Spending Allocation under Nominal Uncertainty: A Model of Effective Price Rigidity*

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Abstract

This paper reverses the typical firm-centered view on the source of comovement of output and inflation through cyclical markups, assuming instead that consumers face greater frictions than firms in responding to relative price changes. We model consumers as uncertain whether the fluctuations in flexible prices posted by local monopolists are aggregate or idiosyncratic in origin. An aggregate increase in local prices may then drive correlated forecasting mistakes, coordinating households' decisions to reallocate spending from a local monopolistic to a competitive market at the cost of individual-specific shopping effort. Our new channel of frictional shopping features positive output-inflation comovement and countercyclical average markup paid by households, even if prices are fully flexible and firm markups are unresponsive to nominal shocks. Moreover, a distortion in competition arises, in that firms respond strategically to consumers' confusion over the source of posted price volatility, resulting in lower demand elasticity and higher firm markups when consumers perceive higher nominal uncertainty.

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1 Introduction

At least since Phelps (1970) and Lucas (1972), most of the literature on the sources of comovement between output and inflation has been firm-centric, modeling firms as relatively more constrained than households in responding to monetary shocks. The typical mechanism relies on two cornerstones. On the one side, firms cannot fully pass on a rise in nominal wages to final prices because of some combination of physical and informational frictions, resulting in lower firm markups.[1] At the same time, since households typically buy a representative consumption basket, they are trivially informed about the aggregate price level, hence, they correctly evaluate the opportunity cost of expanding demand.

Yet, both these elements came under scrutiny recently. First, there is no consensus, both in terms of theory and measurement, on the cyclical behavior of firm markups (see the discussion in Burstein et al. (2020)). Second, survey evidence shows that households make large and persistent errors in assessing aggregate inflation and the opportunity cost of consumption (Coibion and Gorodnichenko, 2015; Angeletos et al., 2020).

Motivated by these facts, this paper proposes a new channel for inflation-output comovement based on households' correlated mistakes in assessing the opportunity cost of consumption, which does not require, although can complement, counter-cyclical firm markups and nominal price rigidity. Our model relies on two building blocks. First, households form their view of the evolution of the aggregate price level by extrapolating from the prices posted at the firms they shop at, i.e. by *learning from prices*. This is consistent with recent evidence that consumers' inflation perception mostly relies on their shopping experience (D'Acunto et al., 2021).[2] Second, shopping is costly, so consumers' sampling of price quotes is limited,

[1] Typical physical frictions are menu costs (Golosov and Lucas (2007), Midrigan (2011), Alvarez et al. (2016)) whereas informational frictions stem from dispersed information (Woodford (2003), Angeletos and La'O (2009), Mackowiak and Wiederholt (2009)). Most of the empirical work at the micro-level has investigated firms' pricing in the attempt to differentiate between these two different versions of the same firm-centric view. See Alvarez et al. (2018) and Baley and Blanco (2019) for a review.

[2] In the Bank of England Inflation Attitudes Survey in 2016, nearly half the respondents (41%) declared that change in stores' prices is the primary factor leading them to change inflation expectations, while media reports (21%) got less attention. The findings of the Chicago Booth Expectations and Attitudes Survey are similar (D'Acunto et al., 2021): again, one's personal shopping experience is reported as the most important factor in inflation expectations, before family and friends, TV/radio, newspapers, etc. Mosquera-Tarrío (2019) provides evidence that when there is high uncertainty about the aggregate level of prices, households' inflation expectations are closely bound to changes in the consumer's actual consumption basket.

justifying cross-sectional variation in individual perceptions of inflation.[3] Hence, consumers must exert shopping effort to *reallocate spending* towards low-markup sellers, but, in evaluating the likely gains from reallocation, they only rely on the inflation perceived through market exposure at local sellers.

We nest this learning-by-shopping mechanism in an otherwise frictionless macroeconomic framework. Firms' pricing is fully flexible. Each consumer is initially matched to a local firm. Consumers observe the price of a good at their local firm at no cost, as representative of local market exposure. Based solely on the information deriving from this local price, each consumer decides whether to reallocate expenditure – at an individual-specific real cost – to another seller where they can buy the same good at a lower price.[4] In practice, households know that competitive suppliers are offering a lower price than their local vendor, but they are unsure how much. Moreover, households exerting shopping effort obtain a new price quote, becoming better informed about aggregate inflation than stayers. Crucially, in seeking to gauge the benefit of effort, consumers are uncertain over the level of the competitive prices they could get outside their local market, because they do not know whether the local price change they are exposed to is due to aggregate or idiosyncratic causes.

This framework is used to analytically characterize two main set of results. First, we show that inflationary nominal shocks reduce the average markup paid by households and can cause an increase in output, even if firms fully pass-through cost shocks to their price keeping posted markups invariant. An inflationary nominal shock induces a correlated misperception across locations of an increase in local prices relative to the aggregate price level and nominal wage. On the one hand, this misperception causes a positive comovement between inflation and the *extensive* margins of aggregate demand: as local prices spike up, more consumers are induced to exert shopping effort, effectively resulting in a markup gain from switching to competitive

[3] Kaplan and Menzio (2015) document that, although visiting one additional store would lead to about −0.6% in individual CPI, households concentrate their shopping in only about 2.3 shops visited on average per quarter.

[4] We interpret the shopping effort as the cost of purchasing from a distant discount store associated to longer trips or worse amenities. The marketing literature emphasizes the role of shopping costs, such as travel distance, in determining where consumers shop, see Bell et al. (1998). Hausman and Leibtag (2007) show that Wal-Mart supercenters offer many identical items at an average price about 15%–25% lower than traditional supermarkets. Alternatively, one can interpret the shopping effort as the search cost incurred to identify the location of the seller posting the lowest price, for instance because of variation in the set of firms offering products on sale.

sellers. On the other hand, non-switching consumers reduce consumption along the *intensive* margins as they perceive a fall in their real wages. Both effects reduce the market share of local monopolists in favour of low markup sellers resulting in a lower effective markup paid by households. Important to our contribution, a positive output-inflation comovement occurs when households' *learning from prices* is strong enough in relation to the elasticity of intertemporal substitution (EIS), that is, their inflation perception is sufficiently sensitive to inflation in local prices, to induce a sufficiently strong response of aggregate demand along the extensive margins. A non-zero elasticity of demand along the extensive margins is indeed necessary for a positive inflation-output comovement. We show that for a large and empirically relevant combination of parameters, the effective markup and output comove negatively in response to nominal shocks. For instance, an EIS smaller than one is a sufficient condition.

In our second set of results we show that higher consumers' inflation uncertainty increases firm markups.[5] To the extent that consumers interpret a local price change as partly due to an aggregate change in inflation, they are less likely to switch seller; that is, due to uncertainty, the elasticity of demand decreases. The firm internalizes the *signaling power* of its own posted price in shifting consumers' expectations when deciding about pricing, resulting in higher (but constant) markups. Our paper provides a new rationale for policies to reduce inflation uncertainty, for instance in the form of nominal stabilization, relying on an attenuation of this distortion in competition.

Finally, while we conduct our main analysis in a framework without nominal price rigidity, we also study a more general model that allows for firm price rigidity and cyclical markups, and use it to emphasize the potential complementarity with our mechanism in engendering the output-inflation comovement. Moreover, we use retailer scanner data to validate a distinctive prediction of the model, namely that the difference between posted price inflation, a measure obtained by weighting prices posted by different sellers equally over time, and the effective inflation paid by households, which is instead affected by households' reallocation of expenditure across retailers, increases in response to inflationary monetary shocks.

[5]This result contributes to the recent debate about the possibility that firms might exploit consumers' nominal confusion to increase markups. Since the beginning of the COVID-19 crisis, consumers' inflation uncertainty quickly rose to the highest levels in the last 40 years. In this context, a debate regained strength on the possibility that firms might exploit consumers' nominal confusion to set markups above normal (*price gauging*) reinforcing inflationary pressures; *greedflation*, media tagged it.

Review of the Literature Our setting overturns the basic island logic of Lucas (1972), modeling informational frictions on the side of households rather than firms, while preserving the inflation-output comovement. Various scholars have motivated informational frictions either by market segmentation (Lucas, 1972; Lorenzoni, 2009; Angeletos and La'O, 2009) or by some form of inattention (Sims, 2003; Mankiw and Reis, 2002; Woodford, 2003; Mackowiak and Wiederholt, 2009; Matějka, 2016; Kohlhas and Walther, 2021). In all these models, the degree of aggregate rigidity at the macro level is heightened to the degree of informational rigidity at the micro level. In our model, instead, aggregate price rigidity is the result of the aggregate behavior of households seeking to protect their perceived purchasing power against price hikes, and obtains even if the pass-through of cost shocks to firm prices is immediate and full.

More recently, the literature has discussed the impact of household uncertainty in the context of models with supply frictions. Mackowiak and Wiederholt (2015) assume that both households and firms make decisions under rational inattention. In their model, for given level of attention, mistakes in spending allocation across different consumption varieties do not correlate with inflation and aggregate consumption because consumers process information about relative prices of varieties separately from aggregate shocks. Learning about aggregate and idiosyncratic price variation from the same source, posted prices, is instead at the core of our mechanism. A similar weakening of output response to interest rate shocks is obtained by introducing discounting in the intertemporal Euler equation through information frictions as in Angeletos and La' O (2020); Angeletos and Lian (2018); Andrade et al. (2019); Gabaix (2020). Imperfect learning by households about future wealth has been introduced in the literature on noisy news as an explanation for demand-driven business fluctuations (Lorenzoni, 2009; Jaimovich and Rebelo, 2009). In contrast to all these studies, in our model, the cyclicality of the product market wedge relies on households' nominal confusion, so changes in the precision of households' information have a direct impact on the degree of non-neutrality of monetary policy.

Amador and Weill (2010) and Gaballo (2018) have considered firms learning from prices in fully competitive markets, where the strategic use of information asymmetries across agents through *signaling power* is impossible. Chahrour and Gaballo (2020) and Angeletos and Lian

(2019) also present models where households learn from prices, but these models are in real units, so that inflation and nominal shocks have no role. Gutiérrez-Daza (2022) studies a framework where consumers learn from prices but are better informed on nominal wages than on inflation. Differently from all these papers we emphasize the effects of inflation shocks on expenditure reallocation between high and low markup sellers.

Nominal stabilization is optimal in our framework. The policy prescription is similar to the one obtained in the New-Keynesian literature, but the transmission is very different: here nominal stabilization has large first order effects on welfare by reducing the level of firm markup instead (or on top) of reducing firms' dispersion of markup responses to shocks in that literature (Angeletos et al., 2016).

Benabou and Gertner (1993) study the effect of consumers' inflation uncertainty on firm markups when Bayesian learning is introduced in a conventional equilibrium search paradigm. As in our work, after having observed the price posted by their matched seller, consumers evaluate if to incur a cost to learn about the distribution of prices in the economy and reallocate expenditure. Differently from our setting, consumers' demand for given prices is exogenous, there are no aggregate shocks, and the search cost is homogeneous across households, so that the equilibria they study feature either nobody or everybody searching: there is no dispersion in inflation perceptions and consumption. In this environment, Fishman (1996) shows that the pass-through of inflationary cost shocks to firm prices can be incomplete, resulting in a countercyclical firm markup but acyclical reallocation of customers across sellers. We instead emphasize the opposite comovement: firm markups are acyclical whereas the expenditure share across sellers comoves with monetary shocks.

A large macroeconomic literature following the seminal work of Atkeson and Burstein (2008) has emphasized the relationship between market concentration and markups; higher markups result from the ability of large firms to influence the average price paid by consumers in the economy. In our model, monopolistic firms cannot affect the price paid by consumers in the competitive market, but they can influence their perception because of household uncertainty. This *signaling power* increases markups.

Inspired by Phelps and Winter (1970), a number of scholars, notably Rotemberg and Woodford (1999) and Ravn et al. (2006), have pointed out the importance of modelling the buyer-seller relationship to assess the propagation of monetary shocks over the business cycle. The availability of detailed individual data on shopping behavior has now stimulated a growing body of literature modelling the impact of shopping dynamics on the macro-economy. In particular, Kaplan and Menzio (2016) and Coibion et al. (2015) argue that more households switch to low markup sellers when unemployed is higher. We instead emphasize a different, and potentially complementary, mechanism where nominal shocks impact the perceived benefits of switching sellers. In their papers nominal shocks have no direct effect on the cost-opportunity of switching unless, through built in nominal rigidity, they impact unemployment first. In our model, nominal shocks have an effect on output even if firm prices are fully flexible. Moreover, in Kaplan and Menzio (2016) and Coibion et al. (2015), cyclical store switching reduces the cyclicality of aggregate markups to nominal shocks because shopping efforts comove with unemployment. In our framework the shopping effort comoves with inflation surprises, so it can be procyclical conditional on demand shocks or countercyclical conditional on supply shocks. This property of our model is consistent with evidence on the cyclical behavior of shopping time by US households documented by Petrosky-Nadeau et al. (2016).

Finally, in the industrial organization literature, Janssen and Shelegia (2015) study firm optimal pricing in the context of a double marginalization problem where a single manufacturer trades with uninformed downstream firms which learn from strategic prices; Cabral and Fishman (2012) show that this framework can microfound posted price stickiness. We instead model consumers learning from firms in general equilibrium, so that the correlation in posted prices is induced by the stochastic properties of wage inflation and emphasize effective price rigidity.

In Section 2 we present the model and characterize its equilibrium. Section 3 discusses the predictions of the model for the comovement between inflationary shocks and output. Section 4 analyzes the impact of inflation uncertainty on markups and welfare. Section 5 extends the basic setting allowing for nominal price rigidity. Section 6 presents evidence on the comovement between effective inflation and inflation in prices posted by retailers. Section 7 concludes.

2 The model

The economy consists of a continuum of locations ("islands") $j \in [0, 1]$, each inhabited by a unitary mass of households indexed by $i \in [0, 1]$ and one local monopolist indexed by j.[6] All firms sell the same consumption good. Households supply labor and trade bonds in centralized competitive markets, while shopping for consumption in a frictional product market. We model households' frictional behavior by making two changes in an otherwise standard setup. First, households purchase a homogeneous consumption good in a segmented product market and face nominal uncertainty about its price in different locations. In particular, having observed the posted price in their location j, the local market, the household decides whether or not to exert shopping effort to buy in a distant competitive market, whose price cannot be observed otherwise. Second, centralized competitive labor and capital markets do not open until the product market closes. This means that when they make consumption decisions, households are uncertain about aggregate prices, including their own wages. At the end of the period, uncertainty vanishes as households observe all prices, and supply adjusts to satisfy the level of consumption demand.

These assumptions are convenient for modeling household nominal uncertainty due to incomplete observation of product price dynamics, which lies at the core of our contribution. In practice, this setup captures the major influence that local retail prices may have on household expectations because of the frequency of individuals' exposure to shopping prices by comparison with wage revisions or financial contracts. There are two main reasons for positing one-period uncertainty. First, the novelty of our mechanism relates to the microfoundations of the non-neutrality of money rather than to its gradual propagation.[7] Second, by narrowing our framework down to the essentials, we get full closed-form tractability of our economy, allowing transparent characterization of its equilibrium and policy implications.

We lay out the plan of this section. We first describe households' preferences and budget constraint, then the timing and information structure of their maximization problem. Next,

[6]The choice of modelling local markets as constituted by a single firm is consistent with the evidence put forward by Kaplan and Menzio (2015) that consumers concentrate their shopping in a very small number of stores, on average 2.3 per quarter. We relax this assumption in Appendix B.4.

[7]Persistent uncertainty could be simply introduced by assuming, for example, that changes in inflation are due to persistent and temporary components that are indistinguishable to households.

we discuss the firm optimization problem and close the economy with a monetary policy rule that determines nominal wage inflation.

Households Household i living in location j chooses consumption, $c_{ijt} \geq 0$, labor supply, $\ell_{ijt} \geq 0$, bond demand, $b_{ijt} \geq \underline{b}$, and shopping effort, $s_{ijt} \in \{0, 1\}$, to maximize her preferences,

$$E\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{c_{ij\tau}^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} - \varphi\, \ell_{ij\tau} - \psi_{ij}\, s_{ij\tau}\right) \Big| \, \Omega^u_{ijt}\right], \tag{1}$$

with $\beta \in [0, 1)$, $\gamma > 0$ and $\varphi > 0$ denoting, respectively, the time discount factor, the elasticity of intertemporal substitution and the disutility of supplying one extra unit of labor; Ω^u_{ijt} is the information available to the household. The household can purchase the same consumption good from a local or a distant location.[8] The variable ψ_{ij} is the household specific disutility of shopping (shopping cost) and represents the cost of buying from a distant location, which affects utility linearly; it can be interpreted literally as the commuting cost of shopping in a distant location, and we will refer to it as the shopping cost.[9] The shopping cost can also include differences in amenities across sellers, so we do not restrict it to necessarily take positive values. The household has to satisfy the following budget constraint in each period,

$$c_{ijt}\, \mathcal{P}_{jt}(s_{ijt}) + \frac{b_{ijt}}{R_t} = W_t\, \ell_{ijt} + b_{ijt-1} - T_t, \tag{2}$$

where R_t is the risk free nominal rate on bonds, W_t is the nominal wage and T_t is a lump-sum government transfer. The price of consumption, $\mathcal{P}_{jt}(s_{ijt})$, depends on where the household decides to shop,

$$\mathcal{P}_{jt}(s_{ijt}) = \begin{cases} P_t & \text{if} \;\; s_{ijt} = 1 \\ p_{jt} & \text{otherwise} \end{cases}, \tag{3}$$

with P_t denoting the price of consumption in the distant competitive market, whereas p_{jt} is the price of consumption in the local market.

[8]In Appendix B.4 we consider an extension of the model where the household purchases a bundle of differentiated varieties for consumption.

[9]Alternatively it can be interpreted as the search cost to find the location with the lowest price on offer, and can be justified by a version of our model where the identity of sellers offering products in promotions changes over time.

In each location, the shopping cost is distributed across households according to the distribution

$$G(\psi) = 1 - \kappa\left(1 - \frac{\psi}{\Psi}\right)^{-\frac{\lambda}{1-\gamma}}, \tag{4}$$

with scale parameters $\kappa > 0$ and $\Psi = \frac{\varphi^{\gamma-1}}{\gamma-1}$, and shape parameter governed by $\lambda > 0$.[10] The support of the distribution is $[\underline{\psi}, +\infty)$ if $\gamma \leq 1$ and $[\underline{\psi}, \Psi]$ otherwise, with $\underline{\psi}$ satisfying $G(\underline{\psi}) = 0$. We notice that $G(\psi)$ is a generalized Pareto distribution when $\kappa = 1$.[11] This specification of $G(\psi)$ ensures that the price elasticity of demand faced by firms due to customers reallocating across stores can be invariant to firm characteristics in equilibrium, a crucial property for analytical tractability. At the same time, the distribution $G(\psi)$ is flexible enough to allow us to control, through the parameter κ, the mass of households purchasing from local sellers on average, and its price elasticity through the parameter λ.

Timing and information At the beginning of each period, productivity and aggregate nominal shocks realize and all firms post prices before households make their decisions. Within a period, after prices are posted, different markets open and close at different stages.

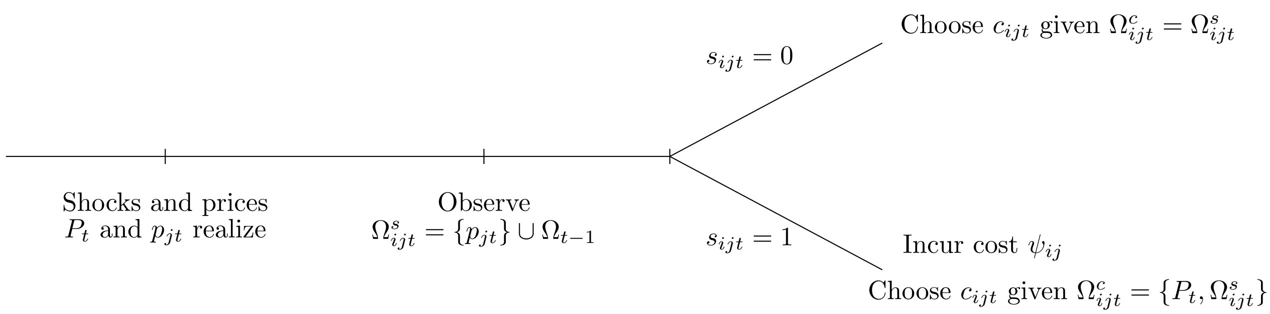
The product market opens first. In this market, households make their shopping and consumption decisions sequentially, as illustrated in Figure 1. The information set available to households evolves depending on their choices. In particular, Ω^u_{ijt} denotes the information available to household i in location j at time t, with $u \in \{s, c\}$ indexing households' information set at the stage of the shopping choice (s) and the consumption choice (c). Initially, each household i in location j decides *where* to shop after observing the price of the consumption good posted in location j, p_{jt}.[12] Therefore, the household ij chooses its shopping effort, s_{ijt}, subject to $\Omega^s_{ijt} = \{p_{jt}\} \cup \Omega_{t-1}$, or simply Ω^s_{jt}, where Ω_{t-1} is the history of realizations of the aggregate state, a sufficient statistic of which is P_τ, for all past periods τ up to $t-1$. As the consumers do not observe P_t at this stage, they are uncertain over the level of the competitive

[10] As it will become clear later, $\Psi = \varphi^{\gamma-1}/(\gamma-1)$ corresponds to the period utility the household obtains when purchasing in the competitive market.

[11] See Arnold (2014) for more details. We further note that, when $\kappa = 1$, the distribution converges to a standard exponential distribution with mean $1/\lambda$ as γ approaches 1; this is a useful benchmark with utility logarithmic in consumption and exponentially distributed shopping costs.

[12] In Section 5 we will extend the model to allow for additional information available to households at this stage.

Figure 1: The timing of the household shopping and consumption problem



prices they could get outside of their local market. They only get to observe P_t upon exerting the shopping effort ψ_{ij}, i.e. only in case $s_{ijt=1}$.[13] After the shopping decision is taken, households choose *how much* to consume. Consumption c_{ijt} is chosen under a potentially larger information set than the shopping decision, i.e. subject to $\Omega^c_{ijt} = \left\{\mathcal{P}_{jt}(s_{ijt}), \Omega^s_{jt}\right\}$, because the household observes the price in the competitive market in case $s_{ijt} = 1$. When the product market closes, the labor and financial markets open simultaneously: the household chooses b_{ijt} and ℓ_{ijt} given the (common) information set $\Omega_t = \{P_t\} \cup \Omega_{t-1}$, where we have posited that observation of market prices W_t and R_t provides perfect information about the aggregate state summarized by P_t.

Firms All firms produce and price the consumption good frictionless under perfect information. We will relax this assumption in Section 5. Each local firm in island j transforms one unit of labor into z_{jt} units of the consumption good, with z_{jt} denoting labor productivity; z_{jt} varies independently across islands and over time according to a log-normal distribution, i.e. $\ln z_{jt} \sim N(\mathrm{z}, \sigma_z^2)$. The local firm chooses the price p_{jt} that maximizes expected profits in each period,

$$p_{jt} = \underset{p}{\operatorname{argmax}} \; k_{jt}(p), \tag{5}$$

[13]In practice, we shall think of P_t as the price of a consumption bundle. Even if consumers have access to on-line information about prices of single goods in distant locations, computing the price of a consumption bundle and identifying the location that sells it at the lowest price entails a non-negligible cognitive and time cost. In Appendix B.4 we will formally model the shopping decision with a consumption bundle.

with

$$k_{jt}(p) \quad = \quad \mathcal{N}_{jt}(p)\,\mathcal{C}_{jt}(p)\,\left(\frac{p}{W_t} - \frac{1}{z_{jt}}\right) \tag{6}$$

where $k_{jt}(p)$ are profits scaled by the nominal wage W_t. Total consumption demand in the island depends then on the mass of households who shop in island j, namely $\mathcal{N}_{jt}(p)$, and on the quantities of consumption demanded by each of them, $\mathcal{C}_{jt}(p)$. We notice that each household i purchasing in island j faces the same prices of consumption, labor and savings and chooses the same consumption $c_{ijt} = \mathcal{C}_{jt}(p)$.

There is a distant market where a representative firm sells the same consumption good of local firms but operate under perfect competition. The production technology of the competitive firm transforms one unit of labor into one unit of output so that the marginal cost of production is W_t and productivity is normalized to one. A zero profit condition in each period requires this firm to price at marginal cost of production, so that the price of the consumption bundle in the distant market is

$$P_t = W_t. \tag{7}$$

Thus, p_{jt}/P_t – i.e. the price of the consumption good sold in the local market relative to the price of the same sold in the distant market – is the relative price in island j, the key object of the household shopping decision.

Monetary policy We assume that monetary policy targets nominal wage inflation, i.e. $\Pi_t \equiv P_t/P_{t-1}$, coinciding with competitive price inflation as given by (7). In particular, it implements the following AR(1) process

$$\ln \Pi_t = \chi \ln \Pi_{t-1} + \pi_t, \tag{8}$$

where the innovation to inflation π_t is normally distributed, $\pi_t \sim N(0, \sigma_\pi^2)$. We could alternatively assume that the monetary authority targets any other nominal variable in the economy, such as the overall price level. The nominal wage is especially convenient for exposition as it directly affects the marginal cost of every firm. Shocks to wage inflation can originate either from shocks to demand or to monetary policy. The Appendix A.1 offers a micro-foundation for (8) in terms of a monetary policy target on real balances of bonds, B_t/P_t, where π_t originates from shocks to the propensity to save, and $B_t = \int_0^1 \int_0^1 b_{ijt} didj$ is the government net supply of bonds in the economy. The parameter χ can be used to match the correlation between forecasts of future inflation with estimates of current inflation, a robust feature of the data.

2.1 Equilibrium

In our economy households make inference from the price of the consumption good, which is an equilibrium object. As usual in the literature dealing with endogenous information, we focus on log-linear equilibria, i.e. equilibria where each endogenous variable has a log-linear distribution in equilibrium. In our case, this requires guessing that the endogenous distribution of local prices, p_{jt}, is log-normal so that the optimal inference rule is log-linear and check that indeed local prices are log-normally distributed in equilibrium. We next state our definition of equilibrium.

Definition 1. *Given the past price level and inflation* $\{P_{t-1}, \Pi_{t-1}\}$*, a distribution of bond holdings* $\{b_{ijt-1}\}_{i,j\in[0,1]\times[0,1]}$*, the realizations of aggregate and idiosyncratic shocks,* π_t *and* $\{z_{jt}\}_{j\in[0,1]}$ *respectively, a log-normal equilibrium is a collection of log-normally distributed prices* $\{P_t, W_t, R_t, \{p_{jt}\}_{j\in[0,1]}\}$*, and quantities* $\{s_{ijt}, c_{ijt}, b_{ijt}, \ell_{ijt}\}_{i,j\in[0,1]\times[0,1]}$ *at time* t *such that:*

- *in each island* j*, each household* i *chooses* $\{s_{ijt}, c_{ijt}, b_{ijt}, \ell_{ijt}\}$ *to maximize the expected utility* (1) *subject to the sequence of budget constraints in* (2)*;*

- s_{ijt} *and* c_{ijt} *are chosen first conditional to* Ω^s_{ijt} *and* Ω^c_{ijt} *respectively,* b_{ijt} *and* ℓ_{ijt} *are chosen at the end of the period under perfect information;*

- *in each island* j*,* p_{jt} *solves the firm problem in (5);*

- W_t *and* P_t *are determined, respectively, by equations* (7) *and (8);*

- R_t *and* T_t *guarantee the equilibrium in the bond market, consistently with the monetary policy target in* (8)*;*

- $\int_0^1 s_{ijt}di > 0$ *holds almost surely in all islands.*

In equilibrium, households and firms optimally solve their problem conditional on the information available to them at that moment. We restrict our attention to equilibria where the mass of households who exert shopping effort is strictly positive almost surely in all islands, i.e. $\int_0^1 s_{ijt}di > 0$. This condition ensures that firm profits are continuously differentiable in a neighbourhood of the profit maximizing price. From now on, without loss of generality, we will focus on equilibria where all households have the same wealth, $b_{ijt} = B_t$ for all i, j in each period.[14] In this equilibria, because of linear labor disutility, heterogeneity of households' consumption is fully accommodated by contemporaneous variation in the supply of labor ℓ_{ijt}, so that the household budget constraint in (2) is satisfied in any period. We next discuss the maximization problems faced by households and firms in equilibrium.

Household maximization Let $V_{jt}(\psi_{ij}; s_{ijt})$ denote the present discounted value of utility in (1) of household i in island j, as a function of the individual shopping cost, ψ_{ij}, and of the shopping behavior in period t, s_{ijt}. Formally, households shop at the local firm if

$$E\left[V_{jt}(\psi_{ij}; 0) - V_{jt}(\psi_{ij}; 1) \,|\, \Omega_{jt}^s\right] \geq 0. \tag{9}$$

Given $V_{jt}(\psi_{ij}; 0)$ is unaffected by the shopping cost, while $V_{jt}(\psi_{ij}; 1)$ falls linearly as a function of ψ_{ij}, the optimal shopping policy is a cutoff rule: household i buys in island j, i.e. $s_{ijt} = 0$, if and only if $\psi_{ij} > \hat{\psi}_{jt}$, with $\hat{\psi}_{jt}$ being the shopping effort of the marginal household for whom (9) holds as an equality. Using (4), the fraction of households consuming locally is then given by $\mathcal{N}_{jt}(p) = 1 - G(\hat{\psi}_{jt})$.

In choosing consumption, households trade-off the marginal benefit of one unit of consumption with its expected cost. In particular, consumption is given by

$$c_{ijt}^{-\frac{1}{\gamma}} = \varphi\, E\left[\frac{\mathcal{P}_{jt}(s_{ijt})}{W_t} \,\Big|\, \Omega_{ijt}^c\right]. \tag{10}$$

The cost of one extra unit of consumption on the right-hand side of (10) is the expected

[14] The irrelevance of the path of bonds for consumption allocations originates from the assumption of linear disutility in labor supply.

utility loss from the extra hours the household will have to work to pay for consumption. It depends on the marginal disutility of one extra unit of labor, φ, and the expectation of how much labor it takes to pay for one unit of consumption, i.e. the price of consumption relatively to labor. Crucially, while the relevant price of consumption, $\mathcal{P}_{jt}(s_{ijt})$, is known at the time of the consumption choice (that is, once the shopping location is set), the nominal wage, W_t, is not observed yet. Households form expectations about the nominal wage using the consumption price itself as a signal, i.e. they also learn about their wage from posted prices. In the competitive market the consumption price is perfectly revealing of the nominal wage, in the local market it is not. Conditional on $s_{ijt} = 0$, (10) gives the per-capita local demand for consumption, denoted by $\mathcal{C}_{jt}(p)$.

In the second stage of the period, households choose on labor and savings. Because the disutility in labor is linear, the equilibrium nominal wage satisfies the following equation,

$$1 = \beta \, R_t \, W_t \, E_t \left[\frac{1}{W_{t+1}} \right], \tag{11}$$

so that the household (once at this stage, perfectly informed) is indifferent about the combination of labor and borrowing with which to finance its consumption expenditure in the product market to satisfy the budget constraint in (2). This further implies that the distribution of wealth across households is irrelevant to consumption decisions and that heterogeneity of wealth does not affect aggregate labor supply. Furthermore, by comparing (10) and (11), we note that the current wage, which the household needs to forecast when deciding on consumption, is in fact a sufficient statistics of the intertemporal opportunity cost of current consumption.

The equilibrium path of the nominal interest rate R_t consistent with (8) is the one that solves the household's saving problem in equation (11), $\ln R_t = -\ln \beta + \chi \, \Delta \ln \Pi_t - .5 \, \sigma_\pi^2$, while $T_t = B_{t-1} - \frac{B_t}{R_t} - P_t K_t$ is the fiscal transfer that guarantees the equilibrium in the bond market for any given path of government debt B_t, and given aggregate real profits $K_t = \int_0^1 k_{jt} dj$. In practice, because the economy aggregates as a representative household economy at the time of the saving and the labor decisions, the particular path taken by B_t and T_t is irrelevant to equilibrium consumption.

Firm maximization Let us turn now to the problem of local firms in (5). To find the optimal price, let us write the first order condition of real profits with respect to p_{jt}:

$$\left[\frac{1}{\mathcal{N}_{jt}(p)}\frac{\partial \mathcal{N}_{jt}(p)}{\partial p_{jt}} + \frac{1}{\mathcal{C}_{jt}(p)}\frac{\partial \mathcal{C}_{jt}(p)}{\partial p_{jt}}\right]\left(p_{jt} - \frac{W_t}{z_{jt}}\right) + 1 = 0. \tag{12}$$

This is a standard pricing function where the pricing problem depends on the elasticity of demand. Nevertheless, in our model, the elasticity of demand is the solution to a non-trivial fixed point problem. On the one hand, equations (9) and (10) depend on the local price both directly, because a change in local prices affects the nominal cost of local consumption, and indirectly because it impacts the expectations of the aggregate price and wage. On the other hand, when local firms decide their price, they take the demand schedule of households implied by equations (9) and (10) as given, but internalize the effect of their posted price on demand through both channels.

We notice that the firm lacks commitment power on price posting both over time and across states; this is a typical pricing protocol in the macroeconomic literature studying the comovement of inflation and output (e.g. Golosov and Lucas (2007), Mackowiak and Wiederholt (2009)), and a feature of most product markets. Therefore, the firm can neither promise lower prices in the future to increase demand today, nor commit to a mapping from a given realization of $\{W_t, z_{jt}\}$ to p_{jt}. The latter matters for allocations here because households are not perfectly informed about the realization of $\{W_t, z_{jt}\}$ and, as discussed extensively, make inference based on observed variations in p_{jt}.[15] The mapping that households use to take consumption decisions is the one consistent with the rational expectations distribution of equilibrium local prices. We next discuss such mapping.

Equilibrium guess We guess that the logarithm of equilibrium local prices, $\ln p_{jt}$, is normally distributed. Given this guess, and that π_t is normally distributed by assumption, household posterior distribution of π_t conditional on observing p_{jt} at the time of the shopping

[15] By committing to a mapping from states to prices, the firm could control the ex-ante distribution used by households to make inference, trading-off gains in different states. Because of lack of commitment, local firms end up in a coordination failure with households; we discuss in detail this result in Section 4.

decision, i.e. conditional on Ω^s_{jt}, is normal with mean and variance given by, respectively,

$$E[\pi_t \,|\, \Omega^s_{jt}] = \omega \times (\ln p_{jt} - E[\ln p_{jt}|\Omega_{t-1}]), \quad \text{and} \quad V(\pi_t \,|\, \Omega^s_{jt}) = (1-\omega)\, \sigma^2_\pi \equiv \mathcal{S}, \tag{13}$$

where the coefficient $\omega \in (0,1)$ captures the elasticity of households' inflation expectations with respect to local prices as the result of the inference to be solved for in equilibrium. ω measures the scope of households' *learning from prices.*

Equilibrium characterization The next proposition characterizes the equilibrium, focusing on three key equilibrium objects: household shopping and consumption policies, and optimal firm pricing.

Proposition 1. *There exists a unique log-normal equilibrium such that, at time t:*

(i) household i buys locally if her ψ_{ij} is larger than

$$\hat{\psi}_{jt} = \Psi - \Psi\, e^{(1-\gamma)\left(\ln\mu + (1-\omega)(\pi_t - \ln z_{jt}) + \frac{1}{2}\mathcal{S}\right)},$$

so that the mass of households purchasing in island j is given by

$$\mathcal{N}_{jt}(p_{jt}) = \kappa\, e^{-\lambda\left(\ln\mu + (1-\omega)(\pi_t - \ln z_{jt}) + \frac{1}{2}\mathcal{S}\right)}, \tag{14}$$

with $\mathcal{N}_{jt}(p_{jt}) < \kappa$ a.s. $\forall\, j$;

(ii) the consumption of household i initially matched to an island j is

$$c_{ijt} = \begin{cases} \mathcal{C}_{jt}(p_{jt}) \equiv C^*\, e^{-\gamma\left(\ln\mu + (1-\omega)(\pi_t - \ln z_{jt}) + \frac{1}{2}\mathcal{S}\right)} & \textit{if} \;\; \psi_{ij} > \hat{\psi}_{jt} \\ C^* \equiv \varphi^{-\gamma} & \textit{otherwise} \end{cases}; \tag{15}$$

(iii) the optimal price posted by the firm in island j is

$$p_{jt} = \mu\, \frac{W_t}{z_{jt}},$$

where μ is the equilibrium markup given by

$$\mu = \frac{(\gamma + \lambda)(1 - \omega)}{(\gamma + \lambda)(1 - \omega) - 1} \quad \textit{with} \quad \omega = \frac{\sigma_z^{-2}}{\sigma_z^{-2} + \sigma_\pi^{-2}}, \tag{16}$$

provided $\omega < \omega^{max} \equiv 1 - 1/(\gamma + \lambda)$, and z sufficiently small relative to σ_z.

Proof. See Appendix A.2. ☐

Proposition 1 shows how households' uncertainty about the origins of fluctuations in the local price affects optimal choices by both firms and households. To get a better appraisal of the mechanism, note that, in equilibrium, we get

$$\ln p_{jt} - E\left[\ln P_t | \, \Omega^s_{jt}\right] = \ln p_{jt} - E\left[\ln W_t | \, \Omega^s_{jt}\right] = \ln \mu + (1 - \omega)(\pi_t - \ln z_{jt}),$$

that is, households do not know whether an increase in $\ln p_{jt}$ is due to changes in aggregate inflation π_t or local productivity z_{jt}, so a positive shock to π_t increases households' perceived relative price and reduces perceived real wage less than one-to-one, by a factor $1 - \omega < 1$. The coefficient ω depends on the volatility of the fundamental shocks, σ_π^2 and σ_z^2. With a higher ω, households attribute indeed a large share of the movements in local prices to aggregate conditions.

Irrespective of the underlying driving shock, a perceived increase in relative price, $\ln p_{jt} - E\left[\ln P_t | \, \Omega^s_{jt}\right]$, shrinks the mass of households purchasing in island j with constant elasticity λ, according to (14). At the same time, a perceived decrease in the (log of the) real wage, $E\left[\ln W_t | \, \Omega^s_{jt}\right] - \ln p_{jt}$, decreases consumption in the local market, with an elasticity equal to γ, according to (15).

It is worth remarking right-away that inflation perceptions of switchers and stayers are different. Because switchers observe another price draw, they will in general have more information than stayers about the aggregate price level. In our model, this new price draw is perfectly informative about inflation, so that a shock to π_t is reflected in their expectations one-to-one. Shocks to inflation are reflected into perceptions of stayers with an elasticity of $\omega < 1$ as from (13). Because of this variation between switchers and stayers, households perceiving higher inflation have higher consumption not only because they pay lower markups after having switched to the competitive sellers but also because they perceive a higher real wage and lower real interest rate than stayers.[16]

From the perspective of firms, a higher ω implies a lower price elasticity of demand, resulting into higher markup. In particular, equations (14)-(15) imply that local firms face a constant demand elasticity equal to $(\gamma + \lambda)(1 - \omega)$ in absolute value, which only depends on second-order moments of the distribution of households' expectations. In other words, despite the elasticity of demand that local firms face is endogenous (it depends on the precision of information available to consumers) its equilibrium value is a constant. Thus, optimal markups are fixed *at the firm-level*, i.e. they do not vary with inflation realizations, although their level changes with the volatility of inflation relative to the one of productivity shocks. Finally, note that, given the nominal marginal cost is log-normally distributed, the profit-maximizing price in (iii) is also log-normally distributed, verifying our conjecture in equation (13). Also, the requirement that z is small enough relatively to σ_z ensures that $\int_0^1 s_{ijt} di > 0$ for approximately all realizations of p_{jt}. To ease exposition, we normalize z $= 0$ from now on, so that log-productivity at local and competitive firms is the same on average.[17]

3 *Effective* aggregate markup and the business cycle

In this section, we gather our main results on the business cycle properties of our economy. We first show that a simple aggregation obtains, deriving aggregate consumption and the effective aggregate markup paid by households. We then study the conditions under which inflation is expansionary and effective aggregate markup is counter-cyclical, two key business-cycle co-movements. We finally show that such comovements are sizeable for plausible calibrations.

Aggregate consumption Let us start with the determination of aggregate consumption to evaluate its response to a change in inflation. In the aggregation across islands, the impact of any local productivity disturbance vanishes, and households' inflation perceptions effectively

[16]Consistent with this prediction, D'Acunto et al. (2021) document that households with higher inflation expectations have higher expectations of wage growth and are less likely to save.

[17]From a theoretical perspective, the particular choice of z that ensures accuracy of our equilibrium is irrelevant. From a quantitative perspective, the choice of z puts bounds on the level of the volatility σ_z which are largely slack in realistic calibrations. See Appendix A.2 for a detailed discussion.

comove with an innovation in inflation. The mechanism mirrors the one in the celebrated Lucas (1972) islands' model, but here nominal confusion is all on the side of households, rather then of firms. Aggregate consumption is defined as

$$C_t = \int_0^1 \mathcal{N}_{jt}\mathcal{C}_{jt} + (1-\mathcal{N}_{jt})\, C^* dj, \tag{17}$$

with $\mathcal{N}_{jt}$ and $\mathcal{C}_{jt}$ being, respectively, the mass of households purchasing locally and the local per-capita consumption, according to (14)-(15). The functional forms of such equilibrium policies ensure a simple aggregation in closed form.

Proposition 2. *Let* $\bar{\mathcal{N}} \equiv \kappa e^{-\lambda\,(\ln\mu + \frac{1}{2}\mathcal{S} - \frac{\lambda}{2}(1-\omega)^2\sigma_z^2)}$ *and* $\overline{\mathcal{NC}} \equiv \kappa e^{-(\gamma+\lambda)\,(\ln\mu+\frac{1}{2}\mathcal{S}-\frac{\gamma+\lambda}{2}(1-\omega)^2\sigma_z^2)}$, *be, respectively, the mass of households buying local and total local consumption in terms of competitive per-capita levels, both evaluated at* $\pi_t = 0$*. Aggregate consumption is given by*

$$C_t = \left(1 - \bar{\mathcal{N}}\, e^{-\lambda\,(1-\omega)\,\pi_t} + \overline{\mathcal{NC}}\, e^{-(\gamma+\lambda)\,(1-\omega)\,\pi_t}\right) C^*. \tag{18}$$

Inflation shocks are expansionary on output, $\partial C_t/\partial\pi_t > 0$*, only if* $\lambda > 0$*.*

Proof. See Appendix A.3 □

Equation (18) shows that an increase in π_t affects aggregate consumption through two channels: on the one hand, it lowers consumption of non-switching consumers with elasticity γ; on the other, it increases the mass of switchers with elasticity λ. The latter end up consuming more of the former, i.e. $C^*/\mathcal{C}_{jt} \approx \mu^\gamma > 1$ almost surely, because of the lower markup paid and higher real wage perceived after a move in the competitive sector.

We immediately see that a necessary condition for positive inflation-consumption comovement is $\lambda > 0$. Without any dynamics in extensive margins, aggregate consumption would fall as local consumers decrease their consumption in response to a perceived increase in the real local price, which happens when surprise inflation hits.

To better evaluate the mechanism, it is useful to look at the elasticity of aggregate consumption with respect to π_t, which for expositional clarity we report up to first-order terms (see Appendix A.3 for the full closed-form expression). It obtains as:

$$\frac{\partial \ln C_t}{\partial \pi_t}\Big|_{\pi_t=0} \approx (\lambda\,\mu^{\gamma} - \lambda - \gamma)\,(1-\omega)\,\bar{\alpha}, \tag{19}$$

where $\bar{\alpha} = \bar{\mathcal{N}}\,\bar{\mathcal{C}}/\bar{C}$ is the market share of local markets computed at $\pi_t = 0$. The term $\lambda\mu^{\gamma} - \lambda - \gamma$ has a simple interpretation. Following a surprise hike in inflation, total consumption from switchers increases as their mass increases with elasticity λ and each of them catches a consumption gain of $\mu^{\gamma} - 1 > 0$, as their real wage effectively increases; at the same, total consumption by local consumers falls with an elasticity γ with respect to the perceived lower real wage.

Thus, aggregate consumption increases with inflation when the consumption gains from switchers is larger then the consumption losses from stayers. In particular, the semi-elasticity of aggregate consumption to π_t is positive if and only if local markups are sufficiently high, i.e. $\mu > (1+\gamma/\lambda)^{\frac{1}{\gamma}}$, which holds for a sufficiently high scope of learning from prices, ω.

Aggregate markup The relevant measure of purchasing power in this economy is the *effective* price index of the optimal consumption basket. Formally, the average price paid by households, the effective consumption price index $\mathcal{P}_t$, is defined as

$$\mathcal{P}_t\,C_t = \left(\int_0^1 p_{jt}\,\mathcal{N}_{jt}\,\mathcal{C}_{jt}\,dj + \int_0^1 P_t\,\left(1-\mathcal{N}_{jt}\right)\,dj\right)\,C^{*}, \tag{20}$$

where each price paid is weighted by the quantities purchased at that price. Such quantities depend on consumption demand per customer, captured by C^* and $\mathcal{C}_{jt}\,C^*$, at the competitive and monopolistic sellers respectively, and on the mass of customers at each monopolistic seller, captured by $\mathcal{N}_{jt}$. Using (20) we can derive the effective average markup paid by consumers relatively to the nominal wage in closed form as stated by the following proposition.

Proposition 3. *The effective markup paid on average by consumers is given by*

$$\mathcal{M}_t \equiv \frac{\mathcal{P}_t}{W_t} = 1 - \alpha_t \,+\, \alpha_t\,\mu\,e^{\mathcal{Q}}, \tag{21}$$

with $\mathcal{Q} \equiv -[(\lambda+\gamma)(1-\omega) - .5]\,\sigma_z^2$ *and*

$$\alpha_t \equiv \frac{\overline{\mathcal{NC}}\, e^{-(\lambda+\gamma)\,(1-\omega)\,\pi_t}}{1-\,\bar{\mathcal{N}}\; e^{-\lambda\,(1-\omega)\,\pi_t} + \overline{\mathcal{NC}}\, e^{-(\lambda+\gamma)\,(1-\omega)\,\pi_t}}, \tag{22}$$

being the market share of local sellers.

Proof. See Appendix A.4. □

$\mathcal{M}_t$ is a measure of the aggregate wedge between the marginal product of labor and the real wage obtained by consumers on average in our economy. It is equivalent to the inverse of the average hourly real wage perceived by households. It obtains as a weighted average between the markup posted by local firms, μ, which in the aggregation gets associated to a Jansen inequality term $e^{\mathcal{Q}}$, and the unitary markup paid in the competitive market. The term α_t is the market share of local sellers, which obtains as the average consumption in local markets relative to total consumption.

In general, the local market share α_t always co-moves negatively with innovations in inflation, thus entailing a counter-cyclical effective markup whenever inflation is expansionary on consumption. To better appreciate the cyclical properties of the local market share, let us look at the elasticity of α_t with respect to inflation computed at $\pi_t = 0$, which for expositional convenience we report here up to first-order terms (see Appendix A.4 for the full closed-form expression). We have:

$$\frac{\partial \ln \alpha_t}{\partial \pi_t}\Big|_{\pi_t=0} \approx -\left[\lambda + \bar{\alpha}\,\lambda\,(\mu^\gamma - 1) + \gamma\,(1-\bar{\alpha})\right]\,(1-\omega) < 0\;. \tag{23}$$

As inflation increases, households perceive a lower relative price in the competitive sector by a factor $1-\omega$, and the market share of local sellers falls for three reasons. First, the mass of customers buying there falls with elasticity λ; second, those consumers that reallocate increase expenditure by a factor $(\mu^\gamma - 1)$ relative to local sellers; finally, those consumers that stay, reduce demand with elasticity γ because of the perceived lower real wage.

Let $\bar{\mathcal{M}}$ denote the aggregate markup when $\pi_t = 0$. By combining (21) and (23) we immediately obtain that

$$\frac{\partial \ln \mathcal{M}_t}{\partial \pi_t}\Big|_{\pi_t=0} = \frac{\bar{\mathcal{M}}-1}{\bar{\mathcal{M}}} \times \frac{\partial \ln \alpha_t}{\partial \pi_t}\Big|_{\pi_t=0}, \tag{24}$$

i.e. the effective markup paid on average in the economy comoves negatively with inflation surprises thanks to the negative comovement of the market share of local sellers. The negative comovement between effective markup and inflation implies an incomplete pass-through of wage inflation shocks to the effective price of consumption, *even if* firms pass wage inflation fully through posted prices.

Accounting for expansionary inflation So far we have shown that inflation always negatively comoves with the aggregate markup, whereas it positively comoves with aggregate consumption with a sufficiently large firm markup μ. Here we use the equilibrium value of the firm markup in (16) to uncover the conditions under which inflation is expansionary on consumption, and, therefore, the aggregate effective markup is countercyclical. The following Proposition summarizes our result.

Proposition 4. *Given* (16)*,* (19) *and* (23)*, there always exists a threshold* $\bar{\omega}(\gamma, \lambda) \leq \omega^{max}$ *such that:*

$$\frac{\partial \ln C_t}{\partial \pi_t} \geq 0 \quad \text{and so} \quad \frac{\partial \ln \mathcal{M}_t}{\partial \ln C_t} \leq 0 \quad \text{if and only if} \quad \omega \geq \bar{\omega}(\gamma, \lambda) \tag{25}$$

where $\bar{\omega}(1, \lambda) = 0$ *for any* λ*. In particular,* $\bar{\omega}_\gamma > 0$ *always holds, whereas* $\bar{\omega}_\lambda \geq 0$ *with* $\gamma \leq 1$ *and* $\bar{\omega}_\lambda < 0$ *with* $\gamma > 1$*.*

Proof. See Appendix A.5. □

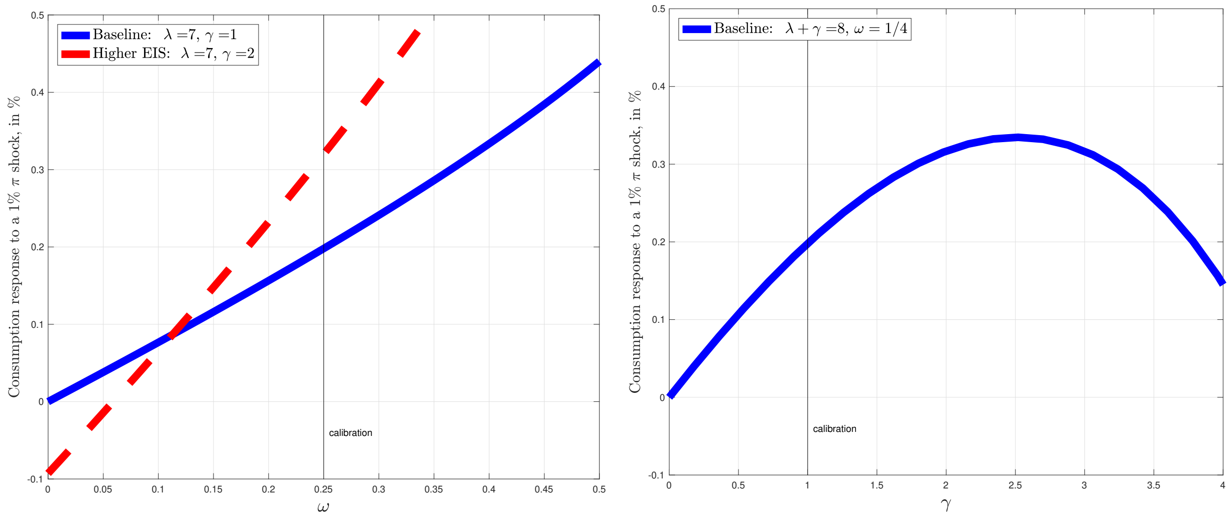
Proposition 4 implies that $\gamma \leq 1$ is a sufficient condition for inflation to be expansionary, and so, aggregate effective markups counter-cyclical. When $\gamma > 1$, there always exists a sufficiently high (and feasible, as $\bar{\omega}(\gamma, \lambda) < \omega^{max}$) level of consumer learning, ω, such that (25) holds.

To get a first pass on the quantitative relevance of the overall consumption-inflation comovement that one can obtain in our economy, we calibrate the model according to available empirical evidence. We choose $\gamma = 1$ and $\lambda = 7$ in our baseline case. The parameter γ governs the intertemporal elasticity of substitution, which the macroeconomic literature typically assumes close to 1.[18] The parameter λ governs the price elasticity of demand firms face with respect to demand reallocation towards the competitors. The value $\lambda = 7$ is the estimate of the elasticity of the customer base to a change in prices obtained from scanner data of a major US traditional supermarket (see Paciello et al. (2019)). We calibrate κ so that the market share of competitive sellers, $1 - \bar{\alpha}$ in our model, is equal to 1/3, roughly corresponding to the combined retail market share of e-commerce and Walmart in the US.[19] We set $\omega = 0.25$ to target a markup in local stores of $\mu = 1.2$, so that competitive sellers post prices at a discount of 20% on average relative to local ones; this dispersion in prices is consistent with the evidence in Hausman and Leibtag (2007) showing that Wal-Mart supercenters offer many identical items at an average price about 15%–25% lower than traditional supermarkets. We notice that $\omega = 0.25$ is also consistent with estimates of the correlation between expected and experienced inflation by D'Acunto et al. (2021). Using equations (19) and (23)-(24), it is immediate to compute the predicted impact response of aggregate consumption and markup to an unexpected 1% increase in π_t at our baseline calibration, which amount to 0.2% increase in consumption and 0.72% reduction in effective markup relative to no shock.

Figure 2 proposes two different exercises to assess the role of the consumer learning, ω, and the relative importance of extensive versus intensive margin of demand, i.e. λ versus γ, in the comovement between consumption and inflation. In the left panel, we plot the response of aggregate consumption to a 1% inflation shock for different values of ω on the horizontal axis, by keeping fixed $\bar{\alpha} = 0.66$ and for given values of $\lambda = 7$ and $\gamma = 1$ (solid blue line) or $\gamma = 2$ (red dashed line). Consistent with Proposition 4, inflation is always expansionary on consumption for a any $\omega > 0$ at $\gamma = 1$. A strictly positive but small level of consumer learning $\omega > \bar{\omega}(2, 7) \approx 0.06$ is needed for inflation to be expansionary despite a relatively high value of the EIS, i.e. $\gamma = 2$. The size of the comovement between consumption and inflation is more sensitive to changes in ω at $\gamma = 2$ because the consumption increase from store reallocation, captured by the term μ^{γ}, is more elastic to changes in local markups induced by the variation

[18] See Hall (1988) and more recently Thimme (2017) for a review.
[19] Source: www.statista.com.

Figure 2: The response of aggregate consumption to nominal shocks



(a) The role of consumers learning

(b) The role of demand elasticity composition

Note: We report the impact response of consumption to a 1% shock to π_t, in % deviation from steady state at our preferred calibration. The left panel explores the working of (19) at different combinations of λ and γ, as ω changes on the horizontal axis, by varying κ so that $\bar{\alpha} = 0.66$ in all simulations. The right panel shows how consumption response changes as the composition of demand elasticity varies, by keeping $\lambda + \gamma$ fixed and varing γ, with $\omega = 0.25$ and $\bar{\alpha} = 0.66$ in all simulations.

in ω. As a result, the positive comovement between consumption and inflation is stronger at $\gamma = 2$ than at $\gamma = 1$ at our baseline calibration of $\omega = 0.25$.

The right panel of Figure 2 proposes a different exercise where the overall elasticity of demand, $(\lambda + \gamma)(1 - \omega)$, is kept constant, whereas its composition is changing. In particular, we let γ vary on the horizontal axis and adjust λ to keep the overall elasticity (and hence the firm level markups) invariant. We note that the impact of γ in terms of an increase of demand elasticity on consumption prices largely dominates the decrease in markup for low values of γ, whereas the opposite occurs as γ becomes sufficiently high. So we conclude that higher (lower) intensive (extensive) margins elasticity does not necessarily mean a lower comovement between inflation and consumption in our model. The reason is that a higher γ increases the effect on demand of paying a lower markup when switching stores. This effect can be stronger than the effect on the comovement from the reduction in the elasticity of demand along the extensive margin, λ. For instance if we go from $\gamma = 1$ to $\gamma = 2$ and, consequently, from $\lambda = 7$ to $\lambda = 6$, the comovement between inflation and consumption increases from 0.2 to about 0.3 for each percentage point of surprise inflation.

4 *Posted* markups and inflation uncertainty

In this section, we discuss the forces that make firm markups increasing in consumer uncertainty about inflation. We will show how uncertainty creates an incentive for firms to raise markups, which, nevertheless, ends up in a coordination failure as higher markups harm both consumers' purchasing power and, perhaps more surprisingly, monopolists' profits.

Let us start by looking more closely at how consumers' nominal uncertainty affects the pricing policy of firms. According to (16), the optimal markup is increasing in the elasticity of consumers' expectations to local prices, as measured by ω. It is easy to note that ω increases when inflation volatility – or equivalently consumers' ex-ante uncertainty on inflation – σ_π, raises. Intuitively, when local price variation is explained more by aggregate shocks, consumers assign a higher chance to inflation innovations when they see a price change. In fact, the possibility that local prices may increase because of economy-wide inflation rather than local conditions makes consumers more reluctant to exert shopping effort in response to a hike in local price. Thus, the elasticity of consumers' demand with respect to local prices flattens, creating an incentive for firms to charge higher markups.

However, counter-intuitively, in equilibrium greater consumer uncertainty entails lower firm profits despite higher markups. Proposition 5 illustrates formally the result.

Proposition 5. *Let $\bar{k} \equiv E[k_{jt}(p_{jt})]$ denote expected firm profits evaluated at equilibrium pricing $p_{jt} = \mu \times W_t/z_{jt}$ with μ given by (16); we have*

$$\frac{\partial \mu}{\partial \omega} > 0 \quad \text{and} \quad \frac{\partial \bar{k}}{\partial \omega} < 0,$$

provided $\omega \neq 0$. The negative effect of ω on profits is larger when demand elasticity, $\gamma + \lambda$, is higher, but does not depend on its composition, i.e. on the ratio γ/λ.

Proof. See Appendix A.6. □

At the core of the proposition is the fact that firms cannot signal a change in markup independently from a change in price, i.e. they lack commitment in setting markups, which leads to a coordination failure with consumers. Let $\mathcal{D}_{jt}(\tilde{\mu}, \mu^e)$ denote the demand of firm

j at time t as a function of a given (possibly off-equilibrium) markup $\tilde{\mu}$ and of consumers' expected markup μ^e,

$$\mathcal{D}_{jt}(\tilde{\mu}, \mu^e) \equiv C^* \kappa \, e^{-(\lambda+\gamma)(\ln \mu^e + (1-\omega)(\ln \tilde{\mu} - \ln \mu^e + \pi_t - \ln z_{jt}) + \frac{1}{2}\mathcal{S})}. \tag{26}$$

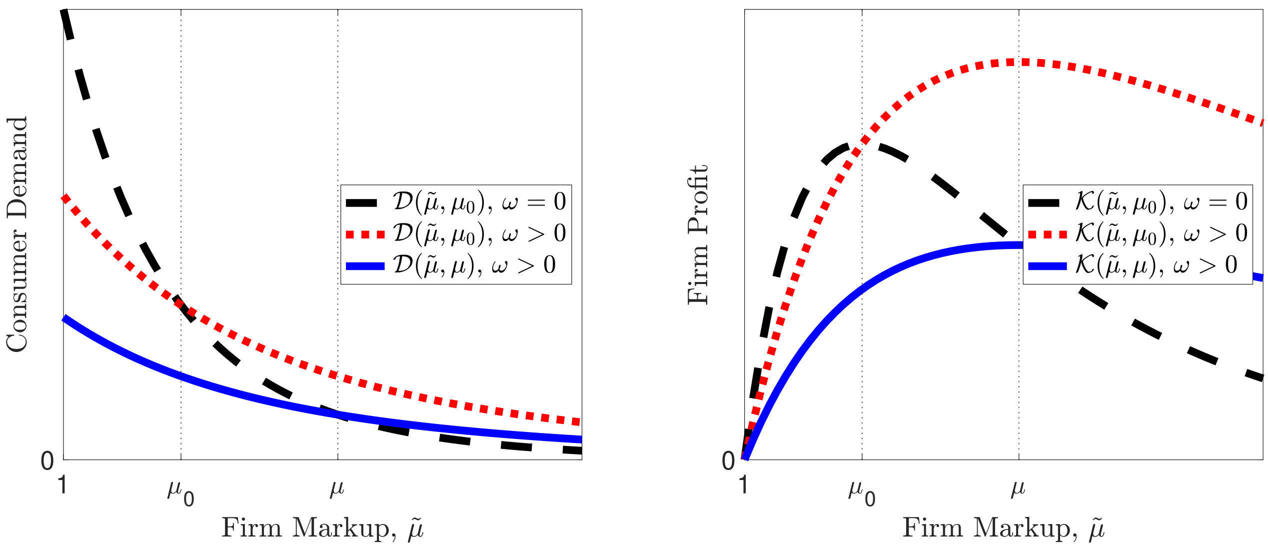
Let $\mathcal{K}_{jt}(\tilde{\mu}, \mu^e) \equiv \mathcal{D}_{jt}(\tilde{\mu}, \mu^e) \times (\tilde{\mu} - 1)/z_{jt}$ denote the associated profits. In practice, $\mathcal{D}_{jt}(\tilde{\mu}, \mu^e)$ accounts for the possibility that consumers may miss-interpret a change in the local price due to an off-equilibrium change in markup as one due to inflation; the higher ω, the higher the chances consumers assign to a general change in inflation. In equilibrium, consumers' expectations and firms' chosen markup coincide at $\mu^e = \tilde{\mu} = \mu$ given by (16), so to obtain the equilibrium demand and profits, i.e. $\mathcal{D}_{jt}(\mu, \mu) = \mathcal{N}_{jt}(p_{jt})\mathcal{C}_{jt}(p_{jt})$ and $\mathcal{K}_{jt}(\mu, \mu) = k_{jt}(p_{jt})$. Next we use a numerical example to discuss why a higher ω is associated to lower equilibrium profits.

The left and right panels of Figure 3 plot, respectively, $\mathcal{D}_{jt}(\tilde{\mu}, \mu^e)$ and $\mathcal{K}_{jt}(\tilde{\mu}, \mu^e)$ evaluated at $\pi_t = 0$ and $\ln z_{jt} = 0$, as a function of $\tilde{\mu}$ on the horizontal axis. The dashed black line plots $\mathcal{D}_{jt}(\tilde{\mu}, \mu^e)$ and $\mathcal{K}_{jt}(\tilde{\mu}, \mu^e)$ when $\omega = 0$ and $\mu^e = \mu_0$, with μ_0 being the markup that satisfies the equilibrium condition in (16) at $\omega = 0$.

Suppose the firm chooses $\mu = \mu_0$ when consumers are uncertain, i.e. $\omega > 0$, and expect $\mu^e = \mu_0$ (dotted red line). Such a strategy cannot be an equilibrium, as a firm setting discretionary prices has an incentive to increase markups. Because of their uncertainty, consumers will interpret any increase in markup above μ_0 – in proportion to ω – as a change in aggregate inflation. This leads to a flattening of consumers' demand (left-hand panel) and to a higher profit curve for markups greater than μ_0 (right-hand panel), as denoted by red dotted curves. The latter show the possibility of increasing profits by raising the markup to μ. However, in a rational expectation equilibrium, consumers' expectation must correctly evaluate the actual markup, so that $\mu^e = \mu$. This implies a downward shift in demand and in firms' profits from the dotted red to the solid blue curve. In the resulting equilibrium profits are lower than if ω were zero.

To sum up, inflation uncertainty causes consumers to confuse an off-equilibrium move in markups with a positive inflationary shock, flattening their demand schedule. This induces

Figure 3: Pricing and consumers' uncertainty.



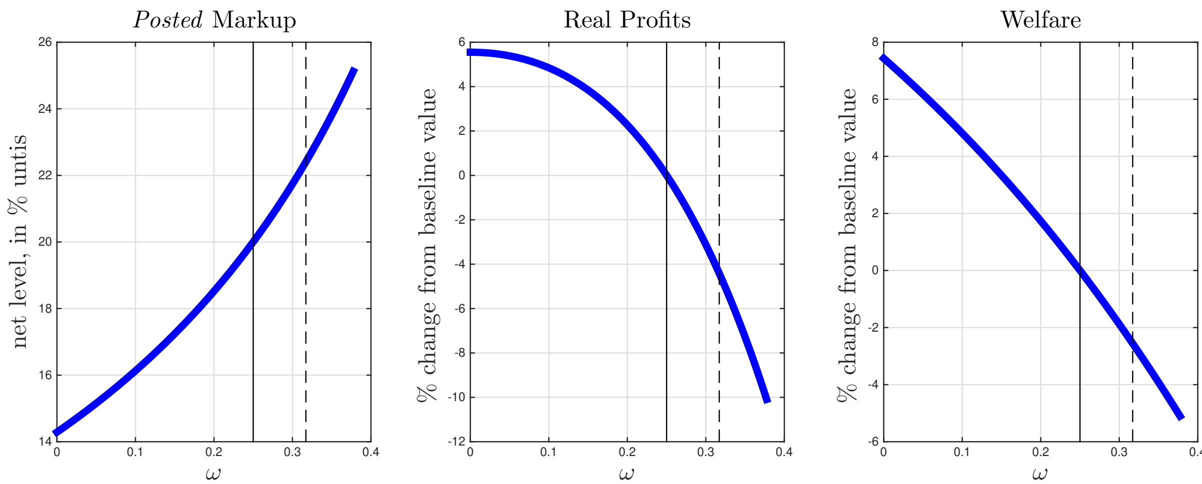
Note: We illustrate consumer demand $\mathcal{D}(\tilde{\mu}, \mu^e)$ and firm profits $\mathcal{K}(\tilde{\mu}, \mu^e)$ for $\mu^e = \mu_0, \omega = 0$ in dashed black, $\mu^e = \mu_0, \omega = 0.5$ in dotted red and $\mu^e = \mu, \omega = 0.5$ in solid blue, where μ_0 and μ are the equilibrium markups of (16) in the case $\omega = 0$ and $\omega = 0.5$ respectively. In all curves we fix: $\lambda + \gamma = 7$, $\pi_t = 0, \ln z_{jt} = 0, \kappa = 1, \mathcal{S} = 0$. All values are chosen with the sole purpose of maximizing the visual sharpness of the figure.

firms to increase markups as they cannot commit not to exploit a lower demand elasticity. In equilibrium, consumers' demand shifts downwards (maintaining the same elasticity), compressing firm profits.

Finally, we discuss the quantitative relevance of consumers' inflation uncertainty for equilibrium markups, profits and welfare. We set $\lambda = 7$ and $\gamma = 1$ in all panels of Figure 4. At our baseline calibration we set $\sigma_\pi^2 = 0.35\%$, which matches the quarterly volatility of CPI inflation in the US since 2010, and σ_z so that $\omega = 0.25$ (vertical solid line); κ is set so that $\bar{\alpha} = 0.66$ at the baseline calibration. We vary consumers' ex-ante uncertainty about inflation, σ_π, which implies a variation of ω as measured on the horizontal axis. The left panel plots the net level of markups at local sellers in percentage units, which at baseline takes value 20%. The central panel plots real profits of local sellers as a percentage change from baseline values. The right panel reports the percentage variation in welfare from baseline level, expressed in units of a consumption equivalent gain on the vertical axis.[20] Figure 4 shows that large first

[20] We report in Appendix B.2 the derivation of welfare and its properties, and in Appendix B.2.2 the details about consumption-equivalent measure.

Figure 4: The effect of consumers' uncertainty on markups, profits and welfare



Note: The figure shows how markups, real profits and welfare vary with consumers' uncertainty, σ_π^2 through variations in ω. We set $\lambda = 7$, $\gamma = 1$ and σ_z so that $\omega = 0.25$ at baseline nominal uncertainty value of $\sigma_\pi = 0.35\%$, as denoted by the vertical solid line. The dashed vertical line corresponds to a counterfactual of a 50% increase in σ_π, giving $\omega = 0.3171$. κ is set so that $\bar{\alpha} = 0.66$ at the baseline calibration. The welfare is reported in difference from the baseline and in units of consumption equivalent change; the definition of welfare is in Appendix B.2.

order welfare losses obtain from inflation uncertainty due to its impact on firm markups. For instance, we can use Figure 4 to evaluate the effect of a 50% increase in inflation volatility, σ_π^2, which entails an increase in ω from 0.25 to 0.3171 (dashed vertical line).[21] Markups of local monopolists increase by 2.5 percentage units, from 20 to 22.5%. At the same time, real profits decrease by about 5% with respect to baseline, which is the result of a larger steady state fraction of switchers in response to higher markups. Finally, because of higher markups, higher consumers' uncertainty has substantial effects on welfare, amounting to a loss of roughly 2.5% of baseline consumption.

5 Introducing *Posted* Price Rigidity

So far we have studied a framework where firms are perfectly informed and pricing is frictionless. In this section, we allow for the traditional channel of price rigidity at the firm

[21] Such an increase has been measured in the Survey of Consumers by the New York Fed on the sample February 2020 to June 2022 with respect to the historical average, from January 2013 to January 2020, in relation to one year ahead inflation.

level introducing firms' uncertainty. We show that our mechanism reinforces the traditional channel.

We assume that firms know their productivity, as in the baseline case, but may be uncertain on wages; this uncertainty affects the extent to which posted prices reflect the actual realization of wages, which households discount in their weighting of the local price. Moreover, this new setting breaks the equivalence between the price in the competitive market and the nominal wage, so that the value of shopping and consuming in the competitive market, in units of labor, is uncertain to households. In practice, we extend our baseline framework assuming that the only information that firms (for simplicity, both local and competitive) have is a common signal about the aggregate state: $\vartheta_t = \pi_t + u_t \in \Omega_t^f$ with $u_t \sim N(0, \sigma_u^2)$, i.i.d. over time. The assumption of a common signal is functional to preserving perfect competition in the distant market. As firms maximize expected profits, a local price solves now $p_{jt} = \text{argmax}_p \; E[k_{jt}(p) \,|\Omega_{jt}^f]$ (as derived in Appendix B.3.2), whereas the competitive price is the expected nominal wage, i.e. $P_t = E[W_t|\Omega_t^f] = \delta\, \vartheta_t$ with $\delta = \sigma_u^{-2}/(\sigma_u^{-2} + \sigma_\pi^{-2})$ being a measure of the uncertainty of firms (both local and distant) ranging from no ($\delta = 0$) to full information ($\delta = 1$).

We also assume that the information set of consumers in island j, at the time of the shopping choice, includes exogenous island-specific information about the current aggregate state summarized by an exogenous signal: $\Omega_{jt}^s = \{p_{jt}, \theta_{jt}, \Omega_{t-1}\}$, with $\theta_{jt} = \pi_t + \nu_{jt}$, where $\nu_{jt} \sim N(0, \sigma_\nu^2)$ is an i.i.d. noise across islands. Let us denote by $\rho = \sigma_\nu^{-2}/(\sigma_p^{-2} + \sigma_\nu^{-2} + \sigma_\pi^{-2})$ the weight put by local consumers on their exogenous signal, with σ_p^{-2} being the precision of local prices. The overall precision of the information available to local consumers is then $\zeta \equiv \delta\omega + \rho$ ranging from no ($\zeta = 0$) to full ($\zeta = 1$) information. For what concerns consumers in the competitive market, they still observe the competitive price P_t, but, in contrast to the basic setting, it does not reveal full information about the nominal wage. Therefore, let $\tilde{\rho}$ denote the elasticity of consumers' inflation expectations to a change in the competitive price P_t. The rest of the model is unchanged.

Proposition 6. *A first-order approximation of the elasticity of aggregate consumption to an* *inflation shock is,*

$$\frac{\partial \ln C_t}{\partial \pi_t}\Big|_{\pi_t=0} \approx \underbrace{\left[\lambda\mu^{\gamma} - \gamma - \lambda\right]\bar{\alpha}\,(\delta - \zeta)\,\pi_t}_{\text{local information gap}} + \underbrace{\left[\bar{\alpha}(\lambda\mu^{\gamma} - \lambda)\,\mu^{\gamma-1}\,\zeta \,+\, (1-\bar{\alpha})\,\gamma\,\tilde{\rho}\right](1-\delta)\,\pi_t}_{\text{competitive price rigidity}}. \qquad (27)$$

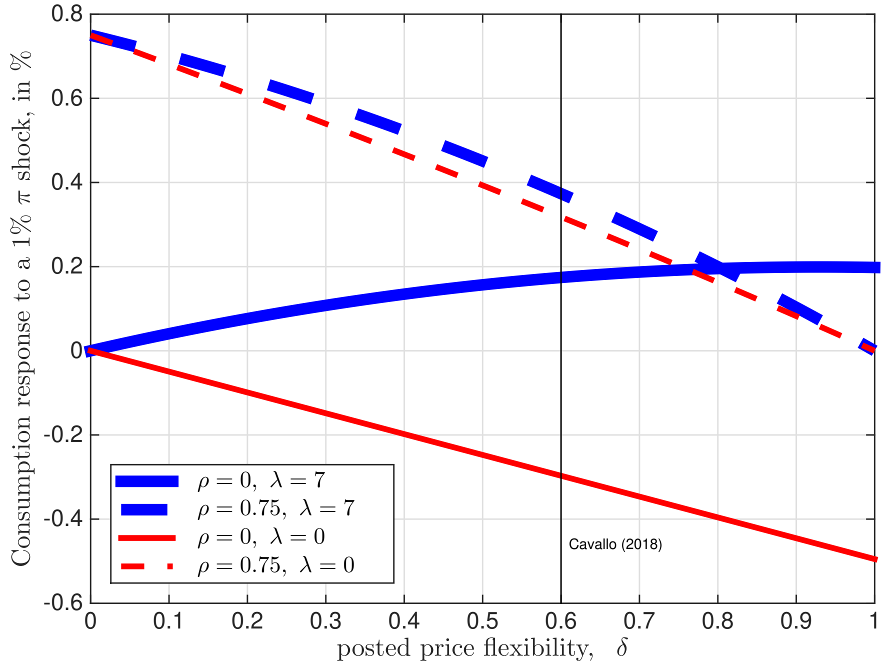
Proof. See Appendix B.3. □

The proposition emphasizes how the overall effect on aggregate consumption obtains as the sum of two effects. Equation (27) is the equivalent to (19) when firms are imperfectly informed and households have access to an exogenous signal on the aggregate price level. The first term, tagged *local information gap*, is the analogous of (19) except for the information gap $\delta - \zeta$. The information gap captures the uncertainty of consumers *relative* to local firms. In particular, the term $\delta - \zeta$ measures the elasticity of the relative local price perceived by the consumer, to a change in the local price p_{jt} originating from a change in aggregate nominal marginal cost. The basic setting (19) obtains as $\sigma_\nu \to \infty$ and $\sigma_u \to 0$, i.e. firms have perfect information ($\delta = 1$) and households only observe local price signals ($\rho = 0$). In this new setting, households could instead be better than firms in predicting the aggregate state: this happens when the exogenous signal is sufficiently precise so that $\zeta > \delta$.

The second term, tagged *competitive price rigidity*, captures instead the traditional effect of incomplete pass-through to competitive prices, scaled by $1 - \delta$. Importantly, this term unambiguously reinforces consumption-inflation comovement. In particular, price rigidity in the competitive market works through two channels. The first is relative to extensive margins. Intuitively, an expected increase in real wages raises the expected utility from shopping competitive relative to local by a factor equal to the ratio of these utilities in steady state, i.e. $\mu^{\gamma-1}$. This induces further switching and so larger aggregate consumption by the usual net consumption marginal gain $\bar{\alpha}\,(\mu^{\gamma} - 1)$ times the elasticity λ. The second component, which is relative to intensive margins, accounts for the increase in aggregate consumption due to a perceived raise in real wages by consumers in the competitive market, so it is proportional to the spending share in the competitive market, $(1 - \bar{\alpha})$, times the elasticity γ.

Overall, equation (27) is useful to understand how our mechanism complements to the traditional price rigidity á la Lucas (1972), typically embedded in workhorse New Keynesian models. We emphasize this complementarity in Figure 5. The figure plots a percentage

Figure 5: The interaction of *posted* and *paid* price rigidities.



Note: The figure shows the consumption response to a 1% inflation shock, measured as a percentage change from steady state, as a function of posted price rigidity δ for different combination of consumers' information ρ and importance of spending reallocation λ. We fix $\gamma = 1, \mu = 1.2, \bar{\alpha} = 0.66$. The vertical line denote the value of posted price flexibility of $\delta = 0.6$ measured in Cavallo (2018).

response of consumption from steady state to a 1% shock in inflation as a function of posted price flexibility δ. We fix $\gamma = 1$, $\lambda = 7$, $\mu = 1.2$ and $\bar{\alpha} = 0.66$ according to our preferred baseline calibration. We highlight the value $\delta = 0.6$, consistent with estimates by Cavallo (2018).[22] The solid bold line denotes the case in absence of any exogenous signal, i.e. $\rho = 0$. Our previous result of a response of about 0.2% obtains in case of full posted price flexibility $\delta = 1$. As price rigidity increases, going on the left along the x-axis, the response decreases achieving zero in case of unresponsive posted prices at $\delta = 0$; in this case firms do not know anything and consumers do not learn anything ($\zeta = \tilde{\rho} = 0$) so that aggregate consumption does not move. The dashed bold line denotes the case with exogenous information such that when firms are fully informed also households are, formally: $\rho = 0.75$, so that $\zeta = 1$ when $\delta = 1$. In this extreme case, the response of aggregate consumption is zero and monotonically

[22] Cavallo (2018) estimates the short-run and long-run pass-trough of the nominal exchange rate to retail prices in the US for the period 2013-2017 using online data from a large number of multi-channel retailers.

increases as firms get less informed (going left on the x-axis) and so posted prices more rigid.

We report the counterfactual in which we switch off our extensive margin mechanism, i.e. $\lambda = 0$ with thin red lines. This benchmark is particularly interesting because demand only responds to inter-temporal substitution incentives, popular in the new-Keynesian narrative of demand driven business cycles. Interestingly, notice that, when households are better informed than firms (the dashed thin line lies above zero), the consumption inflation comovement is positive, whereas it is negative when firms are better informed than households (the solid thin line lies below zero). The motivation is straightforward. Without any fluctuation in extensive margins, wage inflation needs to be associated to a fall in the perceived real wage by consumers, so that households take advantage of the reduction in markups at local sellers due to the incomplete price pass-through and increase their demand. In the spirit of the canonical Lucas' islands model, in models based on intensive margins only, consumers are typically fully informed on the current price level, i.e. $\rho = 0.75$ in our case, so that the effects of nominal shocks on aggregate demand are maximal, for given posted price rigidity.

Finally, and most importantly, the distance between the bold blue and the thin red is the additional effect introduced by our channel, which is always positive both in the solid and the dashed cases. In this sense, our channel working through fluctuations in extensive margins of aggregate demand reinforces the traditional channel based on intensive margins of aggregate demand and rigid posted prices in any configuration. The impact of our channel is maximal when firms are fully informed and consumers only learn from their prices ($\rho = 0$ and $\delta = 1$), whereas it is always zero when firms are completely uninformed ($\delta = 0$) or consumers completely informed ($\zeta = 1$).

6 Empirical evidence

One distinctive prediction of our model is a counter-cyclical behavior of the gap between paid and posted price inflation. In this section, we use scanner data to estimate the comovement between paid and posted prices in response to monetary shocks and check whether it falls within the range of our model predictions.

Let $\mathcal{P}_t^{pos}$ denote the average price paid by households across all sellers in our economy, with the price posted by each seller weighted by its steady state market share: $\mathcal{P}_t^{pos} \equiv (1 - \bar{\alpha})\, P_t + \bar{\alpha}\, \int_0^1 p_{jt} dj$. Let $\pi_t^{eff} \equiv \ln \mathcal{P}_t - \ln \bar{\mathcal{P}}$ and $\pi_t^{pos} \equiv \ln \mathcal{P}_t^{pos} - \ln \bar{\mathcal{P}}^{pos}$ denote, respectively, effective and posted price inflation due to the realization of nominal shocks in period t, i.e. π_t. Using the equilibrium pricing for p_{jt} and the definition of effective price index and markups in equations (20)-(21), it is immediate to show that

$$\pi_t^{eff} - \pi_t^{pos} \approx \frac{\bar{\mathcal{M}} - 1}{\bar{\mathcal{M}}} (\ln \alpha_t - \ln \bar{\alpha}) \approx -\frac{\bar{\mathcal{M}} - 1}{\bar{\mathcal{M}}} \frac{\mu}{\mu - 1} \left(\frac{\lambda}{\lambda + \gamma}\, \bar{\alpha}\, \mu^{\gamma} + 1 - \bar{\alpha} \right) \pi_t^{pos} \tag{28}$$

where we have used $\pi_t^{pos} = \pi_t$ due to the full pass-through of inflation shocks to posted prices.[23] In our model, when inflation rises (falls) unexpectedly, effective inflation increases (decreases) less than posted price inflation as consumers reallocate a larger (smaller) share of expenditure from high- to low-price sellers.[24] We test whether (28) holds in the data.

Operationally, we adopt the same methodology and data used by Coibion et al. (2015) and studied further by Gagnon et al. (2017) to measure paid and posted prices.[25] The data on transaction prices come from Information Resources Inc. ("IRI") which includes weekly price and quantity information from 2001 to 2011 on items, each item defined as the interaction of an Universal Product Code (UPC), pertaining to one of 31 product categories, and a seller pertaining to about 2,000 supermarkets and drugstores in 50 markets in the U.S.[26] Each combination of product category and market is a "stratum". Let $p_{mscj,t}^{pos}$ be the price posted for an item in month t, with m, s, c and j denoting respectively the market, the store, the product category and the UPC; it is obtained as $p_{mscj,t}^{pos} = TR_{mscj,t}/TQ_{mscj,t}$, where $TR_{mscj,t}$ and $TQ_{mscj,t}$ are, respectively, the total revenues and quantities sold in month t. The effective price paid on average for UPC j across all stores s in market m is obtained as

[23]We notice that in the basic version of our model $\pi_t^{pos} = \pi_t$ because firms pass-through one-to-one cost shocks to prices. Nevertheless, Appendix B.1.1 shows that this relationship holds also when firms have an incomplete, but uniform, cost pass-through to posted prices $\delta \in [0, 1]$, so that $\pi_t^{pos} = \delta\, \pi_t$.

[24]In standard frameworks with CES demand structure and dispersion in the price pass-through of inflationary shocks (as in conventional New-Keynesian models), effective inflation increases less rapidly than posted price inflation when inflation rises, but decreases more rapidly than posted price inflation when inflation falls, because consumers are always chasing firms with lowest prices in both cases. See Section 2.1 of Chapter in Galí (2015) for details on the log-linearization of effective price inflation in New-Keynesian models.

[25]In our baseline specification we adopt the sample used by Gagnon et al. (2017), which argue in favor of a less extreme trimming of observations than Coibion et al. (2015).

[26]The product categories include housekeeping, personal care, food at home, cigarettes and photographic supplies.

$p^{eff}_{mscj,t} = \sum_{s \in S_m} TR_{mscj,t} / \sum_{s \in S_m} TQ_{mscj,t}$, where S_m is the set of stores in market m. Posted and effective price inflation are obtained, respectively, as

$$\pi^{pos}_{mc,t} = \sum_{(j,s) \in I_{mc}} w_{mcjs,t}\, \pi^{pos}_{mcsj,t}, \quad \pi^{eff}_{mc,t} = \sum_{j \in J_{mc}} w_{mcj,t}\, \pi^{eff}_{mcj,t},$$

where $\pi^{pos}_{mcsj,t}$ is the monthly inflation associated to $p^{pos}_{mscj,t}$, I_{mc} is the combinations of UPCs and stores in stratum $\{m, c\}$, $\pi^{eff}_{mcj,t}$ is the monthly inflation associated to $p^{eff}_{mscj,t}$, and J_{mc} is the set of UPCs j sold in the stratum $\{m, c\}$. The item and UPC weights, $w_{mcjs,t}$ and $w_{mcj,t}$, are assumed to change infrequently relatively to the frequency of price variation. In order to isolate the effect of posted price changes on inflation from the effects of reallocation across stores, $w_{mcjs,t}$ is computed at the yearly frequency using the revenue share of store s in a stratum (m, c) in a year. The weight of each UPC j, $w_{mcj,t}$, is computed at the yearly frequency, as in the case of average posted prices. In both cases, UPCs within a category are weighted uniformly.[27]

Motivated by the relationship in (28), we estimate the following relationship:

$$\pi^{eff}_{mc,t} - \pi^{pos}_{mc,t} = \varrho\, \pi^{pos}_{mc,t} + \iota\, u_{m,t} + f_t + h_{mc} + error_{mc,t}, \tag{29}$$

where $u_{m,t}$ is the seasonally-adjusted unemployment in market m and month t, h_{mc} are time and stratum fixed effects, $\pi^{pos}_{mc,t}$ is instrumented by US monetary shocks so to capture the response of the inflation gap to contemporaneous inflationary shocks; f_t are time fixed effects present only in the OLS regression. US monetary shocks are estimated by Romer and Romer (2004) from the residuals of a regression of the federal funds rate on lagged values and the Federal Reserve's information set. As these estimates are available only until 2007, estimates refer to the sample truncated at 2007.[28] In columns (1)-(3) of Table 1, we report estimates (29). The estimate of ϱ is consistently negative and statistically significant, ranging from -0.56 in the OLS regression to -0.66 conditional on monetary shocks; its value is essentially

[27] In Appendix B.1 we report estimates for alternative weighting assumptions.

[28] The time series extended to 2007 is made available by Johannes Wieland at https://doi.org/10.3886/E135741V1. We note that, while monetary shocks are a natural instrument for our analysis and are the only drivers of inflation fluctuations in our model, (28) would hold also in response to other aggregate shocks that would cause unexpected aggregate variation in inflation.

Table 1: Gap between effective and posted price inflation

Dependent variable	$\pi^{eff}_{mc,t} - \pi^{pos}_{mc,t}$			$\Delta \ln s_{mc,t}$
	(1)	(2)	(3)	(4)
Inflation, $\pi^{pos}_{mc,t}$	-0.56^{***}	-0.65^{***}	-0.66^{***}	3.68^{***}
	(0.02)	(0.22)	(0.23)	(0.93)
Unemployment, $u_{m,t}$			-0.55^{***}	2.31^{***}
			(0.14)	(0.58)
Stratum F.E.	Yes	Yes	Yes	Yes
Month F.E.	Yes	No	No	No
IV regression	No	Yes	Yes	Yes

Note: In columns (2)-(4) we instrument $\pi^{pos}_{mc,t}$ with the monthly time series of US monetary shocks from 2001 to 2007, and estimate (29) on a monthly basis.

unaffected by controlling for unemployment which, consistently with Coibion et al. (2015), has a negative effect on the measure of the inflation gap. Finally, in column (4) we run the same regression but we replace the dependent variable with the growth rate of retail sales in each market m and category c, to verify that positive inflation shocks are indeed associated to higher sales. A 1% increase in posted prices conditional on a monetary shock is associated to a 3.68% increase in nominal revenues on average. In Appendix B.1 we report results for different sample specifications, as suggested by Gagnon et al. (2017), and show that the estimate of ϱ is consistently negative across the different specifications. Finally, we can compare the estimates of ϱ in columns (2) and (3) of Table 1 to its value predicted by (28) at our baseline calibration, i.e. $\mu = 1.2, \lambda = 7, \gamma = 1$ and $\bar{\alpha} = 2/3$. The calibrated model predicts a value of ϱ equal to -0.73, so well within one standard deviation of the point estimate.

7 Conclusions

What is the source of output-inflation comovement? To answer this question, a long tradition in macroeconomics puts firms' pricing frictions at the center of business cycle models. At first glance, this may appear inevitable in the formalization of the Keynesian logic. One needs firms as price setters in order for demand, not supply, to determine the actual quantity produced in equilibrium. Higher inflation translates into higher perceived real income, because inflation increases less than nominal income due to sticky posted prices. This paper, instead, sets out a novel theory in which the evolution of effective consumer prices, not posted prices, is the engine of the New-Keynesian logic. This approach overturns the standard island setting of Lucas by assuming that uncertainty about the aggregate versus the idiosyncratic origin of fluctuations in posted prices affects demand but not supply decisions.

The results of the paper have been obtained in a static environment where the customer base of a firm is the same at the beginning of each period. This assumption has allowed an analytical characterization of the problem. In future work we plan to extend our analysis to a framework where consumers switch seller persistently so that nominal shocks can have persistent effects on output through spending reallocation.

We have chosen to place our main friction on the demand side of final good markets. Nothing would prevent thinking that a similar mechanism could also be in play for firms on the demand side of an intermediate good market. In this case, final price rigidity would be the result of the cumulative effect of these frictions at different market layers. We do not pursue this line of modelling for simplicity, but also because we want to stress the relevance of households' frictional behavior suggested by the data. Our choice is also functional to the kind of empirical test we can perform: in the last section we will use scanner data on final goods market to validate a distinctive prediction of our model; as far as we know, data of similar quality are not available for intermediate good markets. Moreover, we have been working in a setting with flexible pricing. Incomplete price pass-through of cost shocks are contemplated in our analysis as the result of imperfectly informed firms. Introducing nominal price rigidity due to physical frictions in price adjustment might complement our mechanism reinforcing positive output-inflation comovement. We have abstracted from modeling a feedback from unemployment to the cost opportunity of shopping together with our mechanism. The data suggests that both channels might be at work and a quantitative macroeconomic analysis cannot avoid modeling them both. We leave this analysis to future research.

On the empirical front, we have exploited retailer level data to measure effective price inflation paid by households and compare it to a counterfactual inflation where retailers' market shares are kept constant. Exploiting matched retailer-consumer data, future research should focus on studying the extent to which fluctuations of retailers' market shares are driven by consumer store switching along the extensive margin as opposed to demand reallocation in continuing relationships.

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A Appendix

A.1 Mutual fund and monetary policy

There is a representative mutual fund operating in a competitive financial market. The mutual fund owns all firms in the economy, borrows in one period nominal bonds, B_t, from households, and lends in one period nominal bonds, Q_t, to the government. The flow budget constraint of the mutual fund is such that

$$\frac{B_t - Q_t}{R_t} + P_t K_t = B_{t-1} - Q_{t-1}, \tag{30}$$

where $K_t = \int_0^1 k_{jt}\, dj$ is the aggregate dividend, and $B_t = \int_0^1 \int_0^1 b_{ijt}\, di\, dj$ in equilibrium. There are exogenous shocks to the supply of assets to households which evolves according to

$$\Delta \ln B_t = \chi\, \Delta \ln B_{t-1} + b_t, \tag{31}$$

with $b_t \sim N(0, \sigma_b^2)$ i.i.d over time. The government finances the net repayment of debts to the mutual fund with a lump-sum tax on households,

$$T_t = Q_{t-1} - \frac{Q_t}{R_t}. \tag{32}$$

Monetary policy controls the equilibrium nominal interest rate R_t in the bond market through Q_t to achieve a given target for real supply of assets to the private sector,

$$\ln \frac{B_t}{P_t} = \phi\, \ln B_t. \tag{33}$$

This target effectively determines the path of the nominal price level in the economy, $\ln P_t = (1 - \phi)\, \ln B_t$. The case $\phi = 1$ corresponds to the case of full price level stabilization. Combining equations (31)-(33) we obtain the implied policy target for inflation,

$$\ln \Pi_t = \chi\, \ln \Pi_{t-1} + (1 - \phi)\, b_t. \tag{34}$$

This specification of monetary policy ensures that innovations to inflation are Gaussian distributed, $\pi_t = \ln \Pi_t - \chi\, \ln \Pi_{t-1} \sim N(0, \sigma_\pi^2)$, with $\sigma_\pi^2 = (1 - \phi)^2 \sigma_b^2$. This assumption, together with the functional form of distribution of shopping preferences $G(\psi)$, ensures the existence of Gaussian equilibrium which we can characterize analytically. Moreover, the parameter $\rho > 0$ allows to capture the relationship between inflation perceptions and expected inflation observed in the data.

We notice that the effect of an increase in B_t is isomorphic to an increase of the discount factor β, a popular reduce form way of modeling demand shocks. In this sense, we will refer to an increase in B_t as to a negative demand shock. In particular, changes in R_t transmits to demand in the product market by affecting the equilibrium nominal wage through equations (10)-(11), and the equilibrium price level P_t through (7).

A.2 Proof of Proposition 1

Consumption policy. Conjecture that p_{jt} is log-normal distributed. In this case, households optimally forecast the log P_t according to a linear rule, as in (13). It follows that the elasticity of $E[P_t|\Omega_{jt}^s]$ to p_{jt} is constant and given by ω. Given the conjectured log-normality of price and price (wage) expectations, we can write the policy function for consumption as

$$c_{ijt} = \begin{cases} \varphi^{-\gamma}\, e^{-\gamma\,(\ln p_{jt} - E[\ln W_t|\Omega^s_{ijt}] + \frac{1}{2}\mathcal{S})} & \text{if} \;\; s_{ijt} = 0 \\ \varphi^{-\gamma} & \text{if} \;\; s_{ijt} = 1 \end{cases}. \tag{35}$$

Equilibrium mass of switchers. We next show that the optimal threshold to shop is also log-normally distributed and given by equation (14). We first compute the following object:

$$\Delta_{jt} \equiv E\left[\frac{c_{ijt}^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} - \varphi\, \ell_{ijt} \Big| \Omega^s_{ijt}, s_{ijt} = 0\right] - E\left[\frac{c_{ijt}^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} - \varphi\, \ell_{ijt} \Big| \Omega^s_{ijt}, s_{ijt} = 1\right]$$

where we substitute in the household budget constraint, i.e.

$$\ell_{ijt} = \frac{1}{W_t}\left(\mathcal{P}_{jt}(s_{ijt})\, c_{ijt} + T_t + \frac{B_t}{R_t} - B_{t-1}\right),$$

the government budget constraint, $T_t + \frac{B_t}{R_t} - B_{t-1} = -K_t$, the optimal consumption policy (10) and the price $\mathcal{P}_{jt}(s_{ijt})$ paid by the customer depending on the shopping decision, to obtain

$$\begin{aligned} \Delta_{jt} &= E\left[\frac{\left(\varphi^{-\gamma}\, e^{-\gamma\,(\ln p_{jt} - E[\ln W_t|\Omega^s_{ijt}] + \frac{1}{2}\mathcal{S})}\right)^{1-\frac{1}{\gamma}} - \varphi^{1-\gamma}}{1 - \frac{1}{\gamma}} \,|\, \Omega^s_{ijt}\right] + \\ &- E\left[\frac{\varphi}{W_t}\left(p_{jt}\,\varphi^{-\gamma}\, e^{-\gamma\,(\ln p_{jt} - E[\ln W_t|\Omega^s_{ijt}] + \frac{1}{2}\mathcal{S})} - P_t\,\varphi^{-\gamma}\right) |\Omega^s_{ijt}\right] \\ &= \varphi^{1-\gamma}\, \frac{e^{-(\gamma-1)\,(\ln p_{jt} - E[\ln W_t|\Omega^s_{ijt}] + \frac{1}{2}\mathcal{S})} - 1}{\gamma - 1}. \end{aligned}$$

The threshold for the shopping cost, $\hat{\psi}_{jt}$, at which the household is indifferent if shopping or not is given by $\Delta_{jt} + \hat{\psi}_{jt} = 0$, that is:

$$\hat{\psi}_{jt} = \frac{\varphi^{1-\gamma}}{\gamma - 1}\left(1 - e^{-(\gamma-1)\,(\ln p_{jt} - E[\ln W_t|\Omega^s_{ijt}] + \frac{1}{2}\mathcal{S})}\right).$$

By using $\Psi = \varphi^{1-\gamma}/(\gamma - 1)$, the definition of G, and the shopping threshold, we obtain

$$\mathcal{N}(p_{jt}) \;=\; 1 - G(\hat{\psi}_{jt}) = \kappa\left(1 - \frac{\hat{\psi}_{jt}}{\Psi}\right)^{\frac{\lambda}{\gamma-1}} = \kappa\, e^{\lambda\,(\ln p_{jt} - E[\ln W_t|\Omega^s_{ijt}] + \frac{1}{2}\mathcal{S})}. \tag{36}$$

Optimal pricing. The firm pricing problem in equations (5)-(6) depends on firm demand $\mathcal{N}(p_{jt}) \times \mathcal{C}(p_{jt})$, characterized by a constant elasticity equal to $-(\lambda+\gamma)$ with respect to a change in perceived relative price $\ln p_{jt} - E[\ln W_t|\Omega^s_{jt}]$. Given the guess of log-normality, equation (13) implies that $E[\ln W_t|\Omega^s_{jt}]$ has a constant derivative, ω, with respect to $\ln p_{jt}$. It follows that the elasticity of firm demand to a change in p_{jt} is constant and equal to $-(\lambda + \gamma)(1 - \omega)$, so that the optimal price that solves the problem in (5)-(6) is given by equations in point (ii) of the proposition, provided that $(\lambda + \gamma)(1 - \omega) > 1$, a condition verified if $\gamma + \lambda > 1 + \frac{\sigma^2_\pi}{\sigma^2_z}$.

From expected prices to expected inflation. Households' policies (35) and (36) are expressed in terms of the posterior distribution of the logarithm of the price level (or wage, as P_t and W_t are identical according to (7)). In fact, a given distribution of posterior beliefs about the current inflation innovation having mean $E[\pi_t \,|\, \Omega_{ijt}^{(\cdot)}]$ and variance $V(\pi_t \,|\, \Omega_{ijt}^{(\cdot)})$ implies a distribution of posterior beliefs about the current competitive price level with mean:

$$E[\ln P_t \,|\, \Omega_{ijt}^{(\cdot)}] = E[\ln P_t \,|\, \Omega_{t-1}] + E[\pi_t \,|\, \Omega_{ijt}^{(\cdot)}], \tag{37}$$

and variance $V(\ln P_t \,|\, \Omega_{ijt}^{(\cdot)}) = V(\pi_t \,|\, \Omega_{ijt}^{(\cdot)})$. The term $E[P_t \,|\, \Omega_{t-1}] = P_{t-1} + \chi \ln \Pi_{t-1}$ is readily interpretable as the expectation of the current price level in the competitive market conditional on past information. By substitution, using the equilibrium pricing function, we then get the policies (14)-(15) in the proposition, in terms of inflation and local disturbances directly.

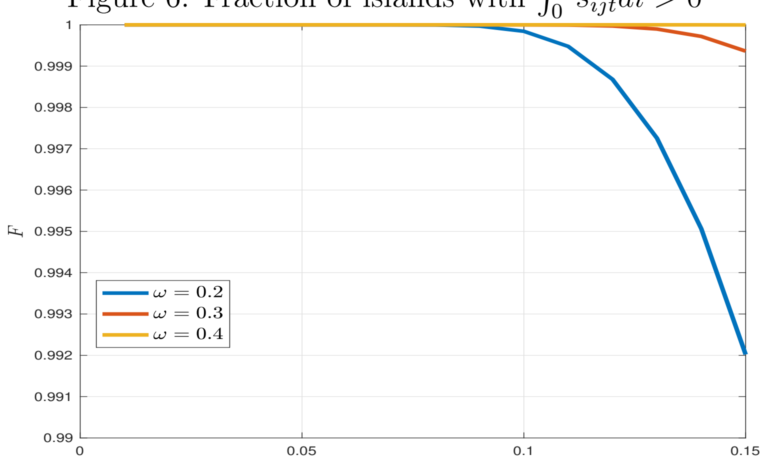
Ensuring $\int_0^1 s_{ijt} di > 0$ almost surely in all islands. Finally, the assumption that z is sufficiently small relative to σ_z guarantees that $\hat{\psi}_{jt} > \underline{\psi}$ in all islands with probability arbitrarily close to 1, so that the firm demand is continuously differentiable at the optimal price in (16) almost surely in all islands. In particular, let

$$F \equiv Pr(\hat{\psi}_{jt} > \underline{\psi}) = Pr\left(\mathcal{N}(p_{jt}) < \kappa\right) = Pr(\ln p_{jt} - E[\ln W_t | \Omega_{jt}^s] + 0.5\mathcal{S} < 0),$$

where $\ln p_{jt} - E[\ln W_t | \Omega_{jt}^s] + 0.5\mathcal{S}$ is normally distributed with mean $\ln \mu - \mathrm{z} + \frac{1}{2}\mathcal{S}$ and variance $(1-\omega)^2(\sigma_\pi^2 + \sigma_z^2)$. We notice that ω, μ and $\mathcal{S}$ are given and finite once we choose finite values of σ_π^2 and σ_z^2, and do not depend on z. Hence, z can be decreased so that F increases to a value arbitrarily close to 1.

We next provide a quantitative assessment of the constraints the requirement $F \approx 1$ puts on the level of volatility of the economy. Let $\sigma_z = \tilde{\sigma}_z \times \hat{\sigma}$ and $\sigma_\pi = \tilde{\sigma}_\pi \times \hat{\sigma}$, for given $\tilde{\sigma}_z$ and $\tilde{\sigma}_\pi$, and the scaling parameter $\hat{\sigma}$. We notice that $\omega = \tilde{\sigma}_\pi^2/(\tilde{\sigma}_z^2 + \tilde{\sigma}_\pi^2)$ is independent of σ. We then set $\mathrm{z} = 0$, $\lambda = 4$, $\gamma = 1$ and compute the value of F corresponding to different combinations of ω and $\sigma^2 = \sigma_z^2 + \sigma_\pi^2$, using $\mu = (\lambda+\gamma)(1-\omega)/((\lambda+\gamma)(1-\omega)-1)$ and $\mathcal{S} = (1-\omega)\,\sigma_\pi^2$, with $\sigma_\pi^2 = \sigma^2(1+1/\omega)$. We plot the results of this exercise in in Figure 6. It shows that our equilibrium characterization is accurate in a neighbour of our baseline calibration even when $\mathrm{z} = 0$ provided $\sigma < 0.1$.

Figure 6: Fraction of islands with $\int_0^1 s_{ijt} di > 0$



A.3 Proof of proposition 2

The **aggregate consumption** is given by:

$$C_t = C^* \left(1 - \int \mathcal{N}(p_{jt})\, dj\right) + \int \mathcal{N}(p_{jt})\mathcal{C}(p_{jt})\, dj.$$

We have

$$\frac{C_t}{C^*} = 1 - \int e^{-\lambda\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)}\, dj \;+\; \int e^{-(\lambda+\gamma)\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)}\, dj$$

$$= 1 - \bar{\mathcal{N}}\; e^{-\lambda\,(1-\omega)\,\pi_t} + \overline{\mathcal{NC}}\, e^{-(\lambda+\gamma)\,(1-\omega)\,\pi_t}$$

with

$$\bar{\mathcal{N}} \;\equiv\; \kappa e^{-\lambda\,(\ln\mu + \frac{1}{2}\mathcal{S} - \frac{\lambda}{2}(1-\omega)^2\sigma_z^2)} \tag{38}$$

$$\bar{\mathcal{C}} \;\equiv\; e^{-\gamma\,(\ln\mu + \frac{1}{2}\mathcal{S} - \frac{\gamma}{2}(1-\omega)^2\sigma_z^2)} \tag{39}$$

$$\overline{\mathcal{NC}} \;\equiv\; \kappa e^{-(\gamma+\lambda)\,(\ln\mu + \frac{1}{2}\mathcal{S} - \frac{\gamma+\lambda}{2}(1-\omega)^2\sigma_z^2)} = \bar{\mathcal{N}}\,\bar{\mathcal{C}}\, e^{\gamma\,\lambda\,(1-\omega)^2\sigma_z^2}. \tag{40}$$

Let us calculate now the elasticity of consumption to inflation at $\pi_t = 0$, which obtains as

$$\left.\frac{\partial \ln C_t}{\partial \pi_t}\right|_{\pi_t=0} \;=\; \frac{\lambda\,(1-\omega)\,\bar{\mathcal{N}} - (\lambda+\gamma)\,(1-\omega)\,\overline{\mathcal{NC}}}{1 - \bar{\mathcal{N}} + \overline{\mathcal{NC}}} = \left(\lambda\,\frac{\bar{\mathcal{N}}}{\overline{\mathcal{NC}}} - \lambda - \gamma\right)(1-\omega)\,\underbrace{\frac{\overline{\mathcal{NC}}}{1 - \bar{\mathcal{N}} + \overline{\mathcal{NC}}}}_{=\bar{\alpha}}$$

where

$$\frac{\bar{\mathcal{N}}}{\overline{\mathcal{NC}}} = \mu^{\gamma} e^{\gamma\,(\frac{1}{2}\mathcal{S} - \frac{\gamma}{2}(1-\omega)^2\sigma_z^2)} e^{-\gamma\,\lambda\,(1-\omega)^2\sigma_z^2}. \tag{41}$$

A.4 Proof of proposition 3

The effective price is defined as

$$\mathcal{P}_t\, C_t = P_t\, C^* \left(1 - \int \mathcal{N}(p_{jt})\, dj\right) + \int \mathcal{N}(p_{jt})\mathcal{C}(p_{jt}) p_{jt}\, dj.$$

We then have:

$$\frac{\mathcal{P}_t}{P_t}\frac{C_t}{C^*} = 1 - \int e^{-\lambda\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)}\, dj \;+\; \frac{1}{P_t}\int e^{-(\lambda+\gamma)\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right) + \ln p_{jt}}\, dj$$

$$= 1 - \bar{\mathcal{N}}\; e^{-\lambda\,(1-\omega)\,\pi_t} + \overline{\mathcal{NC}}\,\mu e^{\mathcal{Q}} e^{-(\lambda+\gamma)\,(1-\omega)\,\pi_t}$$

where we have used (38)-(40), $\mathcal{Q} \equiv -((\lambda+\gamma)(1-\omega) - .5)\,\sigma_z^2$, and

$$-(\lambda+\gamma)\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right) + \ln p_{jt} - \ln P_t = -(\lambda+\gamma)\left(\ln\mu + (1-\omega)\pi_t - (1-\omega)z_{jt} + \frac{1}{2}\mathcal{S}\right) + \ln\mu - z_{jt}.$$

Finally, using the equilibrium relation $P_t = W_t$ we can define the effective aggregate markup as

$$\mathcal{M}_t \equiv \frac{\mathcal{P}_t}{W_t} = \frac{1 - \bar{\mathcal{N}}\, e^{-\lambda\,(1-\omega)\,\pi_t} + \overline{\mathcal{NC}}\, e^{-(\lambda+\gamma)\,(1-\omega)\,\pi_t}\, \mu e^{\mathcal{Q}}}{1 - \bar{\mathcal{N}}\, e^{-\lambda\,(1-\omega)\,\pi_t} + \overline{\mathcal{NC}}\, e^{-(\lambda+\gamma)\,(1-\omega)\,\pi_t}} = 1 + \alpha_t \left(\mu e^{\mathcal{Q}} - 1\right),$$

with

$$\alpha_t \equiv \frac{\overline{\mathcal{NC}}\; e^{-(\lambda+\gamma)\,(1-\omega)\,\pi_t}}{1 - \bar{\mathcal{N}}\, e^{-\lambda\,(1-\omega)\,\pi_t} + \overline{\mathcal{NC}}\, e^{-(\lambda+\gamma)\,(1-\omega)\,\pi_t}}.$$

The elasticity of the local market share to inflation at $\pi_t = 0$ obtains as

$$\left.\frac{\partial \ln \alpha_t}{\partial \pi_t}\right|_{\pi_t=0} \quad = \quad \frac{\frac{\overline{\mathcal{NC}}}{1-\bar{\mathcal{N}}+\overline{\mathcal{NC}}}\left[-(\lambda+\gamma) - \left(\lambda\frac{\bar{\mathcal{N}}}{\overline{\mathcal{NC}}} - (\lambda+\gamma)\right)\frac{\overline{\mathcal{NC}}}{1-\bar{\mathcal{N}}+\overline{\mathcal{NC}}}\right](1-\omega)}{\frac{\overline{\mathcal{NC}}}{1-\bar{\mathcal{N}}+\overline{\mathcal{NC}}}},$$

where (41) holds.

A.5 Proof of Proposition 4

First of all we work out the conditions under which that $\lambda\mu^\gamma - \lambda - \gamma > 0$, which decides the sign of (19). We have that

$$\left(\frac{(\gamma+\lambda)\,(1-\omega)}{(\gamma+\lambda)\,(1-\omega)-1}\right)^\gamma \quad > \quad \frac{\gamma+\lambda}{\lambda} \tag{42}$$

or

$$\omega \quad > \quad \bar{\omega}(\gamma,\lambda) \equiv \frac{\left(\frac{\gamma+\lambda}{\lambda}\right)^{\frac{1}{\gamma}}\frac{\gamma+\lambda-1}{\gamma+\lambda} - 1}{\left(\frac{\gamma+\lambda}{\lambda}\right)^{\frac{1}{\gamma}} - 1}$$

where $\bar{\omega}(\gamma,\lambda)$ is the value at which the $\partial \ln C_t/\partial \pi_t = 0$, i.e. (42) holds as an equality.

By substitution we note $\bar{\omega}(1,\lambda) = 0$. We then show that:

$$\frac{\partial\bar{\omega}(\gamma,\lambda)}{\partial\gamma} \quad = \quad -\frac{\left(\frac{\lambda+\gamma}{\lambda}\right)^{\frac{1}{\gamma}}}{\gamma^2\left(\left(\frac{\lambda+\gamma}{\lambda}\right)^{\frac{1}{\gamma}}-1\right)^2(\lambda+\gamma)^2}\left(\lambda\left(\ln\left(1+\frac{\gamma}{\lambda}\right)-\frac{\gamma}{\lambda}\right)+\gamma^2\left(\ln\left(\frac{\lambda+\gamma}{\lambda}\right)^{\frac{1}{\gamma}}-\left(\frac{\lambda+\gamma}{\lambda}\right)^{\frac{1}{\gamma}}+1\right)\right) > 0$$

since $\ln x < x - 1$ for any $x \in (0,1)$ and $x \in (1,\infty)$.

Let us now investigate the conditions under which

$$\frac{\partial\bar{\omega}(\gamma,\lambda)}{\partial\lambda} = -\frac{\left(\frac{1}{\lambda}(\lambda+\gamma)\right)^{\frac{1}{\gamma}}\left(\lambda - \lambda\left(\frac{1}{\lambda}(\lambda+\gamma)\right)^{\frac{1}{\gamma}}+1\right)}{\lambda\left(\left(\frac{1}{\lambda}(\lambda+\gamma)\right)^{\frac{1}{\gamma}}-1\right)^2(\lambda+\gamma)^2} > 0$$

or, equivalently,

$$\frac{\lambda+1}{\lambda} \quad < \quad \left(\frac{\lambda+\gamma}{\lambda}\right)^{\frac{1}{\gamma}}. \tag{43}$$

It is easy to prove by substitution that for $\gamma = 1$, (43) holds as an equality. We note

$$\frac{\partial\left(\left(\frac{\lambda+\gamma}{\lambda}\right)^{\frac{1}{\gamma}}\right)}{\partial\gamma} = \frac{1}{\gamma^2}\left(\frac{\lambda+\gamma}{\lambda}\right)^{\frac{1}{\gamma}}\left(\ln\left(\frac{\lambda}{\lambda+\gamma}\right) - \frac{\lambda}{\lambda+\gamma} + 1\right) < 0 \tag{44}$$

as $\ln x < x - 1$ for any $x \in (0, 1)$ and $x \in (1, \infty)$. Therefore, $\bar{\omega}_\lambda > 0$ with $\gamma < 1$ and $\bar{\omega}_\lambda < 0$ with $\gamma > 1$.

A.6 Proof of proposition 5

Using the equilibrium pricing condition, $p_{jt} = \mu\, W_t/z_{jt}$, we can rewrite equilibrium real profits (6) as

$$k_{jt}(p_{jt}) \quad = \quad \mathcal{D}_{jt}(\mu, \mu)\frac{1}{z_{jt}}(\mu - 1)$$

which already embeds the equilibrium condition $\mu = \mu^e$. By taking expectations we have

$$\bar{k} = C^* \kappa\, e^{(1-\omega)^2\, \frac{(\lambda+\gamma)^2}{2}(\sigma_\pi^2+\sigma_z^2)+\frac{(\lambda+\gamma)}{2}\mathcal{S}}\ (\mu - 1)\mu^{-(\lambda+\gamma)}.$$

It is then immediate to show that (i) $\partial\bar{k}/\partial\omega < 0$, because $\partial\bar{k}/\partial\mu < 0$ given that $\lambda + \gamma > 1$, and $\partial\mu/\partial\omega > 0$ from (16), and that (ii) the effect of a change in $1 - \omega$ on $\bar{k}$ is scaled by $\lambda + \gamma$.

B Extensions (for online publication)

In this section, we implement some robustness to the empirical analysis in the main text, derive welfare and present some extensions to our baseline framework.

B.1 Empirical analysis: robustness

We discuss here the methodology for the estimates of Section 6. We use the same data and methodologies of Coibion et al. (2015) and Gagnon et al. (2017), denoted as CGH and GLSS respectively, to construct measures of effective and posted price inflation. Data on transaction prices comes Information Resources Inc. ("IRI") and includes weekly price and quantity information from 2001 to 2011 on items, each item defined as the interaction of an Universal Product Code (UPC) pertaining to 31 product categories and a store pertaining to about 2,000 supermarkets and drugstores across 50 U.S. markets.[1] We use the methodology detailed in the Online Appendix and the codes made available in the Data Set of GLSS to construct the measures of effective and posted price inflation. In particular, each combination of product category and market is a "stratum", corresponding to the level at which we, as well as CGH and GLSS, evaluate the variation in posted and price inflation. In each stratum there is a collection of weekly "price trajectories", each corresponding to an item, and containing information on the number of units sold, total revenues as well as a promotion flag and other characteristics.[2]

Table 2: Gap between effective and posted price inflation

Dependent variable	$\pi^{eff}_{mc,t} - \pi^{pos}_{mc,t}$			
	(1)	(2)	(3)	(4)
Inflation, $\pi^{pos}_{mc,t}$	-2.41^{**}	-2.48^{**}	-2.17^{**}	2.24^{**}
	(1.08)	(1.15)	(0.99)	(1.05)
Unemployment, $u_{m,t}$		-2.505^{***}		-2.29^{***}
		(1.08)		(0.99)
Stratum F.E.	Yes	Yes	Yes	Yes
Month F.E.	No	No	No	No
IV regression	Yes	Yes	Yes	Yes
Market weights	Yes	Yes	No	No
Common weights	No	No	Yes	Yes

Note:We instrument $\pi^{pos}_{mc,t}$ with the monthly time series of US monetary shocks from 2001 to 2007, and estimate (29) on a monthly basis.

In our baseline regression of Table 1, UPCs have been weighted uniformly In Table 2, the weights $w_{mcj,t}$ used to aggregate effective price changes across UPCs are computed either

[1] The product categories include housekeeping, personal care, food at home, cigarettes and photographic supplies.

[2] All observations pertaining to private labels are dropped. We refer to the GLSS for further details.

using the revenue shares of each UPC in a stratum in a year, "Market-specific" weights, or the revenue share in the category across all markets in a year, "Common" weights. Similarly, the weights $w_{mscj,t}$ used to aggregate posted price changes across items are computed either using the revenue shares of each item in a stratum in a year, or the revenue share of the item in the category across all markets in a year. Crucially, weights are computed at a yearly frequency so that the reallocation of consumer spending from high-price to low-price stores is captured by $\pi^{eff}_{mc,t}$ but not $\pi^{pos}_{mc,t}$.

Table 2 reports estimates of equation (29) for different weighting schemes. The regression is obtained on the GLSS' sample which uses a less invasive method to control for outsize price adjustments, and is the same sample used in Table 1. We refer to CGH and GLSS for a discussion about the advantages and disadvantages of the different approaches.

B.1.1 Effective versus posted price inflation

We derive equation (28). Let us generalize the firms' pricing by allowing an incomplete, but common, pass-through of wage inflation shocks to posted prices. In particular, let us assume that both P_t and p_{jt} have a contemporaneous elasticity of δ to changes in W_t. The index of posted prices evolves as

$$\mathcal{P}^{pos}_t = P_{t-1}\,\Pi^{\chi}_{t-1}\,e^{\delta\,\pi_t}.$$

We recall that P_t rvolves as $P_t = P_{t-1}\,\Pi^{\chi}_{t-1}\,e^{\pi_t}$. The effective and price prices can then be expressed as

$$\frac{\mathcal{P}_t}{P_t} = e^{-(1-\delta)\,\pi_t}\left[1\,+\,\alpha_t\,(\mu-1)\right], \quad \frac{\mathcal{P}^{pos}_t}{P_t} = e^{-(1-\delta)\,\pi_t}.$$

We can write

$$\ln\left(\frac{\mathcal{P}_t}{\mathcal{P}_{t-1}}\frac{P_{t-1}}{P_t}\right) \approx (\alpha_t-\alpha_{t-1})\,(\mu-1)-(1-\delta)\,\pi_t, \qquad \ln\left(\frac{\mathcal{P}^{pos}_t}{\mathcal{P}^{pos}_{t-1}}\frac{P_{t-1}}{P_t}\right) \approx -(1-\delta)\,\pi_t.$$

Then, combining these results,

$$\ln\left(\frac{\mathcal{P}_t}{\mathcal{P}_{t-1}}\right) - \ln\left(\frac{\mathcal{P}^{pos}_t}{\mathcal{P}^{pos}_{t-1}}\right) = \pi^{eff}_t - \pi^{pos}_t \approx \frac{\bar{\alpha}(\mu-1)}{1+\bar{\alpha}(\mu-1)}\,(\ln\alpha_t - \ln\alpha_{t-1}).$$

B.2 Welfare

Given that we posit that shocks last for only one period, without any loss of generality we can take the unconditional expectation of the present-flow utility of a household as our measure of welfare. Dropping the time subscripts, welfare is thus defined as

$$\mathcal{W} \equiv E\left[u(P) + \mathcal{N}(p_{jt})(u(p_{jt})-u(P)) - \int_0^{\hat{\psi}(p_{jt})}\psi g(\psi)d\psi\right]$$

where $u(\mathcal{P}_{jt}(s_{ijt})) = (c^{1-\gamma^{-1}}_{ijt}-1)/(1-\gamma^{-1}) - \varphi\,\ell_{ijt}$ denotes utility net of switching costs with $\Delta_{jt} \equiv E[u(p_{jt})-u(P_t)|\Omega^s_{jt}]$ being the utility gain expected by households at the time of their shopping choice. Let $\tilde{\mathcal{W}} \equiv \Psi - \gamma/(\gamma-1)$ be the value of the welfare in a *perfect competition benchmark* with no shopping disutility, i.e. one in which all households buy effortlessly from the competitive market so that profits are zero. We can now state a new proposition.

Proposition 7. *Welfare measured in deviations from the perfect competition benchmark is given by*

$$\mathcal{W} - \tilde{\mathcal{W}} \;=\; \varphi\,(\mu^* - 1)\left(E\left[\mathcal{D}(p_{jt})\frac{p_{jt}}{P_t}\right] - \tilde{C}\right) + \varphi\, E\left[\mathcal{D}(p_{jt})\left(\frac{p_{jt}}{P_t} - \frac{1}{z_{jt}}\right)\right] < 0$$

where $\mu^* = (\gamma+\lambda)/(\gamma+\lambda-1)$ *is the markup in the absence of signaling power.*

Proof. See Appendix B.2.1. □

The proposition defines welfare as the sum of two components. The first is the difference between the expected real local demand and the perfect competitive level $\tilde{C}$ at $\tilde{\mathcal{W}}$. This difference is negative and impacts on welfare proportionally to the net markup with no signaling power $\mu^* - 1$. The second term is average real profits, which are positive, although not sufficiently to reverse the overall sign of $\mathcal{W} - \tilde{\mathcal{W}}$.

Both components go to zero as the elasticity of demand goes to infinity, i.e. $\lim_{\gamma+\lambda\to\infty} \mu^* = 1$ at which we also have $\lim_{\gamma+\lambda\to\infty} K_t = 0$. In this case, market power is absent because either agents can shop in the competitive market at no cost (case $\lambda \to \infty$) or else they prefer saving everything then consuming to any price above the competitive price (case $\gamma \to \infty$). As a result, infinite demand elasticity entails our welfare benchmark $\tilde{\mathcal{W}}$.

B.2.1 Proof of proposition 7

Let us define

$$u(\mathcal{P}_{jt}(s_{ijt})) \equiv \frac{c_{ijt}^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + \varphi\,\mathcal{L}(\mathcal{P}_{jt}(s_{ijt}))$$

with c_{ijt} as in (15) and $\mathcal{L}(\mathcal{P}_{jt}(s_{ijt})) = \frac{1}{W_t}\left(\mathcal{P}_{jt}(s_{ijt})\,c_{ijt} + T_t + \frac{B_t}{R_t} - B_{t-1}\right)$. Welfare is given by

$$\mathcal{W} \equiv E\left[u(P) + \mathcal{N}(p_{jt})(u(p_{jt}) - u(P)) - \int_{\underline{\psi}}^{\hat{\psi}(p_{jt})} \psi g(\psi) d\psi\right].$$

Given that $\mathcal{N}_{jt}$ is measurable with respect to Ω^s_{jt}, we can write $E[\mathcal{N}_{jt}\,(u(p_{jt}) - u(P))] = E[\mathcal{N}_{jt}\,E[u(p_{jt}) - u(P)|\Omega^s_{jt}]]$ where

$$\Delta_{jt} \equiv E[u(p_{jt}) - u(P)|\Omega^s_{jt}] = \Psi\,\left(e^{-(\gamma-1)\,(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S})} - 1\right),$$

with $\Psi = \varphi^{1-\gamma}/(\gamma-1)$, so that

$$\mathcal{N}_{jt}\Delta_{jt} = \kappa\Psi\left(e^{-(\gamma+\lambda-1)\,(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S})} - e^{-\lambda\left(\ln p_{jt} - E\left[\ln P_t|\,\Omega^s_{jt}\right] + \frac{1}{2}\mathcal{S}\right)}\right).$$

Recall that $\mathcal{N}_{jt} = \kappa e^{-\lambda(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S})}$ so that

$$\mathcal{N}_{jt}\Delta_{jt} + \Psi\,\mathcal{N}_{jt} = \kappa\,\Psi e^{-(\gamma+\lambda-1)\,(\ln p_{jt} - E[\ln P_t|\Omega^s_{jt}] + \frac{1}{2}\mathcal{S})} = \Psi E\left[\mathcal{D}(p_{jt})\frac{p_{jt}}{P_t}\right], \tag{45}$$

with $\mathcal{D}(p_{jt}) \equiv N_{jt}(p_{jt})\mathcal{C}_{jt}(p_{jt})$. Let us calculate now the cumulative shopping effort cost

$$\begin{aligned} -\int_{\underline{\psi}}^{\hat{\psi}(p_{jt})} \psi g(\psi) d\psi &= -\int_{\underline{\psi}}^{\hat{\psi}(p_{jt})} \frac{\lambda}{\gamma-1}\frac{\psi}{\Psi}\,\kappa\left(1-\frac{\psi}{\Psi}\right)^{\frac{\lambda}{\gamma-1}-1} d\psi \\ &= -\frac{\Psi}{\frac{\lambda}{\gamma-1}+1}\left(1-\mathcal{N}_{jt}\right) - \frac{\frac{\lambda}{\gamma-1}}{\frac{\lambda}{\gamma-1}+1}\Delta_{jt}\mathcal{N}_{jt}, \end{aligned}$$

where we exploited the fact $\hat{\psi}_{jt} = -\Delta_{jt}$. We know that

$$u(P_t) \;=\; \Psi - \frac{\gamma}{\gamma-1} + \varphi\, K_t$$

where note $E[K_t] = E[k_{jt}]$. Once defining $\tilde{\mathcal{W}} \equiv \Psi - \gamma/(\gamma-1)$, we can use (45) to write

$$\mathcal{W} - \tilde{\mathcal{W}} - \varphi E[K_t] \;=\; E\left[\Delta_{jt}\mathcal{N}(p_{jt}) - \frac{\Psi}{\frac{\lambda}{\gamma-1}+1}\left(1-\mathcal{N}(p_{jt})\right) - \frac{\frac{\lambda}{\gamma-1}}{\frac{\lambda}{\gamma-1}+1}\Delta_{jt}\mathcal{N}(p_{jt})\right]$$

and finally

$$\mathcal{W} - \tilde{\mathcal{W}} = \varphi(\mu^* - 1)\left(E\left[\mathcal{D}(p_{jt})\frac{p_{jt}}{P_t}\right] - C^*\right) + \varphi E\left[\mathcal{D}(p_{jt})\left(\frac{p_{jt}}{P_t} - \frac{1}{z_{jt}}\right)\right].$$

B.2.2 Consumption-equivalent welfare measure

Suppose a welfare measures $\bar{\mathcal{W}} = \mathcal{W}_{baseline} + k$ where $\mathcal{W}_{baseline}$ is a convenient reference. Let us denote

$$\mathcal{K}(C^*, C) \equiv E\left[(1-\mathcal{N}(p_{jt}))\frac{(C^*)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + \mathcal{N}(p_{jt})\frac{C(p_{jt})^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}\right]$$

where notice $\mathcal{K}(x\,C^*, x\,C) = x^{1-\frac{1}{\gamma}}\mathcal{K}(C^*, C)$. We want to find the g_c such that

$$\mathcal{K}(g_c\,C^*, g_c\,C_{baseline}) - \mathcal{K}(C^*, C_{baseline}) = k$$

where $C_{baseline}$ is local consumption corresponding to $\mathcal{W}_{baseline}$. We get

$$g_c = \left(\frac{k + \mathcal{K}(C^*, C_{baseline})}{\mathcal{K}(C^*, C_{baseline})}\right)^{\frac{\gamma}{\gamma-1}}.$$

B.3 Uncertain firms

The signaling power in this version of the model is given by:

$$\omega = \frac{1}{\delta}\frac{\frac{1}{\sigma_u^2+\sigma_z^2/\delta^2}}{\frac{1}{\sigma_u^2+\sigma_z^2/\delta^2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}, \tag{46}$$

where,

$$\delta = \frac{\sigma_u^{-2}}{\sigma_u^{-2}+\sigma_\pi^{-2}} \quad \text{and} \quad \rho = \frac{\sigma_\nu^{-2}}{\frac{1}{\sigma_u^2+\sigma_z^2/\delta^2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}}.$$

denote the weight put on the exogenous signals by firms and households respectively, and

$$\mathcal{F} \equiv V(\pi|\Omega_t^f) = \frac{1}{\sigma_u^{-2}+\sigma_\pi^{-2}} \quad \text{and} \quad \mathcal{S} \equiv V(\pi|\Omega_{j,t}^s) = \frac{1}{\frac{1}{\sigma_u^2+\sigma_z^2/\delta^2}+\sigma_\nu^{-2}+\sigma_\pi^{-2}},$$

denote the forecast errors of consumers and firms. We also define $\zeta \equiv \delta\omega + \rho$ the overall elasticity of households' relative price expectations to inflation innovations. Our baseline version of the model obtains with $\sigma_u \to 0$ and $\sigma_\nu \to \infty$, i.e. exogenous firms' signal are fully informative, whereas households' exogenous signal are completely uninformative.

B.3.1 Derivation of the policies.

The consumption policy in this case is:

$$c_{ijt} = \begin{cases} \mathcal{C}_{jt}(p_{jt}) \equiv \varphi^{-\gamma}\, e^{-\gamma\,(\ln p_{jt} - E[\ln W_t|\theta_{jt},p_{jt}] + \frac{1}{2}V(W_t|\theta_{jt},p_{jt})} & \text{if} \quad s_{ijt}=0 \\ C_t^* \equiv \varphi^{-\gamma}\, e^{-\gamma\,(\ln P_t - E[\ln W_t|\theta_{jt},P_t] + \frac{1}{2}V(W_t|\theta_{jt},P_t)} & \text{if} \quad s_{ijt}=1. \end{cases}$$

In particular, we have:

$$\ln p_{jt} - E\left[\ln W_t|\, p_{jt},\theta_{jt}\right] \;=\; \ln\mu - z_{jt}(1-\omega) + \frac{1}{2}\mathcal{V} + (\delta-\zeta)\,\pi_t + \delta\,(1-\omega)\,u_t - \rho\,\nu_{jt}$$

and

$$\begin{aligned} E[\ln W_t - \ln P_t|p_{jt},\theta_{jt}] &= E[(1-\delta)\,\pi_t + \delta\, u_t|p_{jt},\theta_{jt}] \\ &= (1-\delta)(\omega\,\delta\,(\vartheta_t - z_{jt}/\delta) + \rho\,\theta_{jt}) + \delta\,\rho_u\,\underbrace{(\vartheta_t - z_{jt}/\delta - \hat{\rho}\theta_{jt})}_{\tilde{\vartheta}_{ij}}, \end{aligned}$$

with

$$\hat{\rho} = \frac{\sigma_\nu^2}{\sigma_\nu^2+\sigma_\pi^2} \qquad \rho_u = \frac{\sigma_{\tilde{\vartheta}}^{-2}}{\sigma_{\tilde{\vartheta}}^{-2}+\sigma_u^2} \qquad \text{and} \qquad \sigma_{\tilde{\vartheta}}^{-2} = \left(\frac{1}{\sigma_\nu^2+\sigma_\pi^2}+\frac{\sigma_z^2}{\delta^2}\right)^{-1},$$

where $\tilde{\vartheta}_{ij}$ is the best information local consumers have on the common noise u, whose precision is $\sigma_{\tilde{\vartheta}}^{-2}$. In practice consumers use their own signal to further refine their inference about the common noise made conditional on observing the local price.

Finally, the real wage expected conditionally to the information set in the competitive market is:

$$E[\ln W_t|\vartheta_t,\theta_{jt}] - P_t \quad = \quad E[\pi_t|\vartheta_t,\theta_{jt}] - E[\pi_t|\vartheta_t] = \underbrace{\frac{\sigma_\nu^{-2}}{\sigma_\pi^{-2}+\sigma_u^{-2}+\sigma_\nu^{-2}}}_{=\tilde{\rho}}(\theta_{jt}-\delta\vartheta_t)\,, \qquad (47)$$

which is nothing else that the Kalman updating of firms' expectations. This passage mathematically captures the fact that consumers are strictly better than competitive firms in forecasting innovations in inflation.

As before, we next derive the equilibrium threshold of shopping effort. We want to compute the following object:

$$\Delta_{jt} = E\left[\frac{c_{ijt}^{1-\frac{1}{\gamma}}-1}{1-\frac{1}{\gamma}} - \varphi\,\ell_{ijt}\Big|\Omega^s_{ijt}, s_{ijt}=0\right] - E\left[\frac{c_{ijt}^{1-\frac{1}{\gamma}}-1}{1-\frac{1}{\gamma}} - \varphi\,\ell_{ijt}\Big|\Omega^s_{ijt}, s_{ijt}=1\right]$$

as we did in A.2. Conjecture that the price function is log-normally distributed. Following the same steps made in A.2 we get

$$\Delta_{jt} = \varphi^{1-\gamma}\left(\frac{e^{\frac{1}{2}\mathcal{W}_1}}{1-\frac{1}{\gamma}} - e^{\frac{1}{2}\mathcal{W}_3}\right)e^{(1-\gamma)\,(\ln p_{jt}-E[\ln W_t|p_{jt},\theta_{jt}])} - \varphi^{1-\gamma}\left(\frac{e^{\frac{1}{2}\mathcal{W}_2}}{1-\frac{1}{\gamma}} - e^{\frac{1}{2}\mathcal{W}_4}\right)e^{(1-\gamma)E[\ln P_t-\ln W_t|p_{jt},\theta_{jt}]}$$

where we have used the fact $E[E[\ln W_t|P_t,\theta_{jt}]|p_{jt},\theta_{jt}] = E[\ln W_t|p_{jt},\theta_{jt}]$, and $\mathcal{W}.$ indicate Jansen inequality terms. We can then write down:

$$\mathcal{N}(p_{jt}) \quad = \quad 1 - G(\hat{\psi}_{jt}) = \kappa\left(1+\frac{\Delta_{jt}}{\Psi}\right)^{\frac{\lambda}{\gamma-1}}$$

where we used $\Psi = \frac{\varphi^{1-\gamma}}{\gamma-1}$ and $\hat{\psi}_{jt} = -\Delta_{jt}$. We can conveniently approximate the function to first-order approximation around the deterministic mean $\bar{\mathcal{N}} = \kappa\mu^{-\lambda}$. Let us call $x = \ln p_{jt} - E[p_{jt}|\Omega_{t-1}] - E[\ln W_t|p_{jt},\theta_{jt}]$ and $y = E[\ln P_t - \ln W_t|p_{jt},\theta_{jt}]$ we have:

$$\mathcal{N}(p_{jt}) \approx \bar{\mathcal{N}} + \kappa\frac{\partial(1+\mu^{1-\gamma}e^{(1-\gamma)x}-e^{(1-\gamma)y})^{\frac{\lambda}{\gamma-1}}}{\partial x}\Big|_{(x,y)=0}x + \kappa\frac{\partial(1+\mu^{1-\gamma}e^{(1-\gamma)x}-e^{(1-\gamma)y})^{\frac{\lambda}{\gamma-1}}}{\partial y}\Big|_{(x,y)=0}y$$

$$= \bar{\mathcal{N}} - \lambda\mu^{-\lambda}x + \lambda\mu^{-\lambda}\mu^{\gamma-1}y$$

Therefore:

$$\ln\mathcal{N}(p_{jt}) - \ln\bar{\mathcal{N}} \approx -\lambda\,(\ln p_{jt} - E[p_{jt}|\Omega_{t-1}] - E\,[\ln W_t|\,p_{jt},\theta_{jt}]) - \lambda\mu^{\gamma-1}\,E[\ln W_t - \ln P_t|p_{jt},\theta_{jt}]$$

The first-order impact of π on aggregate consumption $C_t = E_t\,[C^*\,(1-\mathcal{N}_{jt})] + E_t\,[\mathcal{N}_{jt}\,\mathcal{C}_{jt}]\,.$

approximated around its deterministic steady state, is

$$
\begin{aligned}
\ln C_t - \ln \bar{C} \approx\, & \tilde{\alpha}\, \gamma\, \tilde{\rho}\, (1-\delta)\, \pi_t \\
& - \tilde{\alpha}\, \mu^{-\lambda} \left(\gamma\, \tilde{\rho}\, (1-\delta)\, \pi_t - \lambda(\delta - \zeta)\pi_t - \lambda\, \mu^{\gamma-1}\, (1-\delta)\, \zeta\, \pi_t \right) \\
& + \tilde{\alpha}\, \mu^{-\lambda-\gamma} \left(-(\lambda+\gamma)\, (\delta-\zeta)\pi_t - \lambda\, \mu^{\gamma-1}\, (1-\delta)\, \zeta\, \pi_t \right)
\end{aligned}
$$

with $\tilde{\alpha} = 1/(1-\mu^{-\lambda} + \mu^{-\gamma-\lambda}) > 0$, finally giving (27) with $\bar{\alpha} = \tilde{\alpha}\, \mu^{-\lambda-\gamma}$.

B.3.2 Pricing under uncertainty

We find the optimal price in the generic case of firms conditioning on an information set Ω^f_{jt} where we just assume $z_{jt} \in \Omega^f_{jt}$. The main case in the text obtains when Ω^f_{jt} is complete. This step will allow to finally verify our conjecture about the log-normality of the equilibrium pricing of local firms. First of all, let us compute demand elasticity. We have:

$$
\begin{aligned}
\frac{\partial \left(\mathcal{N}(p_{jt}) \mathcal{C}(p_{jt}) \right)}{\partial\, p_{jt}} &= \frac{\partial \left(\kappa\, \varphi^{-\gamma} e^{-\gamma\, (\ln p_{jt} - E[\ln W_t | \Omega^s_{jt}] + \frac{1}{2}\mathcal{S})}\, e^{-\lambda (\ln p_{jt} - E[\ln W_t | \Omega^s_{jt}] + \frac{1}{2}\mathcal{S})} \right)}{\partial p_{jt}} \\
&= \left(\frac{\omega - 1}{p_{ij}} - \lambda \frac{1-\omega}{p_{jt}} \right) \mathcal{N}(p_{jt}) \mathcal{C}(p_{jt}) \\
&= -(1+\lambda)(1-\omega) \frac{\mathcal{N}(p_{jt}) \mathcal{C}(p_{jt})}{p_{jt}},
\end{aligned}
$$

where notice we just exploited an implication of the conjectured log-normality pf prices, that is, the fact that households' expectations have a constant elasticity to prices, ω.

Now, let us write the first order condition of real profits with respect to p_{jt} taking as given past price p_{jt-1}

$$
\frac{\partial E[k_{jt} | \Omega^f_{jt}]}{\partial p_{jt}} = E\left[-(\gamma+\lambda)(1-\omega) \frac{\mathcal{N}(p_{jt}) \mathcal{C}(p_{jt})}{p_{jt}} \left(\frac{p_{jt}}{W_t} - \frac{1}{z_{jt}} \right) + \mathcal{N}(p_{jt}) \mathcal{C}(p_{jt})\, \frac{1}{W_t} \middle| \Omega^f_{jt} \right] = 0. \tag{48}
$$

Therefore, the optimal price is

$$
p_{jt} = \mu \frac{E\left[\mathcal{N}(p_{jt}) \mathcal{C}(p_{jt}) \middle| \Omega^f_{jt} \right]}{E\left[\mathcal{N}(p_{jt}) \mathcal{C}(p_{jt}) \frac{1}{W_t} \middle| \Omega^f_{jt} \right]} \frac{1}{z_{jt}},
$$

where

$$
\mu = \begin{cases} \frac{(\gamma+\lambda)(1-\omega)}{(\gamma+\lambda)(1-\omega)-1} & \text{with} \quad \omega < \frac{\gamma+\lambda-1}{\gamma+\lambda}, \\ +\infty & \text{with} \quad \omega \geq \frac{\lambda+\gamma-1}{\gamma+\lambda}, \end{cases}
$$

because of the assumption that z_{jt} is known to the firm. Note that the case where ω is so large that the markup is infinite, is the degenerate (and uninteresting) case in which no household is shopping locally in any island. Thus, in a log-normal equilibrium the optimal price can be expressed as

$$\log p_{jt} = \ln \mu + E[\ln W_t | \Omega^f_{jt}] - \ln z_{jt} + \frac{1}{2}\mathcal{V}, \tag{49}$$

where

$$\mathcal{V} = V\left(\ln \mathcal{N}(p_{jt}) + \ln \mathcal{C}(p_{jt}) \,\middle|\, \Omega^f_{jt}\right) - V\left(\ln \mathcal{N}(p_{jt}) + \ln \mathcal{C}(p_{jt}) - \ln W_t \,\middle|\, \Omega^f_{jt}\right),$$

depends on the conditional correlation of wages and local demand. When firms price under complete information, which is the main case in the text, $\mathcal{V} = 0$.

B.4 Local competition

Here we want to show how we can extend our model to include an arbitrary number of local producers. We assume that there exists $n_x \geq 1$ local producers, each producing a variety of the consumption good with a production technology linear in labor characterized by common productivity z_j.[3] Now c_{ijt} denotes a consumption bundle consisting of an aggregate over the n_x varieties,

$$c_{ijt} = n_x \left(\frac{1}{n_x}\sum_{x=0}^{n_x} c_{xijt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{50}$$

with $x \in \{1, .., n_x\}$ indexing a consumption variety and $\epsilon > 1$ being the elasticity of substitution across varieties. The household can purchase the same bundle from a local market at a price p_{jt} or a distant location, at a price $P_t = W_t$.[4]. Below we show that the elasticity of demand is given by

$$\xi_x = \epsilon\left(1 - \frac{1}{n_x}\right) + (\gamma + \lambda)(1 - \omega)\frac{1}{n_x}, \tag{51}$$

with ω still given given as in (iii) of Proposition 1. The elasticity of demand depends on the degree of substitutability across varieties, ϵ, the more so, the larger the number of varieties in the island. A price increase in a local variety alters the convenience of the local basket more, reducing demand of the local basket by a factor $(\lambda + \gamma)(1 - \omega)/n_x$. When comparing this case with the model with only one variety in a island, $n_x = 1$, we notice that equilibrium markups are higher or lower depending on whether ϵ is smaller or larger than $(\lambda + \gamma)(1 - \omega)$. It is important to note that this extended setting does not imply a degree of households' confusion smaller than the one with local monopolists. Given the presence of island-specific labor productivity shocks, the price of a local basket remains a noisy signal of competitive prices. Thus, all our results based on a given value of the local markup – notably, the one on the counter-cyclical effective markups – hold true. Effectively, introducing competition at the local level, allows for two additional parameters n_x and ϵ to set markups independently from the elasticity of extensive margins, λ, or intertemporal substitution, γ.

[3]In an alternative interpretation, each producer is an imperfectly substitutable shopping venue, selling an homogeneous consumption good; in this case households effectively have preferences for shopping locations.

[4]In the distant location, there is an infinite number of producers supplying the same variety with price equal to marginal cost.

We nest derive the equilibrium value of ξ_x. In this extended setting, conditionally on shopping in island j, the optimal allocation of consumption expenditures is such that $c_{xijt} = c_{ijt}\,(p_{xjt}/p_{jt})^{-\epsilon}$, where p_{xjt} is the price of variety x and p_{jt} is the price of the optimal consumption bundle in the island,

$$p_{jt} = \left(\frac{1}{n_x}\sum_{x=0}^{n_x} p_{xjt}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}.$$

Let us detail now the problem of the firm. Each local firm $x \in \{1, 2, ..., n_x\}$ in island j transforms one unit of labor into $1/z_{xjt}$ units of the consumption variety x, with z_{xjt} denoting labor productivity specific to variety x in island j, which is possibly correlated across varieties within an island. The local firm x chooses the price p_{xjt} that maximizes expected profits

$$p_{xjt} = \underset{p}{\operatorname{argmax}} \left\{ \mathcal{N}_{jt}(p)\,\mathcal{D}_{xjt}(p)\,\left(\frac{p}{W_t} - \frac{1}{z_{xjt}}\right)\right\}, \tag{52}$$

with $\ln z_{xjt} = \ln z_{jt} + \ln \eta_{xjt}$, being composed by an island specific component $\ln z_{jt} \sim N(0, \sigma_z^2)$ and a variety specific component $\ln \eta_{xjt} \sim N(0, \sigma_\eta^2)$, where $\mathcal{D}_{xjt}(p) = \int c_{xijt}\, di = \int c_{ijt}\,(p/p_{jt})^{-\epsilon}$ denotes the local demand for variety x. Given the guess that p_{xjt} are normally distributed, consumers' posterior distribution of inflation innovations after observing $\Omega_{jt}^{s,n_x} = \{p_{xjt}\}_{x=1}^{n_x}$ is Normal with mean

$$E[\pi_t \,|\, \Omega_{jt}^{s,n_x}] = \omega_x \left(\sum_{x=1}^{n_x} \ln p_{xjt} - E[\ln p_{xjt}|\Omega_{t-1}]\right) = \sum_{x=1}^{n_x} \omega_x(\pi_t - \ln z_{jt} - \ln \eta_{xjt}).$$

The weights ω_x must satisfy the following orthogonality condition: $\sigma_\pi^2 - \omega_x\left(\sigma_\pi^2 + \sigma_z^2 + \sigma_\eta^2\right) - \omega_x\,(n_x - 1)\,(\sigma_\pi^2 + \sigma_z^2) = 0$ or

$$\omega_x = \frac{\sigma_\pi^2}{n_x(\sigma_\pi^2 + \sigma_z^2) + \sigma_\eta^2} = \frac{\omega}{n_x}\frac{n_x(\sigma_\pi^2 + \sigma_z^2)}{n_x(\sigma_\pi^2 + \sigma_z^2) + \sigma_\eta^2}$$

with ω given by (iii) in Proposition 1. Total demand for variety x is therefore given by

$$\mathcal{D}_{xjt}(p_{xjt}) = \underbrace{\kappa\, e^{-\lambda\left(p_{jt} - E[P_t|\Omega_{jt}^s] + \frac{1}{2}\mathcal{S}_x\right)}\, e^{-\gamma\left(p_{jt} - E[P_t|\Omega_{jt}^s] + \frac{1}{2}\mathcal{S}_x\right)}}_{\mathcal{N}_{jt}(p_{jt})\mathcal{C}_{jt}(p_{jt})} \left(\frac{p_{xjt}}{p_{jt}}\right)^{-\epsilon},$$

where $\mathcal{S}_x = V(\pi_t|\Omega_{jt}^{s,n_x})$. In equilibrium, we have that local markups are always constant and expressed now by $\mu = \frac{\xi_x}{\xi_x - 1}$, where the elasticity of demand is given by

$$\xi_x = \left(1 - \frac{1}{n_x}\right)\epsilon + (\gamma + \lambda)\left(\frac{1}{n_x} - \omega_x\right). \tag{53}$$

We note $\lim_{\sigma_\eta \to 0} \omega_x = \omega/n_x$ features the simple case presented in the main text.