in 2016, the FDC and the DoJ jointly published "Antitrust guidance for human resources professionals," which explicitly stresses that the labor markets will be treated as any other market regarding antitrust matters. The EU has no specific labor market regulation, but anti-competitive practices in the labor market have been used as motivating elements of prosecution in recent cases (see, e.g., the PVC floorings cartel case in France: `http://www.autoritedelaconcurrence.fr/user/standard.php?id_rub=663&id_article=3044&lang=en`). See, also, Sections 5.2 and 5.3 for a brief discussion of antitrust issues in labor markets.

[20] Formally, the risky task would be assigned as long as $\mu_1 + e^{FB} - \frac{1}{2}(e^{FB})^2 + E\max\left(\mu_2 + e^{FB} - s, 0\right) \geq s + \max\left(\mu_1 + e^{FB} - s, 0\right)$, a condition that does not depend on α.

[21] Formally, the risky task is now assigned whenever $\mu_1 + e^{FB} + (1-\alpha)E\max(\mu_2 + e^{FB} - s, 0) \geq s + (1-\alpha)\max(\mu_1 + e^{FB} - s, 0)$, a condition that is more likely to hold as α decreases.

Since we primarily want to stress the impact of the lack of commitment on task allocation, the most relevant benchmark against our result is the one in which workers cannot commit to effort, but firms can commit to task allocations. In this case, firms would commit to assigning the risky task as long as this is efficient – i.e., whenever

$$\mu_1 + e_1^*(\alpha) - \frac{1}{2}e_1^*(\alpha)^2 + E \max(\mu_2, s) \geq 2s. \tag{7}$$

As one can see, α then affects welfare only through its positive impact on effort. Since the left-hand side of (7) is increasing in μ_1 and α, this condition can be rewritten

$$\mu_1 \geq \mu^*(\alpha),$$

where $\mu^*(\alpha)$ is decreasing in α. This also implies that a higher α, by increasing effort, makes learning unambiguously more valuable and, hence, increases welfare. Figure 2 illustrates the role of the ability to commit on task allocation. It depicts the regions in which the risky task is assigned with and without commitment on task allocation. It is

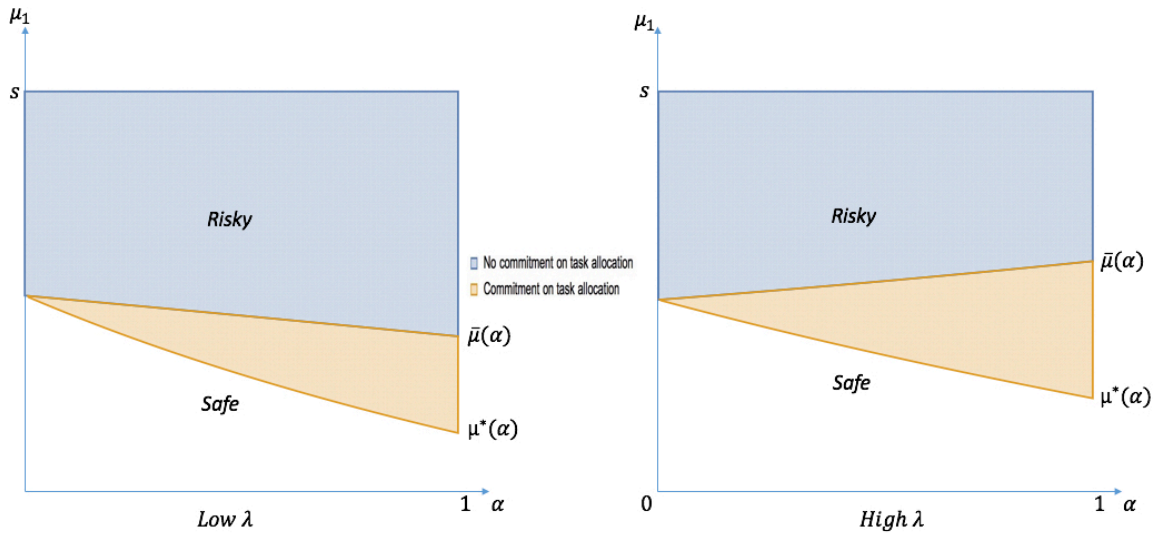


Figure 2: Task assignments with and without commitment on task allocation
Parameter specification: $\lambda = 0.2$ *(left panel);* $\lambda = 0.7$ *(right panel)*

easy to check that $\mu^*(\alpha) < \overline{\mu}(\alpha)$ for all α: since firms capture only a fraction of the value of information, there is a set of reputations μ_1 for which the risky task is efficient but, nevertheless, firms pick the routine task. This corresponds to the intermediate region where $\mu_1 \in (\mu^*(\alpha), \overline{\mu}(\alpha))$. This region – and, hence, the inefficiency – expands with

α.[22] This suggests that being able to commit to task allocations is more valuable when there is more labor market competition. In a competitive environment, firms should, thus, attempt to design strategies to alleviate this commitment problem. One such commitment strategy is to narrow down the portfolio of tasks to reduce the uncertainty that employees face regarding staffing decisions. This can be achieved through a strategy of expertise (specialization on some risky tasks), or through outsourcing routine tasks. For instance, McKinsey typically outsources all of its presentations for business pitching to India.[23] Our model suggests that these policies may be an optimal way for firms to respond to intense labor market competition.

4 Endogenous competition

In this section, we examine how firms can strategically choose the extent to which they want to expose themselves to or insulate themselves from labor market competition. In line with the idea that firms can endogenously create monopsony power within the employment relationship, we assume that firms are competitive in period 1, but can, through their choice of α, design how much to lock in their current employees (for instance, by managing exit opportunities or outplacement, by imposing non-compete clauses, by making employee performance more or less visible to outsiders, and so on).

We assume that firms can separately decide on the extent of monopsony power α (to which they can commit) and task assignments (to which they cannot). This seems an entirely appropriate assumption in thinking of α as explicit contractual terms such as non-compete agreements. More broadly, however, the distinction between the choice of α and the task-allocation decision may be more blurred, especially if one thinks of α as the visibility of performance. Indeed, some tasks may be intrinsically more visible to potential rival employers because, for example, they are performed on a more "visible" site. However, as mentioned in the introduction, the case in which visibility and tasks are bundled has been studied in the literature (see, e.g., Waldman (1984, 1990); Milgrom

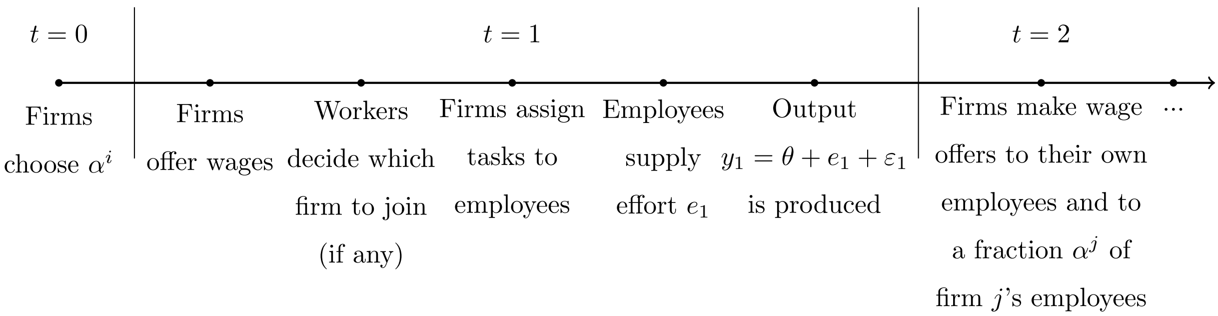
[22]In particular, when $\lambda \geq \lambda^*$, an increase in α reduces the extent of experimentation ($\overline{\mu}(\alpha)$ increases), while more experimentation would be desirable ($\mu^*(\alpha)$ decreases). Notice that when $\alpha = 0$, the firm can appropriate the full social value of information, so commitment is irrelevant.

[23]See https://timesofindia.indiatimes.com/business/india-business/Now-its-offshoring-of-presentations/articleshow/856257.cms

and Oster (1987)), and it is relevant to address the cases in which they are or can be made distinct. In addition, the fact that the extent of competition might vary with tasks is already an inherent feature of our model, since we assume that workers always have bargaining power on the safe task.

More generally, the key distinction between the choice of α and the task-allocation decision that we want to stress is commitment. We envision α as a broad policy taken at the firm level (e.g., outplacement effort, promotion criteria, organization of contacts with clients, and so on) that would be costly to overturn – for example, for reputational motives – while task assignments are individual and harder to monitor.[24]

We assume that firms simultaneously and non-cooperatively choose their policies α^i at no cost.[25] The timeline of the game with endogenous α is as follows.



Notice that, for the sake of concreteness, we assume that firms choose their α in a first step and then offer period 1 wages, but we would derive identical results if these decisions were taken simultaneously.[26] Finally, we consider the case in which there exists some value of $\alpha \in [0, 1]$ such that $\mu_1 \geq \overline{\mu}(\alpha)$.[27] This implies that α^*, the degree of competition that maximizes welfare, is uniquely defined on $[0, 1]$. We derive the following Proposition:

[24]To underline the importance of reputational concerns in the case of outplacement policies, Gilson and Mnookin (1989) mention the case of Cravath, a leading US law firm that "because of the need to maintain a continual flow of referral work to power the outplacement process, consciously must decide not to expand to the point where it can undertake all the work that comes to it" (page 583).

[25]Notice that the cooperative and non-cooperative solutions coincide here, as there are no externalities between firms. If a firm decides to let other firms make job offers to its own employees, it does not generate any value to competitors, since all the rents from competition then accrue to workers. In this way, the level of competition chosen could also be interpreted as the cooperative outcome determined through, for example, a self-regulatory body. Besides, introducing constraints on the set of feasible α or any kind of cost would leave our results qualitatively unchanged.

[26]Which timing assumption is appropriate depends on the interpretation one has in mind: if one views α as an organizational design decision, it makes sense to think of firms first designing the environment of the organization, and then setting wages; another version of the timing, where α and wages are offered simultaneously, better fits the interpretation of α as contractual terms such as non-compete clauses.

[27]Otherwise, no worker can ever be assigned to a risky task, so firms' choices are inconsequential.

Proposition 3. *The equilibrium is constrained-efficient: in equilibrium, some firms choose* α^* *and only firms that choose* α^* *are active – i.e., attract workers.*

Proof. In the Appendix. □

Constrained efficiency follows from the fact that firms are (perfectly) competing on the choice of α and, hence, have to choose the level of α that maximizes welfare if they want to appeal to workers. To be attractive to workers, a firm must be able to credibly assign the risky task, which requires that $\mu_1 \geq \overline{\mu}(\alpha)$. Once this is secured, worker utility is maximized when α is as high as possible (because u_r increases in α; see Lemma 2). Therefore, competition pushes firms to set the highest α consistent with $\mu_1 \geq \overline{\mu}(\alpha)$, which is precisely the one that maximizes total welfare – i.e., α^*. Thus, a consequence of Proposition 2 is that the career pattern chosen by firms reflects the extent of learning opportunities in the market (the value of information): when performance is a good indicator of talent (λ is small), firms optimally choose to leave all the surplus to workers to maximize effort incentives ($\alpha^* = 1$); when performance is a poor indicator of talent (λ is large), firms must sometimes retain a share of future surplus to make sure that they have a proper incentive to assign the risky task ($\alpha^* < 1$). Furthermore, since the minimal reputation necessary to be assigned the risky task $\overline{\mu}$ is decreasing in the noise-to-signal ratio λ, and in α when λ is large, the share that firms must retain, $1 - \alpha^*$, increases in λ. This suggests that limits on competition (e.g., non-compete clauses) should be more widespread when it is more difficult to learn about talent.

4.1 Heterogeneous workers

The result that active firms all choose the same level of competition α^* reflects the fact that the welfare-maximizing degree of competition is the same for all workers, given that all workers are *ex ante* identical. In what follows, we relax this last assumption and assume that there are two types of workers who differ in their initial reputation μ_1. A fraction β of workers have initial reputation μ_1^H, while the remaining $1 - \beta$ start with reputation $\mu_1^L < \mu_1^H$. We make the following two assumptions:

Assumption 1: The initial reputation $\mu_1 \in \left\{\mu_1^L, \mu_1^H\right\}$ is observable, and firms can make different wage offers to workers with different μ_1.[28]

[28]That wage offers can be contingent on the reputation of an individual worker was already implicit in

Assumption 2: α is determined at the firm level – i.e., firms cannot offer a different α to workers with different μ_1.

One interpretation is that μ_1 captures a worker's level of education, which determines his first wage offer.[29] In turn, α captures the career prospects that workers expect from joining the firm. These expectations reflect workers' perceptions regarding the firm's human resources practices – whether these perceptions are shaped by the observation of contracts already in place (e.g., wage-tenure profiles of existing employees, non-compete clauses, etc.), or by the firm's reputation or culture. Under Assumption 2, the choice of α is driven not only by the desire to attract workers, but also by the pool of workers that a firm can attract.

As before, we restrict attention to the interesting case in which there exists a value of α such that $\mu_1^L \geq \overline{\mu}(\alpha)$. Otherwise, an L-worker can never be assigned a risky task and, thus, is irrelevant to firms, so, in effect, our analysis of homogeneous workers applies. Let α_L^* and α_H^* denote the welfare-maximizing levels of competition when $\mu_1 = \mu_1^L$ and when $\mu_1 = \mu_1^H$. We derive the following result:

Proposition 4. *There are two cases:*

- *Case 1:* $\lambda > \lambda^*$ *and* $\overline{\mu}(0) < \mu_1^L < \overline{\mu}(1)$.

  In this case, $\alpha_L^* < \alpha_H^* \leq 1$. *In equilibrium, some firms choose* α_L^*, *and attract only L-workers; some firms choose* α_H^*, *and attract only H-workers. Firms that offer* $\alpha \notin \{\alpha_L^*, \alpha_H^*\}$ *are inactive.*

- *Case 2:* $\lambda < \lambda^*$ *or* $\lambda > \lambda^*$ *and* $\mu_1^L > \overline{\mu}(1)$.

  In this case, $\alpha_L^* = \alpha_H^* = 1$. *In equilibrium, some firms choose* $\alpha = 1$ *and only firms that choose* $\alpha = 1$ *are active.*

In each of the two cases, the equilibrium is constrained-efficient: that is, each worker joins a firm that, in equilibrium, offers the level of α that maximizes welfare given his education level. However, Proposition 4 yields the additional prediction that firms offer

the above analysis, as, in period 2, workers receive offers that depend on their individual performance in period 1.

[29] In a related analysis, DeVaro and Waldman (2012) allow for *ex ante* observably heterogeneous workers. They argue, and show empirically, that promotion decisions (to which firms cannot commit) depend on a worker's type.

differentiated career paths when performance is a poor indicator of talent. The intuition is as follows: a firm always creates more surplus for workers when α increases, *as long as the worker is assigned the risky task*. In the case in which the noise-to-signal ratio is low ($\lambda \leq \lambda^*$), a higher α increases the range of workers that can be assigned the risky task, so a higher α unambiguously raises a firm's attractiveness to all types of workers. The choice of α here is akin to a choice of "vertical quality": all firms should make the same choice to maximize quality, which results in a Bertrand-like equilibrium.

However, when $\lambda > \lambda^*$, increasing α decreases the set of workers to whom the risky task is assigned, so increasing α has an ambiguous effect: a higher α creates more surplus for workers that are good enough to be assigned the risky task; however, it may result in the exclusion of low-quality workers, as the firm cannot commit not to assign them the routine task if they accept a job offer. In particular, when the choice of α determines whether or not low-quality workers can be attracted – that is, when $\overline{\mu}(0) < \mu_1^L < \overline{\mu}(1)$ – the choice of α is akin to a location choice, and the equilibrium is such that firms horizontally differentiate by focusing on different segments of the labor market.[30]

Notice, also, that our differentiation result rests on the assumption that α is chosen at the firm level and cannot be made contingent on the worker's education level (Assumption 2). To the extent that firms are able to offer different α to different workers (for instance, through contractual clauses or differentiated career tracks), Proposition 3 would immediately extend: each firm would offer to each worker the level of α that maximizes welfare given his initial reputation. Accordingly, different career paths and wage profiles would arise within the same firm for workers with different education levels. In any case, the main prediction of Proposition 4 is that heterogeneity among workers endogenously generates the coexistence of different career paths (within or across firms) when performance is a poor indicator of talent.[31] The role of Assumption 2 is, therefore, only to underline that, whenever there are constraints that make it costly for firms to offer different career prospects to different kinds of workers, differentiation arises across but not within firms. For instance, if one interprets α as corporate culture, we should expect different firms to

[30]Notice that our result that workers self-select into firms can hold even if we slightly relaxed Assumption 1 and assumed that employers learn μ_1 only after hiring the worker but before assigning tasks. Indeed, firms' beliefs about a worker's expected ability matter only at the task-assignment stage.

[31]In Section 5, we relate the emergence of dual career tracks in professional service firms to the fact that increased demand forced them to recruit associates from a broader and lower-quality pool of applicants.

develop and sustain different reputations regarding the careers they offer.

Another empirical prediction arising from Proposition 4 has to do with differences in career profiles across industries or occupations: in industries or occupations in which performance is highly informative about talent ($\lambda < \lambda^*$), we expect firms to offer the same career opportunities to all employees. In contrast, in industries or occupations in which performance is a relatively poor indicator of talent ($\lambda > \lambda^*$), Proposition 4 suggests that different types of workers are offered different career paths. When there is differentiation across firms, another prediction is that firms also differ in terms of output per worker. The difference in average production across firms is driven by both a sorting effect and an incentive effect.[32] Indeed, the differential treatment of workers has an impact on their outputs that goes beyond their intrinsic productivities. Specifically, low-ability workers are punished three times: first, because they are endowed with lower expected ability; second, because this lower ability generates lower incentives to supply effort holding α fixed – and, hence, a lower surplus;[33] and third, because low-ability workers face reduced opportunities to prove themselves ($\alpha_L < \alpha_H$), as firms' lack of commitment in task allocation limits the possibilities for steep career paths, which, again, lowers effort and, thus, surplus. This "triple jeopardy" is reminiscent of the Matthew effect (Merton, 1968). Note that this result that less-skilled employees exert less effort holds even though talent and effort are independent in the production technology (the efficient level of effort is independent of talent). This highlights that the positive relationship between talent and effort that emerges in our model arises from the various commitment problems, and is further amplified by the endogenous choice of α. This idea is reminiscent of Zabojnik and Bernhardt (2001), who argue that inter-industry wage differentials are not driven by employees' different intrinsic characteristics, but by the fact that incentives to invest in human capital vary according to the characteristics of the employer.

[32] In relation to this point, Lazear (2000) notes in a paper about explicit incentives that wage contracts endogenously affect the pool of workers, and he documents this sorting effect empirically.

[33] In our model, returns to effort are increasing with reputation (hence, $\frac{\partial e_1^*}{\partial \mu_1} > 0$) because the talent-intensive task is assigned to workers with higher reputations only. This property is general in such a two-period model with task assignments, as Martinez (2009) shows.

5 Discussion

5.1 Professional service firms

As we suggest in the introduction, a prominent application of our model is to professional service firms. This is an important application in its own right;[34] moreover, our analysis applies more widely to firms in which human capital is the main asset, and talent and career management are, thus, key.[35] In one of the leading practitioner books on professional services, Maister (2012) suggests that "the ability to attract, develop, retain and deploy staff will be the single biggest determinant of a professional service firm's success" (p.189). Moreover, in their recruitment activities, such firms stress that jobs are stepping stones to broader careers. That is, such jobs provide opportunities for workers to prove their capabilities. Although these development opportunities arise, in turn, through staffing decisions, Maister (2012) points to massive inefficiencies in staffing. In particular, he estimates that partners spend up to 50% of their time on tasks that could be performed by juniors, and views this "underdelegation problem" as the key management challenge for professional service firms. This suggests that discretion in task allocation creates a major efficiency concern and, thus, that these firms should thus strive to design career management strategies to cope with this commitment problem.

5.1.1 Labor market strategies

In line with our analysis, these firms seem to promote an intermediate level of labor market competition. On the one hand, most professions are self-regulated through professional organizations, which have a tendency to tame competition (often in the name of preserving the trustworthiness of the profession). This can take the form of barriers to entry (formal or not) or implicit agreements not to solicit clients, poach from competitors, or compete on

[34]There is considerable discussion on what precisely constitutes a professional service firm and on the size of this sector. By all means, it is significant: Mawdsley and Somaya (2015) report that, in the US, the professional and business services (PBS) sector is significantly larger than the manufacturing sector in terms of share of GDP. Moreover, GDP value added by the PBS sector grew by 260% between 1991 and 2011, compared to only 77% for manufacturing. There are also dramatic differences in terms of employment growth.

[35]This is certainly the case for technology companies, for instance. Indeed, the Google IPO prospectus states (p. 13): "Our performance is largely dependent on the talents and efforts of highly skilled individuals. Our future success depends on our continuing ability to identify, hire, develop, motivate and retain highly skilled personnel for all areas of our organization."

prices, restrictions on qualifications, etc. (Von Nordenflycht, 2010).[36] Corollary 1 suggests that such entry barriers or limits on competition set by professional organizations may actually be warranted from a welfare perspective. On the other hand, professional service firms seem to promote some competition through outplacement activities, for example. As Maister (2012) points out, management of exit opportunities is a key strategic lever to attract workers and motivate them to exert effort: "A firm that is truly committed to (and actively works at) placing its 'alumni' (not passed-over partnership candidates) in good positions can respond to the career progress needs of all of its juniors, and thereby create a highly motivational atmosphere" (p.173).

In addition, in line with Proposition 4, there is evidence of horizontal differentiation between firms. First, as Hansen, Nohria, and Tierney (1999) document, some firms (e.g., Ernst&Young, Andersen Consulting) have specialized in addressing routine problems by reusing codified knowledge in a more standardized way, while others (e.g., McKinsey, Bain) have specialized in "out-of-the-box" problem solving. While such strategies look very much like product market differentiation, it seems clear that specializing in high-level strategic problems can, in effect, provide some commitment power to staff consultants on challenging tasks that enable them to showcase their skills, thus allowing firms to attract the most talented employees. Focusing strictly on human resources practices, there is also evidence that firms with different cultures coexist on the market, as Broderick (2010) emphasizes: "The career development process varies significantly across firms and (...) is driven by values and culture. At one end of the spectrum are the "sink or swim" firms (...) that see little or no need to spend dollars on development. (...) At the other end of the spectrum are the firms that take the nurturing and development of their people very seriously. These are the firms that consistently recruit the top students in the class or the best industry and service experts in their fields. These are also the businesses with the highest retention rate of top performers and the highest scores on all the "Best Places to Work" charts" (p.47). Thus, firms seem to strategically sustain different cultures in terms of career paths. Furthermore, firms that offer better opportunities to move on are able to consistently attract and motivate the best talent, in line with our results.

[36]For example, in the UK, qualifying as a solicitor requires a "period of recognised training" of at least two years through a training contract. See https://www.lawsociety.org.uk/law-careers/becoming-a-solicitor/qualifying-with-a-degree/period-of-recognised-training/ retrieved on March 10, 2020."

5.1.2 The evolution of up-or-out

One of the most noteworthy evolutions of human resources practices in professional services in past decades is the progressive movement away from the up-or-out organizational model. While up-or-out used to be almost universal in professional services, a variety of permanent associate (non-partner) positions have been created since the 1980s, which constitutes a violation of the basic principle of up-or-out (Gilson and Mnookin, 1989; Malos and Campion, 1995; Gorman, 1999; Galanter and Henderson, 2008; Malhotra, Morris, and Smets, 2010). Some firms have even abandoned up-or-out contracts altogether. Morris and Pinnington (1998), for instance, show that as far back as the 1990s, two thirds of UK law firms were no longer using up-or-out contracts. Gorman (1999) and Gilson and Mnookin (1989) analyze the evolution of up-or-out in law firms and propose two main explanations for the erosion of the model. First, Gorman (1999) documents a substantial increase in the complexity and knowledge intensity of large law firms' work. She relates this evolution to the fact that corporate clients direct most of their routine work to their in-house legal departments and resort to outside counsel only for unusually complex problems such as litigation, hostile takeovers, high-stakes transactions, etc. Second, Gilson and Mnookin (1989) argue that the heightened importance of matters involving law and regulation in corporations generated a dramatic increase in the demand for legal services; in this context, they suggest that the reduced ability of firms to outplace the large cohort of newly recruited associates explains the lower recourse to up-or-out.[37]

Relatedly, Gilson and Mnookin (1989) also point that "the business (...) appears to be growing so fast that the traditional sources of new associates – elite national law schools –no longer provide an adequate supply. (...) Major firms may have begun to hire from a different and perhaps less capable pool associates than in the past" (page 589). That such a change in the composition of employment has triggered the creation of non-partner positions is fully in line with Proposition 4, which states that the presence of a heterogenous pool of employees results in a dual career system with employees of higher expected quality being offered steeper career paths. There is indeed evidence

[37]This analysis is corroborated by a partner at a UK law firm, who reports: "We started to loosen the system and exceptions to the rule occurred more and more (...) we could no longer place people...if you can't do that you can't really have up-or-out...you can't just drop good loyal employees on the street because they are aren't partner material" (quoted by Morris and Pinnington (1998), page 16).

that employees with less education are treated differentially, in the sense that they face worse partnership prospects and have lower promotion ratios.[38] Interestingly, as discussed above, this evolution has taken place in a context in which the work has become more technical, and, so, plausibly more difficult to assess. This is precisely the instance in which our model predicts differentiation (high λ), and, in fact, there is evidence that firms with more complex and challenging work do make greater use of permanent employees (Gorman, 1999). Finally, it is also interesting to note that, in this dual system, there is an association between the career paths that different employees follow and the tasks that they perform. As Gorman (1999) points out, "[S]enior attorneys appear to practice in complex, challenging areas of law, but this is not the case for permanent associates, whose work appears to be more routine" (page 662).

Notice that, although in our model, firms do not explicitly choose whether they offer up-or-out contracts, it is possible to interpret α as capturing the extent of up-or-out. First, if one interprets α as outplacement efforts (or the reputation the firm has for making such efforts), then α indirectly captures up-or-out: as we have seen above, the growing difficulty for firms to outplace their employees mechanically makes up-or-out harder to sustain.[39] Second, while employees denied partnership are asked to leave the firm only in exceptional instances, they typically leave by themselves, thereby enacting up-or-out (Malhotra, Morris, and Smets, 2010). In such a context, a higher visibility of performance (e.g., more frequent client contact) or stronger outplacement efforts increase the probability of finding a good position outside the firm, so a higher α *de facto* generates a stricter implementation of up-or-out in practice. In line with this idea, the literature largely documents how the movement away from up-or-out did not mean substituting it with another career system, but rather adjoining an extra career track based on permanent non-partner positions to the initial up-or-out organization. In line with our interpretation, this suggests the existence of a dual career system in which up-or-out is formally in place but is only selectively implemented (in variable proportions across firms).

[38] Gilson and Mnookin (1989) write that "resources might be allocated differentially between populations with dramatically different likelihoods of success, resulting in an implicit second track for local law school graduates" (page 592). See, also, DeVaro and Waldman (2012).

[39] See, also, above Footnotes 24 and 37 on the key role played by the ability to outplace denied employees in enacting up-or-out.

5.2 Non-compete agreements

Firms can also directly affect the extent of labor market competition contractually, notably through covenants not to compete or non-solicitation clauses. Empirical evidence on the use and effects of these clauses is emerging.[40] Certainly, non-compete covenants appear to be widespread: Starr, Prescott, and Bishara (2019) document their use in the US market and report that approximately 18 percent of U.S. workers have signed such clauses. Garmaise (2011) notes that non-compete agreements can encourage firm-sponsored general training (which we can interpret as assignment to the risky task for the purpose of learning) but decrease managers' incentives to invest (which, in our model, is captured by effort). More generally, our model predicts that non-compete clauses should be more prevalent in occupations for which output is less easily measurable. This seems consistent with the findings of Starr, Prescott, and Bishara (2019), who document that the occupations in which noncompetes are found most frequently are architecture and engineering (36%) and computer and mathematical jobs (35%), while farm, fishing, and forestry have the lowest incidence (6%).

On the normative side, our result that some monopsony power within the employment relationship is sometimes warranted suggests that such clauses may improve efficiency by promoting an efficient allocation of tasks. In our model, such restrictions on competition are justified only by the lack of commitment on task allocation and when the production (or learning) technology makes it difficult to learn about employees' skills. A policy implication is thus that antitrust authorities should be more inclined to enforce these clauses in occupations in which performance is noisier and in which lack of commitment on task allocation is particularly severe. This suggests that the recourse to non-compete clauses for some low-skill workers – for instance, McAdams (2019) reports a nontrivial number of non-competes in the contracts of sandwich makers, dog walkers, and warehouse workers – is more likely to be an anticompetitive device designed to suppress labor market competition than to be an instrument to promote learning or other specific investments. In a similar vein, the no-poaching agreements signed between several Silicon Valley tech firms seem more likely to be socially justified than those signed by fast food franchisees.[41]

[40] McAdams (2019) provides a recent survey on non-compete agreements.

[41] In 2010, the DoJ settled with major high-tech firms from the Bay area – including Apple, Google, Intel, eBay, and Adobe – over their no "cold call" agreements. In June 2017, a class action antitrust suit

5.3 Changes in the nature of tasks

As mentioned above, there have been important changes in the nature of tasks performed within professional service firms. This is not unique to professional services. More generally, there is considerable discussion of the future impact of artificial intelligence and automation on jobs (Agrawal, Gans, and Goldfarb, 2018; Autor, 2015; Frey and Osborne, 2017; Susskind and Susskind, 2015). As Autor (2015) suggests, computers might "substitute for workers in performing routine, codifiable tasks while amplifying the comparative advantage of workers in supplying problem-solving skills, adaptability, and creativity" (page 5). In this context, our paper suggests that new technologies might have an ambiguous impact on firms' labor market policies. On the one hand, the substitution of workers by computers for routine tasks seems to reduce the discretion in task assignment: for instance, a junior associate at a law firm may be less wary than in the past of being assigned routine tasks, such as collecting information from casebooks or libraries. On the other hand, if the new skills central to performance are adaptability or creativity, it might be more difficult for employers to learn about these skills from observing performance – in the language of our model, λ gets larger. Depending on which of these two effects dominates, one might expect employers to choose more- or less-aggressive retention policies.

More broadly, while it is clear that the changes in the nature of tasks will have a significant and durable impact on the demand for labor, our analysis highlights how these changes will also impact opportunities for learning within firms, as well as the career concerns that such possibilities for learning create. In such a context, a central challenge from a policy perspective, in particular for antitrust policy, is to understand the interplay between learning within firms and competition between firms. In spite of some recent cases, labor antitrust enforcement is still muted compared to product market antitrust enforcement. In addition, while the antitrust authorities have recently become much more active in the definition of a clear antitrust doctrine for labor markets, the antitrust approach of labor markets has so far been primarily inspired by the approach of product markets.[42] However, an important difference between labor and product markets

was filed against McDonalds based on the no-poach terms in its franchise agreement. Several fast food chains are also involved in related antitrust cases.

[42]See also Footnote 19, and Naidu, Posner, and Weyl (2018) for a review on labor market issues in

is that relationships between firms and workers involve a great deal of non-contractibility (almost by the definition of employment), while transactions between buyers and sellers are often easier to contract on. In this paper, we considered two sources of incomplete contracting (on effort and task assignment) and highlighted that labor market competition interacts with these two commitment problems in a subtle manner. It seems important, therefore, to identify the circumstances under which limits on labor market competition can be justified by the presence of imperfect contracting. Our paper takes a step in that direction, and suggests that it would be useful to try to address these questions more generally in future research.

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6 Appendix

6.1 Proof of Lemma 1

Denoting $z_1 \equiv \theta + \varepsilon_1$, and noting that z_1 is Normal with mean μ_1 and variance $1+\sigma^2 = \frac{1}{1-\lambda}$, the worker, if engaged in the risky task, faces the following problem in period 1:

$$e_1^* \in \arg\max_{e_1}$$

$$s + \alpha \int_{\frac{s-\lambda\mu_1}{1-\lambda}-(e_1-e_1^*)}^{+\infty} (\lambda\mu_1 + (1-\lambda)(z_1 + e_1 - e_1^*) - s)\sqrt{\frac{1-\lambda}{2\pi}} e^{-\frac{1-\lambda}{2}(z_1-\mu_1)^2}\, dz_1 - \frac{1}{2}e_1^2. \tag{8}$$

Differentiating with respect to e_1 and taking $e_1 = e_1^*$ yields

$$e_1^* = \alpha(1-\lambda) \int_{\frac{s-\lambda\mu_1}{1-\lambda}}^{+\infty} \sqrt{\frac{1-\lambda}{2\pi}} e^{-\frac{1-\lambda}{2}(z_1-\mu_1)^2}\, dz_1.$$

Finally, changing variables in the integral, we can rewrite

$$e_1^* = \alpha(1-\lambda)(1-\Phi(\frac{s-\mu_1}{\sqrt{1-\lambda}})),$$

where Φ denotes the cdf of a Normal distribution with mean 0 and variance 1.

One should check the second-order condition. Differentiating (8) twice yields

$$\alpha(1-\lambda)\sqrt{\frac{1-\lambda}{2\pi}} e^{-\frac{1-\lambda}{2}(\frac{s-\mu_1}{1-\lambda}-(e_1-e_1^*))^2} - 1$$

$$< \alpha(1-\lambda)\sqrt{\frac{1-\lambda}{2\pi}} - 1$$

$$< 0 \text{ for all } (\lambda,\alpha) \in (0,1)^2.$$

6.2 Proof of Lemma 2

Denoting $X \equiv \frac{s-\mu_1}{\sqrt{1-\lambda}}$, one can rewrite

$$e_1^* = \alpha(1-\lambda)\left(1-\Phi(X)\right). \tag{9}$$

Since, in equilibrium, $\mu_2 = \lambda\mu_1 + (1-\lambda)(\theta+\varepsilon_1)$, we derive that $\mu_2 \sim \mathcal{N}(\mu_1, \sqrt{1-\lambda})$. This implies that the continuation surplus when the risky task is undertaken can be rewritten, changing variables in the integral,

$$\begin{aligned} E\max(\mu_2-s,0) &= \int_s^{+\infty} \frac{1}{\sqrt{2\pi(1-\lambda)}}(\mu_2-s)e^{-\frac{(\mu_2-\mu_1)^2}{2(1-\lambda)}}\, d\mu_2 \\ &= \int_{\frac{s-\mu_1}{\sqrt{1-\lambda}}}^{+\infty} \frac{1}{\sqrt{2\pi}}(\sqrt{1-\lambda}\mu+\mu_1-s)e^{-\frac{\mu^2}{2}}\, d\mu \\ &= \sqrt{1-\lambda}\left(\varphi(X) - X(1-\Phi(X))\right), \end{aligned} \tag{10}$$

where $\varphi(.)$ denotes the density of the Normal distribution with mean 0 and standard deviation 1.

Recalling that $u_r(\alpha) = s + \alpha E \max(\mu_2 - s, 0) - \frac{1}{2}e_1^{*2}$, and using (9) and (10), we derive

$$\frac{\partial u_r}{\partial \alpha} = \sqrt{1-\lambda}\,(\varphi(X) - X(1-\Phi(X))) - \alpha(1-\lambda)^2(1-\Phi(X))^2.$$

This has the same sign as

$$\varphi(X) - X(1-\Phi(X)) - \alpha(1-\lambda)\sqrt{1-\lambda}(1-\Phi(X))^2$$

$$> \varphi(X) - X(1-\Phi(X)) - (1-\Phi(X))^2 \text{ for all } (\alpha,\lambda) \in (0,1)^2.$$

To complete the proof, we now show that

$$\varphi(X) - X(1-\Phi(X)) - (1-\Phi(X))^2 > 0 \text{ for all } X.$$

The derivative of this function is $(1-\Phi(X))(2\varphi(X)-1) < 0$, using the fact that $\varphi(X) < \frac{1}{\sqrt{2\pi}} < \frac{1}{2}$ for all X.

One can rewrite the function as follows:

$$-(1-\Phi(X))\left(1-\Phi(X) + X - \frac{\varphi(X)}{1-\Phi(X)}\right).$$

The hazard rate $\frac{\varphi(X)}{1-\Phi(X)}$ behaves like X when $X \to \infty$ (using L'Hopital's rule), so the function tends to 0 when $X \to \infty$. This implies that it is always positive.

We conclude that

$$\frac{\partial u_r}{\partial \alpha} > 0.$$

6.3 Proof of Proposition 1

When $\mu_1 < s$, $\pi_r(\alpha) \geq \pi_s(\alpha)$ reads as

$$\mu_1 + \alpha(1-\lambda)(1-\Phi(X)) + (1-\alpha)\sqrt{1-\lambda}\,(\varphi(X) - X(1-\Phi(X))) \geq s$$

$$\Leftrightarrow -X + (\alpha\sqrt{1-\lambda} - (1-\alpha)X)(1-\Phi(X)) + (1-\alpha)\varphi(X) \geq 0. \tag{11}$$

Let

$$H(X,\alpha,\lambda) \equiv -X + (\alpha\sqrt{1-\lambda} - (1-\alpha)X)(1-\Phi(X)) + (1-\alpha)\varphi(X). \qquad (12)$$

It is easy to check that

$$H_1(X,\alpha,\lambda) = -1 - (1-\alpha)(1-\Phi(X)) - \alpha\sqrt{1-\lambda}\varphi(X) < 0,$$

where subscript i refers to the partial derivative with respect to the i^{th} variable.

Furthermore,

$$H_3(X,\alpha,\lambda) = -\frac{\alpha(1-\Phi(X))}{2\sqrt{1-\lambda}} < 0.$$

In addition, one checks that $H(0,\alpha,\lambda) = \frac{\alpha\sqrt{1-\lambda}}{2} + (1-\alpha)\varphi(0) > 0$ for all α and λ, and one derives $H(1,\alpha,\lambda) < H(1,\alpha,0) = (2\alpha-1)(1-\Phi(1)) + (1-\alpha)\varphi(1) - 1 \approx 0.06\alpha - 0.91 < 0$ for all $\alpha \in (0,1)$.

We conclude that there exists a unique $X^* \in (0,1)$ such that $H(X^*,\alpha,\lambda) = 0$. Accordingly, the firm allocates the risky task if and only if $\mu_1 > \overline{\mu}$, where $\overline{\mu} = s - \sqrt{1-\lambda}X^* < s$.

By the implicit function theorem, one has

$$\frac{\partial X^*}{\partial \lambda} = -\frac{H_3(X^*,\alpha,\lambda)}{H_1(X^*,\alpha,\lambda)} < 0.$$

This implies that

$$\frac{\partial \overline{\mu}}{\partial \lambda} = \frac{1}{2\sqrt{1-\lambda}}X^* - \sqrt{1-\lambda}\frac{\partial X^*}{\partial \lambda} > 0.$$

Using the implicit function theorem again, one has

$$\frac{\partial X^*}{\partial \alpha} = -\frac{H_2(X^*,\alpha,\lambda)}{H_1(X^*,\alpha,\lambda)},$$

which has the sign of

$$H_2(X^*,\alpha,\lambda) = (\sqrt{1-\lambda} + X^*)(1-\Phi(X^*)) - \varphi(X^*),$$

which itself has the same sign as

$$h(\alpha, \lambda) \equiv \sqrt{1-\lambda} + g(X^*),$$

where $g(X) \equiv X - \frac{\varphi(X)}{1-\Phi(X)}$.

We will make use of the following lemma:

Lemma 3. *For all $X \in [0,1]$, $g'(X) > 0$ and $g(X) \in (-1, 0)$.*

Lemma 3 can be proven easily by remarking that g is concave (the hazard rate of the Normal distribution $\frac{\varphi}{1-\Phi}$ is convex), and by using the property (already used) that $\frac{\varphi(X)}{1-\Phi(X)}$ behaves like X when $X \to \infty$ (using L'Hopital's rule).

Given $X^* \in (0,1)$ and $\frac{\partial X^*}{\partial \lambda} < 0$, the lemma implies that, for any value of α, $\sqrt{1-\lambda} + g(X^*)$ is decreasing in λ, positive at $\lambda = 0$, and negative at $\lambda = 1$.

Therefore, for any α, there exists a $\lambda^*(\alpha)$ such that $h(\alpha, \lambda^*(\alpha)) = 0$.

We now show that $\lambda^*(\alpha)$ does not depend on α. Differentiating $h(\alpha, \lambda^*(\alpha)) = 0$ with respect to α, one gets

$$h_2(\alpha, \lambda^*)\frac{\partial \lambda^*}{\partial \alpha}(\alpha) + h_1(\alpha, \lambda^*) = 0.$$

One has $h_1(\alpha, \lambda^*) = g'(X^*(\alpha, \lambda^*))\frac{\partial X^*}{\partial \alpha}(\alpha, \lambda^*)$. But by definition of λ^*, we have $\frac{\partial X^*}{\partial \alpha}(\alpha, \lambda^*) = 0$. Since $h_2 < 0$, we conclude that $\frac{\partial \lambda^*}{\partial \alpha}(\alpha) = 0$. Therefore, λ^* is independent of α.

Finally, we conclude that, for any α, $\frac{\partial X^*}{\partial \alpha} > 0$ if and only if $\lambda < \lambda^*$.

Given that $X^* = \frac{s-\overline{\mu}}{\sqrt{1-\lambda}}$, for any α, $\frac{\partial \overline{\mu}}{\partial \alpha} > 0$ if and only if $\lambda > \lambda^*$.

□

6.4 Proof of Corollary 1

$\overline{\mu}(0) < \mu_1$ implies that $W(0) = W_r(0) = \mu_1 + E\max(\mu_2, s)$, using $e_1^* = 0$ at $\alpha = 0$. In turn, $\mu_1 < \overline{\mu}(1)$ implies that $W(1) = W_s = 2s$.

By definition, $\overline{\mu}(0)$ is such that $\overline{\mu}(0) + E\max(\mu_2 - s, 0) = s$. We derive:

$$\begin{aligned} W(0) - W(1) &= \mu_1 + E\max(\mu_2, s) - 2s \\ &> \overline{\mu}(0) + E\max(\mu_2 - s, 0) - s \\ &= 0. \end{aligned}$$

6.5 Proof of Proposition 3

The proof is a direct consequence of Lemma 2, which states that workers have a higher surplus in firms with higher α as long as the risky task is assigned to them.

Suppose that no firm offers $\alpha = \alpha^*$. If all firms choose $\alpha > \alpha^*$, then it must be the case that $\alpha^* < 1$; that is, we are in the case $\lambda > \lambda^*$ and $\alpha^* = \overline{\alpha}$. In that case, a firm that offers $\alpha > \alpha^*$ attracts no workers, as workers anticipate that they would be assigned the routine task if they were to join the firm. Therefore, a firm could profitably deviate to some $\alpha \leq \alpha^*$ that gives positive profits.

Thus, there must be some firms choosing $\alpha < \alpha^*$. Let $\alpha_0 = \sup\{\alpha, \alpha < \alpha^*\}$. In addition, there must be some firms that make zero profit. Indeed, either $\alpha = \alpha_0$ for all firms, and, therefore, firms must concede all rents to workers; or some firms choose $\alpha < \alpha_0$ (or $\alpha > \alpha^*$) and, thus, cannot attract any worker. A firm that makes zero profit could deviate and secure a positive profit by choosing $\alpha = \alpha^*$. Indeed, it can then offer a wage $\omega = W(\alpha_0) - u_r(\alpha^*)$. First, this allows the firm to attract all workers, as workers can obtain, at most, $W(\alpha_0)$ – i.e., the entire surplus – by going to a α_0 firm. In addition, the profit that firm 1 derives reads $\pi_r(\alpha^*) - \omega = W(\alpha^*) - u_r(\alpha^*) - \omega = W(\alpha^*) - W(\alpha_0) > 0$.

Therefore, in equilibrium, at least one firm chooses $\alpha = \alpha^*$. Some firms may choose $\alpha \neq \alpha^*$ in equilibrium but then will attract no workers.

6.6 Proof of Proposition 4

Let us first remark that, if $\lambda < \lambda^*$, we have $\alpha_L^* = \alpha_H^* = 1$. If $\lambda \geq \lambda^*$, it is easy to see, using that $\overline{\mu}(\alpha)$ is increasing, we either have $\alpha_L^* = \alpha_H^* = 1$ or $\alpha_L^* < \alpha_H^* \leq 1$.

To accommodate worker heterogeneity, let us slightly adapt the notation and denote welfare and utility as $W(\mu_1, \alpha)$ and $u_r(\mu_1, \alpha)$, where $\mu_1 \in \{\mu_1^L, \mu_1^H\}$.

Suppose, first, that $\alpha_L^* = \alpha_H^* = 1$. In that case, at least one firm must choose $\alpha = 1$, exactly as in the proof of Proposition 3.

Suppose that $\alpha_L^* < \alpha_H^*$. For the same reasons as above, it is impossible that neither α_L^* nor α_H^* is offered in equilibrium.

Suppose, now, that, in equilibrium, some firms offer α_L^*, but not α_H^*. Let $\alpha_1 = \sup\{\alpha, \alpha < \alpha_H^*\}$. There must be firms making zero profits (either those that offer an α different from α_L^* and α_1 if there are some, or firms that pool on α_L^* or α_1). For those firms, it would be profitable to set α_H^*. By offering $\omega^H = W(\mu_1^H, \alpha_1) - u_r(\mu_1^H, \alpha_H^*)$ to H-workers, they would attract all of them and make positive profits.

Suppose, finally, that, in equilibrium, some firms offer α_H^*, but not α_L^*. Let $\alpha_2 = \sup\{\alpha, \alpha < \alpha_H^*\}$. Firms that make no profits in equilibrium (as before, there must be some) could benefit from deviating to α_L^*. By offering $\omega^L = W(\mu_1^L, \alpha_2) - u_r(\mu_1^L, \alpha_L^*)$ to L-workers, they would attract all of them and make positive profits.