Myopic Fiscal Objectives and Long-Run Monetary Efficiency*

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Abstract

Is the pursuit of myopic fiscal objectives, such as short-run redistribution or public spending, a threat to long-run monetary efficiency? We answer this question in the context of a textbook overlapping generations model where we introduce a sequence of one-period fiscal authorities that can tax endowment and trade money. Each authority is myopic in that it cares only about current agents' utility and its own consumption, without any concern about the future. Nonetheless, we show that the sequence of fiscally-backed money purchases that maximize the *current* authority's objective also selects a unique equilibrium – one in which money is traded at the efficient *intertemporal* price – as a by-product. In fact, even if authorities are myopic and do not coordinate, policy implementation gets efficiently constrained through markets by private agents' intertemporal choices. Details about the fiscal stance are also crucial. Multiplicity and sub-optimality emerge as fiscal capacity is bounded, inducing authorities to use money trade to generate resources for public consumption.

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1 Introduction

Trading fiat money for goods can occur when people trust that the same trade will be possible at any future date. This confidence, which is the trademark of traditional currencies, has often been rationalized as the result of the commitment by some public authorities to preserve the value of money in the long run. In fact, commitments are ubiquitous in monetary theories. A stream of literature has emphasized the commitment to back money with real resources as a way to prevent extreme events, such as hyperinflations.[1] Other studies have advocated the importance of commitment ability to prevent inefficient nominal fluctuations.[2]

In reality, the presence of political cycles may undermine confidence in governments' long-run commitments. To avoid any such interference, in almost all economies, monetary policy has been delegated to an independent central bank mandated with an explicit objective of long-run price-stability.[3] Still, this solution may not be sufficient to avoid confidence crises when the fiscal authority may de facto reverse the independence of the central bank to finance government deficits, as emphasized by Sargent and Wallace (1981). The recent success of private cryptocurrencies, and consequent central banks' concerns can be interpreted as the by-product of such fears. Many believe that the delegation of monetary decisions to an algorithm fully detached from any sovereign is the soundest protection against any abuse of seigniorage. But, what are the conditions under which myopic fiscal goals may effectively threaten monetary efficiency? And, on the other hand, can private assets such as cryptocurencies ever become a major currency without any authority explicitly committed to back them?

In this paper, we show that myopic redistribution concerns, stemming from one-period utility maximization, may actually sustain the socially efficient inflation rate as a by-product, even when public authorities have large fiscal spending needs and lack commitment ability or an explicit long-run goal. To formally establish our point, we build on a textbook incomplete-market model: the monetary Overlapping Generations Model (OLG) *à la* Samuelson (1958). This model is well known to capture the self-fulfilling nature of the store-of-value role of money – a role of money studied by Wallace (1981b) and, more recently, by Asriyan et al. (2016). As Brunnermeier and Sannikov (2016) argue forcefully, it is also a natural benchmark to capture the redistributive impact of monetary policy (e.g., Doepke and Schneider, 2006; Sterk and Tenreyro, 2018; Auclert, 2019). We enrich this model along two important dimensions.

First, we introduce an alternative to money as a store-of-value: a storage technology with a socially inefficient fixed return, as in Sims (2013), which creates a meaningful private portfolio problem for private agents. By providing a lower bound to real returns, storage allows for closed-form expressions for off-equilibrium paths along which money progressively loses value, as in hyperinflations.[4] Importantly, these two saving vehicles have different consequences for

[1] This can be a commitment to either a fractional currency backing, as in Obstfeld and Rogoff (1983), or future fiscal surpluses, as in the Fiscal Theory of the Price Level (Leeper, 1991; Woodford, 1994, 1995; Sims, 1994, 2013; Bassetto, 2002). See Obstfeld and Rogoff (2017) for a discussion of the two approaches.

[2] This is, for example, the purpose of interest-rate rules satisfying the Taylor principle in New-Keynesian models, to which timeless policy makers should optimally commit.

[3] This is the spirit of delegation to a conservative central banker, as in Rogoff (1985), and, for example, further explored in a principal-agent framework by Walsh (1995), among others.

[4] As noticed by Sargent (1982), among others, during hyperinflation episodes, money may also stop playing

redistribution: whereas money can transfer consumption across generations, storage transfers consumption over time.

Second, we introduce a sequence of one-period authorities that have the power to tax and carry out money market operations – i.e., can buy and sell money. Each authority implements a policy in order to maximize its one-period objective, which includes the utility of present (but not future) households and its own consumption. Thus, authorities are completely myopic as they embody no consideration for the future.

In this model, absent any policy intervention, a monetary equilibrium exists where savings is fully monetary, but other equilibria are also possible.[5] There exists global indeterminacy because a complementarity in monetary savings – the lower future money holdings the lower current money holdings – features equilibria in which a deviation of a generation from full monetary saving leads the real value of money to shrink indefinitely as storage crowds it out. In addition, there exists local indeterminacy in that there are dynamically-stable multiple equilibria in which only money is used. In no equilibrium is the rate of return socially efficient.

Our main result is that the sequence of myopic policy interventions associated with optimal private saving decisions selects a unique equilibrium – one in which money is efficiently traded – even if public interventions do not pursue any intertemporal objective.

In particular, we show that, when authorities implement their optimal policy, a deviation of a generation from full monetary savings to some storage leads to a unique continuation equilibrium where this stock of storage is consumed across generations efficiently, until the full monetary equilibrium is reached again. In contrast to the case without policy interventions, other equilibria in which storage would progressively crowd out money, are not possible. The reason is that lower returns on savings create inequality between generations, pushing the authorities to continuously buy money to guarantee the old the same level of consumption of the young. This induces future generations to use storage at an increasing rate, which, we show, is ultimately non-feasible.

Key to the result is that the current authority cannot control future price levels and, hence, current private saving decisions. In this sense, the market and future authorities impose a socially efficient constraint to the action of the current authority. In fact, any positive amount of current saving in storage is sub-optimal from the point of view of the current authority, as it reduces the amount of resources for current consumption; a myopic but unconstrained social planner would therefore zero any current storage. In contrast, the expectation of policy interventions leads private agents themselves to decide to optimally de-stock their savings in storage, ensuring the efficient intertemporal trade-off of available resources.

Moreover, since constrained-optimal myopic policies select the first-best allocation as the unique equilibrium outcome, they are also the rules to which a long-horizon Ramsey planner,

its other roles, as a medium of exchange and a unit of account. For the same reason, notice that, to analyze hyperinflation situations, we then cannot simply assume that money enters into utility or is part of a feasibility constraint such as cash-in-advance. Also, we are not interested here in the properties of special assets that have intrinsic advantages in becoming dominant means of exchange (see Williamson and Wright, 2010, for an overview).

[5]Since at least Wallace (1978), this multiplicity of equilibria has been interpreted as reflecting the self-fulfilling nature of money and the role of confidence in money exchanges.

maximizing the present value of all one-period authorities' objectives, would commit to implement its desired allocation. We also show that our results hold for any arbitrarily small weight of households' utility in the objective of short-sighted authorities and whatever asset is initially held by households, regardless of whether or not they are public liabilities. In other words, the mere pursuit of short-term consumption equality, no matter how little it weighs in the objective of the authority, guarantees the long-run efficient value to what is already used and held by households as money, as a by-product.

Nevertheless, as we show in the second part of the paper, when the ability to set taxes is limited (e.g., due to distortionary costs of taxation), uniqueness and efficiency may get lost. In this case, the authority faces a trade-off between financing its expenditures and money stability, which prevents it from achieving the first-best. Manipulating the price level is, then, not only a way to equalize consumption levels across generations but also a way to gather real resources for public expenditures. When fiscal capacity is low, the temptation of increasing seigniorage and the presence of an alternative to money – storage – leads to the existence of an additional type of equilibrium. In this additional equilibrium, private agents use storage jointly with money, the real value of which never goes to zero. This is due to the emergence of a Laffer curve of seigniorage. A higher "inflation tax" that the authority imposes in the attempt to increase its own resources induces agents to move savings from money to storage. By doing that, agents reduce the "tax base" from which the authority drains resources. As a result, the authority ends up in a coordination failure with the private sector, with both consuming less compared to the equilibrium in which only money is used, even though the primary fiscal surplus is lower.

These results suggest that the ability to ensure the stability of fiat currencies is crucially determined by the fiscal capacity of governments: countries like Argentina – whose governments suffer from recurrent fiscal problems (see Buera and Nicolini, 2019, among other) – should be more exposed to monetary instabilities compared to others with stronger fiscal capacity as the US. However, this does not mean US governments being necessarily more concerned about monetary efficiency or more forward looking than the ones in Argentina.

Literature review. To the best of our knowledge, this is the first paper showing that short-run fiscal objectives associated with optimal private saving decisions may lead to the efficient determination of the price level. In previous literature, the emphasis on long-run commitments, did not allow to fully appreciate that, when policy implementation occurs through markets, the efficiency of intertemporal prices is assured through private agent choices. Thus, authorities do not necessarily need to care about intertemporal optimality for policy to select the efficient equilibrium.

Our paper relates to a popular literature on the interaction between monetary and fiscal policy, as pioneered by Sargent and Wallace (1981). In the same spirit, we study a framework in which the conduct of fiscal policy is crucial for monetary stability. In contrast to that stream of literature, in our setting, the presence of a fiscal authority is not only a source of danger; on the contrary, it has an active and essential role in preserving monetary stability. Consistent with Wallace (1981b), we show that interventions require fiscal backing. Yet, this requirement does not imply fiscal interventions in equilibrium, but out of equilibrium.

On a one hand, our theory relates to Obstfeld and Rogoff (1983), in that an off-equilibrium intervention is essential to stabilize the money market, and, in principle, there could be no fiscal interventions along the equilibrium (this is the case with a discount factor equal to one, as discussed in the paper). On the other hand, it is worth noting that Obstfeld and Rogoff (1983)[6] demonstrate that the mere ability to commit has so strong consequences that the presence of a fiscal authority may not even be necessary. Anyone endowed with commitment power can use an arbitrarily small redemption value to prevent fiat money from losing value. In our theory, instead, commitment has no role, whereas the ability to raise taxes is crucial.

Nicolini (1996) analyzes the mechanism of Obstfeld and Rogoff (1983, 2017) in a model in which a fiscal authority decides under discretion the implementation of a costly conversion facility for money. In his model, should hyperinflation occur, there is always a period in which the social costs of hyperinflation will exceed the fixed-cost of the conversion facility. As agents anticipate the intervention of the authority, hyperinflation does not occur, although the facility is not implemented along the equilibrium. In contrast to Nicolini (1996), we assume, as in Sims (2013), that agents face a portfolio choice that effectively constrains the authority's plan, in the spirit of Bassetto (2002). Absent such a feature, our model would always exhibit a unique equilibrium, even in the case of limits to fiscal capacity, consistent with Nicolini (1996).

More recent works about the determination of the price level include Benigno (2020) and Hall and Reis (2016), among others. Although all these works deviate from the typical framework of the fiscal theory of the price level, they are also concerned with the commitment to a particular rule for fiscal transfers without inquiring about its optimality and sub-game perfection.

In this respect, we are closer in spirit to Atkeson et al. (2010) and, more generally, to Bassetto (2005), which emphasize how policy implementation is not about committing to unconditional *actions*, but about committing to a strategy leading to feasible actions as a function of private agents' decisions. The optimal policy should then make privately suboptimal those actions that the authority finds undesirable and cannot directly control. In contrast to these papers, we do not assume any form of commitment on the side of the fiscal/monetary authority, consistent with Cochrane (2011)'s discussion of credibility. We share this approach with Barthélemy and Mengus (2021), who investigate the social cost of the commitment required to implement a unique equilibrium in macroeconomic games.

Other papers investigate the effects of monetary policy rules in overlapping generation models by postulating a demand for money, as in Asriyan et al. (2016), to cite a recent example. Another example is Tirole (1985), who considers a situation in which the government forces agents to invest some of their savings in an intrinsically worthless asset (that he labels "gold"). More generally, exogenous motives for money demand obtain by introducing a cash-in-advance or money-in-the-utility-function, as reviewed by Walsh (2010). Limit cases of money-in-the-utility-function economies, as defined in Woodford (2003), feature cashless economies such as the OLG studied by Galí (2014). All these approaches rule out equilibria in which money loses value by assumption; these equilibria are, instead, the only source of multiplicity in our paper.

[6] See, also, Wallace (1981a) for an earlier discussion of a related argument.

Another related stream of literature models money as one possible emerging medium of exchange in search and matching economies (Kiyotaki and Wright, 1989). Also, in those environments, one may formalize the idea that the government's commitment to implement a certain transaction can coordinate agents on the preferred medium of exchange as a unique equilibrium (Aiyagari and Wallace, 1997; Li and Wright, 1998). A natural interpretation of such a commitment is the fact that tax obligations can be carried out in money only as modeled by Starr (1974) and Goldberg (2012), among others. As long as sanctions to tax evasion have a negative utility impact, this is another way to enforce real value to money. In any case, Malmberg and Öberg (2021) theoretically show that the constraint to pay tax in money is, in fact, neither a necessary nor a sufficient condition to ensure price level determination.

Our paper is also connected to the literature on multiplicity of equilibria and seignorage revenues initiated by Bruno and Fischer (1990). In particular, Bruno and Fischer (1990) show that there may exist multiple equilibrium inflation rates consistent with the same seignorage income. In contrast, we show that, in a model in which private agents make portfolio choices, there may exist equilibria with higher inflation rates associated with lower seignorage revenues.

2 A Simple Model of Fiat Money

2.1 Physical environment

We consider an economy populated by equal-sized overlapping generations of atomistic agents and a sequence of short-sighted fiscal authorities. Time is discrete and indexed by $t \in \{1, 2, ...\}$. The consumption good is homogeneous and perfectly divisible and it appears in each period in flows of endowments. We will focus on the case in which endowments are sufficiently high in agents' first period of life, so they have an incentive to save. Saving can occur in two forms.

First, there exists a homogeneous and perfectly divisible asset called money, which is intrinsically worthless. Money exists in an initial physical stock M_0 in the economy. The fiscal authority can hold physical money and can also issue liabilities that are indistinguishable from physical money. Thus, at each time t, we have

$$M_t + M_{g,t} = M_0, \tag{1}$$

where M_t is the stock of money privately owned, whereas $M_{g,t}$ denotes the stock of money held by the authority. Only the latter can be negative, in which case the money held by the private sector must include both physical money and public liabilities.

The alternative to money is to store part of the endowment in a technology with a fixed real return as in Sims (2013). Every quantity of consumption goods stored at time t – namely, S_t – yields θS_t quantity of consumption goods available next period, where $\theta < 1$.

2.2 Households

At each date, a new generation of homogeneous agents is born. Each agent lives two periods and then disappears. Agent $i \in (0, 1)$ born at time t maximizes the following utility function:

$$U_{i,t} \equiv u(C_{i,y,t}) + \beta u(C_{i,o,t+1}), \tag{2}$$

where $C_{i,y,t} \geq 0$ and $C_{i,y,o} \geq 0$ are individual consumption in the first and second period, respectively; $\beta \in (0, 1]$ is the discount factor and $u(\cdot) \in \mathcal{U}$ is the utility function. $\mathcal{U}$ denotes a set of continuous and differentiable functions $u(\cdot) : \mathbb{R}^+ \to \mathbb{R}$ with typical concavity properties – i.e., $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $u'(0) \to \infty$, with $u'(\cdot)$ being a multiplicative function. Standard utility functions such as CRRA and CARA belong to this family.

The budget constraint of an agent i born in period t is:

$$C_{i,y,t} = W^y - T_t - S_{i,t} - \frac{M_{i,t}}{P_t}, \tag{3}$$

$$C_{i,o,t+1} = W^o + \theta S_{i,t} + \frac{M_{i,t}}{P_{t+1}}, \tag{4}$$

where W^y and W^o are endowments in consumption goods available to the agents when young and old, respectively; T_t is a (positive or negative) lump-sum real transfer paid by the young; $S_{i,t} \geq 0$ is the amount of goods stored in the first period; $P_t \geq 0$, is the equilibrium price of consumption in terms of money; and $M_{i,t} \geq 0$ is the quantity of money acquired by i when young at time t. The first generation is born at date 0, lives just one period, owns a stock of fiat money $M_0 > 0$, does not have storage, $S_{-1} = 0$, and has utility function $U_0 \equiv u(C_{o,1})$. Aggregate consumption, storage and monetary holdings are denoted by $C_{y,t} \equiv \int C_{i,y,t} di, C_{o,t} \equiv \int C_{i,o,t}, S_t \equiv \int S_{i,t} di$, and $M_t \equiv \int M_{i,t} di$, respectively.

2.3 The authorities

In analogy with households, we introduce a sequence of short-sighted authorities, each one solving a one-period problem. The authority in office at time t maximizes the following one-period objective function:

$$\mathbb{U}_t \equiv \int u(C_{i,y,t}) di + \int u(C_{i,o,t}) di + \tilde{\lambda} u(G_t), \tag{5}$$

which, it is worth noting, does not include any monetary target. The authority cares, instead, about the current flow of utilitarian welfare – i.e., the utility of the current young and old agents, but also the level of public spending – that is, its own consumption, G_t proportional to $\tilde{\lambda} \geq 0$.[7] Notice also that we assume for simplicity that the authority puts the same weights for the old and the young: what is important in our results is that the authority is sufficiently willing to transfer resources to the old generation/money holders from the young generation/taxpayers.

[7] Note that G_t does not necessarily entail a "waste." The $\tilde{\lambda} u(G_t)$ component can be added to the utility of the agents without any impact on any private choice: in such a case, G_t denotes a public good whose provision is out of the control of the agents (for example, a public health good).

The budget of the authority is written as:

$$T_t + \frac{M_{g,t-1}}{P_t} = \frac{M_{g,t}}{P_t} + G_t. \tag{6}$$

That is, transfers plus the real value of money holdings from the previous period must equal public consumption and the real value of new money holdings. Because of (1), an increase in $M_{g,t}$ corresponds to a decrease in M_t. The budget constraint of the authority must hold in any state of the world – i.e., in and off equilibrium. In fact, the price level P_t is determined in the money market as described below, and taxes adjust to make sure that (6) always holds.

2.4 Money market

We describe in detail here how the market for money works. In the market, money demand expressed in terms of consumption goods has to match the real value of the supply of money.

Formally, let us denote by $m_{i,t}$ the private real demand of money by agent i at time t, which cannot exceed available real resources

$$m_{i,t} \leq W^y - T_t - S_{i,t} - C_{i,y,t}.$$

The aggregate quantity $m_t \equiv \int m_{i,t}\, di$ is, therefore, the quantity of goods owned by the young put up for exchange with money, at time t. In analogy, let us define

$$m_{g,t} \leq T_t - G_t,$$

the public real demand of money – i.e., the quantity of goods that the authority bids in exchange for money, at time t.

The private supply of money at time t is simply M_{t-1} – i.e., the money holdings of the old – as there is no alternative use. Let us further indicate by $M^S_{g,t} \geq 0$ the public supply of money. That is, the authority can be a buyer or a seller of money. We refer to $\Delta_t = \left(m_{g,t}, M^S_{g,t}\right)$ as the position of the authority on the money market.[8]

For a given available nominal supply of money $M_{t-1} + M^S_{g,t}$ and real money demand $m_t + m_{g,t}$ a *market-clearing price* P_t is such that

$$P_t\left(m_t + m_{g,t}\right) = M_{t-1} + M^S_{g,t}. \tag{7}$$

Finally, $M_{g,t} = M_{g,t-1} + m_{g,t}P_t - M^S_{g,t}$ and

$$M_t = m_t P_t \tag{8}$$

[8] Note that one can rewrite the budget constraint of the authority uniquely in terms of the actions that the authority can perform:

$$T_t + \frac{M^S_{g,t}}{P_t} = m_{g,t} + G_t.$$

In words, the authority can tax or use seigniorage revenues to either consume or buy money from the private sector.

define the stocks of money held by the authority and the private sector, respectively, at the end of the trade.

2.5 Timing, market clearing, and equilibrium

Let us now describe the economy as a game between households and the authority. We state timing assumptions and then formally define the strategic space of each actor in the economy and a notion of equilibrium.

Timing. In our economy, all actions taken at a given time t are set simultaneously. Each time is characterized by an aggregate state $\omega_t \equiv \{W^o, W^y, S_{t-1}, M_{t-1}\}$. We define a policy of the authority $\mathcal{P}_t \equiv (T_t, \Delta_t, G_t)$ as a collection of transfers imposed on the young,[9] and money market operations and public consumption that are implemented by the authority at time t.

Actions and continuation policies. A date-t strategy for the authority is a mapping $\sigma_{\mathcal{P},t} : \omega_t \mapsto \mathcal{P}_t$ from aggregate states to an action at date t decided by the time-t authority. We define $\sigma_{\mathcal{P}} \equiv \{\sigma_{\mathcal{P},\tau}\}_{\tau=1}^{\infty}$ as the policy plan of authorities. A date-t strategy for households is a mapping $\sigma_t : \omega_t \mapsto \{S_t, m_t\}$ from aggregate states to a portfolio choice of households at date-t.[10] We define $\sigma \equiv \{\sigma_\tau\}_{\tau=1}^{\infty}$ as the policy plan of households. We define $\sigma^t = \{\sigma_\tau\}_{\tau=t}^{\infty}$ and $\sigma_{\mathcal{P}}^t = \{\sigma_{\mathcal{P},\tau}\}_{\tau=t}^{\infty}$ as the continuation strategies of households and the authorities, respectively, from time t onward.

Equilibrium. Consistent with the literature on macroeconomic games (e.g., Ljungqvist and Sargent, 2018, among others), we use the concept of competitive equilibrium on the private agents' side and require the authority to implement the optimal policy. We restrict this to symmetric equilibria, as this is without any loss of generality (see later remark).

Definition 1. For a given initial quantity of money M_0, aggregate storage $S_0 = 0$ and endowments (W^y, W^o), an *equilibrium* is a set of policy plans $(\sigma, \sigma_{\mathcal{P}})$ such that at any ω_t for $t \geq 1$, $\{S_t, m_t\} = \sigma_t(\omega_t)$ and $\mathcal{P}_t = \sigma_{\mathcal{P},t}(\omega_t)$ are such that:

(i) $\{S_t, m_t\}$ maximizes (2) subject to (3)-(4) for each i, taking prices (P_t, P_{t+1}) and taxes T_t as given;

(ii) $\mathcal{P}_t$ maximizes (5) subject to (6), taking $(\sigma^t, \sigma_{\mathcal{P}}^{t+1})$ as given;

(iii) P_t is determined by (7), M_t by (8);

(iv) market-clearing conditions for money (1) holds.

[9] Here, we implicitly assume that the government cannot implement direct transfers to the old. As will become clear, the absence of such an instrument, by itself, will not prevent the authority from implementing its first-best allocation.

[10] Note that the money acquired by households, M_t, is an equilibrium object, as it depends on the price level. Defining the macroeconomic game with markets in this way is consistent with Bassetto (2002): only in this way, we can make sure that this action is selected consistently with household's' budget constraint off-equilibrium.

In an equilibrium, each individual choice at time t is a best response to the perfect foresight of the aggregate choice of other agents and the authority from time t onwards. As any of these agents are atomistic, this leads them to take price levels and taxes as given. In analogy, the authority at time t sets a best response to the perfect foresight of the aggregate choice of agents from time t onwards and to the policies of future authorities from time $t+1$ onwards. The short-sighted behavior of agents and authorities may lead to possible miscoordinations. On the one hand, agents are subject to coordination failures, as they take (current and future) aggregate actions as given.

2.6 Interpretation and other remarks

On money and storage. We may think about physical money as shells, gold or (with a little abuse of language) bitcoins – i.e., any intrinsically worthless asset whose existence, measure and ownership is readily verifiable by anyone. Thus, money can, but does not necessarily need to, be public liabilities.

Concerning storage, three remarks are in order. First, storage can be interpreted as capital in Tirole (1985): when money is not used, agents overinvest in capital, lowering the return on capital to a dynamically inefficient level; when money is used instead, capital and money yield the same return, which may not be first-best. In our economy, the assumption of a fixed return allows for a closed-form analysis without modifying the key insight that taking resources away from money inefficiently lowers real returns in the economy.

Second, we do not consider other forms of saving assets that could restore dynamic efficiency. In fact, our argument remains valid as far as money is used as a store of value in equilibrium, as is the case, for example, when including capital with stochastic returns as in Brunnermeier and Sannikov (2016).

Third, the aggregate resource constraint

$$C_{y,t} + C_{o,t} + G_t = W^y + W^o + \theta S_{t-1} - S_t, \tag{9}$$

which holds in equilibrium because of Walras' law, shows that private storage decisions affect the availability of resources at a given time: the higher the storage, the lower real resources available.

Therefore, whereas money can transfer consumption across generations, storage transfers consumption across time. Thus, portfolio choices of agents effectively put constraints on the feasibility of fiscal plans, in line with the point put forward by Bassetto (2002). In particular, as we will see later more formally, any positive level of S_t is sub-optimal from the point of view of the current authority as it implies lower resources for current consumption.

On the mapping to infinite-horizon models. It is worth remarking that our modeling choice of short-sighted agents and authorities helps emphasize that our main result does not rely on any long-run optimality (transversality conditions), history-dependent strategies (trigger strategies) or time-inconsistent behavior (commitments). Nevertheless, their insights equally apply to infinite-horizon economies. In Appendix B, we show how our OLG economy in the absence of policy delivers the same allocation of a simple Bewley economy with infinitely-lived agents subject to income fluctuations, as it is well known since Townsend (1980). Later, in Proposition 7, we show that the optimal short-sighted policy is also a solution to the analogous infinite-horizon Ramsey problem; this result can be seen as a corollary of our main result that the sequence of optimal short-sighted policies achieves the first-best in the economy.

3 Optimal portfolios and optimal policy

In this section, we derive the optimal policies of the young and the authority and show how monetary policy gets implemented through the money market.

3.1 Optimal private portfolios

At each period t, the young generation decides how much to save and how to divide the resulting savings between storage and money holdings. We directly state agents' choices as symmetric – we will make some remarks on the possibility of heterogeneous choices at the end of this subsection. Let us denote by ρ_{t+1} the gross per-unit real return on real savings D_t defined as:

$$D_t \equiv S_t + m_t,$$

where m_t is the equilibrium real money holding as defined in (8). For a given $\sigma_{\mathcal{P}}$, the optimal level of real saving D_t is given implicitly by $u'\left(W^y - T_t - D_t\right) = \beta\rho_{t+1}u'\left(\rho_{t+1}D_t + W^o\right)$, whereas the split between money and storage is given by arbitrage between the equilibrium return on money Π_{t+1}^{-1} and storage θ. We can then state the following.

Lemma 1 (Optimal private-sector policy)**.** *For a given arbitrary policy plan $\sigma_{\mathcal{P}}$, the private-sector optimal policy $\sigma_t^* \in \sigma^*$ at any date $t \geq 1$ is given by*

$$\begin{aligned}
&S_t = 0, \quad m_t = D_t && \textit{if} \quad \Pi_{t+1}^{-1} > \theta && \textit{in which case} \quad \rho_{t+1} = \Pi_{t+1}^{-1}, && (10)\\
&S_t + m_t = D_t && \textit{if} \quad \Pi_{t+1}^{-1} = \theta && \textit{in which case} \quad \rho_{t+1} = \theta, && (11)\\
&S_t = D_t, \quad m_t = 0 && \textit{if} \quad \Pi_{t+1}^{-1} < \theta && \textit{in which case} \quad \rho_{t+1} = \theta, && (12)
\end{aligned}$$

where

$$D_t = \frac{W^y - T_t - R(\rho_{t+1})\rho_{t+1}^{-1}W^o}{1 + R(\rho_{t+1})}. \tag{13}$$

with

$$R(\rho_{t+1}) \equiv u'_{-1}\left(\beta\rho_{t+1}\right)\rho_{t+1} \tag{14}$$

and $\rho_{t+1} = \max\{\Pi_{t+1}^{-1}, \theta\}$ where u'_{-1} denotes the inverse of u'. Note that $R(\rho) > \rho$ for any $\rho \in (0,1)$.

Proof. See Appendix A.1 □

Households fix their amount of real savings depending on the present value of their after-tax income and their preferences. As a result, saving choices are purely forward-looking: the young generation makes saving decisions only by looking at future returns; current inflation is not relevant to their current saving decision. When the return on money is greater (resp. smaller) than the return on real storage, agents save everything in money (resp. storage). Money and storage may coexist only insofar as they yield the same return. In particular, given that the return on savings has to be the same across agents, the real value of the saving portfolio has to be the same across households. This condition is consistent with randomization between money and storage at the individual level. We will show later that this possibility does not affect equilibrium allocations.

To make the saving problem of the young non-trivial, we shall maintain that the endowment of the old is sufficiently small. This requirement is captured formally by:

$$W^y > R(\theta)\theta^{-1}W^o;$$

that is, at the minimal saving return θ, the young still have an incentive to save. This assumption captures the essence of overlapping generation models, where the efficient forward transfer of resources through market transactions is prone to inefficient coordination failures.

Remark on homogeneous portfolio choices. As we said, the restriction to symmetric equilibrium is without loss of generality. Since returns on savings are determined by aggregate variables only (Π_{t+1}, θ), objectives are strictly concave, and budget sets are convex, there will be a unique solution to the individual saving problem – i.e., $D_{i,t} = D_t$ for each i. Nevertheless, the allocation of real returns between money and storage is a potential source of within-cohort heterogeneity when both yield the same return – that is, when $\Pi_t^{-1} = \theta$. We show in Appendices A.2 and A.3 that such heterogeneity is immaterial to the characterization of the set of equilibria.

3.2 Constrained-optimal myopic policy

We will now derive the optimal response of the myopic authority. The first step is to note that the budget constraint of the young individual can be rewritten independently of current real money demand $m_{i,t}$ and current taxes T_t, as the following lemma states.

Lemma 2. *The level of consumption by the young is given by:*

$$C_{y,t} = W^y - G_t - \Pi_t^{-1}m_{t-1} - S_t. \tag{15}$$

Proof. See Appendix A.2. □

This is a powerful implication because it shows that the consumption of the young is independent of any return ρ_{t+1}, discount factor β and utility function $u(\cdot)$ and depends only on storing choices, public consumption and real money holdings of the old.

We then show how the *current* authority implements monetary policy determining *current* (but not future!) inflation for given private sector choices. The simultaneous trades of agents and the authority on the money market determine the rate of inflation $\Pi_{t+1} \equiv P_{t+1}/P_t$ between period t and $t+1$. In particular, because of (7), we can state the following.

Lemma 3 (Implementation of Monetary Policy)**.** *For given* (P_{t-1}, m_{t-1}, m_t)*:*

$$\Pi_t = \frac{m_{t-1} + M^S_{g,t}/P_{t-1}}{m_t + m_{g,t}} \tag{16}$$

entails a surjective mapping from Δ_t *to* Π_t*.*

By offering more money on the market ($M^S_{g,t} > 0$), the authority pushes the price level up, producing inflation. In contrast, by demanding money against consumption ($m_{g,t} > 0$), the authority depresses the current price level, reducing inflation. Thus, for given private choices, choosing a position on the money market Δ_t is equivalent to choosing the *current* inflation Π_t. However, the current authority has no control on the *future* inflation Π_{t+1}, which is what matters to current storing choices S_t as shown by Lemma 1.

By plugging (15) in the objective of the authority, we can easily derive the constrained-optimal policy of the authority as stated by the following proposition.

Proposition 4. *For a given portfolio policy* σ_t*, we can rewrite the constrained problem of the authority at time* t *as:*

$$\max_{\Pi_t, G_t} \left\{ u\,\underbrace{\left(W^y - G_t - \Pi_t^{-1} m_{t-1} - S_t\right)}_{=C_{y,t}} + u\,\underbrace{\left(\Pi_t^{-1} m_{t-1} + \theta S_{t-1} + W^o\right)}_{=C_{o,t}} + \tilde{\lambda} u(G_t) \right\}, \tag{17}$$

whose solution is given by:

- $\Delta_t(\sigma_t)$ *is such that* $C_{y,t} = C_{o,t}$*; that is,*

$$\Pi_t(\sigma_t) = \frac{(2+\lambda)m_{t-1}}{W^y - (1+\lambda)(W^o + \theta S_{t-1}) - S_t} \qquad \text{if} \quad \lim_{m_{t-1}\to 0} C_{y,t} \geq \lim_{m_{t-1}\to 0} C_{o,t} \tag{18}$$

$$\Pi_t(\sigma_t) \to \infty \qquad \text{otherwise} \tag{19}$$

according to (16)*;*

- $G_t(\sigma_t)$ *is such that* $G_{y,t} = \lambda C_{o,t}$*; that is,*

$$G_t(\sigma_t) = \frac{\lambda}{1+\lambda}(W^y - S_t - \Pi_t^{-1} m_{t-1}), \tag{20}$$

- *where* $T_t(\sigma_t)$ *is such that* (6) *holds; that is,*

$$T_t(\sigma_t) = \frac{1}{1+\lambda}\Pi_t^{-1} m_{t-1} + \frac{\lambda}{1+\lambda}(W^y - S_t) - m_t, \tag{21}$$

where we define $\lambda = 1/(u')^{-1}(\tilde{\lambda})$*.*

Proof. See Appendix A.3. □

Expression (17) reveals the trade-offs at stake in the policy problem. According to (18), the optimal inflation level is the one that equalizes consumption of the young with that of the old. To increase the price level, the authority raises real resources by taxing the young generation and uses these resources to purchase money from the old, thus redistributing resources to them. A corner solution (19) emerges when the young consume less than the old at the autarky limit, $m_{t-1} \to 0$, in which case the authority would like to choose a negative money return to transfer resources from the latter to the former: given that this is unfeasible, $\Pi_t \to \infty$ obtains. The optimal amount of public consumption (20) is such that the marginal utility of consumption of the young is equal to the marginal utility of public consumption weighted by λ. In the case where $u = \log$, $\lambda = \tilde{\lambda}$. More generally, λ is an increasing function of $\tilde{\lambda}$ so that $\lambda = 0$ when $\tilde{\lambda} = 0$ and $\lambda \to \infty$ when $\tilde{\lambda} \to \infty$. Taxes (21) clear the budget constraint of the authority.

Finally, let us have some words about how private saving decisions interact with the optimal policy as described by Proposition 4. From (17), we can first notice that higher current storage S_t means that the young divert resources, potentially available for current consumption, to the future. Lower overall consumption induces current authority to implement lower redistribution through a higher current inflation rate. An unintended effect of the anticipation of such policy could be thought to push previous generations to further shift their savings from money to storage. Let us turn to description of the equilibrium outcome to see how these forces unfold.

4 Equilibrium

In this section, we characterize the set of equilibria. First, we show that, in the absence of policy interventions, the economy exhibits multiple equilibria. We then demonstrate that the implementation of the constrained-optimal myopic policies leads to a single equilibrium in which money is the only saving asset and yields the efficient intertemporal rate of return. This holds true no matter how much the authority care about its own spending ($\tilde{\lambda}$), but under the assumption that the authority is not restricted in its ability to tax.

To ease presentation, we continue focusing on the case of a time-invariant endowment, and postpone the analysis of the optimal time-consistent policy with time-varying endowment to Appendix C.

4.1 Multiplicity in the absence of policy reaction

Let us first establish the benchmark in the absence of public policies – i.e., with $\mathcal{P}_t = (0, 0, 0)$ at each date t. In this case, by combining Lemma 3 and Lemma 1, we obtain that equilibrium inflation must satisfy:

$$\Pi_{t+1} = \frac{m_t}{m_{t+1}} = \frac{\frac{W^y - R(\rho_{t+1})\rho_{t+1}^{-1}W^o}{1+R(\rho_{t+1})} - S_t}{\frac{W^y - R(\rho_{t+2})\rho_{t+2}^{-1}W^o}{1+R(\rho_{t+2})} - S_{t+1}}, \tag{22}$$

given that $M_t = M_0$, and so $m_t = M_0/P_t$, for any $t \geq 1$. We can then easily check that, absent policy, a continuum of market equilibria exists, as the following proposition states.

Proposition 5. *For any $\{\lambda, \beta\}$ and initial conditions $M_0 > 0$ and $S_0 = 0$, without any policy – i.e., with $\sigma_{\mathcal{P}} = \{0, 0, 0\}$ for any $t \geq 1$ – a multiplicity of equilibria exist.*

In particular:

i) ***Local indeterminacy of monetary equilibria*** *obtains when private-sector policies $\sigma_t^* \in \sigma^*$ given by $\{S_\tau = 0, m_\tau = D_t\}_{\tau=t}^{\infty}$ feature more than a sequence $\{\Pi_{\tau+1} < \theta^{-1}\}_{\tau=t}^{\infty}$ that satisfies (22) converging to $\Pi^* \equiv 1$.*

In the CRRA case $u(\cdot) = (\cdot)^{1-\sigma}/(1-\sigma)$ with $\sigma > 0$, local indeterminacy obtains when:

$$\left| \frac{(1 + \sigma\beta^{1/\sigma}) + \frac{W^o}{W^y}(1-\sigma)}{(1-\sigma) + \frac{W^o}{W^y}(1 + \sigma\beta^{-1/\sigma})} \right| < 1; \tag{23}$$

otherwise, a unique monetary equilibrium exists.

ii) ***Global indeterminacy of asymptotically autarky equilibria*** *exists for each $s \geq 1$, such that, the private-sector policy $\sigma_t^* \in \sigma^*$ is given by $\{S_\tau = 0, m_\tau = D_t\}_{\tau=1}^{s-1}$ with $\Pi_t \leq \theta^{-1}$ for $t \leq s$, and by*

$$S_t = \frac{W^y - R(\theta)\theta^{-1}W^o}{1 + R(\theta)} - \theta m_t, \qquad \text{and} \qquad m_{t+1} = \theta m_t$$

with $\Pi_{t+1} = \theta^{-1}$ for $t > s$, with $P_s \in (P^*, \theta^{-1}P^*)$ and $m_s = M_0/P_s$.

iii) ***An autarky equilibrium*** *exists where the private-sector policy $\sigma_t^* \in \sigma^*$ at any date $t \geq 1$ is given by $m_t = 0$, $S_t = (W^y + R(\theta)\theta^{-1}W^o)/(1 + R(\theta))$ and $P_t \to \infty$.*

Proof. See Appendix A.4 □

Without policy interventions, the model exhibits two different kinds of indeterminacy.

Local indeterminacy emerges when a continuum of monetary equilibria exists where storage is not used, but inflation remains bounded around its steady state. This occurs, as there are local converging paths of inflation satisfying (22) for $S_t = S_{t+1} = 0$ at any t. Intuitively, local indeterminacy obtains when saving choices are sufficiently insensitive to inflation rates, which happens when income effects are sufficiently strong or β is sufficiently small.[11]

Global indeterminacy may arise instead due to the existence of storage equilibria in which money progressively loses value as consumption inequality between the young and the old emerges. The main force behind this kind of equilibria is the complementarity of storage decisions across generations. Suppose that the generation born at time t decides to increase their storage. This reduces their real money demand – i.e., the amount of consumption they are willing to exchange for money. The return on money from period $t-1$ to period t then has to decline. In turn, storage becomes a relatively more profitable investment for the generation

[11] For example, in the CRRA case with risk aversion σ, the condition of indeterminacy is maximal for $W^o = 0$ and reads as $\beta < ((\sigma-2)/\sigma)^\sigma$; that is, it is characterized by a monotonically increasing upper bound on β as σ increases above 2, with the largest bound given by $\lim_{\sigma\to\infty}((\sigma-2)/\sigma)^\sigma \approx 1/e^2 = 0.13534$. This means that, at least in this example, local indeterminacy emerges only for a small discount factor.

born at time $t-1$. In particular, as storage at time t becomes sufficiently large to lower the return on money strictly below θ, it is optimal for the generation born at time $t-1$ to also store goods.

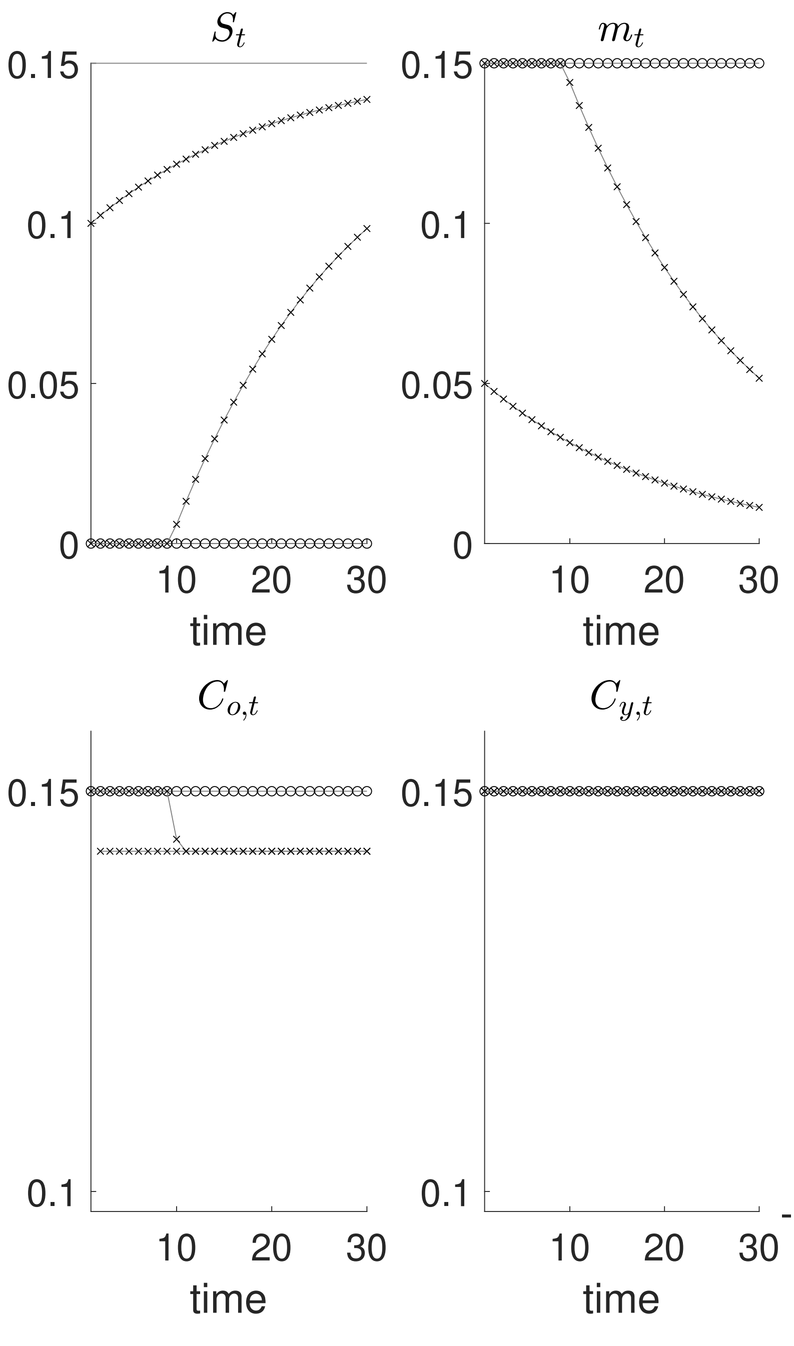

Figure 1 – **Global indeterminacy.** Equilibria with no policy interventions for $u(\cdot) = \log(\cdot)$, $\beta = 1$, $\theta = 0.95$, $W^y = 0.3, W^o = 0, \mathcal{P} = (0, 0, 0)$. Circles denote the monetary equilibrium; cross markers denote two asymptotic autarky equilibria: one that starts at $S_0 = 0.1$ and the other at $S_0 = 0$ with a jump at $S_{10} = 0.006$. Autarky, which is possible in this case, is denoted by a solid line.

In Figure 1, we illustrate a case in which the monetary equilibrium is unique and show its co-existence with storage equilibria. We assume that $u(\cdot) = \log(\cdot)$, $\beta = 1$, $\theta = 0.95$ and $W^y = W = 0.3$ and $W^o = 0$. A *monetary equilibrium* exists where agents never use storage. Agents then perfectly equalize consumption across periods. This equilibrium, which is denoted with a circle marker in Figure 1, is characterized by a constant real demand for money $m_t = W/(1 + R(1))$, constant prices $\Pi_t = 1$, and no storage.

In addition to this equilibrium, there also exist equilibria in which storage and money are both used, and storage progressively crowds out monetary savings. We call this kind of equilibria *asymptotic autarky equilibria*. As storage and money are used at the same time, in these equilibria, $\Pi_t = \theta^{-1}$ holds since arbitrage between the two saving assets must not be possible. Along these paths, real money demand follows the process:

$$m_{t+1} = \theta m_t; \tag{24}$$

that is, lower real money demand today depresses future real money demand, so that storage crowds out money as time goes on. In the end, storage converges to $\lim_{t\to\infty} S_t = W/2$. Given that $M_0/P_t = m_t$ in the absence of intervention and m_t converges to 0, money ultimately has no real value – i.e., $\lim_{t\to\infty} M_0/P_t = 0$. These equilibria are denoted with a cross marker in Figure 1. Importantly, notice that storage can jump in any period from zero to positive since there are positive levels of S_t compatible with $\Pi_t < \theta^{-1}$ for which $S_{t-1} = 0$ is optimal. In the figure we provide an example showing that storage jumps to a positive value at $S_{10} = 0.006$. However, for S_t to be positive, $\Pi_{t+1} = \theta^{-1}$, which implies that $S_{t+1} > S_t$. So, storage can jump from zero to positive at any period, but then it can never go back to zero.

An *autarky equilibrium* exists in the absence of policy interventions. It is represented by a single solid line in Figure 1. In this case, storage is maximal, and the real value of monetary savings is zero, with prices being infinitely large (so that inflation is not defined). Consumption profiles are the same as in an asymptotic autarky equilibrium with storage and money, as the return to savings is the same.

4.2 Equilibrium with Constrained-Optimal Short-Sighted Policies

We now turn to the case in which policy interventions are optimally chosen, as determined by Proposition 4. We first provide a set of equations characterizing the equilibrium outcome and then describe the equilibrium set. This set boils down to the only monetary equilibrium. We finally provide a discussion of why policy interventions lead to a single equilibrium.

Equilibrium Characterization. To start with, let us focus on the equilibrium conditions implied by the private sector. First, by combining the young generation's budget constraint (15) with the optimal level of taxes set by the authority in Proposition 4, we are able to compute the real demand for money at date t:

$$m_t = \frac{W^y + W^o - (2+\lambda)R(\rho_{t+1})\rho_{t+1}^{-1}W^o - (1 + R(\rho_{t+1})(2+\lambda))S_t + \theta S_{t-1}}{(2+\lambda)R(\rho_{t+1})}. \tag{25}$$

Using (18) at date $t+1$, we can recover the actual law of motion for inflation at any state as:

$$\Pi_{t+1} = \frac{1}{R(\rho_{t+1})} \frac{W^y + W^o - (2+\lambda)(R(\rho_{t+1})\rho_{t+1}^{-1})W^o + \theta S_{t-1} - (1 + (2+\lambda)R(\rho_{t+1}))S_t}{W^y - (1+\lambda)(W^o + \theta S_t) - S_{t+1}}, \quad (26)$$

which must always hold in any equilibrium.

Now we investigate the equilibrium set once optimal policy is in play. Formally, this requires the equilibrium allocation to satisfy (26). As shown by the following proposition, this set of equilibria boils down to a unique equilibrium, one in which money is efficiently traded.

Proposition 6 (Global and local price level determination.). *For any $\{\lambda, \beta\}$, given endowments such that $W^y > (1+\lambda)W^o$, and initial conditions $M_0 > 0$ and $S_0 = 0$, there exists a unique equilibrium, in which money is efficiently traded. In such an equilibrium, at any $t \geq 1$:*

(i) $\sigma_t^* \in \sigma^*$ is such that:

$$S_t = 0 \quad \text{and} \quad m_t = \beta \frac{W^y - (1+\lambda)W^o}{2+\lambda};$$

(ii) $\mathcal{P}_t^* \in \sigma_{\mathcal{P}^*}$ is such that:

$$\begin{aligned} \Pi_t &= \frac{1}{u'_{-1}\left(\beta \Pi_t^{-1}\right)\Pi_t^{-1}} = \beta \\ G_t &= \frac{\lambda}{2+\lambda}\left(W^y + W^o\right) \\ T_t &= \frac{1+\lambda-\beta}{2+\lambda}W^y - \frac{1-\beta(1+\lambda)}{2+\lambda}W^o; \text{ and} \end{aligned}$$

(iii) the price level is given by $P_t = M_t/m_t$, where $M_t = \beta M_{t-1}$.

Furthermore, for any $S_t \in (0, (W^y - (1+\lambda)W^o)/(1+R(\theta))]$, a unique equilibrium exists in which consumption is equalized across living agents and storage shrinks over time at the socially efficient rate, reaching the steady state characterized by $(\sigma^*, \sigma_{\mathcal{P}^*})$.

Otherwise, when $W^y \leq (1+\lambda)W^o$, a unique equilibrium exists in which $m_t = S_t = 0$ for all $t \geq 1$ and $\Pi_t = \infty$ for all $t > 1$.

Proof. See Appendix A.5 □

The proposition states that, for the initial condition $S_0 = 0$, the optimal policy eliminates any possible inflation indeterminacy in monetary equilibria. Moreover it fixes the intertemporal rate of return in the monetary equilibrium – i.e., the inverse of inflation – equal to the discount factor β: this is indeed a socially efficient outcome in the spirit of the Friedman rule. To achieve this result, the authority taxes the young generation to buy money at a fixed rate. In particular, both private money holdings and prices shrink at a rate β consistent with a fixed real money demand.

The proposition also states that there exists a unique continuation equilibrium for any *given* positive S_t converging to the unique steady state. This equilibrium is characterized by a level of storage that shrinks towards zero at the socially efficient rate. We show in the proof of Appendix A.5 that any of the paths where $S_t > 0$ has to satisfy a second order differential equation:

$$R(\theta)S_{t+1} - (1 + R(\theta))\theta S_t + \theta^2 S_{t-1} = (R(\theta) - \theta)(W^y + W^o), \tag{27}$$

which does not depend on λ. We also note that the same relation is the first order condition of a unconstrained intertemporal planner problem where money is absent: $\max_{S_t} \{u(c_t) + \beta u(c_{t+1})\}$ subject to $c_t = \theta S_{t-1} - S_t$. Thus, the outcome of myopic and uncoordinated policy interventions is ensuring paths along which any given storage is optimally shared across generations.

The properties of (27) are key to understand the implications of a jump to a off-equilibrium aggregate state $S_t > 0$ and why only one of such path can be an equilibrium. We show that (27) effectively features a saddle-path for any given level of storage, so that only one stable path exists, which leads to the monetary steady state.

Moreover, such a unique path provides for a deflation at the first period after a deviation to positive savings occurs, making the initial deviation form the monetary steady state $S_t > 0$ suboptimal. This result ensures that uniqueness obtains, not because a deviation from equilibrium would prevent the formation of any other equilibrium (as with non-Ricardian policies), but because such deviations are simply not optimal from an individual point of view.

It is also important to notice that, from the point of view of the current authority, any current positive level of storage is sub-optimal as it reduces the availability of resources for current consumption, as shown by (9). Thus, equilibrium allocations are not unconstrained optimal from the point of view of the single authorities, although they are the social first-best (for given public consumption).

Surprisingly, the existence of a unique monetary equilibrium is independent from the size of λ, provided young have savings needs, i.e. $W^y > (1 + \lambda)W^o$.[12] In other words, no matter how little social utilitarian motives matter for optimal policy, there always exists a unique equilibrium providing for the real value of money to be at the socially efficient level. The key intuition for the irrelevance of λ is that the consumption of the government is a fraction of the consumption of the old (which, in this equilibrium, is equal to one of the young), which can always be secured secured through taxes. As a result, whatever the level of λ, the authorities always induce the economy to stay in the monetary equilibrium, in which everyone, authorities included, are better off as overall consumption is larger.[13]

Let us conclude by discussing the autarky situation in which a generation decides to save only using storage. This is not an equilibrium with policy interventions. As we show in the proof of Proposition 6, in an autarky situation, the authority at time t has an incentive to exchange real resources for the the money bought by the young at time $t - 1$ (as a deviation from autarky), no matter how small the deviation is. This leads to an infinite return on money. To see this, suppose that a young individual at time $t - 1$ buys an arbitrarily small but strictly

[12] Notice that this condition is different from the one for which savings are positive in the absence of any policy intervention, that is $W^y > R(\theta)\theta^{-1}W^o$.

[13] This statement generalizes to the case in which the authority gives a sufficiently large relative weight to money holders – i.e., the old generation.

positive amount of money, whereas no one else in either her cohort or the next cohort does – i.e., $m_{t-1} = \epsilon$ and $m_t = 0$, with $\epsilon > 0$ but arbitrarily small. According to (16), any combination $m_{g,t} > 0$ and $M^S_{g,t} = 0$, leads to $\Pi_t \to 0$ and then to an infinite return to money. Because of the profitability of any individual deviation from autarky, autarky cannot be an equilibrium.

Contingent Surplus for Short-run Consumption Equality: an illustration. The key reason that there is now a unique equilibrium, one in which money is the only saving asset, is that private agents have no incentives to deviate and use storage because of out-of-equilibrium policy interventions. These incentives are provided by the balance sheet policy of the authority, whose aim is to sustain consumption equality between the young and the old.

We illustrate this logic in Figure 2. In this figure, we plot the pure monetary equilibrium with circles, but also the continuation of an equilibrium for a *given* S_t with cross markers. The figure is produced with the same parametrization as in Figure 1, except that we now assume that $\lambda = 0.5$. Note that, along the pure monetary equilibrium, because the authority cares about its own consumption, private consumption is lower than in Figure 1, as taxes are raised. On the other hand, the case $\beta = 1$, plotted in the figure, corresponds to a monetary equilibrium in which inflation is equal to one and primary fiscal surplus is zero; that is public spending is completely financed by taxes.

In analogy to Figure 1, we explore a potential equilibrium starting at $S_0 = 0$ with a jump to positive storage at $S_{10} = 0.006$. The dashed line with cross markers denotes the ideal path of storage satisfying (27) that would have sustained such a move. In particular, in analogy to the reasoning in absence of policy, positive storage at time $t = 10$ could be sustained only by a belief of higher storage at time $t = 11$, and so on. Along this path, the increase in storage by the young reduces the real value of private money demand and so generates downward pressure on money return. In this case, the authority reacts by taxing the young to buy money ($m_{g,t} > 0$ and $M^S_t = 0$) in order to sustain its value, and in doing that, it also ensures the consumption level of the old. By Lemma 3, for $\Pi_t = \theta^{-1}$, we get the analogue to (22) with optimal policy interventions:

$$T_t - G_t + m_t = \theta m_{t-1}, \tag{28}$$

where the additional term captures the intervention. In particular, the optimal real surplus decided by the authority in response to past storage choices evolves according to

$$\begin{aligned} T_t &= \left(1 + \lambda - R_t^{-1}\right) \frac{W}{2+\lambda} + \left(1 + R_t^{-1}\right) \left(\frac{S_t - \theta S_{t-1}}{2+\lambda} \right) && (29) \\ G_t &= \frac{\lambda}{2+\lambda} \left(W - S_t + \theta S_{t-1}\right). && (30) \end{aligned}$$

Thus, increasing storage goes along with increasing primary surplus and decreasing (but equally split) consumption. However, storage increases faster than without interventions violating the constraint of positive consumption at some point, which is not possible. So these paths cannot be equilibria.

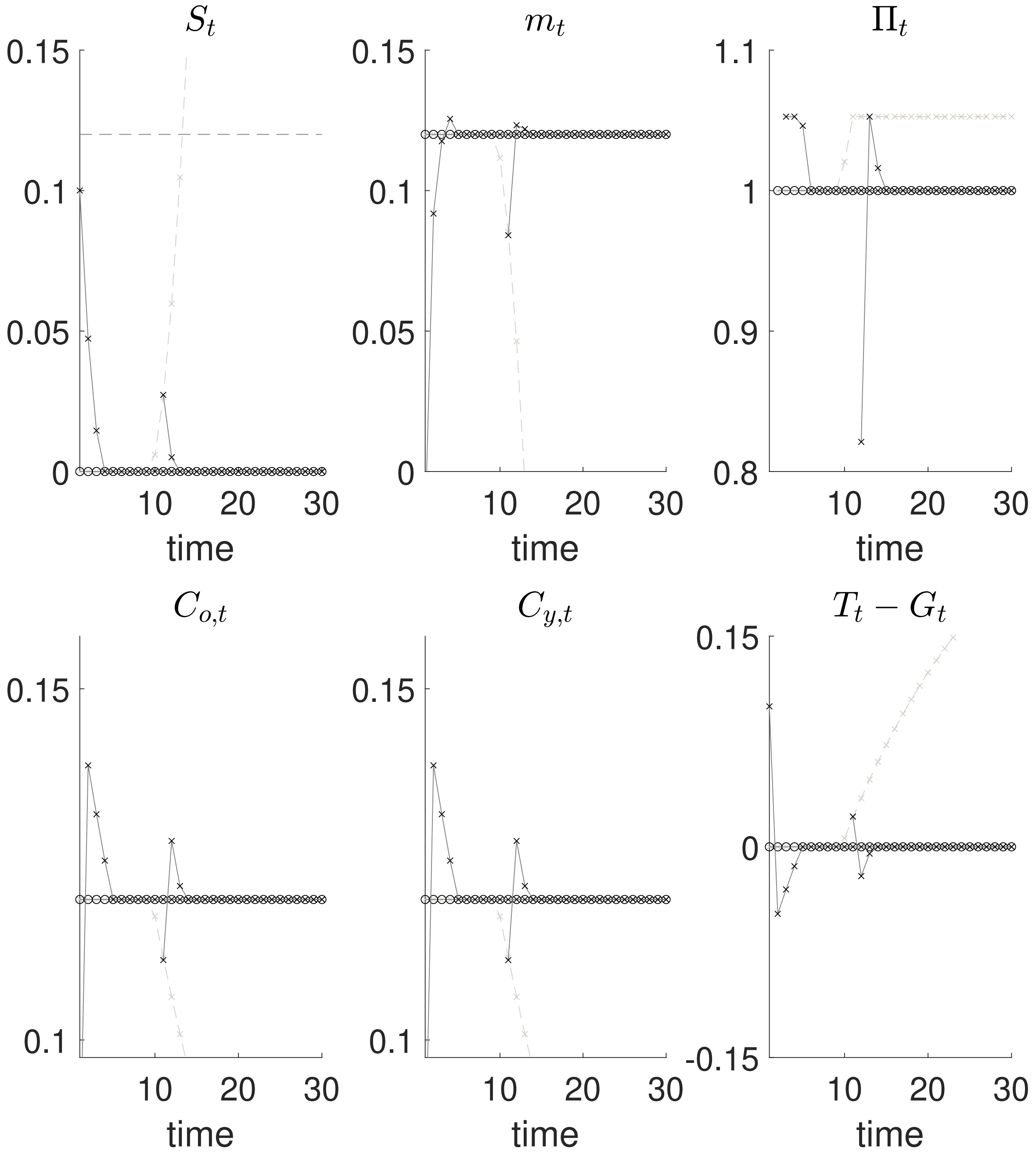

Figure 2 – Equilibria with optimal policy for $\beta = 1$, $\theta = 0.95$, $W = 0.3, \lambda = 0.5$. Circles denote the pure monetary equilibrium; cross markers denote two equilibria with storage: one that starts at $S_0 = 0.1$ and the other at $S_{11} = 0.027$. We also denote with a light grey dashed line the unfeasible path of an equilibrium that starts at $S_0 = 0$ with a jump at $S_{10} = 0.006$, which requires $S_{11} = 0.027$. Autarky, which is not possible in this case, is denoted by a simple dashed line.

As the picture shows, for a *given* positive level of storage at time $t = 11$, there exists a unique continuation equilibrium, denoted by a solid dark line with cross markers, that satisfies

(27) with storage decreasing at time $t = 12$ before converging to zero. Crucially, this implies, according to (26), that an inflation rate Π_{12} from period $t = 11$ to $t = 12$ drops much lower than $\theta^{-1} = 1/0.95$, producing a return on money strictly higher than the one on storage. For that rate of inflation, the young at $t = 11$ would have never optimally chosen to store any unit in storage! By anticipating that no individual would then rationally anticipate $S_{11} > 0$, no jump to positive storage at $S_{11} > 0$ can occur.

Along a path where storage decreases the authority implements a negative surplus: it sells out money in its balance sheet and transfers seigniorage revenues to the young. To understand the optimality of this behavior, notice that a *given* amount of positive storage at time t increases the availability of resources available at time $t + 1$. As stated, storage, in contrast to money, transfers resources across periods, so that there is an optimal decrease in storage that balances the utility of transferring resources to the next generation and the depreciation cost of waiting one more period before consuming. This path is implemented by a sequence of short-sighted policies: as the authority ensures the efficient split of resources between the old and young of two subsequent cohorts at the same period, private-sector choices ensure the efficient split of resources between young and old within the same cohort across two subsequent periods, so that the optimal allocation is achieved.

Finally notice that, in all paths, the authorities' objective of financing their own consumption is completely covered by taxes; thus, primary surplus only results from the implementation of market operations to ensure consumption equality between alive agents. In fact, as Figure 2 shows, in any path in- and out-of-equilibrium consumption equality between young and old is ensured by the policy.

Optimality and Infinite-horizon Policies. The sequence of constrained-optimal myopic interventions leads to the monetary equilibrium in which money is traded efficiently. By implementing optimal short-sighted policies through the market, the authorities effectively select the best allocation according to their short-sighted objective (5). In the following proposition, we show that such short-sighted policies also satisfy optimality conditions when policy is chosen to maximize an analogous long-run objective. We interpret this result as showing that short-run policy objectives may sustain the first-best in a time-consistent way when constrained by private agents' intertemporal considerations.

Formally,

$$\mathbb{W}_t = \sum_{t-1}^{\infty} \beta^{t-1} \mathbb{U}_t \tag{31}$$

is the present value sum of the authority's utility flows. We then have the following.

Proposition 7. *For given ω_{t-1}, the sequence $(\sigma_t, \sigma_{\mathcal{P}}^t)$ solves the date-t Ramsey problem:*

$$\max_{\{C_{o,t}, C_{y,t}, G_t, S_t, M_t, P_t\}_{t\geq 1}} \mathbb{W}_t,$$

subject to the individuals' and authorities' budget constraints (3)-(4) and (6), the individual optimality conditions (10)-(14), and non-negativity constraints $M_t \geq 0$, $S_t \geq 0$, $P_t \geq 0$, at any

$t \geq 1$.

Proof. See Appendix A.6. □

The competitive outcome when policy is fixed according to short-sighted objectives is, therefore, also the best competitive outcome evaluated with long-run objectives. This implies that the constrained short-sighted policy is also the optimal policy under commitment. Hence, the authority never has an incentive to deviate from it, no matter what actions the private sector takes. Said differently, the myopic optimal reaction by the authority coincides with the policy decisions of a forward-looking authority under commitment.

4.3 Discussion: Committing to constant positive surpluses.

This section discusses the relation between the workings of the sequence of short-sighted optimal policies described above and the commitment to constant positive surpluses as analyzed by Sims (2013).[14] This is an important benchmark to determine to what extent fiscal policy should be active in- or out-equilibrium.

We note three main differences.

i) Optimality. The first difference concerns the optimality of our policy. As Sims (2013) emphasizes, committing to a sequence of positive surpluses is generally inefficient. In his case, any positive surplus, no matter how small, distorts the individual intertemporal saving choice by moving inflation away from its optimal value (one), causing inefficient deflation. In our model, public spending and agents' utility are both objectives of the myopic authorities, who seeks to maximize them for any state of the economy, adjusting surplus in response to possible out-of-equilibrium moves.

ii) Out-of-equilibrium behavior. A second important difference is that, in our model, it is the authority's out-of-equilibrium behavior that fixes agents' beliefs in a way that anchors inflation to the optimal value. To appreciate the implications of the mechanism it is instructive to notice that for $\beta = 1$, our short-sighted policies, in contrast to Sims (2013), provide for zero surpluses along the unique equilibrium path, whereas surpluses are optimally different from zero only out-of-equilibrium. To borrow from Bassetto (2005), this is because Sims (2013) implies a commitment to *actions*, whereas in our Proposition 6, the optimal policy is a *strategy*, where the authority's actions are functions of aggregate actions of the private sector.

iii) Absence of long-run restrictions. Finally, in our model, price determinacy does not require any long-run restriction but just a short-sighted social preference for consumption equality. This stands in sharp contrast with the commitment to constant positive surpluses that is beyond Sims (2013) and, more generally, the literature on fiscal dominance, in which price level determination is based on the anticipation of the long-run sequence of positive surplus, to which a benevolent long-living authority should ideally commit.

To better appreciate this last point, we work out the case of an optimal *fixed* surplus in our model, restricting the set of policies that we consider to $\mathcal{P}_t = \left\{\bar{T}, \Delta_t, 0\right\}$, where $\bar{T} > \bar{G} = 0$ (i.e.,

[14] Sims (2013) studies constant positive surpluses in the context of the Fiscal Theory of the Price Level (FTPL). Here, our focus is different as we assume, in contrast with the FTPL that the budget constraint of the authority is always satisfied, in- and off-equilibrium.

$\tilde{\lambda} = \lambda = 0$). The market-clearing condition for money then implies that:[15]

$$m_t = m_{t-1}\Pi_t^{-1} - \bar{T} + \bar{G}. \tag{32}$$

It is easy to note that, when storage is used – that is, with $\Pi_t = \theta^{-1}$ – , an equilibrium in which prices go to infinity ($P_t \to \infty$ – i.e., $m_t \to 0$ for any $t \geq s$ for some $s > 0$) cannot satisfy (32) and so cannot be part of an equilibrium. Of course, any strictly positive level of surplus would work. However, different levels of surplus determine different inflation rates. In particular, by taking $\lambda = 0$, we get the optimal inflation rate $\Pi_t = \beta$ with

$$\bar{T}^* = (1-\beta)\frac{W_y - W_o}{2}, \tag{33}$$

consistent with the equilibrium path envisaged by Proposition 6. We can show that in this case, local indeterminacy is eliminated, as the equilibrium inflation path is determined by:

$$\Pi_{t+1} - \beta \;=\; \Lambda(\Pi_t - \beta), \tag{34}$$

where

$$\Lambda \equiv \frac{W^o + W^y + \sigma\beta\left(W^y - W^o\right)}{\beta\left(W^o + W^y\right) - \sigma\beta\left(W^y - W^o\right)} > 1,$$

for any $\beta \in (0,1]$, $W^y > W^o \leq 0$, $\sigma > 0$ (see Appendix D for details). Therefore, an optimally-chosen fixed surplus is able to fix both local and global indeterminacy in this model, relying on the violation of equilibrium restrictions in the long run, exactly as a Taylor rule would do.

Now, notice how the dynamic nature of (34), in analogy to (22), contrasts with the condition uncovered in our main case (26). In our case, inflation is determined by an entirely static relation for local dynamics, when restricting (26) to $\rho_t = \Pi_t^{-1}$ and $S_t = 0$ at any time. This is because the optimal strategy fixed by the authority, ensuring *current* consumption equality, reacts to simultaneous off-equilibrium moves of the aggregate actions, ensuring that a unique value is compatible with equilibrium optimality. In this sense, the short-sighted optimal policy cuts the dynamic dependence of inflation rates from future outcomes in local dynamics. On the contrary, in (34), as in (22), inflation is determined by a dynamic relation, entailing backward implications from future states. Therefore, even if in the current model, fixing a surplus at an optimal level deals with both global and local indeterminacy, in general, the reliance on long-run restrictions may fail to ensure local indeterminacy in cases in which the short-sighted optimal policy ensures it. This conjecture, whose demonstration is out of the scope of the present study, is a potentially interesting avenue for future research.

[15]Let us first note that under this restriction, Δ_t is constrained: the budget constraint at time t is:

$$\bar{T} - \bar{G} = m_{g,t} - \frac{M_t^S}{P_t}.$$

As a result, when $\bar{T} - \bar{G} > 0$, $M_t^S = 0$ and $m_{g,t} = \bar{T} - \bar{G}$.

5 Limits to guarantees

Equilibrium uniqueness is obtained in Proposition 6 under the assumption that the authority can set taxes contingent on private agents' actions without any constraint (except feasibility). In this section, we discuss how limitations to the authority's contingent tax plan may hamper the uniqueness of the equilibrium and eventually change the set of equilibria. We consider situations in which the authority cannot modify taxes or in which there is a cap on its fiscal capacity – the latter case can be interpreted as an extreme form of distortion from taxation resulting in infinite utility costs when taxes are set above a threshold. We show that when the utility of public spending is sufficiently high relative to fiscal capacity, monetary stability gets lost and multiple equilibria arise.

The main reason is that the inability to adjust taxes leads to a trade-off between monetary stability and the authority's own expenditures. When the authority needs more resources for its own consumption, it cannot resist raising seigniorage, which harms money stability. Importantly, in this setting, the authority still controls the price level, but it can end up in a coordination failure with agents; a Laffer curve of seignorage exists. Key to this result is the simultaneous timing of the portfolio choice of agents: the authority takes as given the private saving choices, which affect the availability of resources. In particular, in the attempt to finance higher public spending, the authority produces a higher inflation rate. A higher inflation rate leads to the reallocation of private savings from money holdings to storage, reducing the total amount of available resources. In such a situation, both actual seignorage revenues and private consumption decrease.

5.1 Fixed taxes

We first explore the equilibrium outcome when the authority cannot change taxes in reaction to saving choices. This situation can be interpreted as one in which the authority commits ex-ante to a given fiscal stance, as well as one in which the authority is a private entity and, therefore, lacks any fiscal power (i.e., ability to generate revenues at will). We show that such a situation leads to a trade-off for the authority between intervening in the money market and financing its own expenditures. When the level of taxes is sufficiently high, the uniqueness of the monetary equilibrium is possible, but money is traded inefficiently. When, instead, the level of taxes is too low, the authority favors its expenditures over purchases of money, thus giving rise to multiple equilibria. In particular, a new equilibrium is possible in which money and storage coexist, but money never loses value.

To ease exposition further, we focus on the case in which only the young receive a constant endowment, so $W^o = 0$, and $W^y = W$.

Optimal policy. Fixing taxes amounts to restricting the policy's space to $\hat{\mathcal{P}}_t = (\Delta_t, G_t, \bar{T})$, where, to maintain the analogy with the benchmark model, taxes on the young $T_t = \bar{T}$ are taken as fixed through time. In contrast to the previous subsection, the authority can back its interventions in the money market only by adjusting its expenditures.

As in the benchmark case, the first step is to rewrite the consumption of the young independently of date-t variables:

$$C_{y,t} = W - \bar{T} - S_t - m_t = \frac{R(\rho_{t+1})(W - \bar{T})}{1 + R(\rho_{t+1})}, \tag{35}$$

and real saving then writes as $S_t + m_t = C_{y,t}/R(\rho_{t+1})$. By combining the authority's budget constraint with the saving equation $S_t + m_t = C_{y,t}/R(\rho_{t+1})$, we obtain spending G_t as follows:

$$G_t = \frac{W + R(\rho_{t+1})\bar{T}}{1 + R(\rho_{t+1})} - S_t - \Pi_t^{-1} m_{t-1}. \tag{36}$$

This equation clarifies that, in case of need of additional resources, the authority has no other option than to drain them via higher inflation, as the other terms are policy-independent. Importantly, this ability to raise resources through inflation depends on the real money holdings of the previous period m_{t-1}: the less the old generation has invested in money (and the more it has invested in storage), the more the authority has to raise inflation to finance its expenditures. There is, then, a complementarity between the use of storage by agents and the incentive to raise inflation by the authority in the next period. Such complementarity does not show up when taxes can be adjusted.

Consumption and portfolio choices are still as described in Section 3.1. Instead, the policy is different. In analogy with the first part of Proposition 4, the optimal policy at date t is given by $\hat{\mathcal{P}}_t^* = \text{argmax}\{\mathbb{U}_t\}$ – i.e., the one that, at each time t, maximizes the flow of the current authority's utility. In contrast to the previous case, the current flow of utility in the authority's objective is now given by:

$$\mathbb{U}_t = u\left(C_{y,t}\right) + u\underbrace{\left(\Pi_t^{-1} m_{t-1} + \theta S_{t-1}\right)}_{=C_{o,t}} + \tilde{\lambda} u \underbrace{\left(\frac{W + R(\rho_{t+1})\bar{T}}{1 + R(\rho_{t+1})} - S_t - m_{t-1}\Pi_t^{-1}\right)}_{=G_t}, \tag{37}$$

where $C_{y,t}$, according to (35), is independent from policy. The solution to this problem, $\hat{\mathcal{P}}_t^* = \{\Delta_t, G_t, \bar{T}\}$, is Markovian and given by

- $\Delta_t(\sigma_t)$ is such that:

$$\Pi_t = \frac{(1 + \lambda) m_{t-1}}{\frac{W + R(\rho_{t+1})\bar{T}}{1 + R(\rho_{t+1})} - \lambda\theta S_{t-1} - S_t} \quad if \lim_{m_{t-1}\to\infty} \lambda C_{o,t} \leq \lim_{m_{t-1}\to\infty} G_t, \tag{38}$$

$$\Pi_t \to \infty \quad \text{otherwise}, \tag{39}$$

  according to (16);

- $G_t(\sigma_t)$ given by (36)

at any t, where, again, $\lambda = 1/u'_{-1}(\tilde{\lambda})$. By losing the ability to change taxes in response to private saving choices, the authority loses the ability to influence the demand of savings and, thus, the consumption of the young. There is now a trade-off in the use of the price for money as an instrument. On the one hand, the authority may reduce consumption inequality by lowering the price for money. On the other hand, it can increase public expenditures by increasing the price for money. Which force prevails depends on the initial level and the importance of public expenditures.

Equilibrium. We now turn to the equilibrium outcome when policies solve (38). In contrast to Section 4, where taxes are unconstrained, we obtain here a multiplicity of equilibria depending on the level of taxes. The following proposition describes these findings.

Proposition 8. *For any β and initial conditions $M_0 > 0$ and $S_0 = 0$, multiple equilibria exist depending on $\{\bar{T}, \lambda\}$.*

(a) *Provided that*

$$\hat{\pi} \equiv (1+\lambda)\frac{W-\bar{T}}{W+\bar{T}} \leq \theta^{-1},$$

an inefficient monetary equilibrium exists such that, for any $t \geq 1$:

(i) $\sigma_t^* \in \sigma^*$ is such that:

$$S_t = 0 \quad \text{and} \quad m_t = \frac{W-\bar{T}}{1+R(\Pi^{-1})}, \tag{40}$$

(ii) $\mathcal{P}_t^* \in \sigma_{\hat{\mathcal{P}}*}$ is such that:

$$\Pi_t = \hat{\pi}, \tag{41}$$

$$G_t = \frac{(1+\lambda)(W+R(\hat{\pi}^{-1})\bar{T}) - (W+\bar{T})}{(1+R(\hat{\pi}^{-1}))(1+\lambda)}; \tag{42}$$

(iii) the price level is given by $P_t = M_t/m_t$ where $M_t = \hat{\pi}M_{t-1}$.

(b) *Furthermore, when*

$$\frac{\bar{T}}{W} < \frac{\lambda\theta}{1+R(\theta)+\lambda\theta},$$

- *a money-storage equilibrium also exists for each $s \geq 1$ such that, (40)-(42) holds for $t < s$, and for any $t \geq s$:*

  (i) $\sigma_t^* \in \sigma^*$ is such that:

$$S_t = \theta S_{t-1} + \frac{W+R(\theta)\bar{T} - \theta(1+\lambda)(W-\bar{T})}{1+R(\theta)},$$

$$m_t = \frac{W-\bar{T}}{1+R(\theta)} - S_t \quad \text{with} \quad \lim_{t\to\infty} m_t = \frac{\theta\lambda(W-\bar{T}) + (R(\theta)-1)\bar{T}}{(1-\theta)(1+R(\theta))} \geq 0;$$

  (ii) $\mathcal{P}_t^* \in \sigma_{\hat{\mathcal{P}}*}$ is such that:

$$\Pi_t = \theta^{-1} = \frac{1}{R(\theta)},$$

$$G_t = \lambda\theta\frac{W-\bar{T}}{1+R(\theta)}; \text{ and}$$

  (iii) the price level is given by $P_t = M_t/m_t$ where $M_t = \theta^{-1}M_{t-1}$.

- *and an autarky equilibrium exists where $\sigma_t^* \in \sigma^*$ is such that $S_t = (W - \bar{T})/(1+R(\theta))$, $m_t = 0$, and $\mathcal{P}_t^* \in \sigma_{\hat{\mathcal{P}}*}$ such that $\Pi_t > \theta^{-1}$, $G_t = \bar{T}$ and $P_t \to \infty$, for any $t \geq 1$.*

Proof. See Appendix A.7. □

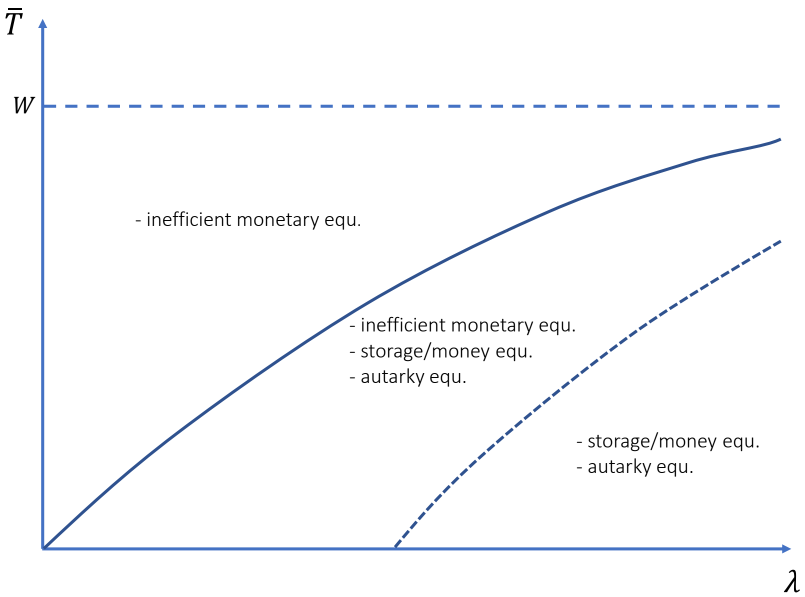

Figure 3 – Equilibria sets in the space $(\lambda, \bar{T})$.

The proposition shows the role of fiscal capacity in nailing down the set of equilibria. The conditions for the existence of equilibria are illustrated in Figure 3. The set of equilibria crucially depends on the level of $\bar{T}$ – the fiscal capacity of the authority – and λ – the importance of public spending. With fixed taxes, the authority systematically uses its seigniorage power to balance the consumption of the old vis-à-vis public expenditure. When taxes are fixed too low relative to the importance of public spending, the authority can only adjust expenditures to purchase money, and, thus, it trades off the welfare gains of money trading against its cost of cutting expenditures. Whenever this equilibrium exists, the possibility of autarky also exists. The trade-off between public spending and agents' consumption does not arise when taxes can be freely set, as, then, the authority has sufficient tools to adjust its expenditures. In such a case, the authority sustains the value of money to improve the total amount of consumption goods available at that time and sets taxes to ensure the fraction that it needs.

It is instructive to remark that, in the extreme case where the authority cannot tax and puts no weight on its spending ($\bar{T} = \lambda = 0$), the set of equilibria in Proposition 8 coincides with the set of equilibria in the absence of policy intervention. This means that in the absence of a fiscal counterpart, the authority cannot do better than the market, in line with Wallace (1981b).

Figure 4 illustrates the different types of equilibria. We use the same parameter values as in Figure 4 but with fixed taxes at $\bar{T} = 0.056$. This level of taxes is compatible with the existence of an inefficient monetary equilibrium and a money-storage equilibrium.

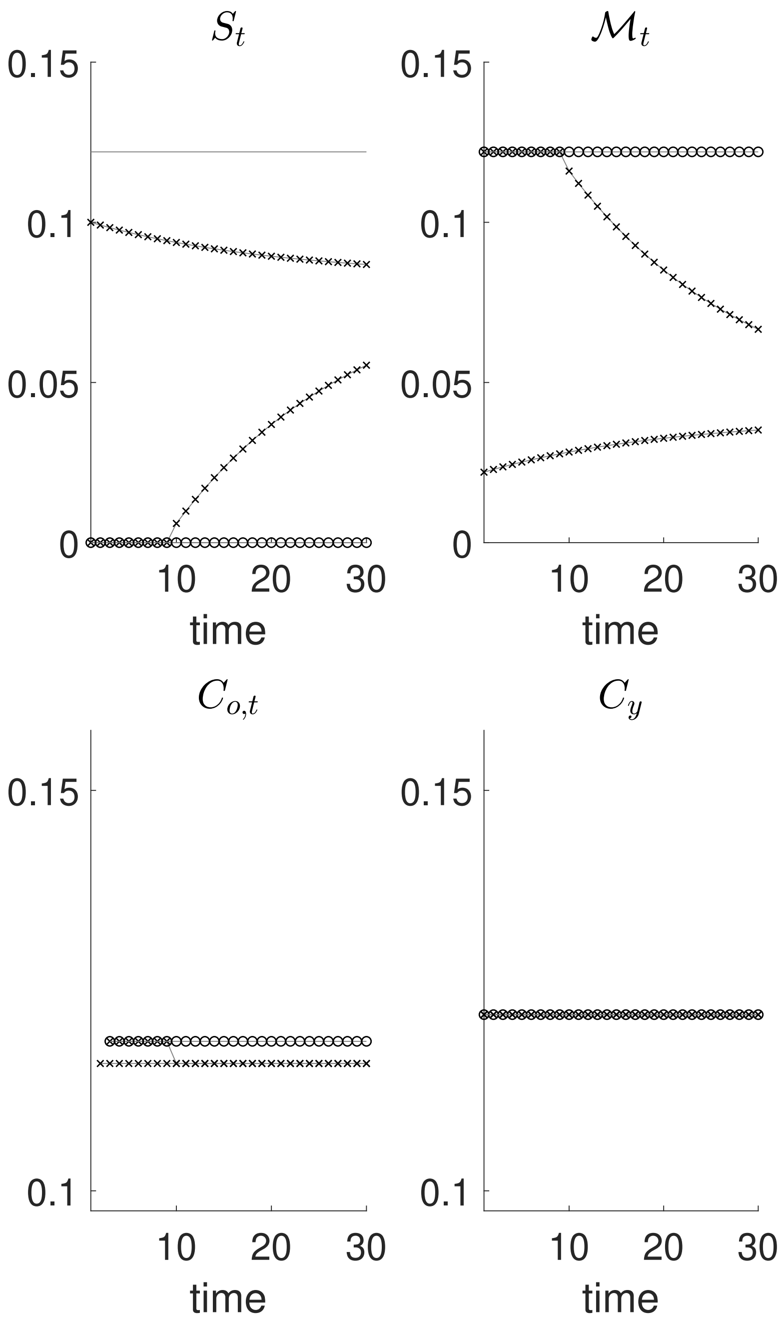

Figure 4 – Equilibria with optimal policy for $\beta = 1$, $\theta = 0.95$, $W = 0.3$, $\lambda = 0.5$ and $\bar{T} = 0.056$. Circles denote the monetary equilibrium; cross markers denote two money-storage equilibria: one that starts at $S_0 = 0.1$ and the other at $S_0 = 0$ with a jump at $S_{10} = 0.006$. Autarky, which is not possible in this case, is denoted by a simple dashed line.

The inefficient monetary equilibrium is denoted by a solid line with circle markers in Figure 4. In this equilibrium, money is the only saving asset, but the level of inflation is generically inefficient, increasing in λ and decreasing in $\bar{T}$. Note that, in this equilibrium, the primary fiscal surplus is negative, indicating that the authority covers part of its spending by creating and selling money – i.e., generating seigniorage.

Money-storage equilibria are denoted by a solid line with cross markers in Figure 4. In these equilibria, money and storage are jointly used, but money never fully loses value. This is possible because, by selling money, the authority makes inflation equal to θ despite the fact that the young keep their real money demand constant. These equilibria obtain for any positive level of storage, whether initial or occurring as a jump from the inefficient monetary equilibrium. Both cases are illustrated in the figure, analoguous to Figures 1 and 2. In particular, the asymptotic convergence point of any equilibrium where storage is used is the steady state of the money-storage equilibrium in this case.

A Laffer curve of seigniorage. In the storage-money equilibrium, inflation is higher than in the inefficient monetary equilibrium; however, the primary fiscal surplus is *less* negative, showing that actual seigniorage revenues are lower. Effectively, in the storage-money equilibrium, the consumption by both the old and the authority is lower. This means that storage-money equilibrium is the result of a coordination failure between private agents and the authority, entailing a sort of Laffer curve of seigniorage.

Let us expand on the reasons behind the coordination failure. Suppose that the young decide to save in storage. This action increases the resources available in the next period and, in particular, the consumption of the old. Since the authority wants to equalize the marginal utility of its own consumption and that of the old, it sells money to drain resources from the old. This increases inflation until it matches the return on storage, making the young indifferent between saving in money or storage, as in the absence of policy. However, whereas in the absence of policy, the equilibrium inflation rate is achieved by subsequent decreases in private money demand, in this equilibrium, it is achieved by subsequent increases in money supply. This is what allows real money demand to stay constant at a level lower than in the inefficient monetary equilibrium. In analogy with the Laffer curve of taxation, we can interpret the lower level of real money holdings as a lower "tax base" of seignorage, which pushes the authority to tax more money holdings to extract resources. This higher "tax rate" corresponds to a higher inflation rate. The expectation of such a higher inflation rate makes the more intense use of storage self-fulfilling and the willingness to tax more through more seignorage self-defeating. The agents would all benefit from being in the monetary equilibrium, but, individually, the optimality of their portfolio may lead them to use storage. The authority would also benefit from being in the monetary equilibrium to expand its tax base and obtain a higher revenue from seignorage, but it cannot because it does not control future authorities' decisions.

This inability to resist taxation through seigniorage also underlies the existence of the autarky equilibrium. When taxes are too low, and, thus, government expenditures are low as well, the government may even have the incentive to drive the price level to negative values so as to tax money holdings. Since negative price levels are not feasible, prices are sent to infinity: such an incentive prevents any credible deflation, which is what the optimal unconstrained policy was able to generate in the absence of private demand for money. As a consequence, autarky can be an equilibrium outcome.

The role of private portfolio choices and sequential authorities. The presence of the storage technology and, thus, of a portfolio choice by agents, together with sequential authorities, is key to the result on multiplicity; but, on the contrary, considering sequential authorities is not sufficient alone. This can be observed in the absence of the storage option: in our model, this is equivalent to setting $\theta = 0$. In this case, Proposition 8 implies that there is only one equilibrium, the inefficient monetary one. The presence of storage choices, instead, affects the availability of resources, which the authority takes as given, as it acts after the private sector moves. Our finding is then consistent with Nicolini (1996), who shows that a sequence of authorities can always implement a single equilibrium, even when paying negligible attention to households' welfare, but only in the absence of a storage technology: this case would correspond to a situation when $\theta = 0$, which, as we underlined, leads also to one equilibrium in our setting. Note, however, that credibility stems from trigger strategies in Nicolini (1996), while it follows from short-run redistribution concerns by the authority in our case.

5.2 Upper bound on taxation

In this subsection, we generalize the above findings by considering a bound on taxation; that is, we explore the case in which taxes are fully flexible conditional on being lower than a certain cap. We show that, when this constraint is sufficiently tight, multiple equilibria can emerge, as characterized in Proposition 8; otherwise, equilibria as described in Proposition 6 hold.

We now assume that taxes on the young generation have to satisfy:

$$T_t \leq \hat{T}, \tag{43}$$

at any t, with $\hat{T} \geq 0$.[16] Given our result that the date-t optimal policy is $\mathcal{P}_t = \text{argmax}\{\mathbb{U}_t\}$ at any t in the unconstrained case and $\hat{\mathcal{P}}_t = \text{argmax}\{\mathbb{U}_t\}$ at any t in the constrained case, we obtain that, at time t, the authority implements $\hat{\mathcal{P}}_t$ when (43) binds and $\mathcal{P}_t$ if not. Leveraging on the results that we have already derived, we can show the following proposition:

Proposition 9. *When*

$$\hat{T} \geq \frac{\lambda + 1 - \beta}{\lambda + 2} W,$$

(i) *the constraint* (43) *does not bind in equilibrium;*

(ii) *a unique equilibrium exists where only money is used as described by Proposition 6.*

Otherwise, when

$$\hat{T} \leq \frac{\lambda + 1 - \beta}{\lambda + 2} W,$$

(i) *the constraint* (43) *always binds in equilibrium; and*

(ii) *the set of equilibria is described by Proposition 8 with taxes T_t fixed at $\hat{T}$;*

[16] Such an upper on taxation may result from an extreme form of distortionary cost of taxation, where it is costless to set taxes up to $\hat{T}$ but arbitrarily large for any higher value.

in particular, only money is used in equilibrium when also

$$\hat{T} \geq \frac{\lambda\theta}{1 + R(\theta) + \lambda\theta} W.$$

Proof. See Appendix A.8 $\square$

When its fiscal capacity is constrained, the authority faces a trade-off between monetary stability and its expenditures, as in the case of fixed taxes (Proposition 8). When the constraint is tight enough, this trade-off results in multiple equilibria, and monetary stability cannot necessarily be ensured.

Indeed, the lack of resources leads the authority to act in conflict with monetary stability: the long-run value of monetary stability is not aligned with the short-run value of expenditures. This misalignment would even make it valuable to commit to keeping fixed government expenditures – a commitment that was not necessary with unconstrained taxes.

When this bound is sufficiently large, $\hat{T}/W \geq (\lambda+1-\beta)/(\lambda+2)$, the monetary equilibrium without money creation is the single equilibrium, as in Proposition 6. In this case, the constraint $T_t \leq \hat{T}$ does not bind in equilibrium. Interestingly, the constraint can bind off-equilibrium, when $(1+\lambda-R(\theta)^{-1})/(2+\lambda) \geq \hat{T}/W \geq (\lambda+1-\beta)/(\lambda+2)$, but without preventing the interventions to be sufficient to rule out other equilibria, in which storage is used.

When the bound becomes tighter, $\hat{T}/W \leq (\lambda+1-\beta)/(\lambda+2)$, the constraint $T_t \leq \hat{T}$ always binds, in- and off-equilibrium. As a result, we are back to a situation as with fixed taxes, described in Proposition 8. In such a situation, when $\hat{T}/W \geq \lambda\theta/(1+R(\theta)+\lambda\theta)$, the monetary equilibrium is still the unique equilibrium, but one in which the authority is creating money to finance its expenditures given that, with taxes only, the level of spending would be suboptimal. It is only when $\hat{T}/W < \lambda\theta/(1 + R(\theta) + \lambda\theta)$ that multiple equilibria may emerge.

6 Conclusion

The question that we have asked in this paper is: to which extent may the pursuit of short-term fiscal objectives sustain long-term monetary stability? The answer provided by this paper is that short-term fiscal objectives can sustain money stability as a by product in so far governments do not have any constraint in taxation, no matter how small is the importance of private agents' utility in the authority objective.

We have generalized and extended the basic framework of Sims (2013)'s overlapping generation model by adding a sequence of short-sighted fiscal authorities with the objective of maximizing current utilitarian welfare and public spending. We have derived their optimal time-consistent use of taxation and money operations. We showed that the optimal policy pursuing short-term consumption equality implies that the authority buys money using tax revenues when private agents deviate from an equilibrium where money is the only saving asset. As a result, there exists a single equilibrium. In this unique equilibrium money is efficiently traded so that the inflation is at the socially optimal value.

We have emphasized that the reasons why we obtain a unique equilibrium differ from the ones for which a commitment to positive surpluses can do as suggested by Sims (2013), consistently with the fiscal theory of the price level. Commitments to positive surpluses achieve a unique equilibrium based on long-run restrictions, in contrast to the time-consistent optimal policy in our model.

Our result holds irrespective of whether money are public liabilities or not; it applies to any asset that it actually *held and used* as money as money by the private sector. In this sense, although public authorities are not committed to provide fiscal-backing to private digital currencies, they could gain it ex-post. This scenario is one in which fundamentally worthless private assets become so widely used as a store of value that a confidence crisis may otherwise trigger major welfare costs.

A Proofs

A.1 Proof of Lemma 1

Proof. Optimal private policies come directly from the first order condition $u'\left(W^y - T_t - D_t\right) = \beta\rho_{t+1}u'\left(\rho_{t+1}D_t + W^o\right)$.

To prove the statement about $R(\rho)$, note that $R(\rho) > \rho$ is equivalent to $u'_{-1}(\beta\rho) > 1$, which is true. Indeed, u' is decreasing since $u'' < 0$. As a result u'_{-1} is decreasing as well. Thus, given that $\beta\rho < 1$, we have $u'_{-1}(\beta\rho) > u'_{-1}(1)$. Given $u'(\cdot)$ are multiplicative, it has to be , $u'(1)u'(1) = u'(1)$ and as a result, $u'(1) = 1$ and $u'_{-1}(1) = 1$. This allows to conclude $u'_{-1}(\beta\rho) > 1$ whenever $\rho < 1$. □

A.2 Proof of Lemma 2

Proof. Use the market clearing condition for money (1) and the equilibrium value of real money holdings (8) into (6) and solve for T_t. Then substitute T_t into (3).

The potential heterogeneity noted in Remark 3.1 does not impact on this result. We can use $D_{i,t} = D_t$ – i.e. $m_{i,t} - m_t = S_t - S_{i,t}$ – to get (15) in case of a cross-sectional heterogeneity in $\{m_{i,t}, S_{i,t}\}$. □

A.3 Proof of Proposition 4

Proof. With symmetric private choices we have $C_{i,o,t} = C_{o,t} = \rho_{t+1}D_{t-1}$ for any i. Hence, given (15) in Lemma 2, it is immediate to show that the optimal $\Pi_t(\sigma_t)$ and $G_t(\sigma_t)$ are the ones that solve (17), i.e. the ones that equalize consumption between the young and the old, with T_t set to satisfy the budget constraint of the authority.

Let us here discuss how the potential heterogeneity noted in Remark 3.1 may impact on this result. This implies that the impact of marginal changes in inflation on the average utility of the old can be written as

$$\frac{\partial \int u(C_{i,o,t})di}{\partial \Pi_t} = -\int u'(C_{i,o,t})\frac{m_{i,t-1}}{\Pi_t^2}\,di$$

which introduces another potential concern for redistribution within the old. However, we note that, along an equilibrium we necessarily have $C_{i,o,t} = C_{o,t} = \rho_{t+1}D_{t-1}$ and so

$$-\int u'(C_{i,o,t})\frac{m_{i,t-1}}{\Pi_t^2}\,di = -u'(C_{o,t})\frac{\int m_{i,t-1}\,di}{\Pi_t^2}$$

which proves that the symmetry in private choices is without loss of generality for the characterization of the equilibrium set.

□

A.4 Proof of Proposition 5

Local indeterminacy. To show existence we work out the CRRA case. The law of motion of inflation in the absence of intervention is:

$$\Pi_t = \frac{m_{t-1}}{m_t} = \frac{1+\beta^{-1/\sigma}\Pi_{t+1}^{1/\sigma-1}}{1+\beta^{-1/\sigma}\Pi_t^{1/\sigma-1}}\frac{W^y - (\beta^{-1}\Pi_t)^{1/\sigma}W^o}{W^y - (\beta^{-1}\Pi_{t+1})^{1/\sigma}W^o}.$$

Note that a fixed point for this law of motion is $\Pi_t = \Pi_{t+1} = 1$. By writing this law of motion as a function $\Pi_t = f(\Pi_t, \Pi_{t+1})$, we can write the following partial derivatives:

$$\frac{\partial f(\Pi_t, \Pi_{t+1})}{\partial \Pi_{t+1}}|_{(\Pi_t,\Pi_{t+1})=(1,1)} = \frac{\beta^{-1/\sigma}\left[(1-\sigma)W^y + W^o\left(1+\sigma\beta^{-1/\sigma}\right)\right]}{\sigma(1+\beta^{-1/\sigma})(W^y - \beta^{-1/\sigma}W^o)}$$
$$\frac{\partial f(\Pi_t, \Pi_{t+1})}{\partial \Pi_t}|_{(\Pi_t,\Pi_{t+1})=(1,1)} = -\frac{\partial f(\Pi_t, \Pi_{t+1})}{\partial \Pi_{t+1}}|_{(\Pi_t,\Pi_{t+1})=(1,1)}$$

Around the fixed point, the dynamic of Π_t and Π_{t+1} is:

$$\Pi_{t+1} - 1 = \frac{1 - \frac{\partial f(\Pi_t,\Pi_{t+1})}{\partial \Pi_t}|_{(\Pi_t,\Pi_{t+1})=(1,1)}}{\frac{\partial f(\Pi_t,\Pi_{t+1})}{\partial \Pi_{t+1}}|_{(\Pi_t,\Pi_{t+1})=(1,1)}}(\Pi_t - 1)$$

which implies:

$$\Pi_{t+1} - 1 = (\Pi_t - 1)\frac{W^y(1+\sigma\beta^{1/\sigma}) + W^o(1-\sigma)}{W^y(1-\sigma) + W^o(1+\sigma\beta^{-1/\sigma})}.$$

or with $X = W^o/W^y$:

$$\Pi_{t+1} - 1 = (\Pi_t - 1)\frac{(1+\sigma\beta^{1/\sigma}) + X(1-\sigma)}{(1-\sigma) + X(1+\sigma\beta^{-1/\sigma})}.$$

The condition for local convergence is then:

$$\left|\frac{(1+\sigma\beta^{1/\sigma}) + X(1-\sigma)}{(1-\sigma) + X(1+\sigma\beta^{-1/\sigma})}\right| < 1$$

Global indeterminacy. It is easy to note that $\Pi_{t+1} = 1 < \theta^{-1}$ and $S_t = 0$ for any t is an equilibrium; one in which money is always used and storage never. We refer to this equilibrium as the *pure monetary equilibrium.*

To check if there exist an equilibrium where storage is used jointly with money we should use the arbitrage condition in (1). For $S_t > 0$ at time t we must have $\Pi_{t+1} = \theta^{-1}$. In this case, (22) obtains as

$$S_{t+1} = \theta S_t + (1-\theta)\bar{S}, \tag{44}$$

with

$$\bar{S} \equiv \frac{W^y - R(\theta)\theta^{-1}W^o}{1+R(\theta)}$$

which implies $S_{t+1} \geq S_t$, given the limit $S_t \leq \bar{S}$ for each date t. Therefore we obtain that, if storage is used in one period, it must necessarily be used on a larger extent next period. In fact, an equilibrium for each initial level of storage $S_1 \in \left[0, \bar{S}\right)$ (S_0 is not an optimal choice, i.e. (22) is not an equilibrium condition for S_0) exists such that storage is always used jointly with money. It is easy to show that in the long run, storage and the real money balance satisfy:

$$\lim_{t\to\infty} S_t = \frac{W}{2} \text{ and } \lim_{t\to\infty} \frac{M_0}{P_t} = lim_{t\to\infty}m_t = 0 \tag{45}$$

for any initial level of storage S_1, where the latter obtains as a consequence of the former because of the expression of D_t in 1. There are equilibria in which storage is always used, prices grows at a rate $1/\theta$ and money loses value in time until it eventually become worthless; let us call them the *asymptotic autarky equilibria.*

Importantly, all asymptotic autarky equilibria do not necessarily feature storage at date-0 and it is possible to construct asymptotic autarky equilibria where storage is not used until a certain date s after which it is always used. In fact, notice that $S_{s-1} = 0$ only requires that $\Pi_s < \theta^{-1}$, that is

$$0 \leq S_s < (1-\theta)\,\bar{S}.$$

Thus, at each date t, after having only used money in past periods, it is possible to start using storage. What is peculiar of the environment with constant endowment is that once storage is used it will used for ever; this is because, for a given S_t, (22) implies a certain S_{t+1} which has the property $S_{t+1} \geq S_t$, given the limit $S_t \leq \bar{S}$ for each t.

Finally, there also exists a *pure autarky equilibrium* defined as one in which $S_t = \bar{S}$ and $m_t = M_0/P_t = 0$ for each t in which money is never used and the price level is infinite and grows at a rate larger than $1/\theta$.

A.5 Proof of Proposition 6

1. Case $W^y > (1+\lambda)W^o$. We first start by the case $W^y \geq (1+\lambda)W^o$. As we will show, this ensures that savings is positive.

To establish our results, we use equations (25) and (26) to investigate how optimally chosen policies affect equilibrium outcomes. These two equations are derived as follows.

We substitute (18) into (21) to get

$$\begin{aligned}
T_t &= \frac{1}{1+\lambda}\left(\frac{W^y-(1+\lambda)(W^o+\theta S_{t-1})-S_t}{(2+\lambda)}\right)+\frac{\lambda}{1+\lambda}(W^y-S_t)-m_t,\\
&= \frac{1}{1+\lambda}\left(\frac{1}{2+\lambda}+\lambda\right)(W^y-S_t)-\frac{W^o+\theta S_{t-1}}{2+\lambda}-m_t,\\
&= \frac{1}{1+\lambda}\left(\frac{1+2\lambda+\lambda^2}{2+\lambda}\right)(W^y-S_t)-\frac{W^o+\theta S_{t-1}}{2+\lambda}-m_t,\\
&= \frac{1+\lambda}{2+\lambda}(W^y-S_t)-\frac{W^o+\theta S_{t-1}}{2+\lambda}-m_t,
\end{aligned}$$

We substitute for T by using the above into $m_t = D_t - S_t$ where D_t is given by (13) and get

$$\begin{aligned}
m_t &= \frac{W^y-\left(\frac{1+\lambda}{2+\lambda}(W^y-S_t)-\frac{W^o+\theta S_{t-1}}{2+\lambda}-m_t\right)-R(\rho_{t+1})\rho_{t+1}^{-1}W^o}{1+R(\rho_{t+1})}-S_t\\
&= \frac{W^y-\frac{1+\lambda}{2+\lambda}W^y+\frac{1+\lambda}{2+\lambda}S_t+\frac{W^o+\theta S_{t-1}}{2+\lambda}+m_t-R(\rho_{t+1})\rho_{t+1}^{-1}W^o-(1+R(\rho_{t+1}))S_t}{1+R(\rho_{t+1})}\\
&= \frac{\frac{1}{2+\lambda}W^y+m_t+\frac{W^o-(2+\lambda)R(\rho_{t+1})\rho_{t+1}^{-1}W^o}{2+\lambda}+\frac{1}{2+\lambda}\theta S_{t-1}+\frac{1+\lambda-(2+\lambda)(1+R(\rho_{t+1}))}{2+\lambda}S_t}{1+R(\rho_{t+1})}\\
&= \frac{\frac{1}{2+\lambda}W^y+m_t+\frac{W^o-(2+\lambda)R(\rho_{t+1})\rho_{t+1}^{-1}W^o}{2+\lambda}+\frac{1}{2+\lambda}\theta S_{t-1}-\frac{1+(2+\lambda)R(\rho_{t+1})}{2+\lambda}S_t}{1+R(\rho_{t+1})}
\end{aligned}$$

and, once solving for m_t we have

$$m_t = \frac{M_t}{P_t} = \frac{W^y+W^o-(2+\lambda)R(\rho_{t+1})\rho_{t+1}^{-1}W^o+\theta S_{t-1}-(1+(2+\lambda)R(\rho_{t+1}))S_t}{(2+\lambda)R(\rho_{t+1})}.$$

To get inflation, we use again (18) to get

$$\Pi_t^{-1}m_{t-1} = \frac{M_{t-1}}{P_t} = \frac{W^y-(1+\lambda)(W^o+\theta S_{t-1})-S_t}{(2+\lambda)}.$$

Combining the last two we get

$$
\begin{aligned}
\Pi_{t+1} &= \frac{M_t}{P_t}\left(\frac{M_t}{P_{t+1}}\right)^{-1} \\
&= \frac{1}{R(\rho_{t+1})}\frac{W^y + (1-(2+\lambda)(R(\rho_{t+1})\rho_{t+1}^{-1}))W^o + \theta S_{t-1} - (1+(2+\lambda)R(\rho_{t+1}))S_t}{W^y - (1+\lambda)(W^o + \theta S_t) - S_{t+1}}.
\end{aligned}
$$

In the rest of this proof, we first show that there exists a pure monetary equilibrium where only money is traded. Then we show that neither an asymptotic autarky nor an autarky equilibrium exists. We use this latter result to investigate the continuation of an equilibrium after a deviation of the private sector (i.e. the private sector starts to use the storage technology) and we show that such a deviation is suboptimal.

a) The pure monetary equilibrium. Let us first show that there exists an equilibrium where only money is traded. More formally, the pure monetary equilibrium where $S_t = 0$ at each t is an equilibrium. This can be easily seen by checking that $S_t = 0$ at any t. In this case we have

$$
\Pi_{t+1} = \frac{1}{R(\Pi_{t+1}^{-1})}\frac{W^y + (1-(2+\lambda)(R(\Pi_{t+1}^{-1})\Pi_{t+1}))W^o}{W^y - (1+\lambda)W^o}
$$

that is

$$
\begin{aligned}
R(\Pi_{t+1}^{-1})\Pi_{t+1}(W^y - (1+\lambda)W^o) &= W^y + (1-(2+\lambda)(R(\Pi_{t+1}^{-1})\Pi_{t+1}))W^o \\
R(\Pi_{t+1}^{-1})\Pi_{t+1}(W^y - (1+\lambda-2-\lambda)W^o) &= W^y + W^o \\
R(\Pi_{t+1}^{-1})\Pi_{t+1}(W^y + W^o) &= W^y + W^o \\
u'_{-1}(\beta\Pi_{t+1}^{-1})\Pi_{t+1}^{-1}\Pi_{t+1} &= 1 \\
u'_{-1}(\beta\Pi_{t+1}^{-1}) &= 1 \\
\beta\Pi_{t+1}^{-1} &= 1 \\
\Pi_{t+1} &= \beta,
\end{aligned}
$$

where $\Pi_{t+1} = \beta < \theta^{-1}$ at any t. The property $u'_{-1}(\beta\Pi_{t+1}^{-1}) = 1$ implies $\beta\Pi_{t+1}^{-1} = 1$ is a property of multiplicative functions. Note that money holdings in this equilibrium are such that $m_t = D_t = \beta\frac{W^y-(1+\lambda)W^o}{2+\lambda}$.

b) Non existence of any equilibria where storage is used. Let us now show that there cannot be an equilibrium where storage is positive at some date $T \geq 0$. Let us proceed by contradiction. Storage is initially at $S_0 = 0$. Suppose, that storage becomes positive at period $t > 1$ so that $S_{t-1} = 0$. If this is part of an equilibrium we should have that necessarily, $\theta = \Pi_{t+1}^{-1}$. In such a case, we can use (25) and (26) to show

$$
\begin{aligned}
\Pi_{t+1} &= \frac{1}{R(\rho_{t+1})}\frac{W^y + (1-(2+\lambda)(R(\rho_{t+1})\rho_{t+1}^{-1}))W^o + \theta S_{t-1} - (1+(2+\lambda)R(\rho_{t+1}))S_t}{W^y - (1+\lambda)(W^o + \theta S_t) - S_{t+1}} \\
R(\theta)\theta^{-1} &= \frac{W^y + (1-(2+\lambda)(R(\theta)\theta^{-1})W^o + \theta S_{t-1} + (1+(2+\lambda)R(\theta))S_t}{W^y - (1+\lambda)(W^o + \theta S_t) - S_{t+1}}
\end{aligned}
$$

$$
\begin{aligned}
(R(\theta)\theta^{-1} - 1)(W^y + W^o) - R(\theta)(1+\lambda)S_t - R(\theta)\theta^{-1}S_{t+1} &= \theta S_{t-1} - (1+(2+\lambda)R(\theta))S_t \\
(R(\theta)\theta^{-1} - 1)(W^y + W^o) - R(\theta)\theta^{-1}S_{t+1} &= \theta S_{t-1} - (1+R(\theta))S_t
\end{aligned}
$$

i.e. that storage S_{t+1} has to satisfy the following second order differential equation:

$$R(\theta)S_{t+1} - (1 + R(\theta))\theta S_t + \theta^2 S_{t-1} = (R(\theta) - \theta)(W^y + W^o).$$

Then necessarily $S_{t+1}t = (1 + R(\theta))\theta S_t + (R(\theta) - \theta)(W^y + W^o) > S_t$, so that $\theta = \Pi_{t+2}^{-1}$ also holds. Standard results on second order difference equations point out that, as $\theta < 1$ and $\theta/R(\theta) < 1$ are the two roots of the associated characteristic equation[17], and so on, finally showing that $\{S_{t+\tau}\}_\tau$ is on a monotonically increasing path converging to $\bar{S} = (W^y + W^o)/(1-\theta)$. Given $\bar{S}$ value higher than available endowments, but consumption cannot be negative, so a contradiction obtains.

c) Non existence of a pure autarky equilibrium. Here we prove that an equilibrium in which real money balance are valueless starting at some date t – i.e. $m_\tau = M_\tau/P_\tau = 0$ starting a $t \geq \tau$ – does not exist.

Indeed, suppose that there exists a date t such that $m_t = 0$. Optimal policy at date-$t+1$ is, according to Proposition 4, to set date $t+1$ inflation at 0. This then implies that the return on money is infinite and exceeds the return on storage.

Such 0 inflation rate is implemented when $m_t = 0$ simply by setting $m_{g,t+1} > 0$ (and thus $M_{t+1}^S = 0$) at date $t+1$, whatever the value of m_{t+1}.

Moreover, this results extends by continuity to any arbitrarily small deviation: suppose that one agent deviates so that $m_t = \epsilon$ with $\epsilon > 0$ arbitrarily small. Date-$t+1$ inflation rate $\Pi_{t+1} = \epsilon/(W^y - (1+\lambda)(\theta S_{t-1} + W^o) - S_t)$ is also arbitrarily close to 0 when ϵ is close to 0 – the denominator in fact is strictly positive as we show below in Lemma 10 for the relevant case $W^y > (1+\lambda)W^o$. As a result, the return on money also exceeds the return on storage θ, thus making the deviation profitable.

Lemma 10. *$W^y - (1+\lambda)(\theta S_{t-1} + W^o) - S_t \geq 0$ with equality if and only if $W^y = (1+\lambda)W^o$.*

Proof. At date 1, we have:

$$S_1 \leq D_1 = \frac{W^y - T_1 - R(\rho_2)\rho_2^{-1}W^o}{1 + R(\rho_2)}.$$

with $T_1 = (1+\lambda)/(2+\lambda)W^y - (W^o - S_1)/(2+\lambda) - D_1$. We then obtain that:

$$S_1 \leq \frac{W^y + W^o - (2+\lambda)R(\rho_2)\rho_2^{-1}W^o}{(2+\lambda)R(\rho_2)} \leq \frac{W^y - (1+\lambda)W^o}{1 + (2+\lambda)R(\rho_2)}$$

which yields the result at date 1.

Suppose that $S_{t-1} \leq \Theta_{t-1}(W^y - (1+\lambda)W^o)$. As before, we obtain that:

$$S_t \leq \frac{W^y - (1+\lambda)W^o + \theta S_{t-1}}{1 + (2+\lambda)R(\rho_{t+1})}$$

and thus that

$$S_t \leq \frac{1 + \Theta_{t-1}\theta}{1 + (2+\lambda)R(\rho_{t+1})}.$$

As a result:

$$\Theta_t \leq \frac{1 + \Theta_{t-1}\theta}{1 + (2+\lambda)R(\rho_{t+1})}.$$

[17] We have shown that indeed $R(\rho) > \rho$ in A.1.

Let us consider then the sequence defined by $\eta_1 = \Theta_1$ and

$$\eta_t = \frac{1 + \eta_{t-1}\theta}{1 + (2+\lambda)R(\theta)}.$$

This sequence converges from below to $1/(1 + (2+\lambda)R(\theta) - \theta)$.

Let us then show that an upper bound to $(1+\lambda)\theta S_{t-1} + S_t$ is $W^y - (1+\lambda)W^o$, that is:

$$(1+\lambda)\theta\Theta_{t-1} + \Theta_t \leq 1$$

Writing this inequality with η_t, we find that it is implied by:

$$\eta_{t-1} \leq \frac{\lambda + 2}{1 + (1+\lambda)(1 + (2+\lambda)R(\theta))}$$

As $\eta_{t-1} \leq 1/(1 + (2+\lambda)R(\theta) - \theta)$, this boils down to comparing

$$\frac{1}{1 + (2+\lambda)R(\theta) - \theta} \leq \frac{2+\lambda}{1 + (1+\lambda)(1 + (2+\lambda)R(\theta))}.$$

which is satisfied as: $1 + (2+\lambda)R(\theta) \geq 1 + (2+\lambda)\theta$. □

d) Uniqueness of the equilibrium continuation starting at a given $S_t > 0$. Suppose that $S_t > 0$ is part of an equilibrium. Let us show that this contradicts date-t agent's optimality condition.

To do this, we shall prove that the equilibrium continuation after a deviation S_t leads to a path for storage $\{S_t, ..., S_{t+n}\}$ so that S_τ with $\tau \in \{1, .., n-1\}$ is decreasing and $S_{t+n} = 0$, taking S_t as given. We show the existence of this path in d.1). We then show in d.2) that such a decreasing path leads money to have a strictly better return than storage at date t. That is $S_t > 0$ cannot be part of an equilibrium with constant endowment: the young at date $t-1$ are therefore better off not investing in storage.

Finally, we show that the path is unique in d.3).

d.1.) The equilibrium continuation features decreasing storage. Let us first show that the continuation equilibrium after storage S_t features decreasing storage.

First, notice that, if such an equilibrium path for storage exists, then storage goes to 0 at some point. More formally, there exists $n > 0$ such that $S_{t+n} = 0$. Suppose that it is not the case and storage is always used, then S_τ converges to $S = W/(1-\theta)$, which is not feasible as we showed in b).

Second, for any n we can construct a path for storage $\{S_t, ..., S_{t+n}\}$ such that $S_{t+n} = 0$ and, at any date $\tau \in \{t+1, ...t+n-1\}$,

$$R(\theta)S_{\tau+1} - (1 + R(\theta))\theta S_\tau + \theta^2 S_{\tau-1} = (R(\theta) - \theta)(W^y + W^o). \tag{46}$$

Indeed, this latter equation defines a linear difference equation of order 2 for S_τ and $S_{t+n} = 0$ as well as S_t define two boundary conditions. As a result, there exists a unique path $\{S_{t+1}, ..., S_{t+n-1}\}$ solving the linear difference equation combined with the boundary conditions.

Our objective here is showing that any potential continuation of equilibrium leads to a decreasing path for storage. More precisely, let us show that the sequence of S_τ is decreasing:

Lemma 11. *Suppose that there exists $t+n$ such that $S_{t+n} = 0$ and for all τ such that $t < \tau < t+n$*

$$R(\theta)S_{\tau+1} + \theta^2 S_{\tau-1} - (R(\theta) - \theta)(W^y + W^o) = \theta(1 + R(\theta))S_\tau$$

then $S_t > S_{t-1} > ... > S_{t+n} = 0$.

Proof. We proceed by iteration. Let us first show that $S_{t+n-2} > S_{t+n-1}$. At date $t+n-1$, we have

$$R(\theta)S_{t+n} + \theta^2 S_{t+n-2} - (R(\theta) - \theta)(W^y + W^o) = \theta(1 + R(\theta))S_{t+n-1}$$

Using the fact that $S_{t+n} = 0$, we can then write:

$$\frac{\theta}{1+R(\theta)}S_{t+n-2} - \frac{R(\theta)-\theta}{\theta(1+R(\theta))}(W^y + W^o) = S_{t+n-1}$$

Given

$$\frac{\theta}{1+R(\theta)} < 1 \quad \text{and} \quad \frac{R(\theta)-\theta}{\theta(1+R(\theta))}(W^y + W^o) > 0,$$

then $S_{t+n-1} < S_{t+n-2}$.

Suppose that that $S_{t+n-1} < S_{t+n-2} < ... < S_\tau$. Let us show that $S_{\tau-1} > S_\tau$. We can write at date τ:

$$\frac{R(\theta)}{\theta}S_{\tau+1} + \theta S_{\tau-1} - \frac{R(\theta)-\theta}{\theta}(W^y + W^o) = (1 + R(\theta))S_\tau$$

□

d.2.) Optimal portfolio decision at date t. We show here that having a decreasing path for storage after a deviation S_t leads to a return on money at date t so that $S_t > 0$ is suboptimal.

Suppose indeed that $S_t > 0$. For this to happen, we need that the return $\rho_{t+1} = \theta$. Let us show that this is not consistent with households' optimal portfolio decision.

At time t, the return on money after a deviation S_t is (as $S_{t-1} = 0$):

$$\frac{P_t}{P_{t+1}} = R(\rho_{t+1})\frac{W^y - (1+\lambda)(W^o + \theta S_t) - S_{t+1}}{W^y + (1 - (2+\lambda)(R(\rho_{t+1})\rho_{t+1}^{-1}))W^o - (1 + (2+\lambda)R(\rho_{t+1}))S_t}$$

Using Lemma 11, $S_{t+1} < S_t$ and the return satisfies:

$$\frac{P_t}{P_{t+1}} > R(\rho_{t+1})\frac{W^y + W^o - (2+\lambda)(W^o + \theta S_t) - S_t(1-\theta)}{W^y + W^o - (2+\lambda)(R(\rho_{t+1})\rho_{t+1}^{-1})W^o - (1 + (2+\lambda)R(\rho_{t+1}))S_t}$$

Suppose that $\rho_{t+1} = \theta$ that would be consistent with $S_t > 0$. We have:

$$\frac{P_t}{P_{t+1}} > R(\theta)\frac{W^y + W^o - (2+\lambda)(W^o + \theta S_t) - S_t(1-\theta)}{W^y + W^o - R(\theta)\theta^{-1}(2+\lambda)(W^o + \theta S_t) - S_t}$$

As $R(\theta)\theta^{-1} > 1$, $S_t > 0$ and $\theta < 1$, the right-hand term is strictly larger than θ: there is there an arbitrage possibility with money, a contradiction with household's optimal portfolio decision.

d.3.) Uniqueness of the equilibrium continuation after S_t. Let us show that there exists a unique continuation of an equilibrium after S_t.

In what we describe above, for every integer n', we can build a unique sequence solving (46) that we denote $\{S_t^{n'}, ..., S_{t+n'}^{n'}\}$ so that $S_{t+n'}^{n'} = 0$, with n' being the number of periods that storage needs to get back to 0. However, in an equilibrium, the sequence should also be such that $S_{t+n'} = 0$ is optimal, which requires $S_{t+n'-1}$ to satisfy $\theta^2 S_{t+n'-1} < (R(1) - \theta)(W^y + W^o)$, i.e. that money return is strictly higher than return on storage.

Let us show that this implies that there exists a unique n, such that storage goes back to 0, that is $S_{t+n} = 0$ and $\theta^2 S_{t+n-1}^n < (R(1) - \theta)(W^y + W^o)$. To this purpose, let us show the following lemma:

Lemma 12. *S^n_{t+n-1} is decreasing with n.*

Proof. First, let us show that $S^1_t = S_t > S^2_{t+1}$. Indeed, $\theta^2 S_t - (R(\theta) - \theta)(W^y + W^o) = \theta(1 + R(\theta))S^2_{t+1} > S_t$.

Let us extend this proof to n. To this purpose, let us note that:

$$\theta^2 S^n_{t+n-2} - (R(\theta) - \theta)(W^y + W^o) = \theta(1 + R(\theta))S^n_{t+n-1}$$
$$\theta^2 S^{n+1}_{t+n-1} - (R(\theta) - \theta)(W^y + W^o) = \theta(1 + R(\theta))S^{n+1}_{t+n}$$

As a result, $S^{n+1}_{t+n} < S^n_{t+n-1}$ if and only $S^{n+1}_{t+n-1} > S^n_{t+n-2}$. Let us investigate whether $S^{n+1}_{t+n-1} > S^n_{t+n-2}$. To this purpose, let us note that:

$$R(\theta)(S^{n+1}_{t+n} - S^n_{t+n-1}) + \theta^2(S^{n+1}_{t+n-2} - S^n_{t+n-3}) = \theta(1 + R(\theta))(S^{n+1}_{t+n-1} - S^n_{t+n-2})$$
$$(1 + R(\theta))(S^{n+1}_{t+n} - S^n_{t+n-1}) = \theta(S^{n+1}_{t+n-1} - S^n_{t+n-2})$$

We can infer two results from these equations. On the one hand, there exists $A_{t+n-1}(\theta) > 1$ such that $A(\theta)(S^{n+1}_{t+n-1} - S^n_{t+n-2}) = (S^{n+1}_{t+n-2} - S^n_{t+n-3})$. On the other hand, $S^{n+1}_{t+n-1} > S^n_{t+n-2}$ if and only if $S^n_{t+n-3} > S^{n+1}_{t+n-2}$.

Let us proceed by iteration: suppose that there exists $A_\tau(\theta) > 1/\theta$ such that for some τ:

$$R(\theta)(S^{n+1}_{\tau} - S^n_{\tau-1}) + \theta^2(S^{n+1}_{\tau-2} - S^n_{\tau-3}) = \theta(1 + R(\theta))(S^{n+1}_{\tau-1} - S^n_{\tau-2})$$
$$A(\theta)(S^{n+1}_{\tau} - S^n_{\tau-1}) = \theta(S^{n+1}_{\tau-1} - S^n_{t+n-2})$$

We then obtain:

$$\theta^2(S^{n+1}_{\tau-2} - S^n_{\tau-3}) = \theta\left((1 + R(\theta)) - \frac{R(\theta)}{A(\theta)}\right)(S^{n+1}_{\tau-1} - S^n_{\tau-2})$$

As a result there exists $A_{\tau-1}(\theta) > 1$ such that:

$$(S^{n+1}_{\tau-2} - S^n_{\tau-3}) = A_{\tau-1}(\theta)(S^{n+1}_{\tau-1} - S^n_{\tau-2})$$

In the end, we obtain by iteration that

$$S^{n+1}_{t+n-1} - S^n_{t+n-2} = A_{t+n-1}(\theta) \times .... \times A_{t+1}(\theta)(S^{n+1}_{t+1} - S_t)$$

Given that all the As are positive and $S_t > S^{n+1}_{t+1}$, we then obtain that $S^{n+1}_{t+n} < S^n_{t+n-1}$. As a result, S^n_{t+n-1} is a decreasing function of n. □

Given that for all τ, we have

$$\frac{\theta}{R(\theta)}S_{\tau-1} - \frac{R(\theta) - \theta}{R(\theta)\theta}(W^y + W^o) \geq S_\tau,$$

we can find a sufficiently large n' such that $\theta^2 S^{n'}_{t+n'-1} < (R(1) - \theta)(W^y + W^o)$. Using Lemma 12, there exists a unique n such that the sequence of storage decisions $\{S_t, ..., S_{t+n}\}$ is such that $\theta^2 S^n_{t+n-1} < (R(1) - \theta)(W^y + W^o)$ and $\theta^2 S^n_t > (R(1) - \theta)(W^y + W^o)$ at any previous date.

e) Intertemporal efficiency of storage. Let us finally look at the intertemporal efficiency of storage decisions. As we show, the path for storage resulting from optimal policy and private decisions is the unique solution to: for any $t \geq 0$:

$$R(\theta)S_{t+1} - (1 + R(\theta))\theta S_t + \theta^2 S_{t-1} = (R(\theta) - \theta)(W^y + W^o),$$

for some initial S_0 and $\lim_{t\to\infty} S_t = 0$. Let us compare this allocation to what a central planner would do when confronted to an intertemporal allocation problem having access to storage,– in this approach, it is as if the central planner can perfectly redistribute resources within periods – that is:

$$\max_{\{S_t\}_{t\geq 0}} \sum_{t\geq 0} \beta^t u(c_t)$$
$$c_t = \theta S_{t-1} + W^y + W^o - S_t,$$
$$S_t \geq 0,$$
$$c_t \geq 0.$$

The first order condition to this problem is:

$$\beta^t u'(c_t) = \eta_t + \mu_t$$
$$\eta_t = \eta_{t+1}\theta + \Gamma_t$$

with Γ_t the Lagrange multiplier associated with $S_t \geq 0$, η_t the one associated with the budget constraint and μ_t the one associated with $c_t \geq 0$.

When $S_t > 0$ and $c_t > 0$, $\Gamma_t = \mu_t = 0$ and rewriting the first order condition yields:

$$\theta S_{t-1} + W^y + W^o - S_t = u'_{-1}(\beta\theta)\left(\theta S_t + W^y + W^o - S_{t+1}\right).$$

When arranging terms and multiplying both sides by θ, we find:

$$R(\theta)S_{t+1} - ((1 + R(\theta))\theta S_t + \theta^2 S_{t-1} = (R(\theta) - \theta)(W^y + W^o). \tag{47}$$

If this holds in any future period, S_t converges to $(W^y + W^o)/((1 - \theta)$, this leads to $c_t = 0$. In addition, the transversality condition for the central planner problem writes:

$$\lim_{t\to\infty} \beta^t u'(c_t) S_t = 0$$

which implies, as $u' > 0$, that S_t converges to 0. This transversality condition then implies that (47) does not hold in any future period and S_t goes back to 0 in a finite number of periods.

The path of storage resulting from the equilibrium between private decisions and optimal policy is the unique path that solves (47) and also converges back to the monetary equilibrium in which $S_t = 0$. As a result, this path also solves the central planner's problem.

2. Case where $W^y \leq (1 + \lambda)W^0$. Let us first show that $m_t = 0$ for all t.

Suppose instead that $D_t > 0$ for some period t. From Proposition 4, if both $m_t > 0$ then $\Pi_{t+1} < 0$, which contradicts optimality. As a result, $\Pi_{t+1} = \infty$ and $m_t = 0$.

As a result, $T_t = \frac{\lambda}{1+\lambda}(W^y - S_t)$ and

$$D_t = S_t = \frac{W^y + \lambda S_t - (1 + \lambda)R(\theta)\theta^{-1}W^o}{(1 + R(\theta))(1 + \lambda}$$

and then:

$$S_t = \frac{W^y - (1 + \lambda)R(\theta)\theta^{-1}W^o}{1 + (1 + \lambda)R(\theta)}$$

As $W^y - (1 + \lambda)R(\theta)\theta^{-1}W^o \leq 0$, this cannot be and we have $S_t = 0$.

A.6 Proof of Proposition 7

Let us now show that $\left(\sigma^t_{\mathcal{P}^*}, \sigma^*\right)$ leads to the authority's first best allocation. To start with, let us first write the date-t Ramsey problem of the authority:

$$
\begin{aligned}
\max_{\{C_{o,t}, C_{y,t}, G_t, S_t, M_t, P_t\}_{t\geq 1}} & \sum_{t=1}^{\infty} \beta^{t-1} \left(u(C_{o,t}) + u(C_{y,t}) + \tilde{\lambda} u(G_t) \right), \\
\text{s.t. } & C_{o,t} - W^o - \theta S_{t-1} - \frac{M_{t-1}}{P_t} \leq 0, \\
& u'(W^0 - D_t) = \rho_{t+1} u'(W^y + \rho_{t+1} D_t), \\
& \rho_{t+1} = \max\{\theta, \Pi_{t+1}^{-1}\} \\
& \text{if } \Pi_{t+1}^{-1} > \theta, S_t = 0 \text{ and } M_t/P_t = D_t, \\
& \text{if } \Pi_{t+1}^{-1} = \theta, S_t + M_t/P_t = D_t, \\
& \text{if } \Pi_{t+1}^{-1} < \theta, S_t = D_t \text{ and } M_t/P_t = 0, \\
& C_{y,t} - W^y + S_t + \frac{M_t}{P_t} + T_t \leq 0, \\
& \frac{M_{t-1}}{P_t} - \frac{M_t}{P_t} + G_t - T_t \leq 0, \\
& M_t \geq 0, \quad S_t \geq 0, \quad P_t \geq 0.
\end{aligned}
$$

To solve this problem, we look at a relaxed version where we do not include the constraints associated with households' first order conditions: for a given S_0 and M_0,

$$
\begin{aligned}
\max_{\{C_{o,t}, C_{y,t}, G_t, S_t, M_t, P_t\}_{t\geq 1}} & \sum_{t=1}^{\infty} \beta^{t-1} \left(u(C_{o,t}) + u(C_{y,t}) + \tilde{\lambda} u(G_t) \right) \\
\text{s.t. } & C_{o,t} - W^o - \theta S_{t-1} - \frac{M_{t-1}}{P_t} \leq 0 && (\zeta_t) \\
& C_{y,t} - W^y + S_t + \frac{M_t}{P_t} + T_t \leq 0 && (\mu_t) \\
& \frac{M_{t-1}}{P_t} - \frac{M_t}{P_t} + G_t - T_t \leq 0 && (\gamma_t) \\
& M_t \geq 0 && (\eta_t) \\
& S_t \geq 0 && (\omega_t) \\
& P_t \geq 0 && (\epsilon_t)
\end{aligned}
$$

In this problem, we have already take into account the market clearing condition for money.

The first order conditions of this problem are:

$$
\begin{aligned}
C_{o,t}: & \quad \beta^{t-1} u'(C_{o,t}) = \zeta_t \\
C_{y,t}: & \quad \beta^{t-1} u'(C_{y,t}) = \mu_t \\
G_t: & \quad \beta^{t-1} \tilde{\lambda} u'(G_t) = \gamma_t \\
S_t: & \quad \theta \zeta_{t+1} = \mu_t + \omega_{t+1} \\
M_t: & \quad \frac{-\zeta_{t+1} + \gamma_{t+1}}{P_{t+1}} = \frac{-\mu_t + \gamma_t}{P_t} + \eta_t \\
T_t: & \quad \mu_t - \gamma_t = 0 \\
P_t: & \quad (-\zeta_t + \gamma_t) M_{t-1} + (\mu_t - \gamma_t) M_t + P_t^2 \epsilon_t = 0
\end{aligned}
$$

where ζ_t, μ_t, γ_t, ω_t, η_t, ϵ_t are Lagrangian associated to the constraints as showed above.

It is inefficient to use only the storage technology, so that $M_t > 0$. In this case, $\eta_t = 0$ and, combining the focs for T_t and M_t, we obtain that at each t:

$$C_{y,t} = C_{o,t}$$
$$G_t = \lambda C_{o,t}$$

with $\lambda = 1/(u')^{-1}(\tilde{\lambda})$, are optimal as provided by $\mathcal{P}_t^*$. T_t is then the one satisfying the budget constraint. A social planner would equalize consumption of the young and the old generations and choose public consumption as a fraction of them. Note that a solution also exists in which the consumption of the old and the young are not equalized, money is used and $P_t \to \infty$. Those are the same conditions characterizing the authority's optimal time-consistent policy.

Finally, note that portfolio decisions that are solutions to Lemma 1 are also solutions of the first order conditions of the private sector, thus satisfying the constraints of the Ramsey problem. To see this, let us note that

$$\frac{u'(C_{y,t})}{\beta u'(C_{o,t+1})} = \frac{\mu_t}{\zeta_{t+1}} = \rho_{t+1}$$

where ρ_{t+1} is the equilibrium return on savings as defined in the text. This is the same optimality conditions for private saving choices (i.e. on D_t). Let us now turn to portfolio composition. According to the first order condition for S_t, we note that $S_t > 0$, i.e. $\omega_{t+1} = 0$, if and only if $\rho_{t+1} = \theta$. As private agents do, the social planner would use storage only when the return on savings is θ. This demonstrates that $(\sigma^t_{\mathcal{P}^*}, \sigma^*)$ entails the authority's first best allocation.

A.7 Proof of Proposition 8

The proof works as follows. First, as in the benchmark case, we find equations that equilibrium variables have to solve. Then we use these equations to provide conditions under which different equilibria may arise.

Equilibrium characterization. Using (38), we get the actual law of motion of inflation of the real value of savings and inflation as:

$$m_t = \frac{W - \bar{T}}{1 + R(\rho_{t+1})} - S_t \tag{48}$$

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t} = \frac{(1+\lambda)\left(W - \bar{T}\right) - (1 + R(\rho_{t+1}))(1+\lambda)S_t}{W + \bar{T} - (1 + R(\rho_{t+1}))\left(\lambda\theta S_t + S_{t+1}\right)} \tag{49}$$

provided $W + \bar{T} \geq (1 + R(\rho_{t+1}))(\lambda\theta S_t + S_{t+1})$, otherwise we have $m_{t+1} \to 0$ and $\Pi_{t+1} \to \infty$. We are ready now to investigate investigate how optimally chosen policies affect equilibrium outcomes.

The pure monetary equilibrium. The pure monetary equilibrium where $S_t = 0$ at each t is still an equilibrium provided $(1+\lambda)(W - \bar{T})/(W + \bar{T}) < \theta^{-1}$. This can be easily seen by checking that $S_t = 0$ at any t implies $\Pi_{t+1} = (1+\lambda)(W - \bar{T})/(W + \bar{T})$ at any t from (49). In turn, $S_t = 0$ requires that $\Pi_{t+1} \leq \theta^{-1}$, thus implying that $(1+\lambda)(W - \bar{T})/(W + \bar{T})$ does not exceed θ^{-1}. We then obtain m_t from (35) with $S_t = 0$ and G_t from (36).

In case $\Pi_{t+1} > \theta^{-1}$ implies $S_t > 0$, so that a pure monetary equilibrium does not exist in that case.

Existence of asymptotic storage equilibria. We investigate now whether there are equilibria where both money and storage are used. $S_t > 0$ implies $\Pi_t = \theta^{-1}$ at t that, is:

$$S_t = \theta S_{t-1} + \frac{W + R(\theta)\bar{T} - \theta(1+\lambda)(W - \bar{T})}{1 + R(\theta)}$$

Let us first consider the case $\theta(1+\lambda)(W-\bar{T}) < W + R(\theta)\bar{T}$. In such a case, $S_t > 0$ implies $S_{t+\tau} > 0$ for $\tau \geq 1$. However, an equilibrium where $S_t > 0$ for each $t \geq \tau$ requires a sequence $\{S_t\}_{t=1}^{\infty}$ converging monotonically to

$$\bar{S} = \frac{W + R(\theta)\bar{T} - \theta(1+\lambda)(W - \bar{T})}{(1 + R(\theta))(1-\theta)}.$$

As previously noted, to be feasible, $\bar{S}$ should satisfy $\bar{S} \leq \left(W - \bar{T}\right)/(1 + R(\theta))$. As a result, a necessary condition to be an equilibrium is:

$$\bar{T} \leq \frac{\lambda\theta}{1 + R(\theta) + \lambda\theta} W.$$

Otherwise, an equilibrium where money and storage are jointly used does not exist.

Similarly to the case without any policy, all asymptotic storage equilibria do not necessarily feature storage at date-0 and it is possible to construct asymptotic storage equilibria where storage is not used until a certain date s after which it is always used. In fact, notice that $S_{s-1} = 0$ only requires that $\Pi_s \leq \theta^{-1}$, that is

$$0 \leq S_s < \frac{W + R(\theta)\bar{T} - \theta(1+\lambda)(W - \bar{T})}{1 + R(\theta)}.$$

Thus, at each date t, after having only used money in past periods, it is possible to start using storage. Also here once storage is used it will used for ever.

In the case when $\theta(1+\lambda)(W-\bar{T}) > W + R(\theta)\bar{T}$, the sequence of storage S_t converges to a negative value; however this violates the constraint $S_t \geq 0$. Thus, in this case, an equilibrium where storage is used with money does not exist.

Finally, let us note that when condition

$$\bar{T} < \frac{\theta\lambda}{1 + R(\theta) + \theta\lambda} W$$

is satisfied, the condition $(1+\lambda)(W-\bar{T})/(W+\bar{T}) \leq \theta^{-1}$ is also satisfied. Indeed, this latter condition is a decreasing function of $\bar{T}$ and the condition is satisfied for $\bar{T} = \frac{\theta\lambda}{2+\theta\lambda}W > \frac{\theta\lambda}{1+R(\theta)+\theta\lambda}W$.

Existence of pure autarky equilibria. We study here the conditions for the existence of a pure autarky equilibrium – i.e. one in which $m_t = 0$ for any t. Without loss of generality, we consider period 1. Suppose that $m_1 = 0$. The optimal rate of inflation at date 2 is:

$$\Pi_2 = \frac{(1+\lambda)m_1}{\frac{W + R(\theta)\bar{T}}{1+R(\theta)} - \lambda\theta S_1 - S_2}$$

$$\Pi_2 = \frac{(1+\lambda)(1+R(\theta))m_1}{W + R(\theta)\bar{T} - (\lambda\theta + 1)(W - \bar{T})}$$

using the fact that, In autarky, $S_1 = S_2 = (W - \bar{T})/(1 + R(\theta))$. The denominator is strictly positive when:

$$\bar{T} > \frac{\lambda\theta}{1 + R(\theta) + \lambda\theta} W,$$

in which case $\Pi_2 = 0$ when $m_1 = 0$. As a result, agents are strictly better off not to store. Otherwise, when

$$\bar{T} \leq \frac{\lambda\theta}{1 + R(\theta) + \lambda\theta} W,$$

A.8 Proof of Proposition 9

To prove this Proposition, we first determine conditions under which the constraint on T_t may bind. We split this investigation depending on whether storage is used in equilibrium.

(Potential) equilibria without storage. Suppose that storage is never used in equilibrium. From Proposition 6, the unconstrained level of taxes is $(1-\beta+\lambda)/(\lambda+2)W$. Depending on the value of $\hat{T}$, this level of taxes is then constrained by $\hat{T}$. When $\hat{T} \geq (1-\beta+\lambda)/(\lambda+2)W$, there exists a monetary equilibrium as described by Proposition 6. Otherwise, a monetary equilibrium exists under the condition of Proposition 8.

(Potential) Equilibria with storage. Suppose that storage is used at some date. Let us first show the following lemma.

Lemma 13. *At date t, suppose that storage is used and that the constraint binds:*

$$\frac{\lambda + 1 - R(\theta)^{-1}}{2 + \lambda} W + \frac{1 + R(\theta)^{-1}}{2 + \lambda} (S_t - \theta S_{t-1}) \geq \hat{T} \tag{50}$$

Then the constraint binds for $t + 1$.

Proof. Suppose that storage is used at date t. As a result $S_t > 0$ and the expected return is θ.

Given that the constraint binds at date t, we can use $R(\theta)(S_{t+1} - \theta S_t) - \theta(S_t - \theta S_{t-1}) = (R(\theta) - \theta)W$ to replace $(S_t - \theta S_{t-1})$ in the constraint to obtain:

$$\frac{(\lambda + 1 - R(\theta)^{-1})}{2 + \lambda} W + \frac{(1 + R(\theta)^{-1})}{2 + \lambda} \left(\frac{R(\theta)}{\theta} (S_{t+1} - \theta S_t) - \left(\frac{R(\theta)}{\theta} - 1 \right) W \right) \geq \hat{T} \tag{51}$$

As a result, the constraint also binds when:

$$\left(\frac{R(\theta)}{\theta} - 1 \right) (S_{t+1} - \theta S_t - W) \tag{52}$$

This is always satisfied as $R(\theta) > \theta$ and $S_t < W/(1 - \theta)$. $\square$

As a result of Lemma 13, either the constraint never binds during or after storage is used or it always binds. If it never binds along these paths, the proof of Proposition 6 implies that these paths cannot be an equilibrium outcome. If the constraint always binds, Proposition 8 implies that these paths can be equilibrium outcomes only when storage is always used and that:

$$\hat{T} \leq \frac{\theta(1 + \lambda) - 1}{R(\theta) + \theta(1 + \lambda)} W. \tag{53}$$

Finally, note that when $\hat{T} \geq \frac{1-\beta+\lambda}{\lambda+2}$, no such paths can be equilibrium outcomes as:

$$\hat{T} \geq \frac{1-\beta+\lambda}{\lambda+2} W \geq \frac{\lambda}{2+\lambda} W \geq \frac{\theta(1+\lambda)-1}{R(\theta)+\theta(1+\lambda)} W, \tag{54}$$

B Infinite-horizon economy with no policy

In this Appendix, we show a simple model with infinite-horizon agents reproducing the same form of market incompleteness of OLG economies. This mapping is well known since Townsend (1980) – a model that, according to Ljungqvist and Sargent (2018) (chap. 28), "can be viewed as a simplified version" of a stochastic Bewley economy.

Consider a simple endowment economy with two types of infinitely-living agents j initially endowed with two different quantities of a non storable endowment, $q \in \{H, L\}$, with $\bar{q} = \{H, L\}/q$ and $H > L$. Each period the distribution of endowments flips. No credit contracts are possible, however agents can buy an intrinsically worthless asset called money available in a stock M so that $M = \sum_j M_{j,t}$ at any t. The problem of the agent j is :

$$V(q_t) = u(C(q_t)) + \beta V(\bar{q}_{t+1})$$

subject to

$$C(q_t) = q_t - \frac{M_{j,t} - M_{j,t-1}}{P_t}$$

with $M_{j,t} \geq 0$, where $V(q)$ denotes the present value of having a quantity of endowment q, $\beta \in (0,1)$ is a discount factor and $C(q)$ is consumption contingent on having endowment quantity q. In a stationary equilibrium, we have

$$V(q_t) = \frac{u(C(q_t)) + \beta u(C(\bar{q}_{t+1}))}{1-\beta^2}.$$

The first order conditions with respect to $M_{j,t}$ are:

$$-\frac{1}{1-\beta^2} u'(C(q_t)) \frac{1}{P_t} + \frac{1}{1-\beta^2} \beta u'(C(\bar{q}_{t+1})) \frac{1}{P_{t+1}} + \nu_{q,t} = 0 \quad \text{with} \quad \nu_{q,t} M_{q,t} = 0$$

where ν_q is the Lagrangian associated with the no short selling constraint of the type having endowment q.

An equilibrium with stationary consumption levels – $C(q) = C(q_t)$ for any t – exists featuring money trade. In this equilibrium the high endowment type $j = H$ optimally buys all money so that:

$$\frac{P_{t+1}}{P_t} = \beta \frac{u'\left(L + \frac{M}{P_{t+1}}\right)}{u'\left(H - \frac{M}{P_t}\right)}, \quad M_{H,t} = M, \quad \nu_H = 0$$

where $u'(C(H)) = \beta u'(C(L))$ and $P_t = P_{t+1}$, and the low endowment type $j = L$ has a binding short-selling constraint in that

$$\nu_L = \frac{1}{1-\beta^2} \frac{1}{P_{t+1}} \left(u'\left(L + \frac{M}{P_t}\right) \frac{P_{t+1}}{P_t} - \beta u'\left(H - \frac{M}{P_{t+1}}\right)\right) > 0,$$

unless the knife-edge case $\beta = 1$. The departure from optimality obtains as the high-endowment type internalizes the utility of the low-endowment type only intertemporally, whereas a social planner would care about it without any time discount. As the low-endowment type is constrained, the setting reproduces the logic of the OLG model presented in the text.

At least one other solution exists where money is worthless, which is the following:

$$P_t \to \infty, \quad \nu_H = 0, \nu_L = 0,$$

characterizing the allocation in autarky. In this case, there is a larger consumption inequality between households as there are no transfers in autarky. Thus, as in the OLG model, this infinite-horizon economy features inequality as a function of trade in money.

C Fluctuations in endowment

In this section we will look at the case of time-varying endowment. We will initially look at the dynamics in the absence of policy and then study the optimal policy reaction. For simplicity we will restrict to the case of log-preferences.

C.1 Absence of policy

Before exploring how time-varying endowment affect the optimal policy, let us review briefly what changes absent policy, i.e. with $\mathcal{P}_t = (0,0,0)$ at each t. In the case of fluctuations in endowment, we have

$$\Pi_{t+1} = \frac{W_t - 2S_t}{W_{t+1} - 2S_{t+1}}.$$

This modification has important consequences on the set of equilibria. As a first result note that the pure monetary equilibrium may not exists any longer. In fact, $S_t = 0$ for any $t \geq 1$ is not an equilibrium when $\Pi_{\tau+1} = W_\tau / W_{\tau+1} \geq \theta^{-1}$. In this case, there not exist an equilibrium where $S_\tau = 0$ because the return on storage *necessarily* exceeds the one on money; on the contrary, S_τ must be strictly positive.

On the other hand, it is possible now an equilibrium where $S_\tau > 0$ and $S_{\tau+1} = 0$. To see this notice that, after substituting $m_\tau = W_\tau/2 - S_\tau$,(24) now becomes

$$S_{\tau+1} = \theta S_\tau + \frac{W_{\tau+1} - \theta W_\tau}{2},$$

so that, when $\theta W_\tau > W_{\tau+1}$, there exists a value of S_τ, namely

$$0 < \hat{S}_\tau \equiv \frac{\theta W_\tau - W_{\tau+1}}{2\theta} < \frac{W_\tau}{2},$$

such that $S_{\tau+1} = 0$, provided $\hat{S}_\tau < W_\tau/2$. The reason is that, when all savings are in money and the endowment decreases sufficiently fast, the return on money may fall below the return on storage, making storing more attractive instead. In general however, while a jump to positive storage is made necessary in situations of a strong decrease in endowment, the return to pure monetary savings is not. In analogy to the case with constant endowment, a multiplicity of equilibria exists that satisfy (C.1) where storage is positive also at date $t+1$.

C.2 Optimal policy reaction with fluctuations in endowment

We build the optimal policy in this case based on two elements.

First, whenever $S_t > 0$ we have that

$$\Pi_{t+1} = \frac{W_t + \theta S_{t-1} - (3+\lambda)S_t}{W_{t+1} - \theta(1+\lambda)S_t - S_{t+1}} = \theta^{-1},$$

or

$$S_t = \frac{S_{t+1} + \theta^2 S_{t-1} - W_{t+1} + \theta W_t}{2\theta}, \tag{55}$$

that is, the return on money must be the same than the return on storage.

Second, whenever $S_t = 0$ instead optimal saving choices require

$$\Pi_{t+1} = \frac{W_t + \theta S_{t-1}}{W_{t+1} - S_{t+1}} < \theta^{-1},$$

or

$$\theta^2 S_{t-1} < W_{t+1} - \theta W_t - S_{t+1}, \tag{56}$$

that is, the return on money is higher than the return on storage.

These two elements, which do not depend on λ, nail down the unique equilibrium consistent with an optimal policy response. The following proposition characterise the equilibrium path of storage, for arbitrary sequence of endowments, that, for an arbitrary initial S_t at some t, converges to zero storage in a finite time.

Proposition 14. *For any $\{\lambda, \beta\}$, and a given $S_t \in (0, W_t)$ and sequence of endowments $\{W_\tau\}_{\tau=t+1}^{\infty}$, the sequence of $\{S_\tau\}_{\tau=t+1}^{\infty}$ characterising an equilibrium with optimal policy is such that:*

i) given the unique $n^* \in \mathbb{N}$ that satisfies the following inequality:

$$\frac{n^* W_{t+1+n^*} - \sum_{i=0}^{n^*-1} \theta^{n^*-i} W_{t+1+i}}{\theta^{n^*}} < \theta S_t < \frac{(n^*+1) W_{t+2+n^*} - \sum_{i=0}^{n^*} \theta^{n^*+1-i} W_{t+1+i}}{\theta^{n^*+1}},$$

ii) then there are at least n^* successive storage values $\{S_{t+1+n^*-n}\}_{n=1}^{n^*}$ given by:

$$S_{t+1+n^*-n} = \frac{n\theta^n \theta S_{t+n^*-n} + n\theta^n W_{t+1+n^*-n} - \sum_{i=1}^{n} \theta^{n-i} W_{t+1+n^*-n+i}}{(1+n)\,\theta^n},$$

before the pure monetary state $(S_{t+n^*+1}, S_{t+n^*+2}) = (0, 0)$.

Proof. To build our solution let us suppose that there is a t such that $S_t > 0$, $S_{t+1} = 0$ and $S_{t+2} = 0$ and work backwards. According to (55) we get

$$S_t = \frac{\theta^2 S_{t-1} + \theta W_t - W_{t+1}}{2\theta} \tag{57}$$

from which it is obvious that, to get $S_t > 0$ either W_t is sufficiently big or it must be $S_{t-1} > 0$. Moreover, according to (56) it should be

$$S_t = \frac{\theta^2 S_{t-1} + \theta W_t - W_{t+1}}{2\theta} < \frac{W_{t+2} - \theta W_{t+1}}{\theta^2}$$

i.e.

$$\frac{W_{t+1} - \theta W_t}{\theta} < \theta S_{t-1} < \frac{2W_{t+2} - \theta W_{t+1} - \theta^2 W_t}{\theta^2}. \tag{58}$$

If the inequality is satisfied with $S_{t-1} = 0$ then only at t storage is positive in an equilibrium with $S_{t+1} = S_{t+2} = 0$. Otherwise, it must be that also $S_{t-1} > 0$.

Consider then, $S_{t-1} > 0$. Applying iteratively (57), this requires that

$$S_{t-1} = \frac{S_t + \theta^2 S_{t-2} - W_t + \theta W_{t-1}}{2\theta} = \frac{2\theta^3 S_{t-2} + 2\theta^2 W_{t-1} - \theta W_t - W_{t+1}}{3\theta^2} \tag{59}$$

from which it is obvious that either W_{t-1} is sufficiently big or it must be $S_{t-2} > 0$. Because of (58) and (59) it must be that

$$\frac{2W_{t+1} - \theta W_t - \theta^2 W_{t-1}}{\theta^2} < \theta S_{t-2} < \frac{3W_{t+2} - \theta W_{t+1} - \theta^2 W_t - \theta^3 W_{t-1}}{\theta^3}$$

If the inequality is satisfied with $S_{t-2} = 0$ then from $t-1$ to t storage is positive in an equilibrium with $S_{t+1} = S_{t+2} = 0$. Otherwise, it must be that also $S_{t-2} > 0$.

By iterating we have

$$S_{t+1-n} = \frac{n\theta^n \theta S_{t-n} + n\theta^n W_{t+1-n} - \sum_{i=1}^{n} \theta^{n-i} W_{t+1-n+i}}{(1+n)\,\theta^n} \geq 0,$$

which requires that either W_{t+1-n} is sufficiently large or S_{t-n} must be positive. In particular, it should be

$$\frac{nW_{t+1} - \sum_{i=0}^{n-1} \theta^{n-i} W_{t+1-n+i}}{\theta^n} < \theta S_{t-n} < \frac{(n+1)W_{t+2} - \sum_{i=0}^{n} \theta^{n+1-i} W_{t+1-n+i}}{\theta^{n+1}} \tag{60}$$

If the inequality is satisfied with $S_{t-n} = 0$ then from $t-n+1$ to t storage is positive in an equilibrium with $S_{t+1} = S_{t+2} = 0$. Otherwise, it must be that also $S_{t-n} > 0$.

The last step of the proof is to verify that

$$\theta S_{t+1-n} = \theta \frac{n\theta^n \theta S_{t-n} + n\theta^n W_{t+1-n} - \sum_{i=1}^{n} \theta^{n-i} W_{t+1-n+i}}{(1+n)\,\theta^n} < \frac{nW_{t+2} - \sum_{i=0}^{n-1} \theta^{n-i} W_{t+2-n+i}}{\theta^{n+1}},$$

leads to (60). This follows from

$$\begin{aligned}
\theta \frac{n\theta^n \theta S_{t-n}}{(1+n)\,\theta^n} &< \frac{nW_{t+2} - \sum_{i=0}^{n-1} \theta^{n-i} W_{t+2-n+i}}{\theta^n} - \theta \frac{n\theta^n W_{t+1-n} - \sum_{i=1}^{n} \theta^{n-i} W_{t+1-n+i}}{(1+n)\,\theta^n} \\
\frac{n\theta^{n+1} \theta S_{t-n}}{(1+n)\,\theta^n} &< \frac{(1+n)nW_{t+2} - (1+n)\sum_{i=1}^{n} \theta^{n+1-i} W_{t+1-n+i} - n\theta^{n+1} W_{t+1-n} + \sum_{i=1}^{n} \theta^{n+1-i} W_{t+1-n+i}}{(1+n)\,\theta^n} \\
\theta S_{t-n} &< \frac{(1+n)nW_{t+2} - n\sum_{i=1}^{n} \theta^{n+1-i} W_{t+1-n+i} - n\theta^{n+1} W_{t+1-n}}{n\theta^{n+1}} \\
\theta S_{t-n} &< \frac{(1+n)W_{t+2} - \sum_{i=0}^{n} \theta^{n+1-i} W_{t+1-n+i}}{\theta^{n+1}}
\end{aligned}$$

which is the same as (60). So we conclude that our recursive formulation indeed holds at any t for given a n.

Now to recover the expressions in the proof we need to operate an appropriate change of variables to express as a given the initial positive level of storage. In practice, whereas the proof defines a sequence of storage expressed as $\{S_{t-n}, S_{t-n+1}, ...S_{t-1}, S_t, 0, 0\}$ the proposition defines the same sequence relabelling time indexes to be $\{S_t, S_{t+1}, ...S_{t+n^*-1}, S_{t+n^*}, 0, 0\}$. □

We finally state here the solution for a sequence of constant endowments, which are the ones considered in the main text. The uniqueness of such paths is established in A.5 (point d.3).

Corollary 15. *For any $\{\lambda, \beta\}$, and a given $S_t \in (0, W_t)$ and sequence of endowments $W_t = W$ at any t, the sequence of $\{S_\tau\}_{\tau=t+1}^{\infty}$ characterising an equilibrium with optimal policy is such that:*

i) given the unique $n^* \in \mathbb{N}$ that satisfies the following inequality:

$$\frac{n^* - \sum_{i=0}^{n^*-1} \theta^{n^*-i}}{\theta^{n^*}} W < \theta S_t < \frac{(n^*+1) - \sum_{i=0}^{n^*} \theta^{n^*+1-i}}{\theta^{n^*+1}} W,$$

ii) then there are at least n^* successive storage values $\{S_{t+1+n^*-n}\}_{n=1}^{n^*}$ given by:

$$S_{t+1+n^*-n} = \frac{n\theta^n \theta S_{t+n^*-n} + \left(n\theta^n - \sum_{i=1}^{n} \theta^{n-i}\right) W}{(1+n)\,\theta^n},$$

before the pure monetary state $(S_{t+n^*+1}, S_{t+n^*+2}) = (0, 0)$.

D Optimal Fixed Surplus

We show here the details of the derivation of (34). For a fixes fiscal surplus T, real money demand in our model obtains as

$$m_t = \frac{W^y - T - \beta^{-\frac{1}{\sigma}}\Pi_{t+1}^{\frac{1}{\sigma}}W^o}{1+\beta^{-\frac{1}{\sigma}}\Pi_{t+1}^{\frac{1}{\sigma}-1}}$$

in case $G_t = 0$ (i.e. $\tilde{\lambda} = 0$), whereas inflation obtains as:

$$\Pi_t = \frac{m_{t-1}(\Pi_t)}{m_t(\Pi_{t+1}) + T}.$$

To analyse the dynamic properties of equilibrium sequences of inflation rates in a first order neighborhood of the steady state we can differentiate both sides to get:

$$\begin{aligned}
\frac{d\Pi_t}{\Pi_t}(\Pi_t - \beta) &= \frac{d\left(\frac{m_{t-1}}{m_t+s}\right)}{d\Pi_t}(\Pi_t - \beta) + \frac{d\left(\frac{m_{t-1}}{m_t+s}\right)}{d\Pi_{t+1}}(\Pi_{t+1} - \beta) \\
&= \frac{dm_{t-1}}{d\Pi_t}\frac{1}{m_t+s}(\Pi_t - \beta) - \frac{dm_t}{d\Pi_{t+1}}\frac{m_{t-1}}{(m_t+s)^2}(\Pi_{t+1} - \beta)
\end{aligned}$$

where

$$\begin{aligned}
\frac{dm_t}{d\Pi_{t+1}} &= \frac{\partial\left(\frac{W^y - T - \beta^{-\frac{1}{\sigma}}\Pi^{\frac{1}{\sigma}}W^o}{1+\beta^{-\frac{1}{\sigma}}\Pi^{\frac{1}{\sigma}-1}}\right)}{\partial\Pi} \\
&= -\frac{\left(\beta^{\frac{1}{\sigma}}W^y - T\beta^{\frac{1}{\sigma}} + \sigma\Pi^{\frac{1}{\sigma}}W^o + \beta^{\frac{1}{\sigma}}\Pi W^o - \sigma\beta^{\frac{1}{\sigma}}W^y + T\sigma\beta^{\frac{1}{\sigma}}\right)}{\sigma\beta^{\frac{2}{\sigma}}\Pi^{\frac{1}{\sigma}(2\sigma-1)}\left(\frac{1}{\beta^{\frac{1}{\sigma}}}\Pi^{\frac{1}{\sigma}-1} + 1\right)^2},
\end{aligned}$$

for any t. After substituting for the optimal fixed surplus $T = (1-\beta)(W^y - W^o)/2$ for which $\Pi = \beta$ we obtain:

$$\frac{dm_t}{d\Pi_{t+1}} = -\frac{W^o + W^y + \sigma W^o - \sigma W^y}{2\sigma + 2\sigma\beta}$$

and in particular,

$$\begin{aligned}
m_t &= \frac{\beta}{2}(W^y - W^o) \\
\frac{1}{m_t + T} &= \frac{1}{\frac{\beta}{2}(W^y - W^o) + (1-\beta)(W^y - W^o)/2} = \frac{2}{W^y - W^o} \\
\frac{m_{t-1}}{(M_t + T)^2} &= \frac{\frac{\beta}{2}(W^y - W^o)}{\left(\frac{\beta}{2}(W^y - W^o) + (1-\beta)(W^y - W^o)/2\right)^2} = \beta\frac{2}{W^y - W^o}.
\end{aligned}$$

Substituting back and solving for $(\Pi_{t+1} - \beta)$ we finally get

$$\Pi_{t+1} - \beta = \frac{W^o + W^y + \sigma\beta(W^y - W^o)}{\beta(W^o + W^y) - \sigma\beta(W^y - W^o)}(\Pi_t - \beta).$$

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